Learning by abduction: A geometrical interpretation

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Abstract

This paper posits Peirce's logical category of abduction as a necessary component in the learning process. Because of the cardinality of categories, Thirdness (or reason) always contains in itself the Firstness of abduction. In psychological terms, abduction can be interpreted as intuition or insight. The paper suggests that abduction can be modeled as a vector on a complex plane. Such geometrical interpretation of the triadic sign helps to clarify the paradox of new knowledge that haunted us since Plato first articulated it in his Meno dialogue. Some implications for the actual teaching practice are addressed, and the value of practical knowledge affirmed.

1. Introduction

This paper proposes a novel learning theory — or rather, a model — based on Charles Sanders Peirce's triadic logic, or semiotics. This paper also explores a possible solution to the so-called learning paradox. While paradox *per se* cannot be overcome, the very existence of what common sense considers a paradox is an ineliminable feature of triadic semiotics based on a logic of non-non-contradiction. What seems to be a paradox is in fact the Firstness of abduction that is always already present within the Thirdness of the cognitive and inferential process. As such, it is inherent in the semiotic thinking and is a precondition for the meaning production in the learning process.

Peirce's pragmatic maxim establishes the criterion for meaning as production of real effects: 'Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then our conception of these effects is the whole of our conception of the object' (CP 5.402). The meaning and essence of every conception depends, in a pragmatic sense, on the way the latter is being used: it 'lies in

the application that is to be made of it' (*CP* 5.532). The difference in the present experience brings in novelty and change that, instead of eliciting adaptation to the environment, affects and transforms the total organism-in-the-environment situation as a whole: this transformation constitutes learning.

2. Peirce's semiotic categories

At the outset I would like to briefly address three Peircean categories. Logic, for Peirce, 'is a science of the necessary laws of thought, or, better still (thought always taking place by means of signs), it is a general semeiotics, treating not merely of truth, but also of the general conditions of signs being signs' (*CP* 1.444). Peirce's pragmatism, as such, blends logic and psychology and allows for the presensory and preconscious — not limited to sense-data — apprehension of reality upon which, despite its being necessarily vague, people are prepared to act.

The triadic nature of relations between signs leads to Peirce's classifying signs in terms of basic categories of Firstness, Secondness, and Thirdness:

First is the conception of being or existing independent of anything else. Second is the conception of being relative to, the conception of reaction with, something else. Third is the conception of mediation, whereby first and second are brought into relation ... In psychology Feeling is First, Sense of reaction Second, General conception Third, or mediation ... Chance is First, Law is Second, the tendency to take habits is Third. Mind is First, Matter is Second, Evolution is Third. (*CP* 6.7)

Firstness is quality, possibility, freedom. Secondness, as a relation of the First to the Second, is of opposites, physical reality, billiard ball forces, rigid deterministic laws, direct effect, action, and reaction. Thirdness relates seconds to thirds; it is synthesis, communication, memory, mediation. When Peirce conceived of signs in terms of images, that is as an extralinguistic category, he described them in numbers which are cardinal and not simply ordinal, like first, second, or third. Secondness, by definition, contains one *and* two, so as there is firstness in secondness, and there are three in the Thirdness. So Thirdness always already has Firstness in itself. The first precognitive category, called by Peirce a quali-signification — or, a qualitative immediacy of experience — may function in a form of intuition or imagination, that is as a category traditionally left outside the boundaries of formal logic. The immediate

Firstness — a sort of premodern natural attraction — was, together with the Thirdness of mediation, left out as insignificant by the ‘pure reason’ of modernity and substituted by the dualistic sin-signification and instrumental rationality based on conventional logic of excluded middle.

The practical bearings, as Peirce called them, of some possible precognitive reasoning, are not to be ignored. They, fair enough, constitute ‘sensations so faint’ (Peirce and Jastrow 1884, quoted in Hacking 1990: 205) so as to bypass the level of common awareness. It is quite difficult to account for rational conclusions based on such vague and faint first impressions. In a characteristic language, Peirce and Jastrow comment that the ‘insight of females as well as certain “telepathic” phenomena may be explained in this way. Such faint sensations ought to be fully studied by the psychologist and assiduously cultivated by every man’ (Hacking 1990: 206).

3. Abduction according to Peirce

While a psychological study is out of the scope of the present paper (as well as gender-colored expressive language of Peirce!), the philosophical examination of abduction is necessary. We approach abduction from the viewpoint of its functional manifestations. An experiential situation is problematic, that is, it involves tension and conflict, because it encounters the otherness, or Secondness of ‘reaction against my will’ (*CP* 8.144) due to the intervention, sometimes beyond one’s awareness of this action, of the brute facts of human experiences. ‘[T]he surprising fact ... is observed’ (*CP* 5.185) — and an inquiring mind makes a first step toward apprehending the experience by abduction, a peculiar logic of discovery, bordering on as yet uneducated (if education is taken conventionally) guess.

Despite being initially preconscious and necessarily vague, the abductive inference, according to Peirce, belongs to objective logic understood broadly as the ‘laws of thought, ... thought always taking place by means of signs’ (*CP* 1.144). The causal influence embedded in the semiotic process of cognition becomes indirect, seemingly asignifying, and moderated by means of inclusion of the third category that breaks down the direct dyadic cause-effect connection.

Nonetheless the formal, albeit vague, rule of abduction enables mind to reason from the premise to the conclusion; such an inference being described by the following statement: if A is B, and C can be signified by B, then maybe A is a sign of C. The interpretation is triggered by the Firstness of abduction which, tending towards the perceptual judgement,

is a hypothesis bearing statement that asserts its conclusion only conjecturally; yet, according to Peirce (*CP* 5.189), there is a reason to believe that the resulting judgment, under the circumstances, is true. Peirce (*CP* 5.184) was adamant that there is no sharp line of demarcation between abduction and perceptual judgement: one shades into the other along the inferential process. Such reasoning is hypothetical only: the generation and selection of a hypothesis leads to a conjecture different from an inductive inference.

The given premise must entail some empirical consequences; the explanation of the initial perception is achieved by analogical reasoning which unfolds into inferences to the would-be consequences of abductive conclusions eventually leading 'to a result indefinitely approximating to the truth in the long run' (*CP* 2.781), asymptotically merging into the synthetic inference in the process.

Peirce emphasized the role of diagrammatic reasoning saying that 'passing from one diagram to the other, the [reasoner] ... will be supposed to see something ... that is of a general nature' (*CP* 5.148), hence contributing to making one's ideas clear. The purpose of such a diagrammatic mode of expression was indeed to 'depict thought's very movement, its processual character, in terms of interconnecting lines, schemes, figures, abstract mappings. In fact [Peirce] believed that all thought is sign process and hence it is capable of being presented diagrammatically' (Merrell 1995: 51).

The epistemic process, for Peirce, means denial of the Cartesian notion of arriving at propositions that mirror reality. The whole notion of a proposition, whose subject designates reality and whose predicate describes the essence of the said reality, is transformed by Peirce into interpretation of reality and living it out experientially: mimesis turns into semiosis. The abductive guess as a matter of a First borders on intuition; an intuitive knowledge traditionally being a synonym for immediate knowledge. Intuition conventionally has been considered to be the initial perception of an object. For Peirce, however, there is no immediate, that is unmediated, knowledge: signs in a process of semiotic inquiry do mediate all cognition. Perception differs not in kind but only in degree from other forms of human knowledge. What in fact enables our 'perception [to turn] inward upon the objects of conception' (Noddings and Shore 1984: 47) is the firstness of intuition.

The very etymology of the word confirms this: to *in-tuit* means to learn from within, yet 'the parish of percepts ... [is] ... out in the open' (*CP* 8.144) of the experiential world. Affirming the continuity of consciousness, Peirce stressed its temporal character. The cognitive, that is inferential, process of interpretation is a series of thought-signs, and the meaning

of each thought becomes understood in each subsequent thought, creating a process of unlimited semiosis.

No thought is ever instantaneous because it needs an inferential stretch (cf. Dewey 1925 [1958]) for its own interpretation. Yet the immediacy of Firstness is always presented in an instant and, as Firstness, it is *had* prior to every mediative Thirdness, making inference appear to be bordering on association and guessing. Peirce, as long ago as 1868, stated that cognition exists only ‘in the relation of my states of mind at different instants ... In short, the Immediate (and therefore in itself unsusceptible of mediation — the Unanalyzable, the Inexplicable, the Unintellectual) runs in a continuous stream through our lives; it is the sum total of consciousness, whose mediation, which is the continuity of it, is brought about by a real effective force behind consciousness’ (Peirce 1955: 236–237), enabling the recursive process of what Noddings and Shore (1984) call the dual representation.

Signs reiterate, they become signs of signs, or representations. As Peirce (*CP* 5.138) stated, ‘the mode of being of a representamen [a sign] is such that it is capable of repetition’ that is, of creating sensible patterns. For Peirce, the concepts literally take part in the reality of what is conceived, implying holism and a sense of auto-referentiality between the inner and outer realities. Every sign is subject to interpretation by a series of subsequent thought-signs, and the whole triad enveloping the ‘the relation-of-the-sign-to-its-object becomes the object of the new sign’ (Sheriff 1994: 37). As a result of multiple interrelations, signs move from one to another, they grow and engender other signs because the triadic logic leads to signs always already becoming something else and something more, contributing — in the process of their growth — to human learning and further development.

Because ‘consistency belongs to every sign, ... the man-sign acquires information and comes to mean more than he did before’ (Peirce 1955: 249). The value of knowledge is in its practical import, that is, the way we, humans, will act, think, and feel — in short, assign meaning to our own experience — as the pragmatic effect of the said knowledge. The meaning and essence of every conception depends, in a pragmatic sense, on the way the latter is applied: it ‘lies in the application that is to be made of it’ (*CP* 5.532). Pragmatic maxim presupposes the discovery of meaning notwithstanding that the ‘meaning lurks perpetually in the future’ (Merrell 1992: 189).

Everything is a sign: the whole universe, for Peirce, is perfused with signs; yet ‘nothing is a sign unless it is interpreted as a sign’ (*CP* 2.308) by means of triadic relations leading to each successive sign’s becoming an interpretant for the preceding one. What seems to be a paradoxical

statement is derived from the nature of the pragmatic method itself. Abduction does seem to function instantaneously, not because there is no temporal interval of inference, but because the mind remains unaware of when it begins or ends.

Peirce, describing the structure of perceptual abduction, noted that ‘the first premise is not actually thought, though it is in the mind habitually. This, of itself would not make the inference unconscious. But it is so because it is not recognized as an inference; the conclusion is accepted without our knowing how’ (CP 8.64–8.65). Intuition therefore, albeit achieving an intellectual knowledge or the *nous* of the ancients, is not *of* something but *is* something; as an epistemic pragmatic method, it is a process — albeit preconscious — of *knowing*. Peirce considered consciousness to be not a *thought* but just a fleeting and quasi-immediate *feeling* that simply presents itself and is not by itself capable of representation but needs *thought* as Thirdness, the latter being ‘a synonym for representation’ (CP 5.105). In the absence of Thirdness, though, it is indeed the ‘first, present, immediate, fresh, new, initiative, original, spontaneous, free ... Only, remember that every description of it must be false to it’ (CP 1.357).

4. Modeling abduction

Keeping in mind the paradoxical flavor of Peirce’s warning, and remembering that any idea is only a ‘tentative suggestion’ (Dewey 1991: 112), let me tell a dream that I awakened with one morning some time ago. How to explain abduction? How to, rather than looking for any theoretical foundations, go from some form of experience to constructing a working model of the abductive inference? Mathematics helps, and a sort of a Peircean diagram may indeed, as ancient Chinese were saying, be worth more than ten thousand words. I will have to draw a diagram, a familiar Cartesian grid for this purpose. However with a difference.

The grid is not Cartesian in a strict sense, its two coordinate axes being located on a complex plane and marked with imaginary, on a vertical axis, and real on a horizontal axis, numbers respectively (Figure 1). Abduction’s place would be on the vertical axis, it is a jump, a leap in *imagination* after all, so imaginary numbers seem to be the appropriate symbols to signify abduction, especially considering that an imaginary number, as Leibniz described it, is the intermediary between being and nonbeing.

Propositional logical reasoning is marked along the horizontal axis by means of real numbers, in the actuality of the physical world. So in this

Figure 1. *The complex plane*

model, a rational thought coexists with an imagination, insight, or intuition, such logic being represented by means of complex numbers, or the ordered pair on the slice of the flat surface, a plane. Both abductive inference and deduction may be represented by vectors (arrows), or directional forces in the physical sense. The two 'add up' or converge onto a resultant vector on a complex plane. A vector, by definition, has both magnitude and direction, that is, it can be described by both mathematical quality and a physical property.

The resultant vector may be considered to represent new knowledge or rather knowledge different from the preceding, infamous, prior knowledge, because abduction contributes to explicating that what was yet tacit and implicit therefore enabling a *jump to the yet unknown*, onto the succeeding level on a complex plane. Peirce indeed distinguished between what he called an ampliative and explicative form of reasoning, suggesting that the former aims at increasing knowledge while the latter, by contrast, is capable of making hidden or implicit knowledge explicit, of making manifest what is perhaps latent.

True, the addition vector as a whole is not the sum total of its parts, because as a *resultant*, it is not the sum in the arithmetical sense, it is

Figure 2. *The resultant vector*

in-between, necessarily having a partial and fractal quality but pointing nevertheless in the determined direction (Figure 2). Only related in a triadic way, Firstness (feeling), Secondness (action), and Thirdness (reason) together constitute a dynamic structure of experience. Without the Firstness of abduction, all knowledge would remain pretty sequential, because signs would stay at the level of Secondness, perhaps growing in magnitude solely because of arithmetical progression along the horizontal line but not being able to change direction geometrically. It is merely some prior knowledge that would be amplified, but a tacit and preconscious, implicit 'knowledge' would lack any possibility of explication so as to enable a new knowledge — represented now as a vector on a complex plane, pointed to by the end of the arrow — to come into being, to enter cognition. Thirdness — a diagonal, a transversal line — enables the coming into being of the new objects of knowledge as the newly created concepts.

A novel hypothesis might — almost literally, as we can see in Figures 1 and 2 — bring a new direction into the line of thinking, and the semiotic categories of Firstness and Thirdness, the two categories outside the formal logic, functioning only on the margins the latter, are capable of constructing the totally new level of knowledge and the higher level of

organization. Peirce indeed pointed out that the semiotic categories of Firstness, Secondness and Thirdness are the ‘conceptions of complexity’ (CP 1.526).

Abduction, which may take the form of intuition, or insight, or imagination, creates a magnitude along the vertical axis, or *depth* (cf. Dewey 1991: 37), that is an intuitive leap leading towards yet another level of complexity in the knowledge system. John Dewey used this term, depth, ‘with respect to the plane upon which it occurs — the intrinsic quality of the [intellectual] response’ (Dewey 1991: 37) of different people, asserting that ‘one man’s thought is profound while another’s is superficial ... This phase of thinking is perhaps the most untaught of all’ (Dewey 1991: 37). To think means to differentiate, and the degrees of differentiation can be expressed diagrammatically via spatial projection enabling, by means of the laws of projective geometry, also the reduction of dimensions of initially undifferentiated, and seemingly adimensional, field of complex numbers. This field appears to be ‘what the world was to Adam on the day he opened his eyes to it, before he had drawn any distinctions, or had become conscious of his own experience’ (CP 1.302).

The natural world is not limited to its solely mechanical aspect similar to experience as not being reduced to action and reaction taking place at the level of Secondness. For example, being burned, as Dewey asserted, is not yet *an* experience but merely a physical change. While pain as quality is indeed the first, the child’s initial reaction would be to instinctively, therefore mechanically, withdraw the hand. Thirdness enters the process as mediation and learning, it takes time and self-reflection. Understanding and mind denote ‘responsiveness to meanings, ... not response to direct physical stimuli’ (Dewey 1916 [1924]: 315).

Nature is much broader and includes its own virtual dimension which, however, is never beyond experience. If the ‘conception of the role of experience within nature means that “human affairs, associative and personal, are projections, continuations, complications of nature which exists in the physical and prehuman world ...” [as] Dewey writes’ (Campbell 1995: 77), then triadic logic serves as a ‘means of detour’ (Dewey 1934 [1980]: 4) necessary for evaluating and understanding such an experience.

Projection is the means of a temporal connection too; as Dewey says referring to the modes of teaching, ‘projection and reflection, going directly ahead and turning back in scrutiny, should alternate’ (Dewey 1991: 217), and abduction itself, being just a guess or a hypothesis, is a projection of sorts: ‘the mind is in the attitude of *search*, of *hunting*, of *projection*, of *trying this and that*’ (Dewey 1991: 112). The meanings are to be created — Dewey continuously asserted that ‘the work of education is

constructive' (Dewey 1916 [1924]: 315) — and the plane becomes the locus for meaning-making enacted by projection, that is, an abductive, intuitive leap.

The Firstness of abduction is as yet a preconscious presentation that reaches the representation in consciousness at the level of Secondness, along the horizontal line. It will surely have a different magnitude: pain, for example, is directly had, but may be interpreted as a toothache — or, as we said earlier, as an effect of being burned — and hence judged to be a *singularity* of a specific kind (cf. Dewey 1938: 515), as *this* and not that. The diagonal or transversal line will cast its own shadow on the horizontal axis appearing as if from nowhere — because it exists at the level of complexity exceeding the realm of real numbers.

5. Resolving the learning paradox

We remember that the dyadic logic, by virtue of its very principles, excludes the Thirdness of mediation, expressed by the diagonal transversal that came into being as a *resultant*, and thus created a closed figure (Figure 2), an area, an integral. The triangle has closed on itself — but, and this is crucial — at a different level of organization. Complex plane would not be complex without the axis of imaginary numbers, but would remain a Cartesian grid preventing us from understanding how the new objects of knowledge may come into existence.

The infamous learning paradox, which is being questioned again and again, and has been recently, and in the context of educational research, connected with abduction (Prawat 1999),¹ will forever remain a paradox. Going back to Plato's discussions in the *Meno*, Prawat (1999: 48) reminds us that if and when any *new* knowledge is incompatible with prior learning, the latter in fact being a precondition for the understanding of what is new, then there is no base on which to build such a new knowledge. Nevertheless learners somehow acquire the new knowledge.

Let us recall the *Meno* dialogue and the infamous *ens primum cognitum* later formulated by Aquinas, or the problem of being as first known. Plato states the famous paradox in the following way:

Men. And how will you inquire, Socrates, into that which you know not? What will you put forth as the subject of inquiry? And if you find what you want, how will you ever know that this is what you did not know?

Soc. I know, Meno, what you mean; but just see what a tiresome dispute you are introducing. You argue that a man can not inquire either about that which he knows, or about that which he does not know; for he knows, and therefore has

no need to inquire about that — nor about that which he does not know; for he does not know that about which he is to inquire. (Plato: 442)

Meno is puzzled by what Socrates means when he provocatively says that we do not learn, and that what is called learning is pretty much a process of recollection. Is it a method of ‘the dramatic, restless, cooperatively inquiring Plato of the Dialogues, trying one mode of attack after another to see what it might yield’ (Dewey, quoted in Bernstein 1992: 50)? Or are we left with an absurdity because either one knows *a priori* what is it that s/he is looking for, or one does not know what one is looking for and therefore cannot have prior expectations of finding anything.

Presenting Peircean abduction as a means of resolving the dilemma, Prawat approaches abduction mostly at the level of heuristics, that is, as a useful metaphor. He presents abductive inference as such as a metaphorical process by virtue of which ‘it consists of old and new meanings’ (Prawat 1999: 60). Prawat takes up the very issue of metaphor seriously and asserts its central role in the process of abduction. He insists however that abduction represents a move from what is known to the unknown missing on the important problem of the very generation of hypothesis. Noddings, commenting on Prawat’s paper, notices this omission: she says (Noddings 1999: 84) that there is complexity involved in generating any new knowledge. In the same spirit, the geometry of a complex plane, presented in this paper as belonging to the paradigm of complexity, attempts to further clarify the learning paradox by means of interpreting it pragmatically, even if only partially, indeed abductively or conjecturally.

It also helps in some respect to unpack the following enigmatic statement by Peirce concerning a guess at the riddle: Thirdness is ‘governing secondness. It brings information, ... determines an idea and gives it body’ (*CP* 1.537 in Sheriff 1994), that is, it contributes to the objects of knowledge appearing to consciousness in the natural world. Contrary to Prawat’s asserting that abduction is the sole means to the new knowledge, we can see — from the Figures 1 and 2 — that surely ‘abduction is involved, but so are deduction, induction, and language moves’ (Noddings 1999: 84). If abduction were the only reason behind the learning process, no new knowledge would ever come into play because no construction of a closed figure, a triangle of Firstness, Secondness and Thirdness, representing an area — an integrated thinking, literally — on a plane would be possible.

The leap of imagination, a sign of Firstness, if such indeed were to take place, would sink back into the dyadic existence, back to the point of its own departure and, worse, we would not know even this! It wouldn’t make a difference to us because there would not be any

difference potentially capable of making the difference as its own derivative, in the first place. If we imagine positioning ourselves along the resultant vector, that is, in the very midst of the complex plane that traverses both horizontal and vertical axes, there are two aspects that are apparent from either ‘inside’ or ‘outside’: the phenomenal Firstness (vertical) and the physical Secondness (horizontal), as two properties or attributes of the same process (Thirdness). The closed structure is equivalent to the conception that some preconceptual ‘virtual meanings’ inherent in the logic of abduction not only lead to experience and belief but are themselves informed by experience even if the informational input enters the process below the conscious level.

Firstness, by definition, does not refer to anything else. We remember Peirce’s having said that abductive inference bypasses our awareness and the mind remains unaware where and when abduction begins and ends. Difference has to be perceived — felt, seen, heard, touched — in order to make a difference so as to create some integration into reflective thinking. The latter is possible only at the level of Secondness, in the physical world of action, where Newtonian laws are in full power and every action has a reaction.

But without Firstness, Secondness is impossible, both are cardinals — and this Peircean finesse was totally ignored by the science of modernity. So too was Thirdness, which, as we said earlier citing Peirce, governs Secondness, creating the necessary mediation of immediacy, the triadic relation between affects, percepts and concepts that alone is able to create learning. And the triangle simply must close on itself, because ‘a complete, an integral act of thought requires that the person making the suggestion (the guess) be responsible also for reasoning out its bearings upon the problem in hand’ (Dewey 1991: 98), such a problem — or perplexity — as an instance of real experience, having initiated this guess in the first case.

6. Concluding remarks

How can this theorizing help us in an actual educational setting? The teacher’s task in a classroom then, in order to get things moving, will become one of providing the appropriate conditions, as Firstness, under which something new would be produced. A classroom permeated with a creative potential of desire, curiosity, trust, and interest towards discovering something as yet unknown has a possibility to turn into the experimental Deweyan laboratory.

The saying goes that children are natural philosophers, precisely because children have this abductive Firstness of imagination that enables

them to engage in a free play, and here are we, adults, children no more, whose routine conceptual thinking has been reduced to the level of Secondness in the form of solely instrumental rationality. So it seems that in order to create a truly *learning* environment, teachers are to establish the Firstness, perhaps to indeed become Firsts themselves, so as enabling their students to acquire experiential knowledge of the facts, as Secondness, by means of assigning multiple values of meanings, as Thirdness, to their own experience.

The figure of the triangle as embodying complexity is important, and assigning the quality of Firstness to abduction modeled by imaginary numbers does not exhaust its significance. Coincidentally, Garrison invokes a symbol of triangle positing the relation between the student, subject matter and the teacher: 'I think teaching resembles a triangle; you cannot have one unless there are all three sides and they enclose a pedagogical space' (Garrison 1999: 312, 84f.). Indeed, only as a genuine triad, the sign would amount to the Thirdness of 'synthetic consciousness, . . . sense of learning' (CP 1.377).

Philosophically, learning — rather than measured as an outcome of some external educational aim — cannot be anything but an immanent process, directed perhaps towards an (in)famous object of desire. The paradigm of complexity, illustrated by the figure of triangle, presupposes the absence of some abstract and transcendental *telos* — exemplified, for example, in the idealized 'access to college,' as Noddings (1993: 9) reminds us. Instead there is an immanent production of meanings due to inherent presence of abduction. Learning becomes an emergent property of transactions between teachers, students and subject matter(s) even in the absence of traditional instruction as a necessary precursor to learning. It is Thirdness that enters the process as mediation and learning, it takes time and self-reflection; it enables creation of novel meanings, and meaning, for pragmatists, is 'that form in which the proposition becomes applicable to human conduct' (CP 5.425). The value of knowledge is in its practical import, that is, the way we, humans, will act, think, and feel — in short, assign meaning to our own experience — as the pragmatic effect of the said knowledge. Learning is reconceptualized: it is learning by abduction.

Note

1. I admit, with gratitude, that I saw this dream about complex plane the night after having read Prawat's (1999) article 'Dewey, Peirce and the learning paradox' in the American Educational Research Journal.

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