Influence of photosensor noise on accuracy of cost-effective Shack-Hartmann wavefront sensors

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ABSTRACT

A Shack-Hartmann (SH) wavefront sensor (WFS) is used in most modern adaptive optics systems where precision and robustness of centroiding are important issues. The accuracy of the SH WFS depends not only on lenslet quality but also on the measurement accuracy of centroids, especially in low-light conditions. In turn, accuracy depends on light and dark noises that are inevitably present in solid-state photosensors. Using a comprehensive mathematical model of the CMOS photosensor, the accuracy of the Shack-Hartmann wavefront sensor is assessed and analysed for each type of noise.

In this paper, new results regarding the influence of different noise sources from a CMOS photosensor on centroiding in Shack-Hartmann wavefront sensors are presented. For the numerical simulations, a comprehensive mathematical model of photosensor's noise was formulated. The influences of light and dark noises as well as pixelisation factor have been assessed. Analysis of the wavefront sensor's accuracy is provided. Results should be of interest for further development of cost-effective wavefront sensors.

1. INTRODUCTION

Wavefront sensors (WFS) for the adaptive optics have come a long way in the development from the kitchen-made¹ lenslet arrays up to the modern sophisticated devices.² Even early Shack-Hartmann wavefront sensors³ with coarse lenslets were found to be surprisingly accurate.¹ However, the accuracy of the SH WFS depends mainly on the light noises and dark noises, which inevitably present on the images from solid-state photosensors. That is especially true for the low-light conditions⁴ and/or the case of cost-effective wavefront sensors.⁵ Using widely available fast and inexpensive CMOS sensors, it would be possible to build low-cost adaptive optics systems for small telescopes, which are still require adaptive optics to be installed.⁶

Considerable efforts in both analytic and numerical models of the wavefront sensors have been made. However, the results are often either incomplete, or obtained in simplified assumptions. For instance, Cao and Yu have analysed the measurement error of the Shack-Hartmann WFS and obtained the analytic results for the centroid errors due to the readout noise of the CCD and the photon shot noise.⁷ However, the assumption that the centroiding noise is purely Gaussian is not always accurate. There were results published⁸⁻¹⁰ indicating the degradation factor of centroiding accuracy caused by the pixel dimensions. It was reported¹¹ that for the ICCD detector, the centroiding error consists mainly of the inherent photon noise of the photon signals and the readout noise of detector and its associated circuits. Tyler and Fried¹² have obtained the theory expression which evaluates the angular position error when a quadrant detector is used in the Shack-Hartmann sensor. However, Tyler and Fried theory is proven to be only an approximation, and more precise estimation of statistical errors in centroid were elaborated.¹³ Hardy¹⁴ obtained the analytical expressions that can be used to evaluate the angular position error; however, those expressions were obtained under assumption that the photon shot noise of the signal is dominant. The analysis shows that the centroid variance depends on the CCD size for the Poisson noise and truncation effects play a significant part in determining the optimum CCD size.^{15, 16} In many articles (e.g.,¹⁷), it is assumed beforehand that the density distribution of the centroids spot as well as noise is Gaussian. In some light conditions, such assumption is not correct that can lead to inaccurate simulations results.

In this work, results of numerical simulations and analysis of centroiding accuracy for the cost-effective CMOS-based wavefront sensors are elaborated. The analysis of influence of different noise sources from the

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CMOS photosensor on the centroiding robustness in the Shack-Hartmann wavefront sensor is presented. For the numerical simulations, a comprehensive mathematical model of photosensor's noise¹⁸ was used. The influence of light and dark noises as well as pixelisation factor has been assessed. The results may be of interest for adaptive optic engineers who aim to build wavefront sensors for small astronomical telescopes and ophthalmology devices.

2. MODELS USED FOR THE NUMERICAL EXPERIMENTS

The numerical simulations were performed using the complete simulator with laser guide star, the model of the atmosphere turbulence, the model of the Shack-Hartmann wavefront sensor, and the model of the CMOS photosensor.

The model of a guide star We assumed a single sodium laser guide star (LGS), which was mathematically represented as sinc-sinc¹⁹ model:

$$\tilde{U}_{pls}(r_1) = A e^{-\frac{ik\Delta z}{2\Delta z} r_1^2} e^{-\frac{ik\Delta z}{2\Delta z} r_c^2} e^{-\frac{ik\Delta z}{2\Delta z} r_1 r_c} \left(\frac{D}{\lambda \Delta z} \right) \text{sinc} \left[\frac{D(x_1 - x_c)}{\lambda \Delta z} \right] \text{sinc} \left[\frac{D(y_1 - y_c)}{\lambda \Delta z} \right], \quad (1)$$

where A is an amplitude factor, x_c and y_c are coordinates the location of the point source in the $(x_1; y_1)$ plane, λ is the observation wavelength, and Δz is a propagation distance (altitude of the guide star).

The model of an atmospheric turbulence We treat the atmosphere turbulence as discrete layers of phase screens. The light from the guide star was propagated through the turbulence layers using angular spectrum propagation,¹⁹ which is a technique for evaluation of the Fresnel diffraction integral. A multi-layered atmosphere was simulated using the Fourier series method (FT method²⁰). We generated the light field that corresponds to the light of the guide star passed through six turbulence layers. We use the von Kármán²¹ power spectrum model for the simulations of the atmosphere turbulence as follows:

$$\Phi_\phi^{vK}(\kappa) = 0.49 \cdot r_0^{-5/3} \cdot (\kappa^2 + \kappa_0^2)^{-11/6}, \quad (2)$$

where:

- $\kappa = \frac{\omega c}{v(z)}$ is the spatial wave number and $v(z)$ is the wind velocity profile;
- $\kappa_0 = \frac{2\pi}{\mathcal{L}_0}$, and $\mathcal{L}_0$ is the outer scale.

The size of turbulent layers was 1024×1024 pixels. The propagated wavefront is then used as an input for the model of Shack-Hartmann wavefront sensor.

2.1 The model of a Shack-Hartmann Wavefront Sensor

The model of SH WFS has 32×32 lenslets. Each of lenslet contains 32×32 pixels. The propagation of light through the lenslets was performed using "object against the lens" case.²² Our numerical experiments are aimed in study of a cost-effective wavefront sensor, and for that reason, we simulate CMOS sensors using previously developed high-level model.¹⁸ The model consists of the photon shot noise, the photo response non-uniformity (PRNU/gain FPN), the read noise (includes the dark current, the dark shot noise, the dark FPN, the sense node reset noise, the source follower noise) and ADC quantisation noise. The photosensor is assumed to have $5.00 \mu\text{m}$ pixels with the pixel fill factor of 50%, quantum efficiency of 60%, and full well of $20000 e^-$. The noises such as PRNU and dark FPN are varied in the numerical experiments. For the sensor we assume that the surrounding temperature is 300K, the sense node gain is $5.00 \mu\text{V}/e^-$, and clock speed is 20 MHz. Such parameters can be considered as typical for a cost-effective CMOS-based wavefront sensor.

3. THE EFFECT OF THE PIXELISATION AND FOCAL LENGTH OF THE LENSLET ON THE WAVEFRONT SENSORS ACCURACY

The goal of the simulations was to provide better simulation results regarding the accuracy of SH WFS. Therefore, we used the previously developed comprehensive model of the photosensor.¹⁸ First, the dependency of centroids coordinate from the focal distance of the lenslets was evaluated. After the focus length for the simulations was chosen, the pixelisation error impact on the centroiding was evaluated.

3.1 The effect of lenslets focus length on the centroiding

First, we have to choose the focus length of lenslets for the simulation of the Shack-Hartmann wavefront sensor. In our case, the observation wavelength was $\lambda = 0.5\mu\text{m}$ and the diameter of the lenslet $D_{\text{lenslet}} = 200\mu\text{m}$. Typical values for the focal distance of the lenslets in the Shack-Hartmann lenslet are 10-20 mm. Therefore, we varied the focus distance of the lenslet from 1 mm to 50 mm with the step of 1 mm. Numerical simulations were performed with double precision in order to eliminate quantisation error influence. All the noise sources from the simulated photosensor were turned off. The results of the simulations are presented in Fig. 1.

Figure 1. Dependency of the centroids coordinates on the focus length of the lenslet in the simulated Shack-Hartmann wavefront sensor.

From the results (see Fig. 1) of the simulation one can see that the centroid position can be effectively measured using the lenslets with focal distance no more than 20-30 mm. Increasing the focus length further gives no improvements over the accuracy of centroiding. This is constant with typical values of 7-10 mm focal length of the lenslets in Shack-Hartmann sensor.²³ Hence the rest of numerical simulations were conducted for the focus length of the lenslets equal to $f = 10$ mm.

3.2 The impact of the pixelisation error on the centroiding accuracy

The real irradiance distribution of each light spot in SH WFS is measured by discrete pixels of CMOS/CCD photosensors that results in a loss of centroiding information.^{24,25} Therefore, the pixelisation error is defined as the error caused by reducing the real irradiance distribution of each focal spot to a series of discrete measurements that represent the integral of the irradiance distribution over finite boundaries. The impact of the pixelisation error can be a significant limiting factor on the total accuracy in the Shack-Hartmann wavefront sensors.²⁴

The numerical simulations for the estimation of a pixelisation error were conducted as follows. The amount of pixels in each lenslets was varied from 64×64 (initial size) down to 4×4 pixels with step of 4 pixels. The size of the lenslet remains constant, i.e. pixelisation error in this experiment can be considered as a change of the pixel size. The image of the centroids C_{orig} from the simulated Shack-Hartmann WFS was downsampled in the spatial domain using a simple interpolation algorithm by nearest neighbourhood. Such a scaled image that contains the picture of centroids is denoted as C_{scaled} . The centroiding coordinates were calculated using a simple CoG algorithm both for the original centroiding image C_{orig} (64×64 pixels) and scaled image C_{scaled} to estimate the pixelisation error.

Figure 2. Dependency of the normalised root mean squared pixelisation error versus the amount of pixels in the lenslet in one direction for: a) low-light ($600 e^-$), b) intermediate light ($3500 e^-$), and c) high light ($13500 e^-$) conditions.

The pixelisation error was estimated for each lenslet separately for the row and column coordinates. We took and averaged 32 images of the centroids pictures for each size of the lenslet. For each image, the coordinates of the k -th centroid (x_{orig}, y_{orig}) for the original C_{orig} image and the coordinates (x_{scaled}, y_{scaled}) for the scaled C_{scaled} centroiding image were compared:

$$\Delta x(k) = 1 - \frac{x_{scaled}(k)}{x_{orig}(k)} \cdot \eta \quad \Delta y(k) = 1 - \frac{y_{scaled}(k)}{y_{orig}(k)} \cdot \eta,$$

where $\eta = p_{orig}/p_{scaled}$ is the number of pixels in the lenslet (p_{orig}) for the original centroiding image C_{orig} from the simulated SH WFS and p_{scaled} for the scaled image C_{scaled} , respectively. The normalised value of an error was calculated for each $\Delta x(k)$ by:

$$\Delta x_{rms} = \sqrt{\frac{\Delta x(1)^2 + \Delta x(2)^2 + \dots + \Delta x(k)^2}{n}} \quad \Delta y_{rms} = \sqrt{\frac{\Delta y(1)^2 + \Delta y(2)^2 + \dots + \Delta y(k)^2}{n}}$$

where n in this case equals 32 (a number of averaged images of centroids). The results of the numerical simulations are presented in Fig. 2 for low illumination, intermediate illumination, and high illumination case, respectively.

As described above, initially the light field was calculated for the lenslet with 64×64 pixels. From the Fig. 2 one can see that the pixelisation error grows slowly when the number of pixels in the lenslet shrinks from 64×64 to 32×32 pixels. The pixelisation error in this case corresponds to $\Delta x_{rms} \approx 0.002$, or 0.2% rms. The growth of the pixelisation error increases with the decrease of pixels number (i.e., increasing the geometrical size of the pixel). For the case of 12×12 pixels in the lenslet one can observe an abrupt increase of the pixelisation error to $\Delta x_{rms} \approx 0.006 - 0.007$, which is 3 times larger than for the case 32×32 pixels in the lenslet.

It is noteworthy that the noise on the simulated photosensor has a little impact on the pixelisation error; impact of the noise increases only for significantly shrunk lenslet size. From the numerical simulations data one can see that the PRNU (gain FPN) has a noticeable impact only when PRNU is quite large, i.e. 5%.

4. THE IMPACT OF PHOTODIODE NOISE

The numerical experiments were conducted in order to estimate the impact of different noises on the centroiding accuracy. The lenslets of the simulated Shack-Hartmann wavefront sensor have the focus length of $f = 10$ mm.

4.1 The noise model of the photosensor

Main sources of the centroiding errors were considered the photon shot noise, the photo response non-uniformity (PRNU/gain FPN), the read noise, and the quantisation noise. The model of the noise in the photosensor is briefly described below.

4.1.1 Shot noise

The shot noise is inevitable in real photosensors because of quantum nature of a light. The photon shot noise can be described by the Poisson statistics:

$$p_a = \frac{P_I^a \cdot e^{-P_I}}{a!},$$

where p_a is the probability that there are a interactions per pixel and P_I number of interacting photons.

4.1.2 Photo response non-uniformity

The photo response non-uniformity (PRNU), also referred as gain FPN, is due to geometrical variations of pixels size. The PRNU in CMOS photosensors is typically 1-2%,²⁶ it is fixed for a given sensor but varies from sensor to sensor. The PRNU was simulated as the Gaussian noise:

$$I_{PRNU} = I \cdot (1 + \eta \cdot \mathcal{N}(0, 1))$$

where η is a PRNU/Gain FPN factor.

4.1.3 Read noise

We use the definition of a read noise²⁷ as any noise that is not a function of signal: that is the dark current, the dark current shot noise, the dark offset FPN, the sense node reset noise, thermal noise and etc. The read noise affects the accuracy of a wavefront sensor only in low light conditions and to some extent on intermediate-level of light. Then the dark noise became dominated by the photon shot noise and the PRNU.

The dark current The dark current, which depends on temperature and integration time,²⁸ was simulated as:

$$D_R[e^-] = 2.55 \cdot 10^{15} P_A D_{FM} T^{3/2} \exp \left[-\frac{E_{gap}}{2 \cdot kT} \right], \quad (3)$$

where P_A is the pixel's area [cm^2], D_{FM} is the dark current figure-of-merit at 300K [nA/cm^2], E_{gap} is the bandgap energy of the semiconductor which also varies with temperature, k is Boltzman's constant. The dependency of a band gap energy on temperature was estimated by Varshni's empirical expression:²⁹

$$E_{gap}(T) = E_{gap}(0) - \frac{\alpha T^2}{T + \beta}, \quad (4)$$

where $E_{gap}(0)$, α and β are the material-specific constants. The temperature of $T = 300K$ was assumed for the photosensor of the WFS. The parameters of bandgap energy were used for the Silicon: $E_{gap}(0) = 1.1557[eV]$, $\alpha = 7.021 \cdot 10^{-4} [eV/K]$, $\beta = 1108 [K]$.

The dark shot noise The dark current, which is simulated using Eq. 3 and Eq. 4, gives a constant signal; the dark shot noise is inevitable since thermally generated currents arrive randomly. The dark shot noise was simulated using Poisson probability density in the same way as photon shot noise.

The dark Fixed Pattern Noise An offset fixed pattern noise (offset FPN) is dark signal non-uniformity, which is the offset from the average dark current across the sensor array in the absence of light. It has been reported³⁰⁻³² that dark FPN distribution is non-symmetrical and can be modelled by Gamma, Wald or Log-Normal distribution. Therefore, the dark noise was simulated using non-symmetrical inverse Gaussian distribution (also referred as the Wald distribution when the mean value equals to one). Wald distribution is a special case of the Inverse Gaussian distribution,³³ where the mean is a constant with the value one and probability density function:

$$W(\beta; x) = \sqrt{\frac{\beta}{2\pi x^3}} \exp \left[-\frac{\beta(x-1)^2}{2x} \right], \quad (5)$$

where the scale parameter is β , $\beta > 0$; also the mean equals to one, the variance is $1/\beta$, and skewness is $\sqrt{9/\beta}$. According to our previous experiments, the skewness $\beta = 3$ is a good description of dark noise distribution when the integration time is short (less than 1 second).

4.1.4 Quantisation noise

The photosensor of the WFS has the ADC with finite resolution, which produces the quantisation error. Typical ADC resolution of the photosensors in the cost-effective wavefront sensors is 10 or 12 bits. Such a resolution usually gives acceptable results; however, it is of interest how the quantisation in ADC affects the centroiding accuracy.

The quantisation error is assumed to be uniformly distributed between -0.5 and $+0.5$ of LSB and non-correlated with signal. The assumption is reasonable since the signal being quantised is larger than one least significant bit.

4.2 The centroiding error caused by the photon shot noise

In order to estimate the photon shot noise influence on the centroiding accuracy, we performed numerical simulations with double precision (i.e., no ADC quantisation). For each integration time of the photosensor*, there were generated 32 images of the centroids given the same turbulence realisation. The standard deviation of the centroids coordinates was calculated in the number of pixels, i.e., for the lenslet that have 32 pixels, the coordinate of centroid can be 15.54 pixels with standard deviation of 0.1554 pixels. The results of the photon shot noise influence are presented in Fig. 3 as the percentage of the centroid's coordinate, i.e. the standard deviation is divided on the centroid coordinate. The number of electrons in the lenslet was calculated as the mean value of non-zero pixels for each integration time.

Figure 3. The centroiding error versus the mean signal level in the lenslets.

As the integration time increases, the shot noise increases proportionally. However, in different lenslets, centroids are different; hence, the dependency may look different. The analysis of the data in Fig 3 allows to say that the influence of shot noise is from 0.2% to 1.5% of pixel.

4.2.1 Distribution of measurement errors of centroids in case of shot noise

The distribution of centroids coordinates in the presence of the photon shot noise has been evaluated as well. The photon shot noise in these numerical experiments was the only one turned on. The integration time and PRNU factor were fixed, and 1000 frames from the simulated SH WFS were taken. Then the histogram of the centroids distribution was estimated, as shown in Fig. 4.

The results are consistent with assumptions³⁴ that the centroiding errors in the case of dominance of the photon shot noise are Gaussian.

4.3 The influence of Photo Response Non Uniformity

The influence of such type of noise is significant, especially on high illumination levels. In the simulations, we varied the PRNU factor as 1%, 2% and 5%. The procedure of measurement is the same as described in Subsection 4.2. The results are shown on Fig. 5. Comparing results for low level of the PRNU with added

*The “*photosensor*” here is the *numerical model* of the real hardware photosensor, which we implemented in MATLAB as a series of software routines.

Figure 4. Estimation of centroids measurement distribution for the case of the photon shot noise only: a) low-light ($600 e^-$), b) intermediate light conditions ($3500 e^-$), and c) high light ($13500 e^-$).

shot noise (see Fig. 5) and shot noise only (see Fig. 3) one can conclude that PRNU of photosensor makes the error of centroiding more constant, i.e., less dependent from the signal level (integration time) of the centroids in wavefront sensor.

Figure 5. Dependency of centroid position error caused by PRNU and shot noise versus the mean signal level for various PRNU factors: a) 1% PRNU, b) 2% PRNU, and c) 5% PRNU.

The increase of the PRNU factor from 1% to 2% also increases the centroiding error, especially in low-light area (below 3000 electrons). The comparison of Fig. 5a and Fig. 5b shows the increase of the centroiding error from $0.3 \dots 0.6$ % in low-level area of measurements up to $0.4 \dots 0.9$ %. Further increase of the PRNU factor to 5% shows the increase of the centroiding errors to $0.9 \dots 1.7$ %, which makes the wavefront estimation difficult. As above, the exact behaviour of errors in different lenslets may differ, but the overall influence is similar.

Distribution of measurement errors of centroids in case of PRNU noise The distribution of the centroids coordinates has been estimated as well. We considered the case of PRNU factor of 1% and added photon shot noise. The integration time and the PRNU factor were fixed, and 1000 frames from the simulated SH WFS were taken. Then the histogram of the centroids coordinates was estimated with granularity of 50 bins. Since the signal values in lenslets are different, it is possible to estimate the centroiding positions distribution related to the different signal levels. The distribution of centroids coordinates was estimated for low-level conditions (about $500\text{--}600 e^-$), intermediate level of light ($3000\text{--}4000 e^-$), high level of light (more than $10000 e^-$). As above, the full well of the pixel is $20000 e^-$. The centroids coordinates are found using simple CoG algorithm. The results are presented in Fig. 6.

According to our simulations, the distribution of centroids coordinates tends to drift from Gaussian-alike in case of low illumination (see Fig. 6a) to more non-symmetrical distribution (see Fig. 6c) in case of high illumination level. That is, the assumption that in case of shot noise the centroiding error can be modelled as Gaussian³⁴ is generally correct; however, for the case of the intermediate illumination the distribution is non-symmetrical that may lead to inaccurate results.

Figure 6. Estimation of centroids coordinates distribution for the case of the photon shot noise and the PRNU of factor 1% for: a) low-light ($600 e^-$), b) intermediate light conditions ($3500 e^-$), and c) high light ($13500 e^-$).

4.4 The impact of read noise

The dark noise in these simulations consists of the *dark current*, the *dark current shot noise*, and the *dark fixed pattern noise* (simulated with non-symmetrical distribution). In the numerical experiments, the PRNU and the photon show noise were turned off.

4.4.1 Dark shot noise influence

The dark noise influences the centroiding accuracy on low-level and intermediate light levels[†]. Therefore, the integration time was varied to measure the signal levels below 5000-8000 e^- that corresponds to low-light conditions. The dark current shot noise was turned on and the dark signal figure-of-merit was varied as $0.1 nA/cm^2$, $0.5 nA/cm^2$ and $1 nA/cm^2$ in order to simulate different amount of dark noise. The results are summarised in Fig. 7.

Figure 7. Dependency of centroid position error versus mean signal level for the dark current figure of merit D_R : a) $0.1 nA/cm^2$, b) $0.5 nA/cm^2$, and c) $1 nA/cm^2$.

First, the relatively low-noise case with dark current figure of merit of $D_R = 0.1 nA/cm^2$ was considered. As seen in Fig. 7a, if the signal is very low (only 100-300 e^-), the centroiding error can be 1-3%. With increase of the signal (integration time or photons), the error grows toward 5-7% of pixel.

Then the simulations with dark current figure of merit of $D_R = 0.5 nA/cm^2$ were performed. The results presented in Fig. 7b shows that the centroiding errors in this case are more scattered than for the case of $0.1 nA/cm^2$ but of the same order 3-6% of the pixel.

For the relatively large dark current corresponding $D_R = 1 nA/cm^2$, the centroiding error is very large - typically 3-7% of pixel (see Fig. 7c). That is at least 2-3 times more than the photon shot noise and more than 3 times larger than the influence of the PRNU. These results can depend on the centroiding algorithm (which is simple CoG in this case) but should give a relative picture of the dark noise impact on the centroiding error.

[†]We are dealing with the *numerical model* of the photosensor in this paper. However, the comparison of the photosensor model with the hardware sensor shows excellent match of the numerical model and a real hardware.

One may further consider the column FPN that appears specifically in CMOS sensors. The column FPN appears as vertical stripes on the images and may degrade the centroiding accuracy even further.

4.4.2 Dark Fixed Pattern Noise influence

For the numerical simulations, we turned on the dark current (the dark current figure-of-merit $1nA/cm^2$ was assumed), the dark shot noise and varied the dark FPN factor as 10%, 25% and 40% that is typical for CMOS sensors. The results are presented in Fig. 8.

Figure 8. The dependency of the centroids position error versus the mean signal level for the Dark FPN of factor: a) 10%, b) 25%, and c) 40%.

One can compare the behaviour of the centroids standard deviation in two cases: 1) when the dark shot noise only as seen in Fig. 7, and 2) dark shot noise and dark FPN (see Fig. 8). Both simulations were made with the same value of dark current figure of merit $D_R = 0.1nA/cm^2$. The comparison of plot in Fig. 8 give rise to the fact that dark FPN makes the distribution of centroiding errors more dense and it does not set out on a constant level.

4.5 The influence on the ADC quantisation on centroiding errors

In order to compare the influence of the ADC, we turned on the photon shot noise, the PRNU with the factor 1%, the dark current with the dark current figure of merit $D_R = 0.5nA/cm^2$ on temperature $T=300K$ using Eq. 3, the dark current shot noise and the dark FPN of factor 25% (simulated with the Wald distribution, skewness parameter $\beta = 3$). Such parameters are considered typical for cost-effective CMOS photosensors usually employed in wavefront sensors. The ADCs with 12 bit, 10 bit, and 8 bit quantisation were considered. The results were compared with double-precision simulations (no quantisation noise) and summarised in Table 1.

Table 1. Quantisation error caused by the finite ADC resolution (error is given in percent of a pixel).

Light conditions	Double precision (no quantisation)		12 bit quantisation		10 bit quantisation		8 bit quantisation	
	mean error	max error	mean error	max error	mean error	max error	mean error	max error
Low light level	5.2%	8.1%	6.4%	12.8%	22.6%	32.1%	35.9%	65.2%
Mid level of light	3.9%	4.3%	4.4%	5.2%	7.8%	5.4%	10.3%	14.4%
High level of light	3.5%	4.1%	3.9%	4.2%	4.6%	5.0%	6.4	7.2%

One can see from the results in Table 1 that the quantisation errors produced by 10-bit ADC, which are frequently used in cost-effective WFS, may be considered as a sub-optimal solution. Indeed, mean error in centroids coordinates of 22% in low-light conditions can lead to poor wavefront sensing, especially in the case of a dim guide star. On the other hand, the ADC with 12 bit resolution introduces much less quantisation error in the signal from wavefront sensor. Therefore, it should be used in cost-effective WFS instead of commonly used 10-bit ADCs.

5. CONCLUSION

Inaccuracy in the detection of wavefront distortions introduces considerable errors into the wavefront reconstruction and leads to overall performance degradation of adaptive optics system. That is even more important in the case of cost-effective wavefront sensors, which use CMOS photosensors. Therefore, it is important to conduct numerical simulations to find out what types of lenslets and off-the-shelf CMOS sensors can be suitable for the cost-effective adaptive systems for small telescopes or ophthalmology devices.

Assuming that WFS has 64×64 pixels per lenslet, the pixelisation error is shown to grow slowly until the amount of pixels in lenslets are half of original (causes the error of 0.2% rms). The pixelisation error increases abruptly to 0.7% rms (3 times increase) when the number of pixels in the lenslet is a quarter of the original amount. The centroiding errors caused by photon shot noise only and for the case of shot noise and PRNU is of the same order (up to 1.5% of pixel). The PRNU influence on the centroiding accuracy for the PRNU factor of 5%, 2% and 1% is 0.9...1.7 % of pixel, 0.4...0.9 % of pixel and 0.3...0.6 % of pixel, respectively. The results for quantisation errors show that the 10-bit ADCs, which are frequently used in cost-effective WFS, may be considered as a sub-optimal solution. In low-light conditions, the mean error in centroids coordinates for 10 bit ADC is of 22%, whenever for the 12-bit ADC the mean error is only 6%.

The probability distributions of centroiding errors were estimated as well. The distributions of errors for the photon shot noise case are found to be Gaussian. However, for the photon shot noise and PRNU, the distribution of centroids errors tends to drift from the Gaussian-alike in case of low illumination to the non-symmetrical distribution in the case of high illumination.

Although the adaptive optics is increasing in popularity among astronomers, still only about 5% of papers published in the archival astronomical literature rely on data obtained with adaptive optics.³⁵ The reason is that adaptive optics systems still are very expensive and mostly used for large astronomical telescopes. However, it is possible to use widely available fast and inexpensive CMOS sensors; it would be possible to build low-cost adaptive optics systems for small telescopes, which are still requiring adaptive optics to be installed. The results provided in this paper can be a starting point for the design and formulation of specifications for cost-effective adaptive optics systems.

REFERENCES

- [1] Platt, B., "History and principles of Shack-Hartmann wavefront sensing," *Journal of Refractive Surgery* **17**(5), 573–577 (2001).
- [2] Seifert, L., Liesener, J., and Tiziani, H. J., "The adaptive shack-hartmann sensor," *Optics Communications* **216**(4-6), 313 – 319 (2003).
- [3] Shack, R. and Platt, B., "Production and use of a lenticular Hartmann screen," *J. Opt. Soc. Am* **61**(5), 656 (1971).
- [4] Tyson, R., [*Adaptive optics engineering handbook*], CRC Press (2000).
- [5] Widiker, J. J., Harris, S. R., and Duncan, B. D., "High-speed shack-hartmann wavefront sensor design with commercial off-the-shelf optics," *Appl. Opt.* **45**, 383–395 (Jan 2006).
- [6] Brittona, M., Velura, V., Lawa, N., Choib, P., and Penpraseb, B., "CAMERA: a compact, automated, laser adaptive optics system for small aperture telescopes," in [*Proceedings of SPIE, volume 7015, page 29*], (2008).
- [7] Cao, G. and Yu, X., "Accuracy analysis of a Hartmann-Shack wavefront sensor operated with a faint object," *Opt. Eng* **33**(7), 2331–2335 (1994).
- [8] Goad, L., Roddier, F., Beckers, J., and Eisenhardt, P., "National Optical Astronomy Observatories (NOAO) IR adaptive optics program. III-Criteria for the wavefront sensor selection," in [*Proceedings of SPIE*], **628**, 305–313 (1986).
- [9] Kane, T., Welsh, B., Gardner, C., and Thompson, L., "Wavefront detector optimization for laser guided adaptive telescopes," in [*Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*], **1114**, 160–171 (1989).
- [10] Morgan, J., Slater, D., Timothy, J., and Jenkins, E., "Centroid position measurements and subpixel sensitivity variations with the MAMA detector," *Applied optics* **28**(6), 1178–1192 (1989).

- [11] Marcus, S.; Nelson, R. L. R., "Preliminary evaluation of a fairchild ccd-211 and a new camera system," in [*Instrumentation in astronomy III; Proceedings of the Seminar, Tucson, Ariz., January 29-February 1, 1979. Society of Photo-Optical Instrumentation Engineers, p. 207-231.*], (1979).
- [12] Tyler, G. A. and Fried, D. L., "Image-position error associated with a quadrant detector," *J. Opt. Soc. Am.* **72**(6), 804–808 (1982).
- [13] Li, C., Xian, H., Rao, C., and Jiang, W., "Measuring statistical error of Shack–Hartmann wavefront sensor with discrete detector arrays," *Journal of Modern Optics* **55**(14), 2243–2255 (2008).
- [14] Hardy, J. W., [*Adaptive optics for astronomical telescopes*], Oxford University Press, USA (1998).
- [15] van Dam, M. A. and Lane, R. G., "Wave-front slope estimation," *J. Opt. Soc. Am. A* **17**, 1319–1324 (Jul 2000).
- [16] Irwan, R. and Lane, R. G., "Analysis of optimal centroid estimation applied to shack-hartmann sensing," *Appl. Opt.* **38**, 6737–6743 (Nov 1999).
- [17] Li, H., Song, H., Rao, C., and Rao, X., "Accuracy analysis of centroid calculated by a modified center detection algorithm for shack-hartmann wavefront sensor," *Optics Communications* **281**(4), 750 – 755 (2008).
- [18] Konnik, M. V. and Welsh, J. S., "On numerical simulation of high-speed ccd/cmos-based wavefront sensors in adaptive optics," in [*Proceedings of SPIE Optical Engineering + Applications, paper 8149-15*], (2011).
- [19] Schmidt, J. D., [*Numerical Simulation of Optical Wave Propagation, With Examples in Matlab*], Society of Photo-Optical Instrumentation Engineers (2010).
- [20] McGlamery, B. L., "Restoration of turbulence-degraded images," *J. Opt. Soc. Am.* **57**(3), 293–296 (1967).
- [21] Von Kármán, T., "Über laminare und turbulente reibung," *Z. Angew. Math. Mech* **1**(4), 233–235 (1921).
- [22] Goodman, J. W., [*Introduction to Fourier Optics*], McGraw-Hill book company (1968).
- [23] Yoon, G. Y., Jitsuno, T., Nakatsuka, M., and Nakai, S., "Shack hartmann wave-front measurement with a large f-number plastic microlens array," *Appl. Opt.* **35**, 188–192 (Jan 1996).
- [24] Neal, D., Copland, J., and Neal, D., "Shack-Hartmann wavefront sensor precision and accuracy," in [*Proceedings of SPIE*], **4779**, 148–160 (2002).
- [25] Ares, J. and Arines, J., "Minimum-variance centroid thresholding," *4th Iberoamerican Meeting on Optics and 7th Latin American Meeting on Optics, Lasers, and Their Applications* **4419**(1), 652–655 (2001).
- [26] James R. Janesick, [*Scientific Charge-Coupled Devices*], SPIE PRESS Monograph (2001).
- [27] Janesick, J. R., [*Photon Transfer*], SPIE International Society for Optical Engineering (2007).
- [28] Hornsey, R., [*Part III: Noise in Image Sensors*], University of Waterloo (1999).
- [29] Pinault, M.-A. T. E., "On the origin of carrier localization in $ga1 - xin\alpha nyas1 - ygaas$ quantum wells," *Applied Physics Letters* **78**(11), 1562–1564 (2001).
- [30] Baer, R. L., "A model for dark current characterization and simulation," in [*Electronic Imaging Conference, SPIE Vol. 6068, page 37-48*], **6068**, 37–48 (2006).
- [31] Gow, R., Renshaw, D., Findlater, K., Grant, L., McLeod, S., Hart, J., and Nicol, R., "A Comprehensive Tool for Modeling CMOS Image-Sensor-Noise Performance," *IEEE Transactions on Electron Devices* **54**(6), 1321–1329 (2007).
- [32] Dunlap, J. C., Porter, W. C., Bodegom, E., and Widenhorn, R., "Dark current in an active pixel complementary metal-oxide-semiconductor sensor," *Journal of Electronic Imaging* **20**(1), 013005 (2011).
- [33] Shuster, J., "On the inverse gaussian distribution function," *Journal of the American Statistical Association* **63**, 1514–1516 (1968).
- [34] Irwan, R. and Lane, R. G., "Analysis of optimal centroid estimation applied to shack-hartmann sensing," *Appl. Opt.* **38**, 6737–6743 (Nov 1999).
- [35] Hinz, P., Rodigas, T., Kenworthy, M., Sivanandam, S., Heinze, A., Mamajek, E., and Meyer, M., "Thermal Infrared MMTAO Observations of the HRá8799 Planetary System," *The Astrophysical Journal* **716**, 417 (2010).