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Modelling Power Transformers to Support the Interpretation of Frequency Response Analysis

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Abstract—A power transformer will yield a frequency response which is unique to its mechanical geometry and electrical properties. Changes in the frequency response of a transformer can be potential indicators of winding deformation, as well as other structural and electrical problems. A diagnostic tool which leverages this knowledge in order to detect such changes is Frequency Response Analysis (or FRA). To date, FRA has been used to identify changes in a transformer's frequency response but with limited insight into the underlying cause of the change. However there is now a growing research interest in specifically identifying the structural change in a transformer directly from its FRA signature. The aim of this paper is to support FRA interpretation through the development of wideband three phase transformer models which are based on three types of FRA test. The resulting models can be used as a flexible test bed for parameter sensitivity analysis, leading to greater insight into the effects geometric change can have on transformer FRA. The paper will demonstrate the applicability of this modelling approach by simultaneously fitting each model to the corresponding FRA data sets without a priori knowledge of the transformer's internal dimensions, and then quantitatively assessing the accuracy of key model parameters.

Index Terms—power, transformer, frequency, response, analysis, FRA, wideband, model, interpretation, geometry, deformation, sensitivity.

I. INTRODUCTION

THE average age of the world's population of power transformers is increasing [1]. Due to high replacement costs, research into maximizing their longevity is a high priority. To successfully accomplish this in a safe and efficient manner, their condition must be regularly monitored in order to schedule the appropriate maintenance and repair.

When subjected to fault currents, tremendous forces can be placed upon the transformer windings and mechanical structure [2]. Such high levels of mechanical stress can lead to winding deformation which could ultimately lead to transformer failure. The mechanical and electrical integrity of a transformer can also be compromised through poor workmanship and during transportation [3].

Frequency Response Analysis (FRA) is a commonly used diagnostic approach which looks for changes in the frequency response of a transformer. Each FRA test will yield a signature unique to the transformer's mechanical geometry from the perspective of the input and output measurement terminal positions [4]. Changes in the frequency response signature can be indicative of winding deformation. However, to date, there is relatively little understanding of how to interpret the

underlying cause of the change directly from the frequency response [5].

Transformer Frequency Response Analysis was initially proposed by Dick and Erven in 1978 [6]. Since its introduction there has been an interest in advancing the diagnostic benefits associated with FRA by improving our ability to interpret any observed changes. One approach which has been adopted by many researchers has been the use of a transformer model based on its geometric parameters [4], [7], [8]. The rationale behind this approach is that a change in the geometry of a transformer will affect the parameters represented in the model. A sensitivity analysis of the model parameters could therefore be used to assist in determining the root cause behind any change in a transformer's frequency response.

A number of researchers have made significant contributions to this area in recent years. In 2003 Rahimpour et al proposed a detailed single phase transformer model in order to diagnose axial displacement and radial deformation [7]. In 2005 Bjerkan made further advances by proposing a detailed three phase transformer model for the sensitivity analysis of FRA [4]. Articles by Abeywickrama et al between 2005 and 2008 further extended the research area by providing a comprehensive three phase transformer model which included the high frequency behaviour of the transformer's core [8]–[10]. An article by Ragavan et al in 2008 presented an impedance function for a single phase transformer based upon FRA measurement [3]. This work utilised the FRA data to determine model parameters for a simple ladder network directly from the pole and zero locations of the frequency response. In 2009 a three phase transformer model was proposed by Shintemirov et al that utilised genetic algorithms to interpret FRA results at low frequencies [11]. Shintemirov's approach facilitated the determination of a transformer core's parameters. A model proposed by Sofian et al (2010) was used to conduct simulated sensitivity studies on a large three phase autotransformer [12]. Sofian et al investigated the influence that changes to the transformer's structure would have on its frequency response. However despite these advances in the research area, the interpretation of FRA remains subjective and typically based on an experienced technician's assessment [13]. A report from the CIGRE WG A2.26 has called for further research to be conducted in the area of transformer modelling based on geometric parameters in order to support the interpretation of FRA [5].

As highlighted above, researchers have developed comprehensive three phase transformer models for use with FRA [4], [8], [12]. However in each of these cases parameter determination was made via FEA and analytical techniques. This approach requires a priori knowledge of the transformer's

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internal dimensions and material properties. Such information is typically unavailable outside of the laboratory. Other researchers have utilized estimation techniques in order to determine their model parameters, however in the case of [3], [7], [14], [15], they have only utilised a single phase transformer topology which will limit the model's practical application, or a reduced parameter circuit as in the case of [16]. Research by Shintemirov et al estimates parameters directly from FRA and is based on a three phase topology, however it is only aimed at core identification and is not applicable for the entire FRA spectrum [11]. Our research extends on this previous work through the development of a wideband frequency, three phase transformer model, whose parameters are determined directly from FRA. Parameter determination is facilitated by the application of a constrained nonlinear optimisation algorithm to the data sets of three different types of FRA. This approach significantly increases the accuracy of our parameter prediction, and the authors believe that this approach is novel in its application.

To demonstrate the applicability of the FRA modelling approach, FRA data was recorded from an air cooled 1.3MVA 11kV/433V distribution transformer. Since three different types of FRA tests are being considered and each test has three terminal permutations, this results in nine unique FRA data sets. To maximise the accuracy of the modelling approach, the FRA models are simultaneously fitted to all nine sets of data. The accuracy of the approach is verified by quantitatively assessing a number of key parameters against their physically measured counterparts.

In this paper we assume that the transformer is of core type construction with concentric windings. The leakage inductance is considered to be restricted to the axial path between the high and low voltage windings [17].

This paper is structured in the following manner. Section II develops a generic phase model that is used as a building block in later sections. Section III adopts a layered modelling approach in order to build models for three different types of FRA test. Section IV develops the FRA model transfer functions and the corresponding estimation algorithm. In Section V each of the FRA models is fitted to FRA data and the accuracy of the estimated model parameters is assessed. A sensitivity analysis example is then presented. Concluding remarks are given in Section VI.

II. GENERIC PHASE MODEL

The origins of transformer modelling can be traced back to a 1902 paper by Thomas [18]. In this paper Thomas highlighted the need for modelling in order to determine the surge voltage distribution across a transformer's windings. The original ladder network model of a transformer is generally associated with Blume and Boyajian's work of 1919 [19]. In their paper they considered the influence of a transformer's mutual and leakage inductance, the capacitance between adjacent coils, and the capacitance between the winding and ground. Since these foundation papers there have been many significant contributions to the area of power transformer modelling including [20]–[25]. We build upon this previous research

Figure 1. Transformer model for generic phase X

implementing a generic n section lumped parameter three phase transformer model.

There are three terminal permutations associated with each FRA test. For example, for the Capacitive Interwinding test [5], the three terminal combinations are the high voltage terminal A to the low voltage terminal a , the high voltage terminal B to the low voltage terminal b , and the high voltage terminal C to the low voltage terminal c . To represent each of the terminal combinations for this example in a model, it is convenient to have generic phase references for the injection, measurement and open circuited terminals. This has been accommodated in our modelling approach through the use of generic phase referencing. Throughout this paper the generic high voltage terminals are designated X - Y - Z , and the corresponding low voltage terminals are x - y - z . The n section lumped parameter model for generic phase X is given in Figure 1.

Each section of the high and low voltage windings consists of the series combination of an inductive element $\mathcal{L}$ and a resistive element $\mathcal{R}$. To account for the capacitance between these windings, a capacitive element $\mathcal{C}_{Xx}$ couples each equiv-

Figure 2. Magnetic equivalent circuit of a three phase two winding core type transformer

alent winding section. The capacitance between turns and adjacent discs is modelled with the addition of $\mathcal{C}_{SX}$ and $\mathcal{C}_{Sx}$, for the high and low voltage windings respectively. The capacitance between the low voltage winding and ground is given by $\mathcal{C}_{gx}$ and the capacitance between the high voltage winding and the transformer tank walls is given by $\mathcal{C}_{gX}$. Finally, the capacitances $\mathcal{C}_{XY}$ and $\mathcal{C}_{ZX}$ represent the capacitance between adjacent high voltage windings (i.e. A to B and B to C). Note that the magnitude of all shunt capacitances at the winding ends are at half their normal value to reflect their relative distribution.

Each of the inductive, resistive and capacitive elements shown in Figure 1 will now be discussed in detail.

A. Inductive Element

The use of a reluctance model is a convenient method for modelling flux paths within a transformer. This modelling approach is a physical representation, utilising a magnetic circuit based on the transformer's core geometry. In the magnetic circuit, each winding is replaced with a magnetomotive source and each flux path, including those representative of leakage, with a corresponding reluctance.

The proposed magnetic circuit for an FRA injection test is presented in Figure 2. In this figure, $\mathbb{F}_A$ represents the magnetomotive force (mmf) due to the high voltage winding of phase A, $\mathbb{F}_a$ the mmf due to the low voltage winding on phase A and so forth for the other phases. $\mathbb{R}_E$ is the transformer core limb reluctance and $\mathbb{R}_Y$ is the transformer core yoke reluctance. $\mathbb{R}_L$ is the winding leakage flux reluctance. The linear dimensions of the core are l_E for the mean core limb length and l_Y for the mean core yoke width. In order to reduce the level of complexity associated with the model's magnetic circuit, the influence of core joints will not be considered [8].

The core reluctance can be defined in terms of the mean path length l , core cross sectional area A_{CS} , and the core

Table I
TRANSFORMER FERROMAGNETIC INDUCTANCE MATRIX

L	A_j	B_j	C_j	a_j	b_j	c_j
A_i	$\bar{L}$	$\frac{\bar{L}}{2\Gamma}$	$\frac{\bar{L}}{\Lambda}$	$\frac{\bar{L}}{\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{\Lambda\tilde{a}}$
B_i	$\frac{\bar{L}}{2\Gamma}$	$\frac{\bar{L}}{\Gamma}$	$\frac{\bar{L}}{2\Gamma}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{\Gamma\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$
C_i	$\frac{\bar{L}}{\Lambda}$	$\frac{\bar{L}}{2\Gamma}$	$\bar{L}$	$\frac{\bar{L}}{\Lambda\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{\tilde{a}}$
a_i	$\frac{\bar{L}}{\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{\Lambda\tilde{a}}$	$\frac{\bar{L}}{\tilde{a}^2}$	$\frac{\bar{L}}{2\Gamma\tilde{a}^2}$	$\frac{\bar{L}}{\Lambda\tilde{a}^2}$
b_i	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{\Gamma\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}^2}$	$\frac{\bar{L}}{\Gamma\tilde{a}^2}$	$\frac{\bar{L}}{2\Gamma\tilde{a}^2}$
c_i	$\frac{\bar{L}}{\Lambda\tilde{a}}$	$\frac{\bar{L}}{2\Gamma\tilde{a}}$	$\frac{\bar{L}}{\tilde{a}}$	$\frac{\bar{L}}{\Lambda\tilde{a}^2}$	$\frac{\bar{L}}{2\Gamma\tilde{a}^2}$	$\frac{\bar{L}}{\tilde{a}^2}$

permeability μ such that,

$$\mathbb{R}_E = \left[\frac{1}{\mu A_{CS}} \right] l_E \quad (1)$$

$$\mathbb{R}_Y = \left[\frac{1}{\mu A_{CS}} \right] l_Y, \quad (2)$$

where $\mathbb{R}_E$ and $\mathbb{R}_Y$ are directly proportional to the limb and yoke length. By definition, inductance is proportional to the winding flux linkage relative to the current in the winding for self inductance, or the current in another winding for mutual inductance. From (1) and (2), the inductive relationships that exist between winding sections on the three phase transformer core will therefore be dependent upon the core dimensions. It is convenient to specify the self and mutual inductance relationships that exist for each winding section in terms of a common base inductance¹,

$$\bar{L} = \frac{\mu A_{CS} \left(\frac{N_X}{n} \right)^2 (2l_E + l_Y)}{(l_E + l_Y)(3l_E + l_Y)}, \quad (3)$$

and two core dimension constants,

$$\Gamma = \frac{2l_E + l_Y}{2(l_E + l_Y)} \quad (4)$$

$$\Lambda = \frac{2l_E + l_Y}{l_E}, \quad (5)$$

where n is the number of lumped parameter sections within the transformer model and N_X is the number of turns on each high voltage winding. A matrix that represents each of the respective self and mutual inductances can then be generated. This matrix is presented in Table I where i and j are the lumped parameter section numbers and $\tilde{a}$ is the turns ratio.

The model also incorporates a leakage inductance between winding sections. For example, the leakage inductance between sections i and j of generic winding X is given by L_{LXij} .

¹The base inductance relationship is based on the self inductance of an outer limb high voltage winding section (refer Table I).

The permeability of the transformer core as specified in the base inductance relationship of (3) is a complex frequency dependent term. Under low field conditions and the wide frequency spectrum of an FRA test, μ can be defined to be [26],

$$\mu = \mu_0 \mu_s = \mu_0 \left(\mu'_s - j\mu''_s \right) = \frac{k\mu_0\mu_i}{\gamma b} \tanh(\gamma b) . \quad (6)$$

In this relationship μ_0 is the permeability of free space, μ_s is the effective permeability, μ_i is the initial permeability of the core material, k is the lamination stacking factor (which approaches unity) and b is the core lamination half thickness. The propagation constant, γ , is defined as,

$$\gamma = \sqrt{j\omega\sigma\mu_0\mu_i} , \quad (7)$$

where ω is the angular frequency in radians per second, and σ is the lamination material conductivity. As a result of the complex permeability of the core in (6), a winding's inductance and its associated magnetic losses can be represented as an impedance. This impedance can be modelled as the series combination of a frequency dependent inductance, L' , and resistance R'' [27],

$$Z = j\omega L_0(\mu'_s - j\mu''_s) \quad (8)$$

$$= j\omega L' + R'' , \quad (9)$$

where R'' can be considered to be the magnetic loss *resistance* and L_0 is the inductance with unity permeability.

An inductive element $\mathfrak{L}_{Xi}(\omega)$ is now proposed that takes into account the self and mutual inductance relationships described in Table I and the real and imaginary components of (9). $\mathfrak{L}_{Xi}(\omega)$ represents the frequency dependent inductance from section i of generic winding X and is shown in Figure 3(a).

B. Resistive Element

The transformer winding can be considered to have both a DC and a frequency dependent AC resistance. The DC resistance is directly proportional to the resistivity of the conductor and inversely proportional to the winding conductor cross sectional area. The AC resistance is due to the induction of eddy currents within the windings. These induced resistances can be classified into two categories, skin and proximity effect.

An analytical estimate for skin effect can be made using the Dowell Method [28]. If it is assumed that the conductors are closely packed, each layer of a winding will approximate the geometry of a conductor foil. The problem can then be reduced to a one dimensional model [29], i.e.

$$R_S = \frac{R_{DC}\xi}{2} \left[\frac{\sinh \xi + \sin \xi}{\cosh \xi - \cos \xi} \right] , \quad (10)$$

where

$$\xi = \frac{d\sqrt{\pi}}{2\delta} , \quad (11)$$

d is the conductor diameter,

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} , \quad (12)$$

is the skin depth, with f the frequency in Hz, and the permeability and conductivity of the conductor material are given by μ and σ respectively.

Assuming once again that the conductors are closely packed and that each layer of a winding will approximate the geometry of a conductor foil, the AC resistance due to proximity effect for the m^{th} winding layer [29] is,

$$R_P = \frac{R_{DC}\xi}{2} \left[(2m-1)^2 \frac{\sinh \xi - \sin \xi}{\cosh \xi + \cos \xi} \right] . \quad (13)$$

Assuming that the magnetic field from the other conductors is uniform across the conductor cross section, an orthogonal relationship exists between the skin and proximity effect [30]. The two effects can be decoupled and an estimate for the total eddy current losses can be determined through the addition of both effects (10) and (13). By combining both the AC and DC winding resistances and considering them to be sectionally distributed, the series resistance element, $\mathfrak{R}_{Xi}$, is given by,

$$\mathfrak{R}_{Xi}(\omega) = R_{PXi}(\omega) + R_{SXi}(\omega) + R_{DCXi} , \quad (14)$$

where ω is the angular frequency. $\mathfrak{R}_{Xi}$ is presented in Figure 3(c).

C. Capacitive Element

In addition to the displacement current in a capacitor, a dielectric material will also experience losses through conduction and material polarization [8]. For the non-ideal capacitance $\mathfrak{C}$, the circuit admittance is given by [8] [31],

$$\begin{aligned} Y &= j\omega\mathfrak{C} \\ &= G + j\omega C . \end{aligned} \quad (15)$$

It is well known that the admittance of a non-ideal capacitance can be represented as the parallel combination of a conductance G and an ideal capacitance C [17]. The lumped parameter circuit element used to represent each of the non-ideal capacitors used in the transformer model takes the form presented in Figure 3(b). For mathematical convenience an equivalent resistance is substituted for conductance.

Each of the model's capacitance and dielectric loss terms are estimated directly by the algorithm and are not devolved into lower level parameters.

III. MODELLING FOR FRA

The main FRA test types as classified by CIGRE are the End to End Open Circuit, End to End Short Circuit, Capacitive Interwinding, and the Inductive Interwinding test [5]. The End to End Open Circuit test inputs a signal into one end of a winding and measures the response at the other end. It is the most commonly used topology due to its simplicity and its ability to examine individual windings separately. The End to End Short Circuit test is similar to the Open Circuit version, but with a winding of the same phase short circuited. This topology enables the influence of the magnetising inductance to be removed such that the leakage inductance will dominate the low frequency response. At high frequencies the response will be similar to the End to End Open Circuit test [5]. The

Figure 3. Lumped parameter circuit elements. (a) $\mathcal{L}_{X_i}(\omega)$, (b) $\mathcal{C}_{X_i}(\omega)$, (c) $\mathcal{R}_{X_i}(\omega)$

Figure 4. FRA test configurations for a Dyn vector group; (a) High Voltage Winding End to End Open Circuit Test, (b) Low Voltage Winding End to End Open Circuit Test, (c) Capacitive Interwinding Test.

Capacitive Interwinding test inputs a signal into one end of one winding and measures the response at the end of another winding of the same phase. As the test name suggests, this test is particularly sensitive to the interwinding capacitance between windings of the same phase. The Inductive Interwinding test is connected as per the Capacitive Interwinding test, but with the opposite ends of both windings connected to ground. This test will be dominated at low frequencies by the turns ratio of the windings under test.

An interesting article by Satish et al classified the sensitivity of a broad range of FRA test connections [32]. Sensitivity was quantified by counting the number of observed natural frequencies in each of the FRA tests. Of the standard FRA tests considered above, Satish et al considered the End to End Open Circuit test to be in the highest sensitivity category. The End to End Short Circuit, Capacitive Interwinding and Inductive Interwinding tests were each classified in the second tier of sensitivity. In 2006 Jayasinghe et al conducted research on the sensitivity of different FRA measurement connections and their ability to detect different types of faults [14]. In this article the Capacitive Interwinding FRA test was found to be more sensitive to axial displacement and radial deformation than the End to End Open Circuit test. In addition, an article by Ryder et al in 2003 proposes the use of Capacitive Interwinding FRA test as a suitable means of detecting the axial collapse of a 300MVA autotransformer winding [33]. On this basis, and limiting the number of FRA tests considered to three due to computational time, we have decided to focus on the End to End Open Circuit test for both the high voltage and low voltage windings, and the Capacitive Interwinding test.

A. High Voltage End to End Open Circuit FRA Test

Figure 4(a) shows the High Voltage Winding End to End Open Circuit Test on a generic phase Dyn connected transformer. This test involves injecting a swept frequency sine wave into one of the high voltage terminals and recording the output response of another. All of the remaining terminals are left unconnected with respect to the test equipment. Note that the test is repeated for all three terminal pair permutations.

To facilitate FRA modelling, the input terminal for this test sequence is labelled generic terminal X and the output terminal generic terminal Y (terminals Z, x, y and z are unconnected). The modelling approach involves the substitution of the physical phases, A through C, into a prescribed generic counterpart, X through Z, for each of the three FRA test combinations (AB, BC and CA).

Noting that the test transformer is a Dyn connected distribution transformer, the generic phase approach facilitates modelling the phase order differences between Dyn1 and Dyn11 vector groups. With reference to Figure 5, it is noted that the high voltage winding of a Dyn1 vector group has phase sequence A-C-B whereas a Dyn11 has phase sequence A-B-C. By comparing the physical connections of Figure 5 with the generic phase connections of Figure 4(a), the generic phase allocation for each test is established (see Table II).

Figure 5. Dyn vector group topologies; (a) Dyn1, (b) Dyn11.

Table II
GENERIC PHASE ALLOCATION FOR FRA TESTS OF DYN1 AND DYN11 VECTOR GROUPS FOR THE HIGH VOLTAGE WINDING END TO END OPEN CIRCUIT TEST.

Vector Group	FRA Test ($V_{IN}V_{OUT}$)	X/x	Y/y	Z/z
Dyn1	AC	A/a	C/c	B/b
	BA	B/b	A/a	C/c
	CB	C/c	B/b	A/a
Dyn11	AB	A/a	B/b	C/c
	BC	B/b	C/c	A/a
	CA	C/c	A/a	B/b

Table III
GENERIC PHASE ALLOCATION FOR FRA TESTS OF DYN1 AND DYN11 VECTOR GROUPS FOR THE LOW VOLTAGE WINDING END TO END OPEN CIRCUIT TEST.

Vector Group	FRA Test ($V_{IN}V_{OUT}$)	X/x	Y/y	Z/z
Dyn1	an	A/a	C/c	B/b
	bn	B/b	A/a	C/c
	cn	C/c	B/b	A/a
Dyn11	an	A/a	B/b	C/c
	bn	B/b	C/c	A/a
	cn	C/c	A/a	B/b

B. Low Voltage End to End Open Circuit FRA Test

The Low Voltage Winding End to End Open Circuit Test on a generic phase Dyn connected transformer is shown in Figure 4(b). This test injects a swept frequency sine wave into a low voltage terminal and records the output response at the neutral. All other terminals are left unconnected. The test is performed sequentially between each of the low voltage terminals and neutral.

To facilitate the modelling of this FRA test, the low voltage generic terminal x is the source input and the output is the low voltage winding neutral. The modelling approach involves the substitution of the physical phases into their prescribed generic counterparts for each of the three FRA tests (an, bn, and cn). The FRA tests and their corresponding generic phase allocation for the Low Voltage Winding End to End Open Circuit Test is shown in Table III.

C. Capacitive Interwinding FRA Test

Figure 4(c) shows the Capacitive Interwinding Test on a generic phase Dyn connected transformer. This test is conducted between the high and low voltage terminals of a given

Table IV
GENERIC PHASE ALLOCATION FOR FRA TESTS OF DYN1 AND DYN11 VECTOR GROUPS FOR THE CAPACITIVE INTERWINDING TEST.

Vector Group	FRA Test ($V_{IN}V_{OUT}$)	X/x	Y/y	Z/z
Dyn1	Aa	A/a	C/c	B/b
	Bb	B/b	A/a	C/c
	Cc	C/c	B/b	A/a
Dyn11	Aa	A/a	B/b	C/c
	Bb	B/b	C/c	A/a
	Cc	C/c	A/a	B/b

Figure 6. Layered modelling approach.

phase, with the remaining terminals left unconnected. The test is then repeated for the other two phases.

To facilitate modelling of this FRA test, the injection point is the generic high voltage terminal X and the output is the generic low voltage terminal x. The generic phase allocation for each of the three FRA tests (Aa, Bb and Cc) is shown in Table IV.

D. Layered Modelling Approach

To develop a flexible FRA model that can be easily expanded to take into account the different FRA tests and transformer vector group connections, a layered modelling approach has been adopted. In this approach the first layer is three instances (X, Y and Z) of the generic phase model discussed in Section II. The layer above this is the vector group topology of the transformer, which is Dyn in this

case. The outermost layer is the FRA test connection under consideration. The layered modelling approach is shown in Figure 6.

IV. MATHEMATICAL MODEL

A. Normal Tree Conversion

To conduct analysis on a complex circuit, such as the model structures that are being proposed, it is advantageous to convert the models into their *normal tree* form [34]. As an example, the normal tree associated with the High Voltage End to End Open Circuit FRA test is shown in Figure 7.

B. State Space Equation

For system analysis it is convenient to use a state space representation. The state space equations for a network tree [34] are given by,

$$\begin{bmatrix} \dot{\mathbf{q}}(t) \\ \dot{\boldsymbol{\phi}}(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{q}(t) \\ \boldsymbol{\phi}(t) \end{bmatrix} + \mathbf{B}\mathbf{v}_V(t), \quad (16)$$

where

$$\mathbf{A} = - \begin{bmatrix} \mathbb{K}_{CC}\mathbb{C}^{-1} & \mathbb{K}_{C\mathcal{L}}\boldsymbol{\mathcal{L}}_M^{-1} \\ \mathbb{K}_{\mathcal{L}C}\mathbb{C}^{-1} & \mathbb{K}_{\mathcal{L}\mathcal{L}}\boldsymbol{\mathcal{L}}_M^{-1} \end{bmatrix}, \quad (17)$$

$$\mathbf{B} = \begin{bmatrix} \mathbb{K}_{CC}\mathbb{C}^{-1}\mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{VS}^T - \mathbb{K}_{CV} \\ \mathbb{K}_{\mathcal{L}C}\mathbb{C}^{-1}\mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{VS}^T - \mathbb{K}_{V\mathcal{L}} \end{bmatrix}. \quad (18)$$

The state variable $\mathbf{q}(t)$ is defined as,

$$\mathbf{q}(t) = \mathbb{C}\mathbf{v}_C(t) + \mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{VS}^T\mathbf{v}_V(t), \quad (19)$$

which represents the “*net-charge-per-capacitance-tree-branch-cuts*”. The state variable $\boldsymbol{\phi}(t)$ is defined to be,

$$\boldsymbol{\phi}(t) = \boldsymbol{\mathcal{L}}_M\mathbf{i}_{\mathcal{L}}(t), \quad (20)$$

which represents the “*net-flux-per-inductance-link-fundamental-loop*”. In addition, $\mathbf{v}_C(t)$ is the capacitance branch vector, $\mathbf{v}_V(t)$ the voltage source branch vector and $\mathbf{i}_{\mathcal{L}}(t)$ the inductance current link vector. $\mathbb{C}_C$ is the capacitance branch parameter matrix, $\mathbf{C}_S$ the capacitance link parameter matrix, $\boldsymbol{\mathcal{L}}_M$ is the inductance link parameter matrix and $\mathbf{F}_{..}$ is a submatrix that relates the corresponding network tree branch to link coupling. $\mathbb{C}$ is given by,

$$\mathbb{C} = \mathbf{C}_C + \mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{CS}^T. \quad (21)$$

For further details please refer to [34].

C. Transfer Function

An FRA test results in a plot of the frequency response between the recorded output voltage and the injected input voltage. This is effectively the transfer function between the two FRA test terminals of the transformer. With reference to the normal tree model of the FRA test in Figure 7, the FRA input voltage is given by V_{IN} and the FRA output voltage is equivalent to the voltage drop across the 50Ω termination resistor R_T . This voltage drop can be considered in terms of branch capacitor voltage drops. From Figure 7, the output voltage is equivalent to the combined voltage drops across the

branch capacitors C_{gy1} and C_{Yy1} . The transfer function can then be determined from the resulting input and output voltage relationships.

The capacitor branch voltages can be determined from the state space equation of (16) in the following manner. First, take the Laplace transform of the state space relationship,

$$\begin{bmatrix} \mathbf{q}(s) \\ \boldsymbol{\phi}(s) \end{bmatrix} = (\mathbf{sI} - \mathbf{A})^{-1} \mathbf{B}\mathbf{v}_V(s). \quad (22)$$

$\mathbf{q}(s)$ and $\boldsymbol{\phi}(s)$ can be written in terms of their matrix elements,

$$\mathbf{q}(s) = \begin{bmatrix} q(s)_1 & \cdots & q(s)_{6n} \end{bmatrix}^T. \quad (23)$$

$$\boldsymbol{\phi}(s) = \begin{bmatrix} \phi(s)_1 & \cdots & \phi(s)_{6n} \end{bmatrix}^T. \quad (24)$$

With $\mathbf{v}_V(s) = V_{IN}(s)$, we can then define a matrix $\mathbf{P}$ such that,

$$\mathbf{P} = (\mathbf{sI} - \mathbf{A})^{-1} \mathbf{B} = \begin{bmatrix} p_1 & \cdots & p_{12n} \end{bmatrix}^T. \quad (25)$$

The relationship in (25) can be written in block form relative to $\mathbf{q}(s)$ and $\boldsymbol{\phi}(s)$,

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_q & \mathbf{P}_{\phi} \end{bmatrix}^T, \quad (26)$$

where

$$\mathbf{P}_q = \begin{bmatrix} p_1 & \cdots & p_{6n} \end{bmatrix}, \quad (27)$$

and

$$\mathbf{P}_{\phi} = \begin{bmatrix} p_{(6n+1)} & \cdots & p_{12n} \end{bmatrix}. \quad (28)$$

Hence,

$$\mathbf{q}(s) = \mathbf{P}_q^T V_{IN}(s), \quad (29)$$

and,

$$\boldsymbol{\phi}(s) = \mathbf{P}_{\phi}^T V_{IN}(s). \quad (30)$$

Equating (19) with (29) and substituting $V_{IN}(s)$ for $\mathbf{v}_V(s)$,

$$\begin{aligned} \mathbf{P}_q^T V_{IN}(s) &= \mathbb{C}\mathbf{v}_C(s) + \mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{VS}^T V_{IN}(s) \\ \mathbb{C}\mathbf{v}_C(s) &= \mathbf{P}_q^T V_{IN}(s) - \mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{VS}^T V_{IN}(s) \\ \mathbf{v}_C(s) &= \mathbb{C}^{-1} [\mathbf{P}_q^T - \mathbf{F}_{CS}\mathbf{C}_S\mathbf{F}_{VS}^T] V_{IN}(s). \end{aligned} \quad (31)$$

The relationship defined in (31) facilitates the determination of all of the capacitance branch voltages since,

$$\mathbf{v}_C(s) = \begin{bmatrix} v_{Cgx2} & \cdots & v_{Cgx(n+1)} & v_{Cgy1} & \cdots & v_{Cgy(n)} \\ v_{Cgz1} & \cdots & v_{Cgz(n)} & v_{CXx1} & \cdots & v_{CXx(n)} \\ v_{CYy1} & \cdots & v_{CYy(n)} & v_{CZz1} & \cdots & v_{CZz(n)} \end{bmatrix}_{6n \times 1}^T \quad (32)$$

To determine $V_{OUT}(s)$ we will define a matrix $\mathbf{W}$ of dimension $1 \times 6n$ for the summation of the appropriate branch capacitor voltages such that,

$$V_{OUT}(s) = \mathbf{W}\mathbf{v}_C(s). \quad (33)$$

Figure 7. Normal tree representation for a High Voltage Winding End to End Open Circuit FRA Test on a generic phase Dyn connected model.

Table V
COST FUNCTION FRA WEIGHTS FOR THE APPLIED EXAMPLE.

Symbol	FRA Test	Weight
w_H	HV End to End	1
w_L	LV End to End	$\frac{1}{10}$
w_{HL}	Capacitive Interwinding	$\frac{1}{5}$

For the High Voltage End to End Open Circuit FRA Test of Figure 7, V_{OUT} is the combined voltage drops across the branch capacitors C_{gy1} and C_{Yy1} , and hence,

$$\mathbf{W} = \begin{bmatrix} 0_1 & \cdots & 0_n & 1_{(n+1)} & 0_{(n+2)} & \cdots & 0_{4n} \\ & & & 1_{(4n+1)} & 0_{(4n+2)} & \cdots & 0_{6n} \end{bmatrix}_{1 \times 6n}. \quad (34)$$

Substituting (31) into (33),

$$\begin{aligned} V_{OUT}(s) &= \mathbf{W}\mathbf{C}^{-1} [\mathbf{P}_q^T - \mathbf{F}_{CS}\mathbf{C}_s\mathbf{F}_{VS}^T] V_{IN}(s) \\ \therefore \hat{H}(s) &= \frac{V_{OUT}(s)}{V_{IN}(s)} \\ &= \mathbf{W}\mathbf{C}^{-1} [\mathbf{P}_q^T - \mathbf{F}_{CS}\mathbf{C}_s\mathbf{F}_{VS}^T] \end{aligned} \quad (35)$$

D. Estimation Algorithm

A constrained nonlinear optimisation algorithm is utilised to estimate the transfer functions based on FRA data. This algorithm determines the best fit between the proposed models and the corresponding FRA data by finding the model parameters that minimise a cost function. In order to avoid local minima, model parameters are constrained where possible using transformer details that can be obtained through external measurements and knowledge of common manufacturing practices. The cost function, J , is based on the 2-norm between each of the FRA data sets and their corresponding FRA model. A phase dependent model is applied to each of the three phase permutations of each of the three types of FRA test, hence J represents the cumulative total of each 2-norm associated with these tests,

$$J = w_H J_H + w_L J_L + w_{HL} J_{HL}. \quad (36)$$

where

$$\begin{aligned} J_H &= \left\| \log_{10} \left(\frac{\hat{H}_{AC}(s)}{H_{AC}(s)} \right) \right\|^2 + \\ &\quad \left\| \log_{10} \left(\frac{\hat{H}_{BA}(s)}{H_{BA}(s)} \right) \right\|^2 + \\ &\quad \left\| \log_{10} \left(\frac{\hat{H}_{CB}(s)}{H_{CB}(s)} \right) \right\|^2, \end{aligned} \quad (37)$$

$$\begin{aligned} J_L &= \left\| \log_{10} \left(\frac{\hat{H}_{an}(s)}{H_{an}(s)} \right) \right\|^2 + \\ &\quad \left\| \log_{10} \left(\frac{\hat{H}_{bn}(s)}{H_{bn}(s)} \right) \right\|^2 + \\ &\quad \left\| \log_{10} \left(\frac{\hat{H}_{cn}(s)}{H_{cn}(s)} \right) \right\|^2, \end{aligned} \quad (38)$$

$$\begin{aligned} J_{HL} &= \left\| \log_{10} \left(\frac{\hat{H}_{Aa}(s)}{H_{Aa}(s)} \right) \right\|^2 + \\ &\quad \left\| \log_{10} \left(\frac{\hat{H}_{Bb}(s)}{H_{Bb}(s)} \right) \right\|^2 + \\ &\quad \left\| \log_{10} \left(\frac{\hat{H}_{Cc}(s)}{H_{Cc}(s)} \right) \right\|^2. \end{aligned} \quad (39)$$

In (37), (38) and (39) $H_{..}(s)$ represents the FRA data and $\hat{H}_{..}(s)$ the estimated transfer function for the corresponding FRA model. $\hat{H}_{..}(s)$ is defined in (35). Weights can also be given to each type of FRA test. In (36) w_H is the weighting given to the HV End to End Open Circuit FRA Test, w_L is the weighting given to the LV End to End Open Circuit FRA Test, and w_{HL} is the weighting given to the Capacitive Interwinding FRA Test. Due to the dominant influence of the high voltage windings, weights are applied in such a way as to emphasise the HV winding within the cost function. Table V lists the weights given to each of the FRA tests for this example.

V. RESULTS

Using the estimation algorithm described in Section IV-D, each of the nine transfer functions are now estimated with respect to their corresponding frequency response. Analysis of the results for each of the FRA tests will be discussed in the following sections. In addition, since the model parameters are based on physical parameters, analysis will also be conducted on the relative accuracy of key model parameters. The section will conclude with a sensitivity analysis example.

A. High Voltage Winding End to End Open Circuit FRA Test

Bode diagrams for the estimated models of the three High Voltage Winding End to End Open Circuit FRA Tests are shown in Figures 8 to 10. The resonant feature differences between the individual tests are picked up appropriately by the models. The results overall are quite good both in magnitude and phase for frequencies $\leq 1\text{MHz}$.

B. Low Voltage Winding End to End Open Circuit FRA Test

Bode diagrams for the three Low Voltage Winding End to End Open Circuit FRA Tests are shown in Figures 11 to 13. The self resonance for tests an and cn is at approximately 3.2kHz at -11dB whereas for test bn the self resonance is at 2.6kHz at -12dB. This is a result of the inductive disparity [35] between windings on the centre limb with respect to those on the outer limbs. As observed this is successfully emulated by the model. Good fitting results are obtained for frequencies $\leq 1\text{MHz}$.

Figure 8. Estimated transformer model and FRA data for the HV Winding End to End Open Circuit Test measured between the high voltage A and C terminals on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 10. Estimated transformer model and FRA data for the HV Winding End to End Open Circuit Test measured between the high voltage C and B terminals on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 9. Estimated transformer model and FRA data for the HV Winding End to End Open Circuit Test measured between the high voltage B and A terminals on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 11. Estimated transformer model and FRA data for the LV Winding End to End Open Circuit Test measured between the low voltage terminal and neutral of phase A on a 1.3MVA 11kV/433V Dyn1 transformer.

C. Capacitive Interwinding FRA Test

Bode diagrams for each of the Capacitive Interwinding FRA Tests are shown in Figures 14 to 16. The first resonance for tests *Aa* and *Bb* is at approximately 1.5kHz whereas for test *Cc* the first resonance is at 1.8kHz. This variation is due to inductive disparity [35] and is successfully captured by the model. The overall correlation between the model and the three tests for frequencies $\leq 1\text{MHz}$ is excellent.

D. Confirmation of Parameters

To demonstrate the geometric accuracy of the estimated model parameters, a number of parameters which can be readily measured are compared with their estimated counterparts in Table VI. The parameter estimates are quite good with a worst case parameter error of 18%. These results were in conjunction with parameter estimates for initial permeability, $\mu_i = 976$, and lamination material conductivity, $\sigma = 2.1 \times 10^6$.

Figure 12. Estimated transformer model and FRA data for the LV Winding End to End Open Circuit Test measured between the low voltage terminal and neutral of phase B on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 13. Estimated transformer model and FRA data for the LV Winding End to End Open Circuit Test measured between the low voltage terminal and neutral of phase C on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 16. Estimated transformer model and FRA data for the Capacitive Interwinding Test measured between the high and low voltage terminals of phase C on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 14. Estimated transformer model and FRA data for the Capacitive Interwinding Test measured between the high and low voltage terminals of phase A on a 1.3MVA 11kV/433V Dyn1 transformer.

Figure 15. Estimated transformer model and FRA data for the Capacitive Interwinding Test measured between the high and low voltage terminals of phase B on a 1.3MVA 11kV/433V Dyn1 transformer.

For the parameters related to capacitance, an accurate physical measurement is not a straightforward option. The use of an LCR bridge to determine capacitance will require the same modelling considerations and assumptions as used in the FRA model. To provide an estimate for each of the respective capacitances, two dimensional finite element analysis (FEA) was used (FEMM4.2). For the HV to LV winding capacitance, C_{Xxi} , the windings were modelled as a pair of coaxial cylinders. A similar approach was adopted for the LV to core capacitance, C_{gxi} . The capacitance between adjacent HV windings, C_{XYi} , was modelled as two parallel cylinders. The HV winding to tank wall capacitance, C_{gXi} , for the A and C phases, was modelled as a cylinder with a plane on three sides. In the case of the B phase winding, the capacitance was modelled as a cylinder with a plane on two sides only. In each case the winding was assumed to be n sections in length. The HV winding series capacitance, C_{SXi} , was estimated theoretically [36]. The comparative results for the sectional capacitance based on an 8 section model, i.e. $n = 8$, are given in Table VII.

When considered in the context that the parameters have been determined directly from FRA and without use of any *a priori* knowledge of the internal specifications of the transformer, satisfactory estimates have been obtained for each of the capacitance parameters (though a significant disparity is observed in C_{SXi}). It is proposed that the estimation errors associated with these capacitance values are due to small errors in the modelling of complex permeability. Due to the highly non-linear nature of complex permeability, even subtle variations could have a significant influence on the magnetising inductance with frequency. Any errors in inductance will inversely affect the estimated value of a capacitor during the model fitting. It is proposed that the accuracy of the capacitance estimates could be improved by incorporating additional features into the modelling of the transformer's core, such as the influence of core joints.

The overall parameter estimate results of Tables VI and VII are satisfactory and provide support for the physically

Table VI
COMPARISON BETWEEN ESTIMATED AND ACTUAL VALUES FOR KEY MEASURABLE PARAMETERS.

Parameter	Symbol	Units	Estimate	Actual	Error
Core cross sect.	A_{CS}	m ²	0.040	0.036	11%
Core yoke length	l_Y	m	1.1	1.1	5%
Core limb length	l_E	m	1.0	1.2	18%
HV winding turns	N_X	-	786	852	8%

Table VII
COMPARISON BETWEEN THE MODELLED AND FEA/THEORETICAL ESTIMATES FOR THE SECTIONAL CAPACITANCE ($n = 8$).

Sectional Cap.	Symbol	Model Est. (pF)	FEA/Theory Est. (pF)
HV-LV	C_{Xxi}	78	52
LV-Core	C_{gxi}	66	44
HV-Tank (A/C)	C_{gXi}	11	6
HV-Tank (B)	C_{gXi}	10	5
HV-HV	C_{XYi}	10	6
HV Winding	C_{SXi}	81	30

representative nature of the transformer model.

E. Sensitivity Analysis Example - Forced Buckling

Once the models have been calibrated using the baseline FRA data, the models can then be used to conduct parameter sensitivity analysis. This will lead to greater insight into the effects geometric change can have on transformer FRA. An example is provided in this section which artificially changes parameters in order to simulate different degrees of buckling within the low voltage winding of the transformer's A phase.

Since a short circuit places tremendous forces upon transformer windings and mechanical structure, inward radial forces can lead to winding deformation known as buckling which is a common mode of failure [2]. *Forced buckling* in a low voltage winding will lead to an increase in the average distance between the high and low voltage windings [14]. This results in a corresponding reduction in the capacitance between the high and low voltage windings (C_{Aa} in this example). Conversely, since the average distance between the

low voltage winding and the core will decrease, there will be a corresponding increase in the low voltage to core capacitance (C_{ga} in this example). The simulated effect that varying degrees of “forced buckling” will have on the transformer's High Voltage End to End Open Circuit FRA test between phases A and C are shown in Figure 17. In this figure, two cases are presented against the original baseline FRA (0%). The first case is for a 15% decrease in C_{Aa} and corresponding 15% increase in C_{ga} . The second case is for a 25% decrease in C_{Aa} and corresponding 25% increase in C_{ga} .

As demonstrated in this example, the physically representative model supports FRA interpretation by providing an insight into the effect that subtle geometric changes will have on the transformer's frequency response.

VI. CONCLUSION

This paper has presented a novel modelling approach which can be used to simulate FRA tests conducted on three phase power transformers. It is proposed that the resulting models can be used as a tool to support FRA interpretation by providing a flexible test bed for parameter sensitivity analysis.

To demonstrate the modelling approach, models for High Voltage End to End Open Circuit, Low Voltage End to End Open Circuit, and Capacitive Interwinding FRA tests were derived for Dyn connected transformers. To confirm the veracity of the models, FRA tests were performed on a Dyn1 1.3MVA 11kV/433V distribution transformer. The FRA testing procedure resulted in nine unique frequency responses (three connection permutations for each of the three FRA test types). A constrained nonlinear optimisation algorithm was then applied. This algorithm simultaneously estimated each of the model transfer functions for the corresponding FRA data using a common parameter set. The estimation results were very good for each of the FRA tests.

To confirm the physically representative nature of the FRA models, several of the parameters whose value could be accurately determined through internal inspection were compared against their estimated counterparts. These parameters were all within a reasonable tolerance of their actual values, verifying the physically representative nature of the models.

To demonstrate how the modelling approach could be used as a tool to support the interpretation of FRA, model parameters were changed in order to simulate the effect of winding “buckling” on the transformer's FRA.

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Figure 17. Parameter sensitivity simulation for “Forced Buckling” based on a High Voltage End to End FRA test between phases A and C.

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