Steady-State Issues with Finite Control Set Model Predictive Control

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Abstract—Finite Control Set Model Predictive Control (FCS-MPC) is a novel and promising control scheme for power converters and drives. Many practical and theoretical issues have been presented in the literature, showing good performance of this technique. The present work deals with one of the most relevant aspects of any controller, namely, the steady-state operation. As will be shown, basic FCS-MPC formulations can be enhanced to achieve a zero average steady-state error. We focus on a simple H-Bridge power converter with two complementary switches and discuss benefits and drawbacks of our proposal.

Index Terms—Power Electronics, Emerging Control Issues.

NOMENCLATURE

T	: Sampling period.
$\hat{x}^{*k+1}$	: Reference for variable x for $kT \leq t < (k+1)T$.
g^k	: Cost function at instant kT .
$\hat{x}^k$	: Estimated value of x at instant kT .
$\mathbf{x}^k$	: Measured value of x at instant kT .
$\mathbf{S}_i^k$	: Optimum switch state used for $kT \leq t < (k+1)T$.
V_{dc}	: Dc-link voltage.
$\bar{i}_l$	: Average load current value.

I. INTRODUCTION

In the last years, Finite Control Set Model Predictive Control (FCS-MPC) has emerged as a promising control strategy for power electronics converters [1–3]. This control paradigm has shown its flexibility and good dynamic properties being used not only to control the output current waveform but also to handle other aspects, such as switching losses and common mode voltage reduction [4, 5].

An important feature of FCS-MPC is that this strategy uses a linear load model along with a non-linear power converter model, which considers that switching elements can adopt only two states, namely 1 or 0 (ON or OFF). From this fact, converters can apply a finite number of different switching actions (depending on the number of switching elements), and hence one can also obtain a finite number of values for the different electrical variables at each sample period. To track references, a cost function is defined which is evaluated for each possible switch combination. The optimal switching action to be applied is the one which minimizes the cost function. The optimization procedure is realized at each sample time.

In this way the FCS-MPC concept is easy to understand and offers an almost unlimited application field for power converters control. However, practical issues must be considered:

- Computational effort, as the number of switching elements is increased (e.g. multilevel converters [6, 7]).
- Design of cost function especially when more than one target is desired.
- Measurement noise.
- Model parameters mismatch.

Most of these problems have been studied in previous works showing that FCS-MPC is able to deal with them [4, 5, 7, 8]. This work is focussed in one of the most important aspects of controllers, namely, the steady-state error. It will be shown that existing FCS-MPC algorithms give a non-zero steady-state error; moreover, the error is more relevant when lower switching frequency and/or lower magnitude references are used. Two different approaches are proposed to face this problem. The first one is based on the use of different sample and actuation instants. The second one takes into account not only the errors at the sampling instant, but also the intersample value.

To facilitate the understanding of the problem and the proposed solutions, a simple architecture consisting in current control for a four-quadrant chopper converter will be used.

The remainder of this paper is organized as follows: Section II gives a brief review of existing FCS-MPC and defines the steady state error which constitutes the main motivation of this work. In Section III we present our proposed Predictive Control strategies. Section IV shows a simple example and in Section V simulation studies are carried out to show the performance of our proposal. Finally Section VI draws conclusions.

II. FINITE CONTROL SET MODEL PREDICTIVE CONTROL

In the last years, FCS-MPC has emerged as a promising control methodology when compared to traditional PWM strategies [2]. This is due to the fact that one can obtain good performance with regard to the dynamics of the output variables and also because of it is possible to control not only the tracking of the output variables but also other aspects, such as switching losses and common mode voltage reduction.

FCS-MPC operates in discrete time with fixed sample period T and considers that switching elements can adopt just one state within this period. Therefore the controller has just a finite set of possible actions to handle the control variables. Finally, this strategy directly provides the switching actions to be applied to the converter. Consequently, no intermediate modulation stages are required [3].

A. Fundamentals of FCS-MPC

As FCS-MPC works in discrete time, a measurement of the state variables $\mathbf{x}^k$ is taken at each discrete instant k (in some cases, some of these variables can be estimated [9]). These measurements are used by the controller to decide upon the optimal switching action which will be implemented. To account for computational delaying, in standard formulations this switching action at time $k+1$, $\mathbf{S}_i^{k+1}$. It is important to emphasize that the effect of this action will be observed only at the time $k+2$. This delay of one sample period results in the need not only to predict the value of the variables to be controlled at the instant $k+2$ but also to know their values at the instant $k+1$. In the sequel, the process to obtain the values of the control variables at the instant $k+1$ will be called *Estimation Step*. It is defined by:

$$\hat{\mathbf{x}}^{k+1} = F(\mathbf{x}^k, \mathbf{S}_i^k, T) \quad i \in \{1 \dots n\} \quad (1)$$

where n is the number of complementary switches.

The process to obtain the control variable values for the instant $k+2$ will be called *Prediction Step*. It is expressed via:

$$\mathbf{x}^{k+2} = F(\hat{\mathbf{x}}^{k+1}, \mathbf{S}_i^{k+1}, T) \quad i \in \{1 \dots n\} \quad (2)$$

These different stages of the whole algorithm are depicted in Fig. 1.

As previously mentioned, FCS-MPC strategies choose the optimal switching action to be applied at the instant $k+1$, namely $\mathbf{S}_i^{k+1}$, through minimization of a cost function, g^k . For example, it can be defined as the instant difference between the predictive variable $\mathbf{x}^{k+2}$ and the desired reference $\mathbf{x}^{*k+2}$ at the time instant $k+2$, i.e.,

$$g^k = |\mathbf{x}^{*k+2} - \mathbf{x}^{k+2}| \quad (3)$$

B. Steady-State Error

As already stated, FCS-MPC works in discrete time, and thereby its target is to minimize the tracking error e^k produced at each discrete instant k .

Fig. 1. Finite Control Set MPC temporal scheme.

Although, minimizing the discrete error is the controller target one must not forget that the output variables are continuous and the intersample performance must be taken into account. Here, one can define an average error via:

$$\delta_{av}^k = |\bar{x}^k - x^k| \quad (5)$$

where $\bar{x}^k$ is the average value of the control variables defined via:

$$\bar{x}^k = \frac{1}{T} \int_{kT}^{(k+1)T} x(t) \cdot dt \quad (6)$$

As an example, Fig. 2 depicts how FCS-MPC works in steady-state. In this case, three different switch combinations are considered yielding three different possible outputs: x_+ , x_0 and x_- . Fig. 2 shows $x(t)$ produced by applying the optimal switching action sequence which minimizes the discrete error g^k . An important issue depicted in this figure is the fact that the average value of the controlled variable, $\bar{x}(t)$, is not necessarily the same as the reference $x^*(t)$. This issue constitutes the main motivation of the present work.

III. PROPOSED SOLUTIONS

With the intention of achieving that the average error produced by FCS-MPC is reduced, two different approaches are presented in this work. The first one is based on the idea of controlling the output variable at intermediate sample instants. The second one incorporates the continuous-time intersample error into the cost function.

1) *Intermediate Sampling (IS-MPC)*: The use of synchronous current sampling with sinusoidal PWM schemes is wide spread used. By sampling either at the maximum or minimum of the triangular carrier, a good approximation of the control variable average value can be obtained, as shown in Fig. 3.

This idea can be easily adapted to FCS-MPC algorithm, to approximate the average value of $x(t)$. As previously stated, FCS-MPC can only change the value of the switching elements at the beginning of each sample period, $t = kT$; then, the sampling of $x(t)$ is taken at each $t = kT + T/2$ ($\mathbf{x}^{k+\frac{1}{2}}$). Due to this shift of $T/2$, the final estimate of the control variables can be calculated as:

$$\hat{\mathbf{x}}^{k+\frac{1}{2}} = F(\mathbf{x}^{k+\frac{1}{2}}, \mathbf{S}_i^k, T/2) \quad i \in \{1 \dots n\} \quad (7)$$

Fig. 2. Discrete error δ^k and average value $\bar{x}$ produced by FCS-MPC under a Steady-State condition.

Fig. 3. Synchronous current sampling with sinusoidal PWM.

Then, like in traditional FS-MPC algorithm, predictions are calculated by considering the measurement $\mathbf{x}^{k+\frac{1}{2}}$ and the estimate $\tilde{\mathbf{x}}^{k+1}$ of the control variable. For IS-MPC; however, the predictions are evaluated at $t = (k+1)T + T/2$. This gives a better approximation to the average of the output variable, $\bar{x}(t)$, which then gets closer to the reference. Thus, the proper expression for the prediction is:

$$x^{(k+1)+\frac{1}{2}} = F(\tilde{x}^{k+1}, S_i^{k+1}, T/2) \quad (8)$$

This expression is used to evaluate the IS-MPC cost function:

$$g^k = \left| x^{k+2} - x^{(k+1)+\frac{1}{2}} \right| \quad (9)$$

for all possible combinations of S_i^{k+1} . The optimal switching action, S_i^{k+1} , which minimizes (9) is then selected. The temporal scheme for IS-MPC algorithm is shown in Fig. 4.

Note that by measuring in the middle of the sample period, the measurements are almost free of switching noise. Moreover, the two-step prediction of FCS-MPC described in Section II can be reduced to a one-step prediction, and thus avoiding the one-sample-period delay. On the negative side, most calculations must be done in half of the sample period, because of most of estimation and prediction equations depend on the measurements. This implies that in most, the sample period for IS-MPC will be larger than for traditional FCS-MPC.

2) *Integral Error Term (IE-MPC)*: FCS-MPC has shown to get good results when it is applied to power converters. However, in some cases, a poor intersample performance can be obtained, specially when low sample frequency is used and/or low reference is desired.

Considering the traditional FCS-MPC algorithm presented in Section II, one can modify the original cost functions, such as g^k in (3) by adding a second term which considers the

Fig. 4. Intermediate Sampling Model Predictive Control temporal scheme.

intersample performance. This term will be defined as the integral of the error between the reference and the predictive continuous behavior of the control variable within a sample period T , see Section. Taking into account the one-sample-period delay, this integral error is defined via:

$$I^{k+1} = \int_{k(T+1)}^{k(T+2)} (\bar{x}(t) - x(\tilde{x}^{k+1}, S_i^{k+1}, t)) dt \quad (10)$$

This integral, in most of cases, can be explicitly calculated by replacing $\tilde{x}^{k+1}$ by (1) and taking into account that S_i^{k+1} and $\bar{x}(t)$ are constants along the period T , see Section IV for a specific example.

Note that the integral term defined as (10) is not equivalent to a discrete integral (commonly used for PI controllers) which is based on a weighted sum. In (10), a calculation of the continuous integral error along the sample period based on a continuous model is used. This term gives information about the average error for a given switch combination, and hence will make the controller pay attention not only to the final value of the variables, but also to the trajectory used to reach those values.

In addition to considering intersample aspects, the integral error from previous sample times can be also included in the cost function. This gives better dynamic control of the system response. Then, the final expression for the cost function is:

$$g^k = \left| x^{k+2} - x^{k+2} \right| + K_1 \left| I^{k+1} + \sum_{j=0}^m \mathbf{I}^{k-j} \right| \quad (11)$$

where K_1 is the weight term which is a design parameter, and will give a higher/lower importance to the integral term, while the $\mathbf{I}^{k-j}$ terms are the integral errors for the previously applied switch combinations, which were calculated at the previous sampling times. This process is depicted in Fig. 5.

It is worth emphasizing that the proposed cost function structure in (III-2) can be associated with discrete Proportional-Integral (PI) controller, which is divided in a proportional part, which acts over the instantaneous error, δ^k , and provides a fast response, and an integral part which is related with the accumulated error. In this case, the accumulated error considers the continuous behavior of the control variable achieving a better intersample performance.

IV. FOUR QUADRANT CHOPPER EXAMPLE

We will next apply our proposal to a simple power converter topology.

Fig. 5. Model Predictive Control with integral minimization temporal scheme.

A. Topology and model

The four quadrant chopper converter is a simple and flexible buck dc-dc converter. As shown in Fig. 6, it requires four power switches (S_1 , $\bar{S}_1$, S_2 and $\bar{S}_2$) and a dc voltage source V_{dc} .

The load voltage can be easily obtained from:

$$v_l(t) = (S_1 - S_2)V_{dc} \quad S_1, S_2 \in \{0, 1\}. \quad (12)$$

Thus, v_l can reach up to three different values, namely V_{dc} , 0 and $-V_{dc}$.

The load voltage and current are related according to a simple first order differential equation:

$$v_l(t) = Ri_l(t) + L \frac{di_l(t)}{dt} \quad (13)$$

As the switch states are constant during all the sample time, (13) can be exactly expressed via:

$$F(\mathbf{x}^k, S_i^k, t) = i_l^k e^{-t/\tau} + (S_1^k - S_2^k) \frac{V_{dc}}{R} (1 - e^{-t/\tau}) \quad (14)$$

Consequently, the value of the load current at the end of each sample time, assuming a perfect parameter match, is:

$$i_l^{k+1} = i_l^k e^{-T/\tau} + (S_1 - S_2) \frac{V_{dc}}{R} (1 - e^{-T/\tau}) \quad (15)$$

with $\tau = L/R$.

This is the model that will be used in the following sections.

1) *Basic FCS-MPC*: The implementation of a current control based on FCS-MPC is very straightforward from (15). In particular, the estimated and predicted current values are:

$$\hat{i}_l^{k+1} = i_l^k e^{-T/\tau} + (S_1^k - S_2^k) \frac{V_{dc}}{R} (1 - e^{-T/\tau}) \quad (16)$$

$$i_l^{k+2} = \hat{i}_l^{k+1} e^{-T/\tau} + (S_1^{k+1} - S_2^{k+1}) \frac{V_{dc}}{R} (1 - e^{-T/\tau}) \quad (17)$$

whereas, the cost function is defined via:

$$g^k = \left| \hat{i}_l^{k+2} - i_l^{k+2} \right|. \quad (18)$$

The latter is evaluated for the four switch combinations available for the chopper.

Fig. 6. Four-quadrant chopper converter.

2) *Intermediate Sampling (IS-MPC)*: In the proposed IS-MPC method, after taking the measurement, the estimated value of the load current can be obtained from (15):

$$\hat{i}_l^{k+1} = i_l^k e^{-T/2\tau} + (S_1^k - S_2^k) \frac{V_{dc}}{R} (1 - e^{-T/2\tau}) \quad (19)$$

Note that, as stated in section III-1, the estimation horizon is reduced to $T/2$. Then, the predictions are made using (19) as the initial value and $T/2$ as horizon:

$$i_l^{(k+1)+\frac{1}{2}} = \hat{i}_l^{k+1} e^{-T/2\tau} + (S_1^{k+1} - S_2^{k+1}) \frac{V_{dc}}{R} (1 - e^{-T/2\tau}) \quad (20)$$

As the prediction given by (20) is assumed as the average value of the current along the period T , this is the value which is compared with the load current reference in the cost function. We, thus, have:

$$g^k = \left| \hat{i}_l^{k+1} - i_l^{(k+1)+\frac{1}{2}} \right| \quad (21)$$

3) *Use of Integral Error (IE-MPC)*: The integral term for IE-MPC is obtained by replacing (14) in (10):

$$\begin{aligned} I^{k+1} &= \int_0^T \hat{i}_l^{k+1} - F(\mathbf{x}^k, S_i^k, t) dt \\ &= (S_1^k - S_2^k) \frac{V_{dc}}{R} (\tau - T - \tau e^{T/\tau}) + T \hat{i}_l^{k+1} \\ &\quad - \tau i_l^k (1 - e^{T/\tau}) \end{aligned} \quad (22)$$

Note that only the first term of (22) depends on the switch states. Thus most of this expression does not need to be recalculated at each prediction step.

Finally, according to (11) and considering m previous integral errors, the final cost function to be evaluated becomes:

$$g^k = \left| \hat{i}_l^{k+2} - i_l^{k+2} \right| + K \left| I^{k+1} + \sum_{j=0}^m I^{k-j} \right| \quad (23)$$

V. RESULTS

To verify the performance of the proposed solutions, simulation studies were carried out on a H-Bridge converter shown in Fig. 6. A main dc-link voltage source of $V_{dc} = 500[V]$ was used. Electrical load parameters were $R = 10\Omega$ and $L = 10[mH]$. Proposed solutions and standard FSC-MPC strategies were applied using a sample period of $T = 200[\mu s]$. This is a reasonable sample period which allows one to implement the proposed method in a standard digital signal processor. In this case, the load current $i_l(t)$ is the variable to be controlled and the control target is to keep its average value near to the reference.

Since we have considered just one control variable, standard FSC-MPC and IS-MPC as examined in the present work do not require any tuning. On the other hand, as previously stated in section III-2, and according to (23), IE-MPC strategy requires to select the weight factor K and the number of previous integral errors m . To show the effect of these parameters two cases are considered: The first one considers just the current

Fig. 7. Average error as a function of the current reference.

intersample error and the second one takes into account two previous integral errors, $m = 2$, namely i0MPC and i2MPC respectively. For both cases a weight factor of $K = 1.5$ was used.

Fig. 7 shows the average error for the discussed methods. First note that for all the methods a zero steady-state error can be reached only for a finite number of reference points. Moreover, high error values are reached for reference values around zero, 25 A (half of the maximum current) and 50 (maximum current). Indeed for very low and very high reference values the predictive algorithms do not switch, as any action leads to a high instantaneous error. However, as the integral method considers the intersample and accumulate error, switching elements start to commute for a lower reference than standard FCS-MPC. This leads to a lower average error, even if a higher instantaneous error is reached at some point. The IS-MPC also starts to switch at a lower value than FCS-MPC as the value at half of the sample period is considered.

Note that in general, the behavior of i2MPC is the best among the presented method except for a high value of the current reference (over 37 A in this case). On the other side, IS-MPC behavior is not as good as expected being better than FCS-MPC only in the proximities of zero and 50 [A]. Finally, i0MPC have in general a lower performance than i2MPC, except at a high current reference as stated above.

Fig. 8 shows the average error as a function of the current reference value, in this way a better sense of the importance of the average error is presented. From this point of view, the main problem of the predictive techniques are around zero, in this zone the benefits of the use of i2MPC, and even IS-MPC, are clear. For the further results i0MPC will not be considered.

One of the most important benefits of FCS-MPC is the high dynamic behavior that can be reached. Fig. 9 shows the step response from -32 A to 32 A. All methods show a fast response, IS-MPC however, is a little bit faster due to its reduced control delay. Moreover it is possible to see how the current with IS-MPC and i2MPC oscillates around the reference, while for FCS-MPC it mainly commutates firstly under and over the reference (-32 A and 32 A respectively). This effect is highlighted in Fig. 10 and Fig. 11 where the steady-state behavior is shown for a reference of 27 A and

Fig. 8. Porcentual average error as a function of the current reference.

Fig. 9. Step response of the discussed methods.

29.5 A.

For $i_l^* = 25$ A, IS-MPC and FCS-MPC present almost the same behavior, for i2MPC however the i_l waveform is completely different. In this latter case the average value is closer to the reference value, but a higher current ripple is observed as the switches are forced to keep their values for, at least, one sample time. This is the price to be paid in order to reduce the steady-state error.

Finally, Fig. 11 shows how the average error and the current waveform strongly change even for a small reference value fluctuation (from 27 A, shown in Fig. 10, to 29.5 A). For this

Fig. 10. Steady-state behavior at $i_s^* = 27$ [A].

Fig. 11. Steady-state behavior at $i_s^* = 29.5$ [A].

case, the errors of proposed methods are quite similar. The algorithm i2MPC has a slightly better behavior.

VI. CONCLUSIONS

This paper shows that FCS-MPC can not reach a zero average steady-state error, even if optimal models and parameter values are used. To deal with this issue, two different approaches are proposed. For Intermediate Sampling MPC (IS-MPC), unlike standard FCS-MPC, the measurements and the switching action are temporarily shifted by half of sampling time. This allows the control delay to be reduced, and the approximated average value of the control variable to be controlled. The Integral Error MPC (IE-MPC) reduces the average tracking error, by using an analytical expression of the intersampling integral error, by this way the trajectory of the control variable is considered. Moreover past errors can

be also included to ensure a better fit of the average value tracking.

All the presented methods show a high dynamic response, which is one of most important benefits of the predictive technique.

According to the presented results, IS-MPC reaches a low average steady-state error only for low reference values, being its faster response to reference changes its main advantage, due to the control delay is reduced.

IE-MPC algorithm considering two previous integral errors, gives the better steady-state tracking for low and medium reference values. The absolute average error value is increased for high reference values but it is not so significant when it is proportionally compared with the reference value itself.

For the presented schemes the switches maintain their states for an integer number of sample times, hence there is a trade-off between the control signal ripple and its average steady-state error. This effect is more important as the sampling (and hence the switching) frequency is reduced.

ACKNOWLEDGMENT

Pablo Lezana acknowledges the support of the Chilean Research Council (CONICYT) under grant FONDECYT 1085111.

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