Adaptive Channel Estimation Using Least Mean Squares Algorithm for Cyclic Prefix OFDM Systems

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Abstract— Orthogonal frequency division multiplexing (OFDM) delivers high data transmission rate and forms the basis of Beyond 3G. The channel estimation is imperative for the implementation of OFDM. Cyclic Prefix (CP) based block Recursive Least Squares (RLS) channel estimation algorithm has been proposed for OFDM systems but it increases computational complexity. In this paper, we propose a block LMS (Least Mean Squares) channel estimation algorithm which promises less computation but delivers comparable and promising results.

Keywords— Channel estimation, cyclic prefix, OFDM, RLS, LMS.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has gained increasing popularity in both wired and wireless applications [1]. In OFDM multiple relatively slow bit streams are sent simultaneously with one of the bit streams on each of a large number of carriers. OFDM mitigates the impact of fading as the symbols are spread out over a relatively long period of time. OFDM has been used in digital audio broadcasting (DAB) [2], digital video broadcasting (DVB) [3], high-speed Wireless Local Area Networks (WLAN) such as IEEE 802.11a, HIPERLAN/2 [4], IEEE 802.11g and in Wireless Metropolitan Area Networks (WMAN), e.g. IEEE 802.16, generally known as WiMax.

All the above mentioned applications are based on the insertion of the so-called cyclic prefix (CP) which consists of redundant symbols replicated at the beginning of each transmitted block. The insertion of the CP and the inverse fast Fourier transform (IFFT) precoding at the transmitter enables very simple one tap equalization of frequency-selective finite impulse response (FIR) channels. At the receiver end, the CP is discarded to avoid interblock interference (IBI) and each truncated block is fast Fourier transform (FFT) processed- an operation converting the frequency-selective channel into parallel flat-faded independent subchannels, each corresponding to a different subcarrier. Unless zero, flat fades are removed by dividing each subchannel's output with the channel transfer function at the corresponding subcarrier. Hence, knowledge of channel state information is imperative for symbol recovery.

The channel information is usually obtained by using a known sequence (training sequence) or by using *a priori* knowledge of the channel. Under most communication environments, little *a priori* channel knowledge is available. The training sequence, therefore, plays a key role in channel estimation and equalization. For a time invariant channel, only initial training is required. However, for a time varying channel, retraining has to be done periodically to track the channel variations [5], [6], [7]. Otherwise the system performance will degrade. Clearly, such a scheme increases the system overhead and reduces the bandwidth and communication efficiency.

In [8], the CP in OFDM which is normally discarded at the receiver can be seen as a source of constantly sent training sequence. A block recursive least squares (RLS) channel estimation algorithm is proposed in [8] by exploiting the CP present in the OFDM system. This algorithm helps to avoid the system overhead due to retraining as it can adaptively and efficiently track the channel variation without additional training sequence. The algorithm relies on the simple one tap minimum-mean-square-error (MMSE) equalization to estimate the transmitted CP part. In [9], the scheme is reformulated as CP based exponentially weighted block RLS filtering scheme and by replacing the one-tap MMSE equalization with the simple one-tap zero forcing (ZF) equalization. In [10], we investigated the performance of the CP based exponentially weighted block RLS channel estimator and discovered that smaller values of exponential forgetting weightings increased the convergence time and steady state error of the algorithm.

In this paper, we replace the block RLS algorithm in [8] with block least mean squares (LMS) algorithm which reduces the computational complexity and provides comparable results. We present the CP-OFDM system model and the adaptive channel estimation algorithm in Section 2. Section 3 shows the system performance in comparison with RLS algorithm. The system performance degrades with the increase in channel nulls and constellation size, which is similar to block RLS algorithm in [10]. We also investigate the system performance by varying different step size for LMS. The system performs the best with the optimum step size value.

II. THE PROPOSED ADAPTIVE CHANNEL ESTIMATION ALGORITHM

In this section, we review the CP-OFDM system and adaptive channel estimation with block LMS algorithm.

A. CP-OFDM System Model

Fig. 1 The CP-OFDM system with adaptive channel estimation [8].

Fig. 1 depicts the baseband discrete time block equivalent model of a CP-OFDM system with adaptive channel estimation. The system has $m/2$ complex parallel subchannels. The input data are buffered to blocks. Each data block is divided into $m/2$ bit streams then mapped to complex constellation points $X_{i,k}$, $i = 0, \dots, m/2$ at time k . After m -point IFFT on $X_k = [X_{0,k}, X_{1,k}, \dots, X_{m-1,k}]^T$ (here the last $m/2$ samples are just the conjugates of the first $m/2$ samples), the modulated time domain signal becomes $x_k = [x_{0,k}, x_{1,k}, \dots, x_{m-1,k}]^T$.

A CP $x_k^{(f)} = [x_{-v,k}, \dots, x_{-1,k}]^T$ (where $x_{-i,k} = x_{m-i,k}$ and $i = 1, \dots, v$) is appended in front of x_k before transmission through the channel. The channel is usually modelled as a FIR filter with length v . The impulse response of the channel is $h = [h_0, h_1, \dots, h_{v-1}]^T$. At the receiver, the prefix part $y_k^{(f)} = [y_{-v,k}, \dots, y_{-1,k}]^T$ is removed.

The relationship between the prefix part $y_k^{(f)}$, (which is normally discarded at the receiver) and the transmitted signal can be expressed as

$$y_k^{(f)} = A_k h + n_k^{(f)} \quad (1)$$

where

$$A_k = \begin{bmatrix} u_{v,k}^H \\ \vdots \\ u_{2,k}^H \\ u_{1,k}^H \end{bmatrix} = \begin{bmatrix} x_{-v,k} & x_{m-1,k-1} & \dots & x_{m-v,k-1} \\ \vdots & \ddots & \ddots & \vdots \\ x_{-1,k} & \dots & x_{-v,k} & x_{m-v,k-1} \end{bmatrix}$$

and $n_k^{(f)} = [n_{-v,k}, \dots, n_{-1,k}]^T$. This relationship can be used to estimate the channel by exploiting the CP part which can be viewed as the training sequence.

The demodulation is performed only on $y_k = [y_{0,k}, y_{1,k}, \dots, y_{m-1,k}]^T$ by the FFT operation and the demodulated signal is $Y_k = [Y_{0,k}, Y_{1,k}, \dots, Y_{m-1,k}]^T$.

The CP removes the IBI between X_k 's and makes the subchannels independent with each other. Thus, we can write $Y_{i,k} = X_{i,k} H_i + N_{i,k}$, $i = 0, \dots, m-1$, (2)

where $H_i = \sum_{l=0}^v h_l e^{-j2\pi l/m}$ is the channel frequency response and $N_{i,k} \sim N(0, \sigma^2)$ is the noise of the i th subchannel.

According to (2), only standard one-tap equalizer W_i is needed to get the estimation of $X_{i,k}$ from $Y_{i,k}$ i.e. $\hat{X}_{i,k} = Y_{i,k} \cdot W_i$. The ZF one-tap equalizer for the i th subchannel is given by $W_i = 1/H_i$. The decision is then made on $\hat{X}_{i,k}$ to get the final output $\bar{X}_{i,k} = q(\hat{X}_{i,k})$, where $q(\cdot)$ is the decision operation.

B. Adaptive Block LMS Channel Estimation Algorithm

Based on the relationship (1), a block RLS algorithm is proposed in [8] to adaptively estimate the channel then in [9], the method is reformulated as an exponentially weighted block RLS algorithm. In this paper, we replace the block RLS algorithm with block LMS algorithm which reduces the computational complexity as it does not require the knowledge of the cross-correlation and auto-correlation matrix of the training input. The channel estimation at time k with block LMS algorithm is given by

$$\hat{h}(k) = \hat{h}(k-1) + 2\mu \sum_{l=1}^v u_{l,k} e(k) \quad (3)$$

where $e(k) = y_k^{(f)} - \hat{y}_k$ and $\hat{y}_k = \hat{h}(k-1) \sum_{l=1}^v u_{l,k}$.

Based on the above discussion, the channel estimation method using block LMS algorithm can be summarized as follows:

Input: $y_k^{(f)}$ and Y_k

Selecting parameters: μ

Initialization: $k=0$, an initial training process is used to initialize $\hat{h}(0)$.

Computation: $k=1, 2, 3, \dots$

$$1) \hat{H}_{i,k-1} = \sum_{l=0}^v \hat{h}_{l,k-1} e^{-j2\pi l/m},$$

$$\hat{W}_{i,k-1} = 1/\hat{H}_{i,k-1}, i=0, \dots, m-1$$

$$2) \hat{X}_{i,k} = Y_{i,k} \hat{W}_{i,k-1}, i=0, \dots, m-1$$

$$3) \bar{x}_{i,k} = (1/\sqrt{m}) \sum_{l=0}^{m-1} q(\hat{X}_{l,k}) e^{j(2\pi l/m)},$$

$$i = m-v, \dots, m-1$$

$$4) \hat{y}_k^{(f)} = \hat{h}(k-1) \sum_{l=1}^v u_{l,k},$$

$$e(k) = y_k^{(f)} - \hat{y}_k^{(f)}$$

$$5) \hat{h}(k) = \hat{h}(k-1) + 2\mu \sum_{l=1}^v u_{l,k} e(k)$$

In each coefficient update using the LMS and RLS algorithm, the LMS has a complexity of order $O(L)$ while the RLS has a complexity of order $O(L^2)$. Hence, the RLS algorithm is significantly more complex while LMS promises to reduce the computational complexity. Table 1 shows the arithmetic complexity comparison between the two algorithms.

TABLE I
ARITHMETIC COMPLEXITY COMPARISON

	RLS	LMS
Real multiplications	520	80
Real additions	33	20
Real subtractions	182	4
Complex matrix inversion	Involves a $(1+\nu) \times (1+\nu)$ inversion	0

III. SIMULATION RESULTS

We provide a few Monte Carlo simulation results to illustrate the effectiveness of the new algorithm as compared with RLS algorithm. The simulation is carried under the IEEE 802.11a/g environment with 52 subcarriers, $\nu=4$, 4-QAM constellation, SNR=25dB, step size, $\mu=0.01$ for LMS algorithm, and forgetting factors of $\mu_1=0.7$ and $\mu_2=1.0$ for RLS algorithm (the optimum value for the forgetting factors, according to [10]). The transmit power of all used subchannels is equal, that is $\sigma_i^2 = \sigma^2$. The performance is evaluated by the averaged mean square error (MSE) per subchannel, which is defined as $MSE = \sum_{i \in U} \|X_i - \hat{X}_i\|^2 / |U|$ where U is the set of indexes corresponding to the U useful subchannels. The four static finite impulse response (FIR) channels used in simulation are listed in Table 2, whereas the corresponding frequency responses are shown in Fig. 2. Only the first OFDM symbol was sent as pure training sequence to identify the initial channel h_0 for fast convergence. The simulation results were averaged over 300 Monte Carlo runs.

TABLE 2
STATIC FIR CHANNEL MODELS

tap #	Channel A	Channel B	Channel C	Channel D
0	0.3122	0.5312	0.4122	0.2000
1	0.0501	0.5010	0.0501	0.0000
2	0.2056	0.2056	0.3000	0.0000
3	0.2754	0.4754	0.2000	0.0000
4	0.1567	0.8000	0.2567	0.2000

Fig. 2 Frequency responses (magnitude) of FIR channels.

Fig. 3 shows the convergence rate of the algorithm and also its ability to track abrupt changes in channel conditions which in this case happens at data block 101. The channel is having impulse response of Channel A, which remains unchanged for the first 100 data blocks. At data block 101, the channel is switched to Channel B. It can be seen from the Fig. 3 that the convergence rate of RLS is faster than LMS but LMS performs comparably to RLS and actually offers lower MSE.

Fig. 3 Average MSE per subchannel for block RLS and LMS algorithm.

We also test the system performance in terms of Bit Error Rate (BER). The result is shown in Fig. 4. From the figure, it clearly shows that LMS outperforms RLS in BER although both algorithms converge at around the same data block.

Fig. 4 BER for block RLS and LMS algorithm.

We test the performance of the two algorithms in the presence of channel nulls. The result is shown in Fig. 5. The channel is having impulse response of Channel A when we switch to Channel C which carries one spectral null at data block 101 and from Channel C to D at data block 131. Channel D is more severe with two deep nulls across its spectrum. Fig. 4 shows that both algorithms perform poorly due to the noise enhancement caused by the increasing number of spectral nulls. Again, the performance of block LMS algorithm is comparable to that of block RLS in [10].

Fig. 5 Average MSE per subchannel when tracking Channel C and D with spectral nulls.

Fig. 6 shows the performance of the algorithm with different constellation size with Channel A only. Similar with block RLS algorithm performance in [10], block LMS algorithm also shows degradation in performance with increasing constellation size. Therefore, these two adaptive block RLS and LMS algorithm are not suitable for modulation schemes with large constellations.

FIG. 6 AVERAGE MSE PER SUBCHANNEL PERFORMANCE WITH DIFFERENT CONSTELLATION SIZE.

In the last simulation, we examine the performance of the block LMS algorithm by varying the step size of the algorithm with Channel A until at data block 101 when the channel changes to Channel B condition. It is found that, the step size, $\mu = 0.033$ offers the best performance and it is also at the midpoint of the region from the equation in [11] to find the optimum step size which is

$$\mu = \frac{1}{6N\sigma_x^2} \quad (4)$$

where N is the number of taps and $\sigma_x^2 = E[x^2(k)]$ represents the variance of the input signal.

Fig. 7 Average MSE per subchannel performance with varying step size.

IV. CONCLUSIONS

In this paper, we have presented an adaptive channel estimation algorithm using block LMS method for CP OFDM system. This algorithm delivers comparable results as compared with block RLS method in [8] and [10] but greatly reduces the computational complexity. The algorithm was tested for time varying environments with varying number of channel nulls, constellation size and step size. To alleviate the performance degradation problem due to the presence of channel nulls, power loading is recommended to optimize the error rate performance. Due to the poor performance with larger constellation, the scheme may not be suitable for rate adaptation. However, this algorithm can adaptively track the variation of a moderately time-varying channel without increasing the system overhead since no additional training is required.

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