Linear interpolation error in measured surfaces in a dental erosion study

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Abstract:

An extensive palatal erosion study which is currently being undertaken at the University of Dundee, aims to detect erosion through the precise mapping of replicas of children's incisors. All perceived sources of error are being scrutinised closely. The error due to the interpolation of surface positions from discrete point measurements has been investigated in order to gauge its contribution to the erosion estimates. The calculation of the error statistics makes use of covariance functions to express the undulation of the tooth surfaces. It is found that, for the measurement point spacing of 150 μm being used in the current project, the interpolation error for an incisor is typically about 25 μm , to be compared with the root-mean-square error due to tooth impression and replication procedures, assessed as being of order $\pm 3 \mu\text{m}$, and replica measurement, assessed as being of order $\pm 5 \mu\text{m}$. The interpolation error constitutes a surprisingly large proportion of the overall system error. The same error estimation technique can be applied usefully and without difficulty to other research studies concerning erosion or wear in natural or restorative dental surfaces.

Keywords:

interpolation, statistics, dentistry, wear, erosion

1. Introduction

The mapping of teeth replicas and other dental material surfaces is a common operation in laboratory-based dental research projects which investigate the erosion and/or abrasion of natural surfaces and restorative materials. The extensive interest in erosion and abrasion is conveyed by the review of erosion studies in general by Maupomé & Ray (2000) while the variety of measuring methods is indicated by Azzopardi et al. (2000). Wiskott et al. (2000, p218) list the types of measuring instruments used in this work, revealing that the range of devices currently available to the dental researcher is based on optical, mechanical and sometimes electromechanical methodologies. Whatever measuring technique is used, the physically continuous dental surface is then represented by a set of discrete data points, typically arranged in rows and columns across the surface. (The measurement within the columns may be almost continuous sometimes.) Subsequent comparisons of the shapes of the one surface at two different epochs to determine the change due to wear, abrasion or erosion, usually involves comparing two surface models which are derived from repeat measurements of the same tooth, and which are registered using some form of permanent marking. Less commonly, a surface matching technique may be used to compare surfaces to avoid the use of marked control (e.g., Mitchell & Chadwick 1998, Peters et al., 1999, Jovanovski & Lynch, 2000). If the measurements cannot be taken at the same surface points at the successive measurements, interpolation to select intermediate point heights between the measured points is invariably unavoidable. Interpolation errors are then inadvertently created, but their magnitude is rarely scrutinised. A measuring point separation which is too wide can introduce unacceptably large interpolation errors which will influence determinations of erosion or wear. On the other hand, interpolation error can be limited by taking measurements at a fine spacing, but this can represent an inefficient and uneconomical use of resources in terms of the equipment, computer storage space or processing time.

Stimulated by the specific demands of a palatal erosion investigation at the University of Dundee, which involves extensive tooth replica measurement and manipulation of the mapped data with a view to detecting erosion at the 10 μm level, the error due to linear interpolation arising during the use of the dental surface measurements has been estimated. Without such an investigation, it is difficult to appreciate whether or not the interpolation error is inconsequential. The process of estimating the error effect may be of use to other researchers using measured dental surfaces.

This investigation follows a number of steps. In Section 2, it is shown firstly that the interpolation error is related to surface curvature and to point separation, and secondly that the easiest means of evaluating the error is to use a mathematical relationship, which ties the linear interpolation error to a mathematical function known as a surface covariance function. The concept of the covariance function, which characterises the surface texture and which can be derived easily for any surface of research interest, is explained. In Section 3, a number of mapped tooth replica surfaces are used to examine the characteristics of the surface covariance functions for teeth. These provide examples of the error formula's usage, but also provide insight into the typical interpolation error to be associated with dental studies. The experiments finally permit an estimate of the error to be expected from this source in a palatal erosion investigation at the University of Dundee.

2. Materials and Methods

2.1 The linear interpolation error in terms of the surface curvature

For simplicity, our discussion of the interpolation error will be restricted to linear

interpolation and to the two dimensional case, by assuming that a required point on the curve (or surface for the three dimensional case) is interpolated from the two neighbouring measured points in the same row of measurements. If we let the “height” of the surface be y , with the direction along the appropriate row be given by x , we can assume that the height value, $y^*(x)$ at position x is interpolated from the adjacent values at, say, x_1 and x_2 , by:

$$(1) \quad y^*(x) = \alpha_1 y(x_1) + \alpha_2 y(x_2)$$

where α_1 and α_2 are coefficients whose values will in practice add to one.

Determining the statistics of the interpolation error from a function which involves two different values of α is unnecessarily complex, so we again limit our considerations to the case which occurs when the interpolated point is at an equal distance (call it h) from both given points, as in Figure 1. Then

$$\alpha_1 = \alpha_2 = 0.5$$

and also

$$x_1 = x - h$$

$$x_2 = x + h$$

so Equation (1) becomes:

$$(2) \quad y^*(x) = \frac{1}{2} y(x - h) + \frac{1}{2} y(x + h)$$

If $e(x)$ represents the error in the linearly interpolated value at x , then

$$e(x) = y^*(x) - y(x)$$

so

$$(3) \quad e(x) = \frac{1}{2} y(x-h) + \frac{1}{2} y(x+h) - y(x)$$

We can thus estimate $e(x)$ at any point by comparing given data with $y^*(x)$ values evaluated from adjacent points. It is laborious however, as it involves calculations with groups of three adjacent data points.

If the surface profile represented by the $y(x)$ function is a twice differentiable curve, then we can follow a mathematical approach which tells us more about the error. Using Taylor's expansion, we can write:

$$y(x+h) \approx y(x) + h y'(x) + \frac{1}{2} h^2 y''(x) \dots$$

where $y'(x)$ is the first derivative of x with respect to y , and $y''(x)$ is the second derivative of x with respect to y , and

$$y(x-h) \approx y(x) - h y'(x) + \frac{1}{2} h^2 y''(x) \dots$$

so that Equation (3) becomes

$$\begin{aligned} e(x) \approx & \frac{1}{2} [y(x-h) + h y'(x) + \frac{1}{2} h^2 y''(x) \dots] \\ & + \frac{1}{2} [y(x) + h y'(x) + \frac{1}{2} h^2 y''(x) \dots] - y(x) \end{aligned}$$

which reduces to

$$(4) \quad e(x) \approx \frac{1}{2} h^2 y''(x)$$

Equation (4) has two important, if predictable, consequences.

Firstly, Equation (4) verifies that there is a connection between the error caused by linear interpolation and surface texture, as defined now by curvature of the surface, being the second derivative of the curve. Secondly, the interpolation error is shown by Equation (4) to be a function of the separation h .

We cannot be concerned so much with the single error at any one point, but rather we need to study the statistics of the errors. Accordingly, our discussion turns to the *mean square interpolation error*, *MSI error* (or the *root mean square error*, *RMSI error*, derived from it), calculated from a number of points on any one surface, or even on a number of similar surfaces. The MSI error is given by:

$$(5) \quad \text{MSI} = (1/N) \sum \{ e(x) \}^2$$

where N is the number of determinations of $e(x)$ and the sum is taken over all points on the grid.

Furthermore, because Equation (4) has shown that the interpolation error is a function of point separation h , we will now treat MSI as function of h , $\text{MSI}(h)$, defined mathematically as the average of the square of all interpolation errors at that spacing h . Making use of Equations (4) and (5), we can evaluate $\text{MSI}(h)$ in practice:

$$(6) \quad \text{MSI}(h) \approx h^4 (1/4N) \sum \{y''(x)\}^2$$

where the $y''(x)$ values used in the summation for any one $\text{MSI}(h)$ are calculated over distance h , and N is the number of available second derivatives at that spacing h . The units of its square root, the $\text{RMSI}(h)$, are the same as those of y . The practical evaluation of Equation (6) for a given surface clearly needs a number of pairs of points which have separations h , and this may require that distances be "binned" into a limited number of groups. Moreover, its evaluation requires knowledge of the curvature which is usually not available, because the measurements are obtained at discrete points only. It is possible to approximate the curvatures using numerical methods: the second derivative can be computed as the change in the gradient values, which can then be used in Equation (6). This evaluation is as laborious as for Equation (3), as it requires the calculation of the gradient differences taken from many groups of three consecutive data points.

2.2 Surface Covariance Function:

That the statistical interpolation error is influenced by surface texture has now been explained via Equation (6). To quantify texture, a number of functions can be used as alternatives to the second derivative. While it would be possible to use a power spectrum or perhaps a Fourier transform to summarise a surface's component frequencies, a surface behaviour indicator which is easier to utilise is the statistical concept of a *covariance function*, often discussed in statistical texts. A covariance function is a measure of spatial similarity, which is utilised in studies of many physical quantities which are distributed in two-dimensions (such as gravity anomalies), and reference to the concept may also be found therefore in appropriate texts on such physical entities, e.g. Davis (1986). Equation (6) can be converted into a form which incorporates the covariance function and which is easier to evaluate; (see Appendix 1).

Here, the concern is with physical surface heights on dental surfaces. The surfaces of interest are assumed to be defined by a set of measured three-dimensional co-ordinates, where y (as before) is the “height” co-ordinate. We assume that the surfaces have been reduced to a mean height of zero. For any given surface, the *sample* covariance function at spacing h , denoted by $\gamma(h)$, is determined as the mean product of heights at different surface separation distances h :

$$(7) \quad \gamma(h) = (1/N) \sum [y(x) \cdot y(x+h)]$$

surface heights $y(x)$ and $y(x+h)$ being measured points separated by distance h , N denoting the number of pairs of points separated by h .

The units of $\gamma(h)$ are the square of the units used for y . In a manner which is similar to that used to calculate the RMSI(h) error, the practical evaluation of the function for a given surface clearly needs a number of pairs of points which have separations h , and this again may require that distances be binned into suitable groups. Large scale trends in the surface should be removed so that the function represents the statistics of the surface height correlations at shorter distances.

If N is sufficiently large, the values $\gamma(h)$ can form a surprisingly smooth function. A large value of γ occurs when the two surfaces are similar at a given separation distance. In particular, when h is zero, the two surfaces are identical, and γ then has a maximum. This value, $\gamma(0)$, is called the *auto-covariance*. As h increases from zero, the $\gamma(h)$ values normally decrease, as the heights become less correlated with increasing separation, and a covariance function can fall to the value zero at a certain point separation distance, referred to as zero-

crossing separation, when the surfaces heights are not correlated to each other. Following a zero covariance the value of γ can become negative, remain at zero or become positive again. The last occurs if the surface shows some periodicity. The auto-covariance value and zero-crossing distance together can effectively characterise the surface texture. For positive covariance functions, the auto-covariance is a measure of the ‘power’ of the surface, and thus auto-covariance describes the amplitude of the textural undulations. The zero-crossing distance symbolises the rapidity of textural variation within the surface, as it is small for surfaces with little similarity at spacings other than zero. In this sense it is a measure of the self-similarity of the surface.

2.3 Interpolation error and its relationship to surface covariance function

In Appendix 1, it is shown that an alternative expression to Equation (7), giving $MSI(h)$ in terms of statistical covariance values for the surface is:

$$(8) \quad MSI(h) \approx 1\frac{1}{2} \gamma(0) + \frac{1}{2} \gamma(2h) - 2 \gamma(h).$$

(still assuming mid-point interpolation). We now have a means of calculating $RMSI(h)$ which is an alternative to Equation (6). Moreover, it does not require the evaluation of gradients or curvatures over groups of three consecutive points, but instead has the advantage of an easier calculation using the covariance function as determined separately from available or selected pairs of points.

If data is given at spacing $2h$ (as in Figure 1), then there is a practical difficulty, as it can be difficult to evaluate $\gamma(h)$. This can require fitting a curve to the available points of the covariance function. To avoid this problem here, estimates are made of the $RMSI(2h)$, and

then, because the interpolation error MSI is proportional to h^4 by Equation (6), $\text{RMSI}(h)$ is estimated using the relationship:

$$(9) \quad \text{RMSI}(h) \approx \frac{1}{4} \text{RMSI}(2h).$$

In summary, the procedure for estimating the RMS interpolation error for any surface comprises:

1. a determination of the covariance function for various values of h .
2. using Equation (8) to find $\text{RMSI}(h)$
3. estimating the RMSI interpolation error for a chosen distance h of interest by using Equation (9).

It may be of interest to recognise that, in practical two-dimensional interpolation, forms of interpolation equation which are more sophisticated than that in Equations (8) are normally encountered. Also, it is often necessary to take into account the directional dependence of the covariance function. Such complications are not discussed here.

3. Results

3.1 Data

The investigations are based on electroconductive replicas of the palatal aspects of ten permanent maxillary central incisors, selected from an available 250 dental impressions which were recorded as part of an ongoing clinical study. The subjects were school children aged 10 - 11 years attending four secondary schools within the county of Angus of the United

Kingdom. All children have volunteered to participate in the study, and have given informed consent to the taking of appropriate dental castings. Ethical approval was obtained from the Tayside Committee on Medical Research Ethics (Ref 040/99). The ten data sets are derived from the mapping of both incisors from five children. Measurement data exists on a 33x33 grid spaced at 150 μm in both directions. All measurements were undertaken using a surface mapping device that was a development of that previously described by Chadwick *et al.* (1997).

Figure 2 shows a typical incisal replica surface. It may be noted that measurement has stopped before the incisal edge, which would occur beyond the top of the diagram, and that the replica touches on some soft tissue which appears as the high points at the lower edge of the surface.

To provide examples of the use of the equations which estimate the interpolation error, we can take an example of a small section of seven measurements from the left incisor of patient number 1, as shown in Table 1. Linearly interpolating a point at $x=4350$ by using the values at $x=4200$ and $x=4500$ in Equation (2) gives:

$$\begin{aligned} y(4350) &= \frac{1}{2} * (1523 + 1440) \text{ } \mu\text{m} \\ &= 1481.5 \text{ } \mu\text{m} \end{aligned}$$

so that

$$\begin{aligned} e(4350) &= 1481.5 - 1490 \text{ } \mu\text{m} \\ &= - 8.5 \text{ } \mu\text{m} \end{aligned}$$

Alternatively, to evaluate the error $e(4350)$ using Equation (4), we can firstly estimate gradients at midpoints $x=4275$ and at $x=4425$, as the ‘first differences’ in y values over a surface distance of 150 μm :

$$y'(4275) = [y(4350) - y(4200)] / 150$$

$$= (1490 - 1523) / 150$$

$$= -0.22$$

and

$$y'(4425) = [y(4500) - y(4350)] / 150$$

$$= (1440 - 1490) / 150$$

$$= -0.333$$

We then estimate the second derivative at $x=4350$ as the change in the two gradient values over the same distance of $150 \mu\text{m}$:

$$y''(4350) = [-0.333 + 0.22] / 150 \mu\text{m}^{-1}$$

$$= -0.00075533 \mu\text{m}^{-1}$$

By Equation (4):

$$e(4350) = \frac{1}{2} h^2 y''(x)$$

$$= \frac{1}{2} * (150)^2 * (-0.00075533) \mu\text{m}$$

$$= -8.4975 \mu\text{m}$$

The calculations serve to illustrate the lengthy nature of the error evaluations if carried out using Equations (3) and (4), and the advantage of the approach which uses the covariance function.

3.2 Covariance Functions

Covariance functions have been evaluated for the ten tooth replica surfaces, thereby permitting some observations of their typical nature and their variations. Software to compute the covariance functions has been written in-house, but the interested researcher is able to estimate the functions using proprietary software, such as Surfer Surface Mapping System (Golden Software Inc., Golden, Colorado, USA).

The covariance functions are displayed for the ten incisors in Figure 3. The covariance functions have only been evaluated out to $\lambda(1350 \mu\text{m})$ because of the irrelevance of values beyond this spacing to this study. It is seen that the zero-crossing is not approached at this distance. Table 2 shows in more detail the first three elements of the covariance function in the cases which are depicted in Figure 3. It includes the results of using Equations (8) and (9) to estimate the RMSI for measurement point spacing of 300, 150, 100 and 50 μm for these measured surfaces.

The mean RMSI calculated from the ten incisors was 26 μm with a standard deviation of 8 μm . Despite the variation in the covariance function values for the ten teeth as apparent in Figure 3, the variation among the final RMSI for a spacing of 150 μm is surprisingly low.

It is interesting to observe that some similarity in covariance function for the two incisors from the same patient is apparent in Table 2, but the pattern is not consistent over all subjects.

4. Discussion

Utilisation of the RMS interpolation error brings two difficulties. Firstly, the interpretation of the RMSI error as derived here requires a recognition that it is statistical, with the following characteristics.

1. It is based on interpolation error from two points only; interpolation from three or four surrounding grid points will generally give a smaller error.
2. It is based on an assumption of mid-point interpolation.
3. The RMSI error for any replica represents the entire surface and it is an error which will be exceeded as often as not.

Secondly, the effect of an interpolation error on deduced surface changes will depend on the procedures being used for comparing surfaces to detecting differences.

Despite these difficulties, the statistical study provides a useful estimate of the orders of magnitude of errors for the interested researcher. For the major erosion study involving palatal incisors at the University of Dundee, the differences between the replicas are estimated in this work by a mathematical surface matching and difference detection algorithm. The object of the algorithm is to estimate the surfaces' best fit position without the aid of control points, and to simultaneously estimate the change which has occurred from one epoch to the next, an approach which is adopted because the accurate monitoring of the progression of erosion is complicated by the lack of fixed intra-oral reference points from which to make measurements. The measurement point spacing of 150 μm has been estimated here to give a mean RMSI error calculated from ten incisors to be about $\pm 25 \mu\text{m}$ for mid-point interpolation, and this will propagate into the mathematical surface matching algorithm. To this error, the investigators need to add other sources of error, such as that due to the replication procedure (assessed as giving replicas that were within $\pm 3 \mu\text{m}$ of the original test piece, Chadwick et al., 2002) and the measurement (assessed as being of the order of $\pm 5 \mu\text{m}$, Chadwick et al., 1997) to estimate the total error budget, seen as being under 50 μm . It may be noted that annual wear rates for enamel in normal physiological function are quoted in the literature to be of the order of 10 to 30 μm (see Lambrechts et al., 1989; Pintado et al., 1997) so it might be assumed that erosion, as an abnormal occurrence, would cause surface loss greater than this. It therefore appears that reasonable goals for this work are the detection of surface change at magnitudes over about 30 μm .

The expected error in any situation can be investigated by the interested researcher using the procedures followed here, and such an evaluation is recommended because the error may be

larger than imagined. Teeth other than incisors may have more extensive undulations and may require closer spacing of point measurement. In all cases, the use of non-linear interpolation techniques may be used if necessary, to reduce the error due to interpolation when comparing pairs of dental surfaces to detect the changes occurring in them.

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Appendix 1: The relationship between the RMSI error and surface covariance values

Assume a point $y(x)$ is on a curve between two points of value $y(x-h)$ and $y(x+h)$, and that the error, as before, is given by Equation (3):

$$e(x) = \frac{1}{2} y(x-h) + \frac{1}{2} y(x+h) - y(x)$$

The mean square interpolation error for the mean distance h is given by:

$$\begin{aligned} \text{MSI}(h) &= \left(\frac{1}{N} \right) \sum [y(x) - \{ \frac{1}{2} y(x-h) + \frac{1}{2} y(x+h) \}]^2 \\ &= \left(\frac{1}{N} \right) \sum [\{y(x)\}^2 + \{ \frac{1}{2} y(x-h) + \frac{1}{2} y(x+h) \}^2 - 2 y(x) \{ \frac{1}{2} y(x-h) + \frac{1}{2} y(x+h) \}] \\ &= \left(\frac{1}{N} \right) \sum [\{y(x)\}^2 + \{ \frac{1}{4} y(x-h)^2 + \frac{1}{4} y(x+h)^2 + 2 (\frac{1}{2})(\frac{1}{2}) y(x-h) y(x+h) \} - 2(\frac{1}{2})y(x-h)y(x) - 2(\frac{1}{2}) \\ &\quad y(x+h) y(x)] \\ &= \left(\frac{1}{N} \right) \sum \{y(x)\}^2 + \left(\frac{1}{N} \right) \sum \frac{1}{4} y(x-h)^2 + \left(\frac{1}{N} \right) \sum \frac{1}{4} y(x+h)^2 + 2 \left(\frac{1}{N} \right) \sum (\frac{1}{2})(\frac{1}{2})y(x-h) y(x+h) - \\ &\quad 2 \left(\frac{1}{N} \right) \sum \frac{1}{2} y(x-h) y(x) - 2 \left(\frac{1}{N} \right) \sum \frac{1}{2} y(x+h) y(x) \\ &\approx \gamma(0) + \frac{1}{4} \gamma(0) + \frac{1}{4} \gamma(0) + 2 (\frac{1}{2}) (\frac{1}{2}) \gamma(x+h) - 2 \frac{1}{2} \gamma(h) - 2 \frac{1}{2} \gamma(h) \end{aligned}$$

where $\gamma(h)$ is the covariance at the distance h , so that

$$\text{MSI}(h) \approx \{1 + \frac{1}{4} + \frac{1}{4}\} \gamma(0) + 2 (\frac{1}{2}) (\frac{1}{2}) \gamma(2h) - 2 (\frac{1}{2}) \gamma(h) - 2 (\frac{1}{2}) \gamma(h)$$

$$\text{i.e.} \quad \text{MSI}(h) \approx 1\frac{1}{2} \gamma(0) + \frac{1}{2} \gamma(2h) - 2 \gamma(h).$$

Table 1: A series of seven measurements, between measurements at 3900 μm and 4800 μm , at spacing 150 μm , as extracted from row 6 of the data file for the left incisor of patient number 1.

x (μm)	3900	4050	4200	4350	4500	4650	4800
y (μm)	1455	1510	1523	1490	1440	1403	1350

Table 2: Elements of the covariance function and calculated interpolation errors for ten sample incisor tooth surfaces (reduced to mean), with calculated interpolation errors for point spacings of 300, 150, 100 and 50 μm .

Patient	Incisor	$\gamma(0 \mu\text{m})$ ($\mu\text{m})^2$	$\gamma(150 \mu\text{m})$ ($\mu\text{m})^2$	$\gamma(300 \mu\text{m})$ ($\mu\text{m})^2$	MSI(300 μm) by Eqn (8) ($\mu\text{m})^2$	RMSI(300 μm) (μm)	RMSI(150 μm) by Eqn (9) (μm)	RMSI(100 μm) by Eqn (9) (μm)	RMSI(50 μm) by Eqn (9) (μm)
1	Left	770844	759990	746104	9338	97	24	11	3
1	Right	197555	193533	188007	3270	57	14	6	2
2	Left	136251	128544	122241	8409	92	23	10	3
2	Right	132775	124790	117628	8397	92	23	10	3
3	Left	94096	91507	88557	2409	49	12	5	1
3	Right	376633	352620	333380	26400	162	41	18	5
4	Left	535272	525821	512687	7610	87	22	10	2
4	Right	121837	113015	105487	9469	97	24	11	3
5	Left	344937	330336	318364	15916	126	32	14	4
5	Right	253569	240763	229446	13551	116	29	13	3
<i>Mean</i>		<i>296377</i>	<i>286092</i>	<i>276190</i>	<i>10477</i>	<i>102</i>	<i>26</i>	<i>11</i>	<i>3</i>
<i>(SD)</i>		<i>(217660)</i>	<i>(215436)</i>	<i>(212119)</i>	<i>(6902)</i>	<i>(33)</i>	<i>(8)</i>	<i>(4)</i>	<i>(1)</i>

Figure captions:

Figure 1: The interpolated point y^* is at the mid-point between y_1 and y_2 , at distance h from each. Data points are thus assumed to be provided at a separation $2h$.

Figure 2: Typical incisor replica. The incisal edge is towards the top of the diagram, and beyond the measured region. The inclusion of soft tissue in the casting, replication and measurement is shown by the high points at the lower edge of the surface.

Figure 3: Covariance functions for ten incisors from zero to $1350\text{ }\mu\text{m}$, at $150\text{ }\mu\text{m}$ spacings.

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