Adaptive Ordering for Imperfect Successive Decision Feedback Multiuser Detection

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Abstract—A method for selecting the order in which the users are detected in communication systems employing adaptive successive decision feedback multiuser detection is proposed. Systems employing channel coding without the assumption of perfect decision feedback are analyzed. The method is based on the Mean Squared Error (MSE) measurements during a training period for each user. The analysis shows that the method delivers BER performance improvement relative to other previously proposed ordering methods.

Index Terms—Adaptive ordering, multiuser detection, decision feedback.

I. INTRODUCTION

IN SUCCESSIVE Decision Feedback multi-user Detection (S-DFD) [1], [2], users are detected one by one in a successive manner. Consequently, in a multiuser system serving K users, there are $K!$ different ordering possibilities implying $K!$ different (although structurally equivalent) detectors [3]. In previous work on conventional S-DFD [4] users are ordered according to non-increasing powers after matched filtering. An extensive analysis of users' detection order as a design parameter of the DFD with no assumption about perfect feedback has been provided in [3]. A rule for ordering the users according to non-increasing Asymptotic Effective Energies (AEE) has shown that any DFD always outperforms its linear counterpart in AEE for every user, e.g., D-DFD always outperforms the linear decorrelator. Both ordering methods require perfect knowledge of the system parameters such as amplitudes, spreading sequences, and channel.

In previous work on MMSE S-DFD, [5], the users are ordered according to received powers, i.e., the strongest users are detected first. In [6] the problem of the ordering in V-BLAST in a laboratory environment has been analyzed. An ordering method based on the post-detection SNRs has been proposed showing that at each stage during the cancellation process the signal with the largest post-detection SNR among the remaining (non cancelled) signals has to be detected first.

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In this paper we present a new adaptive ordering method which employs an adaptive algorithm and linear multiuser detection (MUD). The method is shown to outperform all previously proposed ordering methods for the analyzed scenarios, delivering an SNR gain for moderate loads. For the overloaded case, a system employing the proposed ordering improves the BER performance, providing lower BER floor values, in comparison with previously proposed ordering methods such as power ordering [5], arbitrary ordering, AEE based ordering [3] or SNR based ordering [6].

In addition, we also show that when all users are received with the same powers it is still possible to improve BER performance applying the adaptive ordering relative to the arbitrary ordering (power based ordering is not applicable for that case). This makes the adaptive ordering more robust in respect to the users' power variations.

II. SYSTEM MODEL

We consider an up-link of symbol-asynchronous CDMA multiuser detection scheme. At the transmitting end all signals are encoded first using Recursive Systematic Convolutional (RSC) Codes [7]. After interleaving and spreading, the users transmit their signals over an unknown multi-path communication channel.

A multi-cell mobile communications system is considered. Two groups of users are recognized in the system: the users within the cell of interest (denoted $1, 2, \dots, K$) and the users from neighboring cells whose signals are strong enough to cause interference within the cell of interest (inter-cell interferers denoted $K+1, \dots, K+K_I$). The total number of users is $K_T = K + K_I$. The received signal is the superposition of all signals plus Additive White Gaussian Noise (AWGN) and for the asynchronous case it can be expressed as [1], [4]

$$r(t) = \sum_{k=1}^{K+K_I} \sum_{i=0}^{L_x-1} s_k(t - iT - \tau_k) x_k(i) + n(t) \quad (1)$$

where i indexes time instants, L_x is the number of symbols transmitted by each user, $s_k(t)$ is the received signature of user k , $x_k(i)$ is the k th user's transmitted symbol, $n(t)$ is AWGN. The k th user received signature waveform is the convolution $s_k(t) = p_k(t) \otimes g_k(t)$, where $p_k(t)$ is the transmitted signature waveform, and $g_k(t) = \sum_{n=0}^{L_k-1} g_{kn} \delta(t - \tau_{kn})$ is the channel impulse response, where τ_{kn} are path delays, g_{kn} are path gains and $\delta(\cdot)$ is the Dirac delta function, while L_k is the length of k th user's channel impulse response. The multi-path taps g_{kn} are assumed to be time-invariant or very slowly time-varying in comparison to the speed of the adaptive algorithm.

The MUD front-end is designed as a bank of linear MMSE filters. Here, we use the *Sliding Window MMSE Filtering* approach as explained in [1], which says that in practice an MMSE filter need not cover the entire received signal vector, but the length needs to cover the useful part of the effective signature vector (including multi-path).

For S-DFD, the k th user's output at time instant i can be expressed as

$$z_k(i) = \mathbf{f}_k^T(i)\mathbf{r}(i) - \mathbf{b}_k^T(i)\hat{\mathbf{x}}_k(i) \quad (2)$$

where $\mathbf{f}_k$ and $\mathbf{b}_k$ are the feedforward and feedback coefficients of user k , respectively, and $\hat{\mathbf{x}}_k(i)$ are the estimated symbols of users $1, 2, \dots, K-1$ obtained during the detection process.

In order to estimate the MMSE S-DFD coefficients, in this paper we use the LMS algorithm. The first stage during the training period is devoted to ordering and will be discussed in more detail in the following section. Once the users' order is determined, the LMS algorithm is used again to estimate the receiver coefficients, as proposed in [1]. While receiving data (coded signals), the S-DFD still operates as an adaptive S-DFD employing the same adaptive LMS algorithm.

In this work we use Maximum a Posteriori Probability (MAP) algorithm [7]. It was observed in [8] that the interference plus noise at the output of the MMSE filters can be assumed to be Gaussian. Therefore, the signal at the time instant i corresponding to the k th user used to feed the channel decoder k can be expressed as $z_k(i) = A_k x_k(i) + \xi_k(i)$, where A_k is a constant and $\xi_k(i)$ is a random variable with variance $\sigma_{\xi_k}^2$. Quantities A_k and $\sigma_{\xi_k}^2$ can be expressed as $A_k = E[x_k(i)z(i)]$ and $\sigma_{\xi_k}^2 = E[z_k(i) - A_k x_k(i)]^2$ so that estimates $\hat{A}_k$ and $\hat{\sigma}_{\xi_k}^2$ can be obtained from the MMSE filter output. The MAP algorithm delivers soft outputs of the i th bit of the k th user defined as the Log-Likelihood Ratio [7] (LLR), $\Lambda(x_k(i)) = \log(p(x_k(i) = +1|\mathbf{z}_k)/p(x_k(i) = -1|\mathbf{z}_k))$. Finally, the expectations (or soft decisions) $\hat{x}_k(i)$ used in the feedback of S-DFD are obtained as $\hat{x}_k(i) = \tanh(\Lambda(x_k(i))/2)$ [9].

III. OPTIMAL ORDERING AND ADAPTIVE ORDERING MMSE ANALYSIS

For the S-DFD with the error propagation the estimation of the k th user symbol after MMSE filtering is $z_k = \mathbf{f}_k^T \mathbf{r}(i) - \mathbf{b}_k^T \hat{\mathbf{x}}_k(i)$, where $\mathbf{f}_k = [f_{k,1}, f_{k,2}, \dots, f_{k,L_f}]^T$ and $\mathbf{b}_k = [b_{k,1}, b_{k,2}, \dots, b_{k,k-1}]^T$ are vectors of k th user feed-forward and feedback coefficients, L_f is the length of the linear filter $\mathbf{f}_k$, and $\hat{\mathbf{x}}_k(i) = [\hat{x}_1(i), \hat{x}_2(i), \dots, \hat{x}_{k-1}(i)]^T$ are soft channel decoders output of the users $1, 2, \dots, k-1$. $\mathbf{r}(i)$ is a part of the entire received sequence $\mathbf{r}$ (eq. (2) in [1]), that covers the i th symbol of the k th user, and it is obtained according to the *Sliding Window* approach in [1]. Furthermore, the soft decoder output contains errors and the output symbols can be expressed as $\hat{x}_k(i) = x_k(i) + \hat{e}_k(i)$, where $x_k(i)$ is the correct symbol value and $\hat{e}_k(i)$ is the error at the decoder output. For the k -th user, the error can be expressed as the difference $e_k(i) = \mathbf{f}_k^T \mathbf{r}(i) - \mathbf{b}_k^T \hat{\mathbf{x}}_k(i) - x_k(i)$. The MSE is obtained as

$$\varepsilon_k = E[|e_k(i)|^2] = \mathbf{f}_k^T \mathbf{R} \mathbf{f}_k - 2\mathbf{f}_k^T \mathbf{S}_{D,k} \mathbf{b}_k - 2\sigma_k^2 \mathbf{f}_k^T \mathbf{s}_k + \mathbf{b}_k^T \mathbf{R}_{\hat{\mathbf{x}},k} \mathbf{b}_k + \sigma_k^2 \quad (3)$$

where $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}(i)^T]$, $\mathbf{R}_x = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2)$, $\mathbf{S}_{D,k} = E[\mathbf{r}(i)\hat{\mathbf{x}}_k^T(i)]$, $\sigma_k^2 \mathbf{s}_k = E[x_k(i)\mathbf{r}(i)]$, and $\sigma_k^2 = E[|x_k(i)|^2]$ is k th user received symbol energy. $\mathbf{R}_{\hat{\mathbf{x}},k} = E[\hat{\mathbf{x}}_k(i)\hat{\mathbf{x}}_k^T(i)] = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_{k-1}^2) + \hat{\mathbf{N}}_k$ where $\hat{\mathbf{N}}_k = E[\hat{\mathbf{e}}_k(i)\hat{\mathbf{e}}_k^T(i)]$ is covariance matrix of decision feedback error, $\mathbf{x}_k(i) = [x_1(i), x_2(i), \dots, x_{k-1}(i)]^T$ and $\hat{\mathbf{e}}_k(i) = [\hat{e}_1(i), \hat{e}_2(i), \dots, \hat{e}_{k-1}(i)]^T$. The MMSE filters $\mathbf{f}_k$ and $\mathbf{b}_k$ can be expressed by

$$\mathbf{f}_k = \sigma_k^2 (\mathbf{R} - \mathbf{S}_{D,k} \mathbf{R}_{\hat{\mathbf{x}},k}^{-1} \mathbf{S}_{D,k}^T)^{-1} \mathbf{s}_k \quad (4)$$

$$\mathbf{b}_k = \mathbf{R}_{\hat{\mathbf{x}},k}^{-1} \mathbf{S}_{D,k}^T \mathbf{f}_k. \quad (5)$$

Combining (4), (5) and (3), we find the following expression for MMSE ε_k

$$\varepsilon_k = \sigma_k^2 [1 - \sigma_k^2 \mathbf{s}_k^T (\mathbf{R} - \mathbf{S}_{D,k} \mathbf{R}_{\hat{\mathbf{x}},k}^{-1} \mathbf{S}_{D,k}^T)^{-1} \mathbf{s}_k]. \quad (6)$$

Equation (6) takes into account error propagation during the previous detection. At each stage during the detection process, the optimal order is determined according to increasing ε_k . This means that, among the undetected users, the user with the smallest ε_k has to be chosen and detected next.

To achieve an optimal ordering we would have to use coded training sequences and to estimate ε_k as described in (6) taking into account error propagation. However, this method is too complicated and it increases the complexity of the ordering process significantly due to use of channel coding. Here, we propose a simple ordering method that determines the users' order according to increasing users' error variances during the training period, employing linear MUD and the adaptive LMS algorithm, which estimates the k th user's linear coefficients according to

$$\mathbf{f}_k(i+1) = \mathbf{f}_k(i) + \mu e_k(i) \mathbf{r}(i). \quad (7)$$

The LMS algorithm calculates the error with each training bit as a difference between correct and the estimated value, i.e. $e_k(i) = x_k(i) - \mathbf{f}_k^T(i)\mathbf{r}(i)$. The k th user's MMSE or error variance is obtained by averaging $|e_k(i)|^2$ during the training process, i.e.,

$$\text{Error Variance } k = \frac{1}{L - \ell'} \sum_{i=\ell'+1}^L |x_k(i) - \mathbf{f}_k^T(i)\mathbf{r}(i)|^2 \quad (8)$$

where $\ell' + 1$ is the training bit from which we start measuring the error variance. Our experience shows that the initial training period should be avoided when determining the error variance since they tend to be unreliable. Once the convergence is achieved, we start measuring error variances. After measuring each user's error variance, the users are then reordered according to non-decreasing error variances.

IV. SIMULATION RESULTS

In this section the simulation results are provided for different scenarios. In the transmitter, users are encoded using four-state (5,7) (in octals) RSC codes. The encoded sequences are interleaved by 2048 bits long random interleavers. The linear filters $\mathbf{f}_k$ are all of the same length $L_f = 21$ chips. The channels $\mathbf{g}_k$ are modelled as five-tap chip-spaced multi-path channels, i.e., $L_k = 5$ with randomly chosen coefficients in each realization. The adaptation constant is chosen to be

Fig. 1. S-DFD BER performance comparing proposed and other ordering methods for the overloaded system $K/N = 1.25$: + - Adaptive (proposed); x - SNR based [6]; * - AEE based [3]; * - Power based [5].

Fig. 3. S-DFD BER performance comparing proposed and other ordering methods for the multi-cell scenario: + - Adaptive (proposed); x - SNR based [6]; * - AEE based [3]; * - Power based [5].

Fig. 2. S-DFD BER performance comparing proposed and arbitrary ordering for the single and multi cell scenarios.

$\mu = 0.01$ in the training period and $\mu = 0.0005$ during detection. The values L and ℓ' in (8) are $L = 500$ and $\ell' = 200$ for Fig. 1, and $L = 1000$ and $\ell' = 200$ for all other figures. Spreading gain is chosen to be $N = 16$.

Fig. 1 shows the adaptive S-DFD BER performance obtained by employing the proposed adaptive ordering and comparing it with other ordering methods given in [6] (SNR based), [3] (AEE based) and [5] (power based) for the overloaded case. All users are received with different powers which are randomly chosen and the difference between the strongest and the weakest user is set to be 3 dB. SNR is determined with respect to the weakest user. The results of Fig. 1 show that the proposed ordering outperforms SNR and AEE-based orderings by shifting the BER floor towards lower values. The improvement in BER floor is around 3 times relative to SNR and AEE based orderings and almost 10 times relative to power based ordering at SNR of 18 dB.

Fig. 2 shows the BER performance obtained by the MSE-

based and arbitrary orderings when all users are received with the same powers. For the equal-power case we analyze both single- and multi-cell scenarios. For the single-cell scenario the number of users within the cell of interest is $K = 6$ and $K = 12$. For the multi-cell scenario, the number of users within the cell is $K = 6$ but there are additional $K_I = 3$ interferers from the neighboring cells. Results in Fig. 2 show that MSE based ordering delivers a significant improvement with respect to both power and arbitrary orderings. The reason for this is that after MMSE detection, different users are generally detected with different error variances. With the adaptive ordering, a user with the smallest error is always detected first, which minimizes the feedback error propagation, which is not the case with the arbitrary ordering. For small number of users arbitrary ordering shows the performance similar to the adaptive ordered detector. However, for moderate loads $\rho = 0.75$ the adaptive ordering delivers improvement of about 5 dB at BER of $2 \cdot 10^{-4}$ relative to the arbitrary ordering. For the overloaded case $\rho = 1.25$ the adaptive ordering provides significant improvement shifting the BER floor for almost two orders of magnitude relative to arbitrary ordering. These observations about BER floor improvement are made for high SNRs ($\text{SNR} > 20$ dB).

Fig. 3 shows the BER performance of a multi-cell scenario where certain number of interfering users are in the neighboring cells. Number of users within the cell of interest is $K = 16$, while number of inter-cell interferers is $K_I = 4$. The simulation set-up, however, differs from the ones used to obtain the results in Fig. 2. Here, for the cases of post-SNR and AEE-based orderings, it is assumed that the inter-cell interferers are perfectly known. This knowledge is taken into account during post-SNRs and AEEs calculations. However, during the detection of information data, those inter-cell users are assumed to be still unknown, i.e., their signals are not used in the decision feedback part of the receiver. The results confirm that the proposed adaptive ordering still outperforms all other orderings and that the main cause of the better performance of the proposed algorithm is accounting for the

Fig. 4. S-DFD BER performance comparing proposed and other ordering methods for two different users loads $K/N = 0.75$ and $K/N = 1$, and hard decisions used in the feedback: + - Adaptive (proposed); > - SNR based [6]; × - AEE based [3]; * - Power based [5].

effects of feedback error propagation, caused not only by inter-cell interference, but also by intra-cell interferers.

The results in Fig. 4 are obtained for the single-cell scenario where hard decisions are used in the decision feedback part. Two sets of BER curves are obtained. The first set of results is obtained for $\rho = K/N = 0.75$, i.e., $K = 12$, and the second set of results obtained for $\rho = K/N = 1$, i.e., $K = 16$. Both sets of curves again confirm better BER performance obtained when the users are ordered according to the proposed algorithm, comparing to all other orderings, i.e., post-SNR based, AEE-based, and power-based. However, it can be noted that the BER performance are significantly degraded with all ordering methods comparing to the soft-decision results. For $\rho = 0.75$, however, the performances of all ordering methods are very close to each other, although the adaptive ordering shows still slightly better BER results

than other orderings. However, for increased number of users ($\rho = 1$), the gap between the proposed adaptive ordering and other orderings becomes more significant. This is not surprising because the feedback error propagation increases with the increasing number of users within the system.

V. CONCLUSION

A new method for determining the ordering in an adaptive S-DFD scheme has been presented. It was shown that significant BER improvement can be obtained by the proposed adaptive MSE based ordering relative to the post-detection SNR based [6], AEE based [3], power based [5] or arbitrary ordering, especially for over-saturated CDMA systems where the number of users exceeds the spreading gain.

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