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Slope stability assessments using finite element limit analysis methods

K. Lim¹, A. J. Li², A. Schmid³, A. V. Lyamin⁴

¹ Corresponding Author, Postgraduate student

School of Engineering

Deakin University, VIC 3217, Australia

kkwl@deakin.edu.au

² Lecturer

School of Engineering

Deakin University, VIC 3217, Australia

a.li@deakin.edu.au

³ Postgraduate student

School of Engineering

Deakin University, VIC 3217, Australia

aksc@deakin.edu.au

⁴ Professor

Centre for Geotechnical and Materials Modelling

The University of Newcastle, NSW 2308, Australia

Andrei.Lyamin@newcastle.edu.au

Contact address for corresponding author:

Kelvin Lim

School of Engineering

Deakin University

Pigdons Road, Geelong Victoria 3217

Abstract

This paper utilized the finite element limit analysis methods to investigate the stability of slopes of various properties and nature. Specifically, this paper investigated a slope with a soft (weak material) band, a post-quake slope and rock slopes. The conventional Mohr-Coulomb failure criterion and the Hoek-Brown failure criterion were utilized for soil and rock slopes, respectively. The Hoek-Brown failure criterion can be applied directly in the finite element limit analysis methods without the need for conversion to the equivalent Mohr-Coulomb parameters. This paper clearly demonstrates the applicability of the numerical limit analysis methods in both soil and rock slopes. It is also significant to note that the results presented in this paper have two distinct solutions – the upper and lower bound solutions. In addition, the failure mechanisms of the slopes are also shown. Prior assumptions of the failure mechanisms are not required for these finite element limit analysis methods, therefore providing a more realistic understanding of slope failures.

Key words: Limit equilibrium methods; Back analysis; Rock slope; Factor of safety; Landslides.

INTRODUCTION

Slope failures are undesirable and investigations of various types of slopes have been performed to prevent slope failures. Slope stability analyses are usually presented in the form of safety factors or limit loads. Many methods have been developed to investigate slope stability and they include the conventional limit equilibrium method, LEM (Bishop 1955; Janbu et al. 1956; Morgenstern and Price 1965; Spencer 1967), the finite element method, FEM (Matsui and San 1992; Griffiths and Lane 1999; Manzari and Nour 2000; Zheng et al. 2005) and the limit analysis method (Michalowski 1995; Donald and Chen 1997; Chen et al. 2003; Viratjandr and Michalowski 2006).

Sometimes the physical properties of the slope and/or the strength parameters cannot be measured accurately in practice, therefore investigations are often carried out to determine or back calculate the soil strength parameters of failed slopes (Duncan and Wright 2005). Traditionally, the LEM or FEM used to perform slope stability analyses is also used to back analyse slope failures. In fact, some FEM-based analyses can also provide better understanding of the soil behaviour and movement during failure (Potts et al. 1997; Troncone 2005).

Hence, encouraged by the need to better understand slope stability, advancement in this field has seen many new methods being developed. For example, the finite element limit analysis methods developed by Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005). In fact, Sloan (2013) highlighted that the methods are applicable to many geotechnical problems particularly slope stability problems. Therefore, it is valuable to verify and demonstrate the use of the numerical limit analysis methods. This paper will adopt the numerical limit analysis methods to investigate three types of slopes: (1) a slope with a soft (weak soil) band, (2) a slope considering earthquake effects and (3) rock slopes.

PREVIOUS STUDY

Back analyses of slope failures have commonly been done to determine the shear strength of the soil that may have caused the failures. For example, Jiang and Yamagami (2006) utilized the Janbu's simplified method (Janbu 1973) to determine the back calculated strength parameters for both circular and non-circular slip surface by matching the actual failure surface to the predicted one. Additionally,

Seed et al. (1990), Eid et al. (2000) and Zhou and Cheng (2014) also performed a LEM-based back analysis on slope failures.

Apart from the LEM, the finite element method (FEM) has also been utilized in slope stability analyses (Matsui and San 1992; Griffiths and Lane 1999; Cheng et al. 2007). Regarding back analyses, Troncone (2005), Chai and Carter (2009), Tang et al. (2009) and Chai et al. (2013) are some of the researchers who used a numerical analysis to investigate slope failures. In fact, the strain softening behaviour of soil and progressive failure of the slopes can be studied comprehensively using the numerical modelling.

As mentioned previously, the limit analysis method has also been utilized to investigate slope stability problems (Michalowski 1995; Donald and Chen 1997; Michalowski 2002; Chen et al. 2003, 2004). Particularly, some of the studies also include the consideration of pore water pressure and seismic 3D effects (Michalowski 1989, 2010). However, while improvements have been made, majority of the limit analysis-based studies are based on the upper bound method, which is not conservative. In addition, the conventional limit analysis method still requires the failure mechanism to be pre-defined.

Apart from the soil-based slope stability problems discussed above, rock slope stability is also a common problem and thus many studies have been performed (Sonmez and Ulusay 1999; Hoek and Marinos 2007; Tsiambaos and Saroglou 2010; Bozzano et al. 2012). Additionally, many criteria have been proposed to estimate the strength of rock masses (Yudhbir et al. 1983; Sheorey et al. 1989; Yu et al. 2002). Merifield et al. (2006) indicated that one of the most prominent rock failure criteria that have been developed and applied for closely jointed rock is the Hoek-Brown failure criterion (Hoek et al. 2002). However, its application in slope stability analyses is fairly limited because most of the current geotechnical engineering programs are still written in terms of the Mohr-Coulomb failure criterion (Li et al. 2008).

Recently, the finite element upper and lower bound limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005) have been intensively developed. An assumption of a potential failure mechanism is not required using these techniques. Although various investigators have utilized the methods in slope stability analyses (Yu et al. 1998; Li et al. 2010; Lim et al. 2015), the methods have yet to be widely adopted in slope designs. Furthermore, the Hoek-Brown failure criterion can be ideally

assigned in the numerical upper and lower bound limit analysis methods (Sutcliffe et al. 2004; Merifield et al. 2006; Li et al. 2009). Therefore, this criterion will be adopted in this study to analyse rock slope problems. Additionally, the study by Hoek et al. (2002), Li et al. (2011) and Qian et al. (2013) showed that the magnitude of the disturbance factor, D found in the Hoek-Brown failure criterion (2002) can significantly influence rock slope stability evaluations. Therefore, back calculations of D will also be performed for the rock slopes investigated in this study.

Problem Definition

The finite element limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005) contain two distinct solutions: the upper bound which is based on the kinematically admissible velocity field and the lower bound which is based on the statically admissible stress field. Fig. 1 shows an example of the mesh configuration of a simple slope with the corresponding boundary conditions for the upper and lower bound limit analysis methods.

The geometries and boundaries of the slopes investigated in this study are constructed according to the original studies. Additionally, it is important to make sure that the boundaries provided are large enough to ensure that the failure mechanisms are not restricted in anyway. The upper and lower bound meshes are automatically re-meshed to better capture the failure mechanisms as well as producing more accurate results (Lyamin et al. 2005). This can be seen in Fig. 1 where the meshes are more concentrated in the region of the failure mechanism.

It is important to note that the optimised solutions of the upper and lower bound limit analyses are obtained with respect to the unit weight. Hence, this study will utilize the strength reduction process formulated by Sloan (2013) to obtain the results in the form of safety factors for some of the cases investigated. The strength reduction method allows for F to be determined through an iterative procedure using Equation (1). By definition, F is obtained when $m = 1$. This strength reduction method to determine F is used in the soil section of this study.

$$m = \frac{\gamma_{ub} + \gamma_{lb}}{2\gamma} \quad (1)$$

where m is the gravity multiplier, γ_{ub} and γ_{lb} are the limit loads obtained from the upper and lower bound simulations respectively while γ in the denominator is the actual unit weight of the case study.

In this study, the Mohr Coulomb failure criterion will be adopted for the soil slope investigations while the Hoek-Brown failure criterion will be utilized for the rock slope analyses. Based on the study by Hoek et al. (2002), the latest version of the Hoek-Brown failure criterion can be expressed as:

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^\alpha \quad (2)$$

$$m_b = m_i \exp \left(\frac{GSI-100}{28-14D} \right) \quad (3)$$

$$s = \exp \left(\frac{GSI-100}{9-3D} \right) \quad (4)$$

$$\alpha = \frac{1}{2} + \frac{1}{6} \left(e^{-GSI/15} e^{-20/3} \right) \quad (5)$$

The above equations show that the parameters m_b , s and α are dependent on the geological strength index (GSI). GSI is the parameter that defines the quality of rock masses. Further information on how to determine its value and the other parameters within the Hoek-Brown failure criterion can be found in Hoek and Brown (1997), Hoek (2000), Marinos and Hoek (2000) and Hoek et al. (2002). Therefore these issues will not be discussed in this paper. The parameter, D , found in the equations above is a disturbance factor that ranges from 0 for undisturbed in situ rock masses and 1 for disturbed rock masses.

Case 1: A slope with a soft (weak) band

Fig. 2 shows the slope with a soft band problem adapted from the study by Cheng et al. (2007). In the original case study, two different slope stability methods, the limit equilibrium method and the FEM-based strength reduction method (SRM) were utilized to investigate the problem. It was demonstrated that the failure surfaces of the problem produced by those two methods agree well with each other. In this paper, the finite element limit analysis methods are adopted to investigate the applicability of the numerical limit analysis methods in this situation (slope with soft band).

Fig. 2 and Table 1 respectively show the slope geometry and properties of the soil. It should be noted that the soil in layer 2 has a thickness of just 0.5m and with $c' = 0$ and relatively small ϕ' . Cheng et al. (2007) further mentioned that conditions similar to this sort of failure (controlled by soft band) has actually occurred in Hong Kong (Fei Tsui Road slope failure).

Fig. 3 shows the failure mechanisms obtained from the numerical upper bound method (plastic zones) as well as those from the LEM and SRM by Cheng et al. (2007). Observation shows that the failure surfaces from the three different methods are very similar. The failures are seen to occur along the soft band at the steepest part of the slope. Therefore, this shows that the finite element limit analysis methods are suitable for dealing with stability problems of slopes with a soft band. Additionally, the upper and lower bound limit loads obtained for this slope (28 m domain width) can be observed in Table 2. Further discussion on the limit loads and the influence of domain widths can be found in the section below.

Different boundary discussion

In order to consider the boundary effect as shown by Cheng et al. (2007) who used the SRM, a domain of 12 m, 20 m and 28 m were investigated. Table 2 shows the obtained limit loads of each domain. The differences observed in the limit loads (upper and lower solutions) are bracketed to within 5% or better. The results also show that the numerical limit analysis methods are not influenced by the domain's sizes. In fact, it is to be noted that the obtained upper and lower bound results - with respect to the limit load - are lower than the in-situ (actual) unit weight. This implies that the slope is unstable. Thus, the strength reduction process described by Sloan (2013) has been used here to obtain the factor of safety (instead of applied load). A brief description of the process can be found in the problem definition section. Table 3 compares factor of safety obtained from this study with those from the SRM and LEM presented by Cheng et al. (2007). From Table 3, the factor of safety from this study is observed to be in between those from the SRM and LEM. The results obtained using the SRM 2 (associated flow rule) are slightly greater than those from SRM 1(non-associated flow rule). In fact, most of the results obtained from the SRM 2 are also greater than that obtained in this study.

Weak bottom layer

In addition to the slope with a soft band problem, Cheng et al. (2007) also investigated the slope in which soils 2 and 3 were interchanged. The numerical limit analysis methods are once again adopted to investigate this slope problem. Fig. 4 shows the failure mechanisms from the different methodologies. The failure surfaces obtained by Cheng et al. (2007) are represented by solid and dashed lines for the SRM and LEM, respectively. It can be observed that the failure surfaces obtained for this soil profile are deeper than those of the soft band problem. However, the most significant observation here is that

all the three methods produce different failure surfaces. The upper bound plastic zones are the smallest (closer to the toe of the slope) while the LEM produces the widest failure surface. In fact, from Fig. 4, it can be seen that the results from the SRM are much closer to the failure mechanisms obtained in this study, compared to the LEM-based result. This may be because the LEM requires an assumption of the failure surfaces. In contrast to the findings obtained by Cheng et al. (2007), Fig. 4 implies that a slope with a weak bottom layer would in fact be a sensitive case. More detailed studies would be needed in order to obtain more accurate results and understandings of the failure loads and mechanisms of such slopes. However, although different failure surfaces are obtained, the factors of safety, F of 1.29, 1.33 and 1.28 obtained respectively from the LEM, SRM and this study are very similar. The results above revealed that this case is sensitive due to the differences in failure mechanism predicted from various methods. Therefore engineers should be careful when designing slopes with similar soil profiles, particularly when considering the magnitude of the slope reinforcement.

Case 2: A post-quake slope

This section of the present paper investigates the post-quake slope found in the case study presented by Shou and Wang (2003). The case study was on the Chiufengershan landslide in Changhuken formation triggered by the 1999 Chi-Chi earthquake in Taiwan. Fig. 5 and Table 4 respectively show the geometry and mechanical properties of the slope. Shou and Wang (2003) indicated that the failure plane shown in Fig. 5 can be well defined and employed the LEM-based sliding-block method to investigate the potential post-quake slope failure. However, as the failure plane is pre-defined, the authors of the present paper found it interesting to use the numerical limit analysis methods - which do not need a pre-defined slip surface - to analyse the problem.

Based on the study by Shou and Wang (2003), the peak strength of the Changhuken shale are $\phi_p = 38.0^\circ$ and $c_p = 4600$ kPa. The residual strength of the shale was measured by shear testing of existing weak planes in samples, and an average residual strength of $c_r = 50$ kPa and $\phi_r' = 30.8^\circ$ were obtained. By considering weathering effects, the average residual strength using 1 month water-bathed samples were $c_r' = 25$ kPa and $\phi_r' = 27.3^\circ$. Detailed strength parameters of the Changhuken shale are listed in

Table 4. Shou and Wang (2003) showed that using the residual strength after certain weathering ($c_r' = 25$ kPa and $\phi_r' = 27.3^\circ$) with dry conditions would produce a factor of safety of 1.67 (LEM). Due to the fact that weathering effects should give a similar influence on the surface layer, a simulation based on the numerical limit analysis methods was performed. The soil properties adopted were $c_r' = 25$ kPa and $\phi_r' = 27.3^\circ$ for the Changhuken shale above potential failure plane (Fig. 5) while the peak strength parameters of $\phi_p = 38.0^\circ$ and $c_p = 4600$ kPa were used for the shale below the potential failure plane. The results obtained are surprisingly different. As mentioned previously, the numerical limit analysis methods results are obtained as a function of the unit weight (γ). The results from the numerical limit analysis methods show that the use of residual strength with weathering effects produces a limit load (γ) less than the actual unit weight, γ prescribed in the study by Shou and Wang (2003). In other words, the slope is deemed unsafe as the limit load is less than the actual γ (in-situ). For further clarification, the factor of safety was calculated using the strength reduction process described by Sloan (2013). The factor of safety of 0.78 obtained in this study clearly indicates that the slope should be unstable. In fact, Fig. 5 shows that the obtained failure surface from the upper bound plastic zones does not follow the potential sliding plane highlighted by Shou and Wang (2003). It would be the reason that the safety factors are so different. In fact, it should be noted that the critical failure surface is automatically generated with the numerical limit analysis methods without the need of a pre-assumed failure surface.

In light of the difference, this study then investigated the slope using the soil residual parameters ($c_r = 50$ kPa and $\phi_r = 30.8^\circ$) without weathering effects. These strength parameters were initially measured by Shou and Wang (2003) using shear testing samples from the field. A similar failure surface to that obtained in Fig. 5 was again obtained here using the residual parameters. It can be observed that the failure surface is still in the middle and steepest part of the slope. However, the results from the numerical limit analysis methods yield a higher limit load than previously obtained using the residual strength after certain weathering. The obtained factor of safety of $F = 1.06$ verify the current slope state. Further investigations were done to include earthquake effects. In this part, the horizontal seismic coefficient (k_h) is an uncertain parameter and is changed gradually in each trial until $F = 1$ was obtained. The findings from the numerical limit analysis methods show that the critical horizontal seismic

coefficient (k_h) for the slope are 0.041 and 0.034 for upper and lower bound solutions, respectively. As the F obtained for the slope without earthquake is only a mere 1.06, the results obtained here prove that a small earthquake coefficient would be sufficient to cause instability in the slope. In fact, the earthquakes coefficient (k_h) in Taiwan could be up to 0.33 (Ou 2006). Hence, based on the conservative LB result of 0.034 obtained here, the slope is considered to pose a very high risk of collapsing.

Case 3: Rock slope stability

A slope failure in a closely jointed rock mass in barite open pit mine

For this part of the present analysis, the slope found in Sonmez and Ulusay (1999) was studied. This rock slope was located at the Baskoyak barite open pit mine, in western Anatolia and was investigated by Sonmez and Ulusay (1999) who back calculated the rock parameters. As Sonmez and Ulusay (1999) used a different measurement to define the rock mass disturbance, this study proposes to back calculate the disturbance factor (D) introduced by Hoek et al. (2002).

This slope has a mean unit weight of 22.2 kN/m³ and a uniaxial compressive strength of the heavily broken part of the schist of 5.2 MPa. Other important parameters of the slope are $m_i = 7$ and $GSI = 16$ (Sonmez and Ulusay 1999; Sonmez et al. 2003). It is important to note that the chart employed for evaluating the GSI was modified by Sonmez and Ulusay (1999) using the Surface Condition Rating (SCR). This chart is similar to that proposed by Hoek and Brown (1997). As indicated by Sonmez and Ulusay (1999), no sign of groundwater was encountered in the previously drilled boreholes or on the pit benches. Thus, the pit slopes were treated as dry for stability assessments.

With the required parameters and geometries of the slope clearly outlined, a back analysis was then carried out to determine the appropriate disturbance factor to be applied for this slope. Disturbance factors of $D = 0.68$ and 0.66 were obtained using the numerical upper and lower bound limit analysis methods, respectively. The obtained magnitudes of D , are in good agreement with the suggestion by Hoek et al (2002) where $D = 0.7$ can be used for mechanical excavation in soft rocks. Additionally, as observed from Fig. 6, the plastic zones (failure surface) obtained from the upper bound method is similar to the actual slip surface (dashed line) presented by Sonmez and Ulusay (1999).

Slope instability in a coal mine in western Turkey

Another example of rock slope instability has been shown here. This rock slope is located at the Kırakdere open pit mine located at the Soma lignite basin, western Turkey. This case study was studied by Sonmez and Ulusay (1999) and Li et al. (2011). The latter authors estimated the factor of safety through the use of an appropriate disturbance factor, D while the former authors performed a back analysis on the slope. The necessary data collected by Sonmez and Ulusay (1999) shows the geometry of the failed slope where a single thin coal seam with a thickness of 4.5 m was overlain by a sequence of compact marl and soft clay beds about 10 m of thickness. Further to that, as indicated by Sonmez and Ulusay (1999), the observation of slope surfaces and available records showed that the groundwater was below the failed marly rock mass and the coal seam acted as an aquifer. The marly rock with a uniaxial compressive strength of 40 MPa and $m_i = 9.04$ has a carbonate content more than its clay content. The observed actual slip surface was of a circular shape. It passed through the compact marl rock mass and along the clay bed, above the coal seam. The three main joint sets which were moderately and closely-spaced, and the bedding planes in the marly sequence resulted in a jointed rock mass. Other parameters required for this investigation were obtained from Sonmez and Ulusay (1999) which include a slope height of $H = 80\text{m}$, a slope angle of $\beta = 60^\circ$ and a Geological Strength Index of $GSI = 37$. As indicated by Li et al. (2011), the approximate unit weight, γ of the slope is 21 kN/m^3 .

Sonmez and Ulusay (1999) noted that the method of excavation used in this case was blasting. Hence, based on the suggestions found in Hoek et al. (2002), D should be between 0.7 and 1.0. Specifically, $D = 1$ is considered to be suitable for production blasting. Hence, using the above slope properties, a back calculation of D was performed. The disturbance factors obtained from the numerical upper and lower bound limit analysis methods are 1 and 0.9, respectively. Therefore, these magnitudes can be seen as close to a production blasting. For back analysis, upper bound result would be more conservative and thus $D = 1$ should be taken into account for this case.

In fact, a closer look at the upper bound plastic zones (Fig. 7) shows that the failure mechanism is not as extended as the one reported by Sonmez and Ulusay (1999). The failure mechanism in this study is found to be concentrated on the lower bench of the slope. The presented slip surface (dashed line) by Sonmez and Ulusay (1999) is larger and extended back- and upwards of the slope. As indicated by Li et al. (2011), the reason for the difference would be because the slope was strongly heterogeneous along

its height. However, it should also be emphasized that the failure surface obtained from the original study was pre-assumed due to the use of the limit equilibrium method while the finite element limit analysis methods (used in this study) do not require the pre-assumption of a failure mechanism. Therefore, the obtained failure mechanism from this study should be the most critical one. The above two examples show that the finite element limit analysis methods can be utilized in rock slope stability analyses. Particularly, the methods allow for the direct utilization of the Hoek-Brown failure criterion without the need for a conversion to the equivalent Mohr-Coulomb parameter. As a result, the inaccuracies due to the conversion can be avoided. Additionally, the finite element limit analysis methods also do not require any pre-assumptions and the failure surface can be located automatically.

CONCLUSION

It is well known that the LEM and FEM have been utilized for most of slope designs while the finite element upper and lower bound limit analysis methods have not been as widely applied. This study clearly shows the applicability of the numerical limit analysis methods for soil and rock slopes. The results obtained in this study were compared with the results from the original studies. It can be seen that some of the results obtained in this study are different from the original studies in terms of the failure mechanism and safety factor. In fact, it should be highlighted that the results obtained using the finite element limit analysis methods should be more realistic than the limit equilibrium method because they do not require the failure surface to be pre-assumed.

In the case of the slope with a soft band, the original study showed that the results obtained from the finite element method (FEM) are highly sensitive. However, this was not the case using the numerical limit analysis methods because it was shown that the presence of the soft band has no effect on the results obtained. In contrast, different failure mechanisms (using the different methods) were obtained for the case with a weak bottom layer. This implies that the case with a weak bottom layer would in fact be a sensitive one. However, the factors of safety obtained by the different methods were in good agreement with each other.

In this study it was also found that the failure mechanisms obtained using the numerical limit analysis methods and the LEM may be significantly different. This would be because the use of the LEM requires an assumption on the failure surface while the numerical limit analysis methods allow the failure surface to be automatically located. Moreover, it was found that the residual strength without weathering should be more realistic in representing the in-situ Changhuken shale strength. Additionally, it was also shown that earthquake effects can be investigated using the numerical limit analysis methods. The results from this study show that the slope is close to the critical state as the critical k_h (0.041 and 0.034 for upper and lower bound methods) are much lower than the required k_h (0.33). In addition, this study also verifies the applicability of the finite element limit analysis methods and Hoek-Brown non-linear failure criterion in rock slope stability assessments. Back calculations of disturbance factors (D) were performed in this study. The disturbance factors D obtained in this study showed good agreement between with those suggested by Hoek et al. (2002). This implies that the suggested value of D by Hoek et al. (2002) is quite reasonable, therefore can be applied to design with more confidence.

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451

Figures

Fig. 1. Mesh configuration and boundary condition: (a) Upper bound; (b) Lower bound

Fig. 2. Scheme of the slope with a soft band (Case 1) adapted from Cheng et al. (2007)

Fig. 3. Failure mechanisms of Case 1 (weak soft band): Upper bound plastic zones (band) and Cheng et al. (2007) (Light solid line: SRM; Dark dashed line: LEM)

Fig. 4. Failure mechanisms of Case 1 (weak bottom layer): Upper bound plastic zones (band) and Cheng et al. (2007) (Solid line: SRM; Dashed line: LEM)

Fig. 5. Failure mechanisms of Case 2: Upper bound plastic zones (band) and Shou (2003) (Dashed line)

Fig. 6. Failure mechanisms of Case 3 (Baskoyak barite open pit mine): Upper bound plastic zones (band) and Sonmez (1999) (Dashed line)

Fig. 7. Failure mechanisms of Case 3 (Kisraktere open pit mine): Upper bound plastic zones (band) and Sonmez (1999) (Dashed line)

Tables

Table 1. Soil properties of the corresponding region

Soil Region	Cohesion (kPa)	Friction angle (°)	Unit Weight (kN/m ³)	Elastic Modulus (MPa)	Poisson ratio
Soil 1	20	35	19	14	0.3
Soil 2	0	25	19	14	0.3
Soil 3	10	35	19	14	0.3

Table 2. Results for domain of 12 m, 20 m and 28 m

Domain Boundary (m)	UB results (kN/m ³)	LB results (kN/m ³)	Difference between LB and UB
12	14.2	13.7	3.5 %
20	14.2	13.5	5 %
28	14.1	13.6	3.5 %

Table 3. Factor of safety comparison

Domain Boundary (m)	Factor of Safety			
	This study	Phase – SRM (1/2)*	Plaxis – SRM (1/2)*	LEM
12	0.86	0.77/0.85	0.82/0.94	0.927
20		0.84/1.06	0.85/0.97	
28		0.87/1.37	0.86/0.97	

*SRM1: Non-associated flow rule; SRM2: Associated flow rule

Table 4. Mechanical properties of the Changhuken shale

Soil parameters	Values
ϕ_p	38°
c_p	4600 kPa
ϕ_r	30.8°
c_r	50 kPa
ϕ'_r	27.3°
c'_r	25 kPa