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Huang, Jinsong; Kelly, Richard; Li, Dianqing; Zhou, Chuangbing; Sloan, Scott; "Updating reliability of single piles and pile groups by load tests". Published in Computers and Geotechnics Vol. 73, p. 221-230 (2016)

Available from: <http://dx.doi.org/10.1016/j.compgeo.2015.12.003>

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Updating reliability of single piles and pile groups by load tests

Jinsong Huang^{1,4}, Richard Kelly^{1,2}, Dianqin Li³, Chuangbing Zhou⁴, Scott Sloan¹

¹ARC Centre of Excellence for Geotechnical Science and Engineering, The University of Newcastle, Callaghan, NSW 2308, Australia

²Coffey, Warabrook, NSW 2308, Australia

³School of Water Resources and Hydropower Engineering, Wuhan University, Wuhan, 430072, China

⁴School of Civil Engineering and Architecture, Nanchang University, Nanchang, 330031, China

Abstract: Pile load tests are used to refine designs and for quality assurance. They can also be used to verify the reliability of piles and pile groups. Stochastic methods have previously been developed to verify the reliability of single piles. A general stochastic method to verify the reliability of pile groups is developed in this paper. The method can be used to assess the reliability of groups where pile tests have been conducted to the ultimate capacity, to below the ultimate capacity but exceeding specified capacity, and where pile tests fail to achieve the specified capacity. In the latter case, the method allows decisions to be made as to whether the reliability of the entire pile group is satisfactory or whether additional piles need to be installed.

Keywords: system reliability; pile group; load test; Bayesian updating

1 Introduction

There are many uncertainties in the design, construction and testing of piles and pile groups (e.g., Li et al. 2013). Geotechnical material parameters are subject to measurement errors and

spatial variability across a site, and are usually assumed based on a small sample of the total ground volume. Analytical or numerical models used for design will thus be a simplification of a complex physical reality. Moreover, construction processes will vary from pile to pile and affect the variability of the ultimate pile capacity. These uncertainties are often managed by performing load tests on a small sample of piles from the total population of constructed piles, or by adopting conservative factors of safety in design when pile tests are not performed. Results from load tests can provide a better estimate of the ultimate pile capacity and also greatly reduce the uncertainty of the ultimate pile capacity, since the error associated with load test measurements is, in principle, much smaller than that associated with predictions depending on the skill of the tester/interpreter.

Although direct load tests can provide a wealth of information for the design and construction of pile foundations and are the most accurate method of determining pile capacity, the pile load test itself involves some degree of uncertainty too. For example, although static load tests are believed to be more accurate than dynamic load tests, there are still uncertainties associated with the interpretation of load-displacement relationships (e.g., Paikowsky 2004).

Nevertheless, confirmation of pile-soil capacity through static load testing is considerably more reliable than capacity estimates from static capacity analyses and dynamic formulas Poulos and Davis 1980. By using direct load tests as part of the design process, an improved knowledge of pile-soil behaviour is obtained that may allow a reduction in pile lengths or an increase in the pile design load, either of which may result in potential savings in foundation costs. With the improved knowledge of pile-soil behaviour, a lower factor of safety (FS) may be used on the pile design load. For example, the FS can be reduced from 3.50 for static analyses based on the standard penetration test (SPT) or the cone penetration test (CPT) to

2.00 for the same analyses that are verified with a sufficient number of static loading tests (ASSHTO, 1998). If reliability-based design is adopted, the resistance factors can be increased (e.g., Poulos 2004, Zhang et al. 2001; Paikowsky 2004). The Australian Standard for Piling-Design and Installation (1995) provides ranges of resistance factors. When static load tests are conducted, resistance factors (0.7 to 0.9) can be used and applied to the maximum load (proof test) or measured capacity. When only static pile analyses are conducted, small resistance factors (0.40–0.65) should be used relating to the source of soil parameters and soil type (SPT in cohesionless soils). Detailed recommendations are provided for resistance factors to be used with the dynamic methods ranging from 0.45 to 0.65 for methods without dynamic measurements, to between 0.50 to 0.85 when utilizing dynamic measurements with signal matching analysis. In the case of the Australian Standard, these resistance factors were generated based on the judgement of an expert panel rather than through statistical calibration.

Calculations of pile and pile group reliability based on load test results have been reported by Kay (1978), Baecher and Rackwitz (1982), Zhang et al. (2001), Zhang (2004), Zhang et al. (2005), Zhang et al. (2006), Zhang et al. (2010) and Zhang et al. (2014). There are two approaches, namely frequentist and Bayesian statistics. Liang and Yang (2006) evaluated the confidence interval of the population mean of pile capacities. It was shown that the required measured capacity decreases with an increase in the number of load tests to achieve the same level of reliability. Most of the work published to date adopts Bayesian methods to update the reliability of single piles. Zhang (2004) developed methods to incorporate the results of proof load tests not conducted to failure into the design of pile foundation. Similar approaches have been applied to steel grid reinforced soil walls by Bathurst et al. (2011). Najjar and Gilbert (2009) assumed that the capacities of individual piles follow a mixed lognormal distribution.

Instead of treating the mean capacity as a random variable, it was assumed that the lower bound of the mixed lognormal distribution is a random variable. It was shown that the lower-bound capacity can cause a significant increase in the calculated reliability for a geotechnical design. The uncertainties in the measured proof load have also been studied. The reliability of large pile groups has been shown to be greater than the reliability of individual piles due to redundancy in the system (e.g., Huang et al. 2015; Zhang et al. 2001). Zhang et al. (2001) used single pile analyses to assess group reliability by adopting a redundancy factor. Paikowsky (2004) suggested that the target reliability for pile group can be reduced to 2.0-2.5 compared to 3.0 for single piles. Pile groups may be able to support the design load when one or more piles are defective (e.g., Poulos 1997). If the reliability of the pile group with defective piles can be quantified, it may be possible to use the defective pile rather than install a replacement pile and increase the size of the pile cap.

Probabilistic methods have not been developed to assess the reliability of pile groups as a function of individual pile test results. Although several studies on the system reliability of pile groups are available (e.g., Zhang et al. 2001), a systematic framework for updating the reliability of pile groups has not been developed. In this study we propose a rigorous framework for updating the reliability of single piles and pile groups. It is assumed that load tests are subject to uncertainty. The proposed framework can be used to assess the reliability of single piles and pile groups subject to general loadings and stratigraphic conditions. The method can be used to assess the reliability of groups where pile tests have been conducted to the ultimate capacity, to below the ultimate capacity but exceeding specified capacity, and where pile tests fail to achieve the specified capacity (or are unknown). In the latter case, the method allows decisions to be made as to whether the reliability of the entire pile group is satisfactory or whether additional piles need to be installed. However, as the purpose of this

paper is to outline a general stochastic methodology, a number of simplifying assumptions have been made. These are:

- 1) The piles in the group are geometrically identical, loaded equally in the vertical direction only and are perfectly plastic.
- 2) Uncertainties relating to design methodology, site characterization, soil properties and construction quality have been quantified or assessed and form prior knowledge.
- 3) The reliability for the piles has been specified and a proper design method has been chosen. A probabilistic assessment of pile capacity has been performed.
- 4) Load tests have been conducted. This could be for design and the load tests are used to refine the design, or at the stage of construction where the load tests are used to make sure the design have a sufficient level of reliability.
- 5) The accuracies of the load test methods, including result interpretations, have been assessed or are known.
- 6) The uncertainties in external loads are not considered, but the proposed methods can be extended to include the uncertainty of loads in a straight forward manner.

The first part of this paper summarises Bayesian updating and the adopted method for performing the numerical computations. The second part of the paper discusses methods adopted in previous studies for the purpose of contrasting these with our proposed framework. The previous studies aimed at updating the mean or lower bound capacity of single piles in a population (as opposed to the capacity of single piles). The third part of the paper presents our new method and example calculations. We show that the uncertainty of the individual pile capacity is reduced by load tests.

2 Bayesian Updating and Monte Carlo Simulations

The design of pile foundation starts usually with limited site investigations and tests. Due to time and budget constraints, designs are usually done by simple methods such as Meyerhof's method Meyerhof 1976 and dynamic methods (wave equation analysis). This information forms prior knowledge for the ultimate pile capacity. Pile tests during construction can then be used to update the prior knowledge. For important and large projects, load tests are also commonly used to confirm that the foundation has enough capacity after the pile foundation has been constructed. From a statistical point of view, to refine or validate design by load tests is very similar to a Bayesian statistical process.

In Bayesian statistics, site investigation information, laboratory test data, *in situ* test data and load tests results are used as evidence to update prior knowledge, predictions and designs. The posterior distribution of the parameters is calculated based on Bayes' theorem:

$$P(\theta | D) = \frac{P(D | \theta)P(\theta)}{P(D)} \quad (1)$$

where θ denotes the parameters to be updated, D the data collected, $P(\theta)$ the prior probability distribution of the parameters, $P(D | \theta)$ the likelihood probability distribution and $P(D)$ the probability distribution of the evidence.

Bayesian statistical methods can be used in a recursive manner. For example, engineering experience and judgement can be incorporated in the prior probability distribution of important engineering parameters. When no such information or strong belief is available, an uninformative prior distribution can be assumed (a uniform distribution). An initial design is performed based on the updated parameters. When load test results become available, the parameters are updated again and the initial design can then be revised. The uncertainties

associated with the updated predictions and designs are usually much smaller than the prior predictions and designs.

In most cases, the posterior distribution cannot be obtained analytically. The Markov Chain Monte Carlo (MCMC) method is commonly used to sample the posterior distribution. There are many variants of MCMC method, but the one proposed by Haario et al. (2006) is adopted in this study. The idea behind the MCMC method is very simple. A Markov chain is formulated so that its invariant distribution is the posterior distribution. The chain is also formulated so that it is irreducible and aperiodic to guarantee convergence. One advantage of the MCMC method is that the normalizing constant in Bayes' formula for the posterior distribution is not required. The basic idea goes back to Metropolis et al. (1953) and was extended by Hastings (1970). In the traditional Metropolis-Hastings algorithm, a large number of samples are rejected if they do not satisfy the acceptance criterion. Because of the evaluation of a complex numerical model in every iteration, this becomes a significant computational challenge for large-scale numerical modelling problems, especially for high dimensional cases because the acceptance rate decreases significantly. The delayed rejection adaptive Metropolis (DRAM) algorithm is chosen in this study because it can achieve a nearly constant acceptance rate which is independent of the dimension. It is based on the combination of the adaptive Metropolis algorithm (Haario et al. 2001) and the delayed rejection algorithm (Tierney and Mira 1999). The adaptive Metropolis algorithms modify the proposed distribution using the information gained from previous steps. Haario et al. (2006) considered a multivariate normal proposal with a mean given by the current value and the covariance matrix C_i . The covariance C_i has a fixed value C_0 for the first i_0 iterations and is updated subsequently as

$$C_i = \begin{cases} C_0 & i \leq i_0 \\ s_d \text{cov}(\theta_0, \dots, \theta_{i-1}) + s_d \varepsilon \mathbf{I}_d & i > i_0 \end{cases} \quad (2)$$

where $\text{cov}(\theta_0, \dots, \theta_{i-1})$ is the covariance of previous samples, ε is a small parameter chosen to ensure C_i does not become singular, $\mathbf{I}_d$ denotes the d -dimensional identity matrix and s_d is a scaling parameter, which depends on the dimensionality, to ensure reasonable acceptance rates of the proposed states. As a basic guideline, Haario et al. (2006) suggest choosing s_d for a model of a given dimension as $2.42/d$. The initial covariance C_0 is taken as the covariance of prior distribution in this study.

In the delayed rejection algorithm (Tierney and Mira 1999), the proposed sample $\theta_i^{(1)}$ is accepted as in Hastings (1970) with a probability

$$\alpha_1(\theta_{i-1}, \theta_i^{(1)}) = \min \left\{ 1, \frac{P(\theta_i^{(1)} | D) q_1(\theta_i^{(1)}, \theta_{i-1})}{P(\theta_{i-1} | D) q_1(\theta_{i-1}, \theta_i^{(1)})} \right\} \quad (3)$$

where $P(\theta | D)$ is the target distribution, q_1 is the first proposal distribution, and θ_{i-1} is the current sample.

If $\theta_i^{(1)}$ is rejected, a second proposal $\theta_i^{(2)}$ is drawn and accepted with probability

$$\alpha_2(\theta_{i-1}, \theta_i^{(2)}) = \min \left\{ 1, \frac{P(\theta_i^{(2)} | D) q_1(\theta_i^{(2)}, \theta_i^{(1)}) q_2(\theta_i^{(2)}, \theta_i^{(1)}, \theta_{i-1}) [1 - \alpha_1(\theta_i^{(2)}, \theta_i^{(1)})]}{P(\theta_{i-1} | D) q_1(\theta_{i-1}, \theta_i^{(1)}) q_2(\theta_{i-1}, \theta_i^{(1)}, \theta_i^{(2)}) [1 - \alpha_1(\theta_{i-1}, \theta_i^{(1)})]} \right\} \quad (4)$$

where q_2 is the second proposal distribution, and α_1 is calculated according to Eq. (3).

If $\theta_i^{(2)}$ is rejected, either a third proposal is invoked or the sample remains at the current position.

3 Assessment of previous methods for updating single pile capacity

Kay (1978) has developed methods for updating the mean capacity of piles. The pile capacity (y) in the population was assumed to be normally distributed with mean μ and standard deviation σ . The standard deviation of the population (σ) was assumed to be known. Kay (1978) called this the “within site variability”. It was further assumed that the mean pile capacity μ is uncertain, and is initially estimated using the prior probability distribution expressed by

$$f'(\mu) \sim N(\mu_0, \sigma_0) \quad (5)$$

where $N()$ represents the Normal distribution, and μ_0 and σ_0 are the mean and standard deviation of μ (mean of mean pile capacity, standard deviation of mean pile capacity). The parameters μ_0 and σ_0 were assumed to be associated with prediction methods (a static or dynamic method).

Suppose n pile capacity tests have been performed and the results are denoted as the vector $\hat{\mathbf{y}}$. The posterior distribution of μ can be obtained by Bayes' rule:

$$f''(\mu | \hat{\mathbf{y}}) = \frac{P(\hat{\mathbf{y}} | \mu) f'(\mu)}{P(\hat{\mathbf{y}})} \quad (6)$$

$$\propto \exp \left(- \frac{\left(\mu - \frac{n\sigma_0^2 \bar{y} + \sigma^2 \mu_0}{n\sigma_0^2 + \sigma^2} \right)^2}{2 \left(\frac{\sigma_0^2 \sigma^2}{n\sigma_0^2 + \sigma^2} \right)} \right)$$

where $\bar{y}$ is the average value of $\hat{\mathbf{y}}$.

A detailed derivation of Eq. (6) can be found in Appendix I.

The posterior mean and standard deviation of μ are

$$\mu_1 = \frac{n\sigma_0^2\bar{y} + \sigma^2\mu_0}{n\sigma_0^2 + \sigma^2} \quad (7)$$

and

$$\sigma_1 = \frac{\sigma_0^2\sigma^2}{n\sigma_0^2 + \sigma^2} \quad (8)$$

Because μ is not known exactly, and all of our knowledge about it is contained in the posterior distribution $f''(\mu|\hat{\mathbf{y}})$, we must integrate Eq. (6) to obtain the posterior distribution of the pile capacity (y)

$$\begin{aligned} p(y|\hat{\mathbf{y}}) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(\mu-\mu_1)^2}{2\sigma_1^2}\right) d\mu \\ &= \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2 + \sigma_1^2}} \exp\left(-\frac{(y-\mu_1)^2}{2(\sigma^2 + \sigma_1^2)}\right) \end{aligned} \quad (9)$$

The detailed derivation of Eq. (9) (e.g., Tang 1971) can be found in Appendix II. Equations. (6) and (9) are close-form solutions and are therefore computationally efficient.

The updated mean pile capacity is μ_1 , which is the same as the posterior mean of μ . The uncertainty of the mean pile capacity has been reduced (i.e., $\sigma_1 < \sigma_0$). The updated predictive standard deviation of the individual pile capacity is $\sqrt{\sigma^2 + \sigma_1^2}$, which is also reduced compared to the prior predictive value $\sqrt{\sigma^2 + \sigma_0^2}$. Therefore, the uncertainty of individual pile capacity is reduced after we have conducted pile capacity tests. It is noted however, that both the prior and posterior predictive standard deviations are larger than σ . This is because the predictive uncertainty includes the uncertainty on the mean pile capacity (

220 σ_1 or σ_0) and the original uncertainty on the individual pile capacity (σ) (e.g., Cowles
221 2013). In the limit where all piles in the population are tested and n is large, σ_1 approaches
222 zero and the posterior predictive standard deviation becomes equal to the population standard
223 deviation σ .

224 The Normal distribution (conjugate distribution) was used in this analysis and the population
225 variance (σ) was assumed to be known. Baecher and Rackwitz (1982) assumed that there are
226 uncertainties in both the mean and standard deviation of pile capacity, i.e., within site
227 variability σ changes from site to site. The prior distribution of μ and σ was taken as the
228 Normal-Inverted Gamma distribution. Using these assumptions, the updated uncertainty of
229 individual pile capacity will be greater than calculated by Kay (1978) because additional
230 uncertainty on σ is included. Zhang (2004) used the same approach as Kay (1978) to
231 calibrate resistance factors under the framework of reliability based design.

232 **4 Updating the capacity of single piles**

233 Instead of looking at the uncertainty of mean pile capacity, we focus on the uncertainty of
234 individual pile capacity. The population pile capacity (y) is assumed to be lognormally
235 distributed Zhang 2004 with prior mean μ and standard deviation σ . It is assumed that a
236 probabilistic design has been conducted and prior estimates of μ and σ have been made.
237 After some tests on pile capacity, μ and σ are again updated. In general, suppose n piles are
238 tested up to a maximum load T (proof test) and m piles have failed to achieve T with known
239 capacities $\hat{y}_i$, $i = 1, \dots, m$.

240 The likelihood distribution for the m failed tests is

241

$$L \propto \exp \left(- \frac{\sum_{i=1}^m (\hat{y}_i - y - \mu_{err})^2}{2\sigma_{err}^2} \right) \quad (10)$$

242

where μ_{err} and σ_{err} are mean and standard deviation of test error. This test error is related to

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the type of test (i.e., static or dynamic) and the method of interpretation (the Davisson

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method, Davisson 1972). For this method the test errors are assumed to be known and must

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be input by the analyst. There is little guidance in the literature about what magnitude of test

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errors to specify. It should be mentioned, however, that the measured capacities are of little

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value if the analyst has not assessed the accuracy of the tests. The selection of test errors

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requires a degree of subjectivity as they depend on factors such as the skill of the tester and

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interpretation of test results.

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The likelihood distribution for the $n - m$ tests that have passed is

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$$L \propto \left(\Phi \left(\frac{\mu_{err} - (y - T)}{\sigma_{err}} \right) \right)^{n-m} \quad (11)$$

252

where Φ is the cumulative distribution function of the standard normal distribution.

253

The posterior distribution of the pile capacity (y) is then

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$$f''(y | \hat{\mathbf{y}}) \propto \exp \left(- \frac{\sum_{i=1}^m (\hat{y}_i - y - \mu_{err})^2}{2\sigma_{err}^2} \right) \left(\Phi \left(\frac{\mu_{err} - (y - T)}{\sigma_{err}} \right) \right)^{n-m} \frac{1}{y} \exp \left(- \frac{(\ln y - \mu_{\ln y})^2}{2\sigma_{\ln y}^2} \right) \quad (12)$$

255

where $\sigma_{\ln y} = \sqrt{\ln \left(1 + \left(\frac{\sigma}{\mu} \right)^2 \right)}$ and $\mu_{\ln y} = \ln \mu - \frac{1}{2} \sigma_{\ln y}^2$.

256 If the capacities of failed piles are not known, such as in the case an untested damaged pile,
 257 the posterior distribution is

$$258 \quad f''(y|\hat{\mathbf{y}}) \propto \left(1 - \Phi\left(\frac{\mu_{err} - (y - T)}{\sigma_{err}}\right)\right)^m \left(\Phi\left(\frac{\mu_{err} - (y - T)}{\sigma_{err}}\right)\right)^{n-m} \frac{1}{y} \exp\left(-\frac{(\ln y - \mu_{\ln y})^2}{2\sigma_{\ln y}^2}\right) \quad (13)$$

259 By directly sampling the posterior distributions Eq. (12) and (13) using the MCMC method,
 260 the mean and standard deviation of pile capacity can be updated.

261 **5 Examples of updating capacity of single pile**

262 *5.1 Updating mean pile capacity using Kay 1978*

263 The pile capacity (y) is assumed to be normally distributed with a mean μ and standard
 264 deviation σ . Following Kay (1978), it is assumed that σ is known ($\sigma = 0.2$) and we are
 265 uncertain about μ . We have some knowledge about μ which is described by the prior
 266 distribution on μ . Let us assume the prior mean and standard deviation of μ are
 267 $\mu_0 = 1.3$ and $\sigma_0 = 0.5$ and suppose five load tests have been conducted. The mean capacity of
 268 tested piles is $\bar{y} = 0.8$. According to Eqs.(7) and (8), the posterior mean and standard
 269 deviation of μ are

$$\begin{aligned} \mu_1 &= \frac{n\sigma_0^2\bar{y} + \sigma^2\mu_0}{n\sigma_0^2 + \sigma^2} \\ 270 \quad &= \frac{5 \times 0.5^2 \times 0.8 + 0.2^2 \times 1.3}{5 \times 0.5^2 + 0.2^2} \\ &= 0.82 \end{aligned}$$

271 and

$$\begin{aligned}\sigma_1 &= \frac{\sigma_0 \sigma}{\sqrt{n\sigma_0^2 + \sigma^2}} \\ &= \frac{0.5 \times 0.2}{\sqrt{5 \times 0.5^2 + 0.2^2}} \\ &= 0.09\end{aligned}$$

272

273 The uncertainty of the mean capacity (μ) has been reduced significantly from $\sigma_0 = 0.5$ to

274 $\sigma_1 = 0.09$ by the five load tests as shown in Fig. 1.

275

276 Fig. 1 Prior and posterior distributions of mean pile capacity (μ)

277 The prior and posterior predictive distributions of the individual pile capacity (y) are shown

278 in Fig. 2. The posterior predictive standard deviation of the individual pile capacities (y) is

279 $\sqrt{\sigma^2 + \sigma_1^2} = 0.22$, which is much smaller than the prior predictive value $\sqrt{\sigma^2 + \sigma_0^2} = 0.54$. It

280 should be noted that however, both the prior and posterior predictive standard deviations are

281 larger than $\sigma = 0.2$, namely the standard deviation of pile capacity. This is because the

uncertainty (σ_1 or σ_0) in the mean capacity (μ) has to be included in the uncertainty of the pile capacity (y).

Fig. 2 Prior and posterior distributions of pile capacity (y)

5.2 Updating pile capacity (unknown capacity of tested piles) using the new approach

The pile capacity (y) is assumed to be lognormally distributed with mean $\mu = 1.3$ and standard deviation $\sigma = 0.65$. Suppose n piles are tested up to a load $T = 1.0$. We assume that the mean and standard deviation of the test errors are $\mu_{err} = 0.0$ and $\sigma_{err} = 0.2$, and that the ultimate capacities of the tested piles are unknown. The posterior mean and standard deviation of pile capacity are obtained using the MCMC method and are shown in Fig. 3 and Fig. 4.. It can be seen from Fig. 3 that the posterior mean capacity increases as the number of positive tests increases. It can be seen from Fig. 4 that the posterior coefficient of variation reaches its minimum when there are both defective and effective piles, as observed by Zhang (2004). It should be noted that the updated standard deviations of individual pile capacity in Fig. 4 are all less than 0.65, namely the standard deviation of pile capacity.

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Fig. 3 Posterior mean pile capacity

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Fig. 4 Posterior standard deviation of pile capacity

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Updating pile capacity (known capacity of tested piles)

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The capacities of failed piles are usually known to a degree of accuracy, and this information

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can be further utilized to reduce uncertainties. In this case, Eq. (12) is applicable. Suppose 3

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out of 4 tested piles failed. The capacities of the three failed piles are 0.3, 0.5 and 0.8. The

posterior mean and standard deviation of pile capacity are shown in Table 1. The posterior standard deviation is much lower than the prior standard deviation.

Table 1 Posterior mean and standard deviation of pile capacity (known capacity of tested piles)

Distribution	Capacity of pile is lognormally distributed
Total number of tests	4
Number of failed tests	3 Capacities of failed piles are 0.3, 0.5 and 0.8.
Prior mean of pile capacity	1.3
Prior standard deviation	0.65
Mean test error	0
Test load	1.0
Standard deviation of test error	0.2
Posterior mean of individual pile capacity	0.67
Posterior standard deviation	0.09

6 Reliability of pile groups

Suppose a group has N piles. The capacity of individual piles (y) is assumed to be a lognomally distributed random variable with mean μ and standard deviation σ . The correlation coefficient between the capacities of piles is ρ . The joint probability density function of N piles is

$$f_y(y_1, \dots, y_N) = \frac{1}{\sqrt{(2\pi)^N |C|} \prod_{i=1}^N y_i} \exp\left(-\frac{1}{2}(\ln y_i - \mu_{\ln y})^T C^{-1}(\ln y_i - \mu_{\ln y})\right) \quad (14)$$

where C is an N by N covariance matrix with diagonal terms equal to $\sigma_{\ln y}^2$ and off-diagonal terms equal to $\rho\sigma_{\ln y}^2$.

Fig. 5 shows a group of nine piles connected by a rigid cap. If the cap is not rigid and/or the pile system is subjected to a general loading condition, the piles will not be identically loaded. A rigorous analysis for the pile group may be performed using software capable of assessing the performance of the group under general loading. However, we restrict our analysis to a vertically loaded group of piles identically constructed. We have assumed that the cap is rigid so that load can be redistributed if defective piles are present.

Fig. 5 Group of piles (shaded pile is tested)

Using a coefficient of group efficiency, the capacity of the pile group is assumed to be

$$q = \eta \sum_{i=1}^N y_i \quad (15)$$

where q is the capacity of the pile group and η is the coefficient of group efficiency.

If the applied load is obtained by the mean capacity divided by Factor of Safety (FS) as $N\mu / FS$, the limit state function for the capacity of the pile group can be defined as:

$$g = \eta \sum_{i=1}^N y_i - \frac{N\mu}{FS} \quad (16)$$

Suppose there is a pile group with the parameters shown in Table 2. Before the cap is constructed, some load tests will be conducted on single piles and the results are used to update the reliability of the pile system.

For a factor of safety equal to two, the probability of failure of a single pile is

$$p_f = P[y < 0.5] = \Phi\left(\frac{\ln 0.5 - \mu_{\ln y}}{\sigma_{\ln y}}\right) = 0.109 \quad (17)$$

The probability of failure of the pile system cannot be obtained analytically. It is not possible to use simple probabilistic methods such as the First Order Reliability Method (FORM), because the joint posterior distribution cannot be sampled easily. In this study, the MCMC method is used to sample the joint posterior distribution. After the capacities of individual piles are sampled, the performance function Eq. (16) is used to evaluate the safety of the pile system. If $g \leq 0$, the pile system failed. Eq. (16) can be replaced by an appropriate numerical model (e.g., finite element model) if one wants to consider more general loading conditions, or the failure of cap has to be considered. Eq.(16) allows for a quick assessment of the system reliability of pile group, which may be desirable over 3D FEM due to time constraints. The results from the Monte Carlo simulations (MCS) are shown in Fig. 6. In Fig. 6, the system redundancy factor is defined as the ratio of the probability of single pile failure to the probability of system failure. It can be seen that the probability of failure of the pile group increases as the correlation between the capacities of individual piles increases. When there is no correlation between the capacities of individual piles, weak piles may be compensated by strong piles. The reduction in the probability of failure could be very significant if a large

358 number of piles are used. This example will serve as a benchmark for the next section where
 359 the probability of failure is updated by load tests.

360 Table 2 Parameters for reliability analysis of pile group

Parameters	Values
Number of piles (N)	9
Mean pile capacity (μ)	1.0
Standard deviation of pile capacity (σ)	0.5
Correlation coefficient (ρ)	0.5
Coefficient of group efficiency (η)	0.9
Factor of safety (FS)	2

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Fig. 6 Probability of failure

7 Reliability of pile groups updated by load tests

Suppose a group has N piles. To be general, suppose n out of N piles are tested up to load T , m piles have failed to achieve T with known capacities $\hat{y}_i$, $i = 1, \dots, m$. The posterior distribution of the pile capacity $(y_1, \dots, y_N)$ is

$$f_y''(y_1, \dots, y_N | \hat{\mathbf{y}}) \propto \exp\left(-\frac{\sum_{i=1}^m (\hat{y}_i - y_i - \mu_{err})^2}{2\sigma_{err}^2}\right) \prod_{i=m+1}^n \Phi\left(\frac{\mu_{err} - (y_i - T)}{\sigma_{err}}\right) \frac{1}{\prod_{i=1}^N y_i} \exp\left(-\frac{1}{2}(\ln y_i - \mu_{\ln y})^T C^{-1}(\ln y_i - \mu_{\ln y})\right) \quad (18)$$

If the pile group consists of only a single pile then Eq. (18) reduces to Eq. (12).

If the capacities of failed piles are not known, such as in the case of untested damaged piles, the posterior distribution is

$$f_y''(y_1, \dots, y_N | \bar{y}) \propto \prod_{i=1}^m \left(1 - \Phi \left(\frac{\mu_{err} - (y_i - T)}{\sigma_{err}} \right) \right) \prod_{i=m+1}^n \Phi \left(\frac{\mu_{err} - (y_i - T)}{\sigma_{err}} \right) \frac{1}{\prod_{i=1}^N y_i} \exp \left(-\frac{1}{2} (\ln y_i - \mu_{\ln y})^T C^{-1} (\ln y_i - \mu_{\ln y}) \right)$$

(19)

7.1 Unknown pile capacity

The pile group presented in the previous section is used again to illustrate how the probability of failure is updated by load tests. The group has 9 piles with the parameters shown in Table 2. It is assumed that the load tests are conducted to mean capacity, i.e., $T = \mu = 1.0$. It is further assumed that $\mu_{err} = 0.0$ and $\sigma_{err} = 0.1$. The capacities of piles that failed to achieve T are assumed to be unknown and, in this case, Eq. (19) is applicable. The updated p_f of the group will depend on the number of tests and the number of tests that failed to achieve T . The MCMC method was used and one million simulations were conducted. The results are shown in Figs 7-10. When the capacities of individual piles are assumed to be uncorrelated, the p_f of the group is shown in Fig. 10. Comparing Fig. 7 and Fig. 10, it can be seen that the p_f of the pile group is significantly higher if the capacities of piles are positively correlated. When the capacities of individual piles are positively correlated ($\rho = 0.5$), p_f becomes higher than 0.0687 (the prior p_f) only when all tests have failed regardless of the number of tests. When all tested piles failed, the p_f of the pile group increases as the number of tests increases. In the most extreme case in Fig. 7 where all 9 piles are tested and failed, the p_f is 0.426 when $\rho = 0.5$. This is because the test load is equal to the mean capacity of individual piles ($T = \mu = 1.0$), which means the total capacity of the group is probably in between 0 and 9.0 when all 9 piles failed. Because $FS = 2.0$, the design load on the group is 4.5. If the

capacities of individual piles are fully correlated, the overall capacity is essentially a random variable with mean of 4.5. The probability that the overall capacity is less than the design load 4.5 (p_f) will be around 0.5. If the coefficient of correlation between individual pile capacities is 0.5, a smaller p_f (0.426 in Fig. 7) will be obtained.

The results in Fig. 7 and Fig. 10 are useful for assessing the reliability of pile groups with defective piles. There are usually two options when there are defective piles, one is to replace the piles and another is to do more tests. For the group shown in Table 2 and $\rho = 0.5$, suppose the target reliability index is at least 3 ($p_f < 0.0014$). If the first load test failed, according to Fig. 7, $p_f = 0.1162$, which is unacceptable. If two more tests are conducted and both pass, p_f is reduced to $2.77e-5$. Two more tests performed immediately after finding the first test failed, to avoid remobilising a pile driving rig, may be less expensive than replacing the first defective pile.

Fig. 7 p_f of pile group updated by load tests (capacity unknown, $\rho = 0.5$)

Fig. 8 Updated mean capacity of defective piles (capacity unknown, $\rho = 0.5$)

Fig. 9 Updated mean capacity of effective piles (capacity unknown, $\rho = 0.5$)

Fig. 10 Updated mean capacity of untested piles (capacity unknown, $\rho = 0.5$)

6.2 Known pile capacity

The capacities of the failed piles are usually known to a degree of accuracy. This information can be further utilized to reduce uncertainties. In this case, Eq. (18) is applicable. The updated p_f of a group will depend on the number of tests, the number of failed piles and the capacities of the failed piles.

Suppose 5 out of 9 piles were tested and 3 piles failed. The capacities of the three failed piles are 0.3, 0.5 and 0.8. The updated p_f of the group is 2.36e-4. The posterior mean of the individual piles are shown in Table 3. For the case where the capacities of the defective piles are not known, the posterior mean of the individual piles are shown in Table 4. Comparing Table 3 and Table 4, the uncertainties of the capacities of all piles are reduced if the capacities of the defective piles are known.

Table 3 Updated capacity of individual piles (capacity known, $\rho = 0.5$)

Pile	1	2	3	4	5	6	7	8	9
Tested capacity	0.3	0.5	0.8	passed		untested			
Posterior mean	0.42	0.55	0.79	1.20		0.83			
Posterior standard deviation	0.08	0.09	0.10	0.25		0.31			

Table 4 Updated capacity of individual piles (capacity unknown, $\rho = 0.5$)

Pile	1	2	3	4	5	6	7	8	9
Tested capacity	unknown			passed		untested			
Posterior mean	0.75			1.28		0.97			
Posterior standard deviation	0.17			0.31		0.37			

If there is no correlation between the capacities of individual piles, the updated probability of failure of the group is less than $1e-6$. The posterior mean and standard deviation of the individual piles are shown in Table 5. Because the capacities of the individual piles are not correlated, the uncertainty of the untested piles is not reduced.

Table 5 Updated capacity of individual piles ($\rho = 0.0$)

Pile	1	2	3	4	5	6	7	8	9
Tested capacity	0.3	0.5	0.8	passed		untested			
Posterior mean	0.38	0.53	0.80	1.44		1.00			
Posterior standard deviation	0.08	0.09	0.10	0.46		0.50			

When the capacities of the defective piles are known, for the pile that failed at load 0.3, the posterior samples generated by the MCMC method are shown in Fig. 11. One million simulations were conducted but only the first 10,000 simulations are shown in Fig. 11. The acceptance is 44.2% which is close to the optimal acceptance rate Gelman et al. 1996. It can be seen from Fig. 11 that the initial one thousand samples should not be used (known as the “burn-in”). The corresponding posterior distributions of the individual pile capacity are shown in Fig. 12.

Fig. 11 Posterior samples of pile failed at load 0.3 ($\rho = 0.5$)

Fig. 12 Posterior distributions of pile capacities (capacity known, $\rho = 0.5$)

7 Concluding remarks

Current methods update the parameters of the distribution of pile capacity by load tests. Instead of treating the mean or lower bound pile capacity as a random variable, this study treats the individual pile capacity as a random variable. The reliability of single piles and pile groups, updated by load tests, is assessed by Bayesian updating and MCMC. It is shown that the system reliability of a pile group is significantly smaller if the individual capacities are positively correlated rather than uncorrelated. The method presented in this paper has the following features

- 1) The uncertainty of an individual pile capacity is reduced by load tests.
- 2) The accuracy of a test method can be taken into account consistently.
- 3) The system reliability of a pile group can be directly updated by load tests.
- 4) The method works for both unknown and known capacities of tested piles.

The method presented in this paper can be used to both refine and validate the initial design of a pile group by load tests. In the case where load tests on individual piles are not favourable, additional load tests may be conducted. If these results are favourable, the reliability of the pile group may be sufficient even when defective piles are found.

The proposed method requires the knowledge of the mean and standard deviation of test errors, which is not available in the literature. Further research is needed to address this issue.

For the purpose of demonstration, all calculations in this paper have been limited to vertically loaded identical piles or pile groups so that an analytical performance function can be used. It should be noted, however, that the proposed framework is applicable to more general cases. The only extra work is to replace the analytical performance function by a suitable method (e.g. the finite element method) for assessing performance or safety.

$$\begin{aligned}
f''(\mu|\hat{\mathbf{y}}) &= \frac{P(\hat{\mathbf{y}}|\mu)f'(\mu)}{P(\hat{\mathbf{y}})} \\
&\propto \exp\left(-\sum_{i=1}^n \frac{(y_i - \mu)^2}{2\sigma^2}\right) \exp\left(-\frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\sum_{i=1}^n \frac{y_i^2 - 2\mu y_i + \mu^2}{2\sigma^2}\right) \exp\left(-\frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\sum_{i=1}^n \frac{-2\mu y_i + \mu^2}{2\sigma^2}\right) \exp\left(-\frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{-2n\bar{y}\mu + n\mu^2}{2\sigma^2}\right) \exp\left(-\frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{n(\bar{y} - \mu)^2 - n\bar{y}^2}{2\sigma^2}\right) \exp\left(-\frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{n(\bar{y} - \mu)^2}{2\sigma^2}\right) \exp\left(-\frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{n(\bar{y} - \mu)^2}{2\sigma^2} - \frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{n(\bar{y} - \mu)^2}{2\sigma^2} - \frac{(\mu - \mu_0)^2}{2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{\mu^2(n\sigma_0^2 + \sigma^2) - 2(n\sigma_0^2\bar{y} + \sigma^2\mu_0)\mu}{2\sigma^2\sigma_0^2}\right) \\
&\propto \exp\left(-\frac{\left(\mu - \frac{n\sigma_0^2\bar{y} + \sigma^2\mu_0}{n\sigma_0^2 + \sigma^2}\right)^2}{2\left(\frac{\sigma_0^2\sigma^2}{n\sigma_0^2 + \sigma^2}\right)}\right)
\end{aligned}$$

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$$\begin{aligned}
p(y|\hat{\mathbf{y}}) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(\mu-\mu_1)^2}{2\sigma_1^2}\right) d\mu \\
&= \frac{1}{\sqrt{2\pi}\sigma} \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{y^2}{2\sigma^2} - \frac{\mu_1^2}{2\sigma_1^2}\right) \int_{-\infty}^{\infty} \exp\left(-\frac{\mu^2}{2}\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right) + \mu\left(\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}\right)\right) d\mu
\end{aligned}$$

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$$\begin{aligned}
&\int_{-\infty}^{\infty} \exp\left(-\frac{\mu^2}{2}\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right) + \mu\left(\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}\right)\right) d\mu \\
&= \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right) \left(\mu^2 - 2\mu \frac{\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}}\right)\right) d\mu \\
&= \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right) \left(\mu - \frac{\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}}\right)^2 \exp\left(\frac{\left(\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}\right)^2}{2\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right)}\right)\right) d\mu \\
&= \frac{\sqrt{2\pi}}{\sqrt{\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}}} \exp\left(\frac{\left(\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}\right)^2}{2\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right)}\right)
\end{aligned}$$

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$$\begin{aligned}
p(y|\hat{y}) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(\mu-\mu_1)^2}{2\sigma_1^2}\right) d\mu \\
&= \frac{1}{\sqrt{2\pi}\sigma} \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{y^2}{2\sigma^2} - \frac{\mu_1^2}{2\sigma_1^2}\right) \frac{\sqrt{2\pi}}{\sqrt{\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}}} \exp\left(\frac{\left(\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}\right)^2}{2\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right)}\right) \\
&= \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2 + \sigma_1^2}} \exp\left(-\frac{y^2}{2\sigma^2} - \frac{\mu_1^2}{2\sigma_1^2} + \frac{\left(\frac{y}{\sigma^2} + \frac{\mu_1}{\sigma_1^2}\right)^2}{2\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_1^2}\right)}\right) \\
&= \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2 + \sigma_1^2}} \exp\left(-\frac{1}{2}\left(\frac{y - \mu_1}{\sqrt{\sigma^2 + \sigma_1^2}}\right)^2\right)
\end{aligned}$$

ACKNOWLEDGEMENT

The authors wish to acknowledge the support of the Australian Research Council in funding the Centre of Excellence for Geotechnical Science and Engineering.

Notation

C	N by N covariance matrix
C_i	covariance matrix of proposal distribution at step i
C_0	initial covariance matrix of proposal distribution

510	$\text{cov}(\theta_0, \dots, \theta_{i-1})$	covariance of previous samples
511	D	data collected
512	FS	factor of safety
513	$f_y(y_1, \dots, y_N)$	joint distribution of pile capacity
514	$f'(\mu)$	prior distribution of mean pile capacity
515	$f''(\mu \hat{\mathbf{y}})$	posterior distribution of mean pile capacity
516	g	limit state function
517	i_0	the maximum steps of initial covariance matrix
518	$\mathbf{I}_d$	d-dimensional identity matrix
519	m	number of failed piles
520	n	number of tested piles
521	N	number of piles in a group
522	$N()$	standard normal distribution
523	$P(\theta)$	prior distribution of the parameters
524	$P(D \theta)$	likelihood
525	$P(D)$	evidence
526	p_f	probability of failure
527	q_1	proposal distribution

528	q_2	proposal distribution
529	s_d	a scaling parameter
530	T	maximum applied load in load tests
531	y	pile capacity
532	$\hat{\mathbf{y}}$	pile capacities obtained by load tests
533	$\bar{y}$	average of $\mathbf{y}$
534	ε	a small value
535	η	coefficient of group efficiency
536	θ	parameters
537	θ_{i-1}	current sample.
538	$\theta_i^{(1)}$	first proposal sample
539	$\theta_i^{(2)}$	second proposal sample
540	μ	mean pile capacity
541	μ_0	prior mean of mean pile capacity
542	μ_1	posterior mean of mean pile capacity
543	μ_{err}	mean test error
544	$\mu_{\ln y}$	mean of underlying normal distribution
545	σ	standard deviation of pile capacity
546	σ_0	prior standard deviation of mean pile capacity

547	σ_1	posterior standard deviation of mean pile capacity
548	σ_{err}	standard deviation of test error
549	$\sigma_{\ln y}$	standard deviation of underlying normal distribution
550	π	target distribution
551	ρ	correlation coefficient of pile capacity
552	Φ	cumulative distribution function of the standard normal distribution
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