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State Estimation for High-speed Multifrequency Atomic Force Microscopy

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Abstract—A fundamental component in the z-axis feedback loop of an atomic force microscope (AFM) operated in dynamic mode is the lock-in amplifier to obtain amplitude and phase of the high-frequency cantilever deflection signal. While this narrowband demodulation technique is capable of filtering noise far away from the carrier and modulation frequency, its performance is ultimately bounded by the bandwidth of its low-pass filter which is employed to suppress the frequency component at twice the carrier frequency. Moreover, multiple eigenmodes and higher harmonics are used for imaging in modern multifrequency AFMs, which necessitates multiple lock-in amplifiers to recover the respective amplitude and phase information. We propose to estimate amplitude and phase of multiple frequency components with a linear time-varying Kalman filter which allows for an efficient implementation on a Field Programmable Gate Array (FPGA). While experimental results for the single mode case have already proven to increase the imaging bandwidth in tapping-mode AFM, multifrequency simulations promise further improvement in imaging flexibility.

I. INTRODUCTION

With the invention of the atomic force microscope (AFM) [1], imaging with nanometer resolution was made possible by coupling the sharp tip of a microcantilever to the surface of a sample through nonlinear tip-sample forces [2]. By utilizing feedback control to perform disturbance rejection, the cantilever maintains a fixed setpoint with respect to the sample while it is being scanned across the surface. Especially when studying dynamic biological processes, tapping-mode AFM (TM-AFM) can provide the required gentle imaging conditions [3] but comes with slow-imaging speeds which is detrimental to capturing dynamics on these time scales [4]. In this mode, the cantilever is commonly excited at its first resonance frequency and the tapping amplitude is maintained by controlling the nanopositioner in its vertical direction. A three dimensional topography image of the sample's surface can be obtained by plotting the controller output against the lateral scan trajectories of the nanopositioner. In order to study the nanomechanical composition of a sample, modern multifrequency AFM (MF-AFM) methods can be employed which extend the imaging channels to multiple frequencies which are commonly associated with either the higher harmonics, generated by the nonlinearity in the tip-

sample force, or with the resonance frequencies of the higher order eigenmodes [5].

High-speed AFM and MF-AFM setups therefore require every component in the z-axis feedback loop as depicted in Fig. 1 to be optimized; a fast cantilever, a fast x-, y- and z-axis nanopositioner, a high-bandwidth z-axis controller and high-speed amplitude and phase estimation are essential [6]. While high-speed estimation methods, such as single wave detectors based on the peak-hold method [7] and real-time integration [8], have been designed particularly for high-bandwidth amplitude estimation, they have no robustness against noise and other frequencies present in the deflection signal. Their inability to obtain phase estimates and to produce accurate amplitude estimates in the presence of other frequency components, makes these methods incompatible with modern nanomechanical characterization methods such as harmonic imaging [9] and bimodal AFM [10]. In contrast, narrowband demodulation performed by a lock-in amplifier is more suitable at rejecting unwanted frequency components, however at the expense of the measurement bandwidth. When phase cancellation is employed, a recently proposed high-bandwidth lock-in amplifier can alleviate the bandwidth constraint on the post-mixing low-pass filter [11].

Recently, we proposed to estimate the cantilever amplitude in TM-AFM using a linear time-varying Kalman filter which shows less high-frequency noise and a higher estimation bandwidth than a commercially available lock-in amplifier [12] which was motivated by early research on tracking power system voltage phasors [13]. Here, we extend our previous work to estimating amplitude and phase of multiple frequencies simultaneously potentially leading to more insights into mode-coupling in multimodal AFM. Our choice of the system representation is especially suitable for a high-speed, highly parallelized implementation on a Field Programmable Gate Array (FPGA) and avoids the numerical complications usually related to estimators for highly resonant structures.

II. SYSTEM MODEL

A. Continuous Model

The modal motions $q_i(t)$ of n cantilever eigenmodes can be described by the set of homogeneous differential equations of simple harmonic oscillators with resonance frequency ω_i given by

$$\begin{aligned} \ddot{q}_1(t) + \omega_1^2 q_1(t) &= 0 \\ &\vdots \\ \ddot{q}_n(t) + \omega_n^2 q_n(t) &= 0. \end{aligned} \tag{1}$$

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Fig. 1. Blockdiagram of the z-axis feedback loop for standard multimodal atomic force microscopy.

The set of equations can be cast into the standard phase variable state-space model by choosing position $x_{2i-1} = q_i(t)$ and velocity $x_{2i} = \dot{q}_i(t)$ as the state variables and choosing position as the output

$$\dot{x} = Ax = \begin{bmatrix} 0 & 1 & & & \\ -\omega_1^2 & 0 & & & \\ & & \ddots & & \\ & & & 0 & 1 \\ & & & -\omega_n^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{2n} \end{bmatrix} \quad (2)$$

$$y = Cx = [1 \ 0 \ 1 \ 0 \ \cdots \ 1 \ 0] x.$$

The canonical form (2) is useful in the analysis and understanding of the state equations, however it's sparse nature makes it generally ill-conditioned for numerical computations. Consider therefore the time-variant transformation

$$T(t) = \text{diag}[T_1(t), \dots, T_n(t)] \quad (3)$$

with

$$T_i(t) = \begin{bmatrix} \cos(\omega_i t) & \sin(\omega_i t) \\ -\omega_i \sin(\omega_i t) & \omega_i \cos(\omega_i t) \end{bmatrix} \quad (4)$$

such that

$$x = T(t)\bar{x}. \quad (5)$$

Then, a time-variant state-space representation can be obtained by

$$\begin{aligned} \dot{\bar{x}} &= \frac{d}{dt}(T^{-1}(t)x) = \bar{A}\bar{x} \\ \bar{y} &= CT(t)\bar{x} = \bar{C}(t)\bar{x} \end{aligned} \quad (6)$$

which converts (2) into the following form

$$\begin{aligned} \dot{\bar{x}} &= 0_{2n}\bar{x} \\ \bar{y} &= [\cos(\omega_1 t) \ \sin(\omega_1 t) \ \cdots \ \cos(\omega_n t) \ \sin(\omega_n t)] \bar{x} \end{aligned} \quad (7)$$

where 0_{2n} is a $2n$ by $2n$ zero matrix. Here, all the dynamics are absorbed in $\bar{C}(t)$ and the individual sinusoidal components can be recovered via

$$\begin{aligned} s_i(t) &= \bar{x}_{2i-1} \cos(\omega_i t) + \bar{x}_{2i} \sin(\omega_i t) \\ &= S_i \sin(\omega_i t + \Phi_i) \end{aligned} \quad (8)$$

where

$$S_i = \sqrt{\bar{x}_{2i-1}^2 + \bar{x}_{2i}^2} \quad (9)$$

$$\phi_i = \arctan\left(\frac{\bar{x}_{2i-1}}{\bar{x}_{2i}}\right). \quad (10)$$

B. Discrete Model

Discretizing (7) exactly for $t = kT_s$, where T_s is the sampling period yields

$$\begin{aligned} x_{k+1} &= I_{2n}x_k \\ y_k &= [\cos \theta_{1,k} \ \sin \theta_{1,k} \ \cdots \ \cos \theta_{n,k} \ \sin \theta_{n,k}] x_k. \end{aligned} \quad (11)$$

where $\theta_{i,k} = \omega_i kT_s$ and I_{2n} is the $2n$ by $2n$ identity matrix. Here, we consider the states $x_{2i-1,k}$ and $x_{2i,k}$, which dictate amplitude and phase of each sinusoidal component, as random variables. If we assume that the state and output equations are only corrupted by zero-mean Gaussian white noise processes w_k and v_k , respectively, then the system representation is in the form of the discrete time-varying (DTV) Kalman filter

$$\begin{aligned} x_{k+1} &= A_k x_k + w_k \\ y_k &= C_k x_k + v_k \\ E[w_k w_k^T] &= Q_k \\ E[v_k v_k^T] &= R_k \\ E[w_k v_k^T] &= 0. \end{aligned} \quad (12)$$

The covariance of the process noise Q_k indicates the model uncertainty and the covariance of the measurement noise R_k relates to the quality of the measurements. Note that Q_k and R_k have an adverse effect on the Kalman gain K_k and therefore directly influence the measurement bandwidth. The recursive Kalman filter implementation then consists of initializing the estimation with a prior estimate of the state $\hat{x}$ and the covariance matrix P

$$\begin{aligned} \hat{x}_{0|0} &= E[x_0] \\ P_{0|0} &= E[(x_0 - \hat{x}_{0|0})(x_0 - \hat{x}_{0|0})^T] \end{aligned} \quad (13)$$

and then iterating between the prediction step

$$\begin{aligned} \hat{x}_{k|k-1} &= A_k \hat{x}_{k-1|k-1} \\ P_{k|k-1} &= A_k P_{k-1|k-1} A_k^T + Q_k \end{aligned} \quad (14)$$

and the measurement update step by correcting the estimated state

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - C_k \hat{x}_{k|k-1}), \quad (15)$$

via calculation of the Kalman gain K_k

$$K_k = P_{k|k-1} C_k^T (C_k P_{k|k-1} C_k^T + R_k)^{-1} \quad (16)$$

and performing the covariance matrix $P_{k|k}$ update

$$P_{k|k} = (I - K_k C_k) P_{k|k-1} (I - K_k C_k)^T + K_k R_k K_k^T. \quad (17)$$

Here, we chose (17) such that the property of symmetry and positive definiteness of the covariance matrix is not lost during computations but other choices are possible [14]. Note, that due to the choice of the system representation, i.e. A_k being the identity matrix, the matrix calculations in the prediction steps (14) are heavily simplified benefiting a high-bandwidth FPGA implementation. Undoubtedly, the computationally most expensive calculation is related to the covariance matrix update (17) therefore parallelizing computations becomes crucial.

III. NUMERICAL RESULTS

A. Error Dynamics of DTV Kalman Filter

In order to get an estimate of the tracking bandwidth of the Kalman filter, we look at the single frequency case ($n = 1$) and define the Kalman filter estimation error $\tilde{x}_k$ and the output error $\tilde{y}_k$ such that

$$\tilde{x}_k = \tilde{x}_{k|k-1} = x_k - \hat{x}_{k|k-1} = A_k \tilde{x}_{k-1|k-1} + w_{k-1} \quad (18)$$

$$\tilde{y}_k = \tilde{y}_{k|k-1} = y_k - \hat{y}_{k|k-1} = C_k \tilde{x}_k + v_k. \quad (19)$$

Then we may determine the error model of the Kalman filter to be

$$\begin{aligned} \tilde{x}_{k|k-1} &= A_k (I - K_k C_k) \tilde{x}_{k-1|k-2} \\ &\quad + w_{k-1} - A_{k-1} K_{k-1} v_{k-1} \\ \tilde{y}_k &= C_k \tilde{x}_k + v_k. \end{aligned} \quad (20)$$

The system is in the form of a linear periodic discrete-time (LPDT) system of the form

$$\begin{aligned} \tilde{x}_{k+1} &= \tilde{A}_k \tilde{x}_k + \tilde{B}_k \tilde{u}_k \\ \tilde{y}_k &= \tilde{C}_k \tilde{x}_k + \tilde{D}_k \tilde{u}_k \end{aligned} \quad (21)$$

with

$$\begin{aligned} \tilde{A}_k &= A_k (I - K_k C_k) \\ \tilde{B}_k &= [I \quad -A_k K_k] \\ \tilde{C}_k &= C_k \\ \tilde{D}_k &= [0 \quad I] \\ \tilde{u}_k &= [w_k \quad v_k]^T. \end{aligned} \quad (22)$$

Hence, the estimation error evolves with dynamics dictated by the eigenvalues λ_k of $\tilde{A}_k$. Specifically, we have

$$\tilde{A}_k = \begin{bmatrix} 1 - K_{1,k} \cos \theta_k & -K_{1,k} \sin \theta_k \\ -K_{2,k} \cos \theta_k & 1 - K_{2,k} \sin \theta_k \end{bmatrix} \quad (23)$$

whose eigenvalues are obtained from

$$\begin{aligned} \lambda_k^2 &+ (K_{2,k} \sin \theta_k + K_{1,k} \cos \theta_k - 2) \lambda_k \\ &+ 1 - K_{2,k} \sin \theta_k - K_{1,k} \cos \theta_k = 0. \end{aligned} \quad (24)$$

Due to the time-varying nature of the Kalman gains and their dependence on the time-varying covariance matrix P , finding an analytical expression for the eigenvalues is

Fig. 2. (a) Single frequency signal (—) and its amplitude estimates, (b) Kalman gains, (c) trace of the covariance matrix P , (d) eigenvalues of $\tilde{A}$ and at $\lambda_k = 1$ (—) and (e) real part of complex eigenvalues of $\Phi_{\tilde{A}}$ for different values of Q (—, —, —). Simulation Parameters: $f_1 = 50$ kHz, $S_1 = 250$ mV, $f_s = 5$ MHz, $Q_{1/2/3} = 10^{-7}/10^{-8}/10^{-9}$.

tedious. However, we notice that the Kalman gains $K_{1/2,k}$ oscillate with equivalent amplitude at the frequency to be estimated, i.e. with ω_1 for the single mode case, are phase shifted by 90° and lead the C_k vector entries by φ . Without loss of generality, we can therefore assume that

$$\begin{aligned} K_{1,k} &= K \cos(\theta_k - \varphi) = D \cos \theta_k + E \sin \theta_k \\ K_{2,k} &= K \sin(\theta_k - \varphi) = D \sin \theta_k - E \cos \theta_k \end{aligned} \quad (25)$$

where $D = K \cos \varphi$ and $E = K \sin \varphi$. Substituting (25) in (24) yields the characteristic equation

$$\lambda_k^2 + (D - 2)\lambda + 1 - D = 0 \quad (26)$$

Fig. 3. (a) Histogram (—) and theoretical probability density function (PDF) (—) of noise process v . (b) Simulated multifrequency signal with modulated amplitude and additive white noise (—) and (c) its power spectral density estimate. (d)-(f) Amplitude reference step (—) and estimated amplitude (—) of each frequency of the signal shown in Fig. 3b. Simulation Parameters: $f_{1/2/3} = 50/130/350$ kHz, $S_{1/2/3} = 250/100/50$ mV, $\sigma = 5$ mV, $f_s = 5$ MHz, $Q_{1/2/3} = 10^{-7}/7 \cdot 10^{-8}/7 \cdot 10^{-8}$.

whose roots are real and time-invariant and are given by

$$\lambda_{k,1/2} = \begin{cases} 1 \\ 1 - D = 1 - K \cos \varphi. \end{cases} \quad (27)$$

We notice that one eigenvalue remains fixed at $\lambda_k = 1$ and the other eigenvalue moves closer to the real axis as the Kalman gain increases. This is illustrated in the simulations shown in Fig. 2 in which three different values of the process noise covariance Q are assumed and the respective Kalman filter aims to track an amplitude step of a single frequency signal $S_1(t) \sin(2\pi f_1 t)$ without additive white noise. Here, the subfigures are color coded and the system shown in blue has the highest Q and the other systems have decreasing Q . As can be seen in Fig. 2(a), the amplitude tracking bandwidth is largest for the largest Q due to corresponding largest Kalman gains shown in Fig. 2(b). This in turn is explained by a higher uncertainty in the model which is expressed by the trace of P shown in Fig. 2(c). Predicted by (27), the eigenvalues of $\tilde{A}_k$ are real with one eigenvalue constant at $\lambda_{k,1} = 1$ which is plotted in Fig. 2(d). The larger Kalman gains cause the second eigenvalue to move towards $\lambda_{k,2} = 0$, resembling a larger tracking bandwidth. Studying the eigenvalues of $\tilde{A}_k$ offers one approach to study the system (22), but it was shown in earlier works that no sufficient stability conditions can be deduced [15]. Instead, one has to look at the eigenvalues of the state transition matrix $\Phi_{\tilde{A}}(j, i) = \tilde{A}_{j-1} \tilde{A}_{j-2} \dots \tilde{A}_i$, where $\Phi_{\tilde{A}}(i, i) = I$. Specifically, the eigenvalues are the characteristic multipliers of the Monodromy matrix $\Phi_{\tilde{A}}(K, 0) = \tilde{A}_{K-1} \tilde{A}_{K-2} \dots \tilde{A}_0$ where K is the period of the LPDT system [16]. Therefore, the real part of the eigenvalues of $\Phi_{\tilde{A}}$ are also plotted in Fig. 2(e) and it can be seen that they quickly converge to zero.

B. Multifrequency Amplitude Estimation

We construct a signal of the form

$$s(t) = \sum_{i=1}^3 S_i(t) \sin(2\pi f_i t) + v \quad (28)$$

where the respective amplitudes $S_i(t)$ and frequencies f_i are related to the oscillation amplitudes, as measured with an optical lever sensor of a standard AFM, and frequencies of the i -th eigenmode. v is a white noise process with standard deviation σ which is obtained from integrating the power spectral density (PSD) of a thermal noise measurement taken with the optical lever sensor (not shown) [17]. From the PSD, a noise density of $1.2 \mu\text{V}/\sqrt{\text{Hz}}$ was calculated which yields a standard deviation of $\sigma = 1.9$ mV for a bandwidth from DC to 2.5 MHz. To demonstrate the effect of white noise rejection of the proposed method, we used $\sigma = 5$ mV and subsequently set the Kalman filter covariance of the measurement noise $R_k = \sigma^2$. The simulation results are shown in Fig. 3 where each amplitude S_i experiences a step change at $t = 2$ ms of varying magnitude. It can be seen that the first mode amplitude is tracked within 2 cycles, the second mode is tracked within 3 cycles and the third mode is tracked within 7 cycles of the respective frequency even under excessive additive white noise. From the time-domain plot, the 10%-90% rise time can be found which can be related to the -3dB bandwidth of a first order system. As such, the tracking bandwidths are 29.6 kHz, 27.2 kHz and 27.0 kHz for the first, second and third mode respectively. Note that the choice of the entries of the Kalman filter covariance matrix Q_k dictates the bandwidth with which each amplitude step is tracked. Here, we tuned the filter for maximum bandwidth in each estimated mode with no overshoot but less aggressive values might be necessary when

Fig. 4. Amplitude (—) estimate, phase (—) estimate and reference (---) of (a) first, (b) second and (c) third mode with amplitude and phase modulation and additive white noise. Simulation parameters: $f_{1/2/3} = 50/130/350$ kHz, $S_{1/2/3} = 250/100/50$ mV, $\sigma = 5$ mV, $f_s = 5$ MHz, $Q_{1/2/3} = 10^{-7}/10^{-8}/10^{-8}$.

other disturbances are present as will be discussed in the next section.

C. Coupling Between Amplitude and Phase Estimation

Next, we consider phase modulation of each frequency component in addition to amplitude modulation, i.e. a signal of the form

$$s(t) = \sum_{i=1}^3 S_i(t) \sin(2\pi f_i t + \phi_i(t)) + v. \quad (29)$$

While a change in oscillation amplitude usually resembles topography features in dynamic AFM, a change in the phase may arise from sample regions with varying elastic properties [18]. Therefore, a phase change is not necessarily correlated with an amplitude change and acts as a disturbance on the amplitude estimation. It can be seen in the simulations shown in Fig. 4 how the amplitude and phase changes are accurately tracked for each of the three frequencies. However, we notice a transient response in the amplitude (phase) estimate if there is a step change in the phase (amplitude). It can be observed that the magnitude of the transient responses in the amplitude estimate due to a phase change are magnified for higher frequency estimates. Here, a tradeoff has to be made between estimation bandwidth and rejecting the coupling between phase and amplitude change. Lowering the Q entries for the higher frequency modes reduces the maximum transient spike due to coupling but comes at the expense of tracking bandwidth.

Fig. 5. (a) Amplitude estimate (—) and reference (---) of the first, second and third mode (from top to bottom) with amplitude modulation only. (b) Phase estimate (—) and reference (---) of the first, second and third mode (from top to bottom) with phase modulation only. Simulation parameters: $f_{1/2/3} = 50/130/350$ kHz, $S_{1/2/3} = 250/150/50$ mV, $\sigma = 5$ mV, $f_s = 5$ MHz, (a) $Q_{1/2/3} = 10^{-7}/7 \cdot 10^{-8}/7 \cdot 10^{-8}$, (b) $Q_{1/2/3} = 10^{-7}/10^{-8}/10^{-9}$.

D. Coupling Between Frequencies

We consider the coupling effect between the estimation of amplitude and phase when the individual modes experience amplitude and phase steps at different points in time. In the context of multifrequency AFM, this can be considered as a practically rather unlikely event since the individual modes are coupled by the same surface. However, due to the difference in frequencies, a higher mode might detect a change in surface properties before a lower mode due to the higher tapping frequency. It can be seen in Fig. 5(a) and Fig. 5(b), that small transient responses can be observed in the higher frequency amplitude and phase estimates, respectively where as the coupling effect seems to reduce with higher frequencies. It was also noticed, that while the amplitude estimate seems to be relatively robust with respect to the added noise and disturbances, the phase estimate becomes increasingly noisy for higher frequencies and as such the tracking bandwidth of the Kalman filter has to be reduced.

E. Robustness to Frequency Deviations

We simulate the robustness of the Kalman filter to 2% deviations in the frequencies whose amplitudes and phases are to be estimated, i.e. $\Delta f_1 = 1$ kHz, $\Delta f_2 = 2.6$ kHz and $\Delta f_3 = 7$ kHz. Note, that these deviations are unnaturally large as the resonant frequencies of the cantilever in dynamic AFM can be determined very accurately prior to an experiment. As can be seen in the time-domain plots in Fig. 6, the amplitude estimates appear increasingly noisy for higher frequencies but the amplitude envelope is still tracked with a 10% error in steady-state for the highest frequency (compare with inset). Here the Q values have to be tuned in order to achieve a desired steady-state error at the expense of a noisy estimate but without sacrificing tracking bandwidth. Further, we notice distinct frequency

Fig. 6. Top: (a) First mode, (b) second mode and (c) third mode with modulated amplitude, additive white noise and 2% frequency deviation (—), amplitude estimate (—), amplitude reference (—) and relative amplitude error in inset. Bottom: Power spectral density estimate of (a) first mode, (b) second mode and (c) third mode amplitude estimate (—). Simulation parameters: $f_{1/2/3} = 0.98 \cdot 50/130/350$ kHz, $S_{1/2/3} = 250/100/50$ mV, $\sigma = 5$ mV, $f_s = 5$ MHz, $Q_{1/2/3} = 3 \cdot 10^{-8}/3 \cdot 10^{-8}/7 \cdot 10^{-8}$.

components in the amplitude estimate as can be seen in the frequency-domain plots in Fig. 6. These distinct components are intermodulation products (IMP) formed by the nonlinear output equation (9) since multiple frequency components are present in the state estimates. These IMP are located at $f_{\text{IMP}} = k_1 f_1 + k_2 f_2 + k_3 f_3$ where k_i are arbitrary integers. For instance, the first two distinct frequencies in Fig. 6(a) are located at $f_{\text{IMP},1} = (f_2 - \Delta f_2) - (f_1 - \Delta f_1)$ and at $f_{\text{IMP},2} = 2(f_1 - \Delta f_1)$.

IV. CONCLUSION

In this work, we propose a linear time-varying Kalman filter to estimate amplitude and phase of multiple frequencies related to cantilever higher eigenmodes for multifrequency Atomic Force Microscopy. We show numerical results, which give insights into the large bandwidth that can be achieved although the analytical treatment is complicated due to the time-varying system representation. We highlight the robustness against frequency deviations as well as the coupling between frequencies and between amplitude and phase estimates. The authors are currently working on the high-speed parallel implementation of the proposed filter on a FPGA.

REFERENCES

- [1] G. Binnig, C. F. Quate, and C. Gerber, "Atomic force microscope," *Phys. Rev. Lett.*, vol. 56, pp. 930–933, Mar 1986.
- [2] D. Sarid, *Scanning Force Microscopy With Applications to Electric, Magnetic and Atomic Forces*. Oxford University Press, 1994.
- [3] Q. Zhong, D. Inniss, K. Kjoller, and V. Elings, "Fractured polymer/silica fiber surface studied by tapping mode atomic force microscopy," *Surf. Sci.*, vol. 290, no. 1-2, pp. L688 – L692, 1993.
- [4] T. Ando, "High-speed atomic force microscopy coming of age," *Nanotechnology*, vol. 23, no. 6, p. 062001, 2012.
- [5] R. García and E. T. Herruzo, "The emergence of multifrequency force microscopy," *Nature Nanotechnology*, vol. 7, pp. 217–226, 2012.
- [6] T. Ando, T. Uchihashi, and T. Fukuma, "High-speed atomic force microscopy for nano-visualization of dynamic biomolecular processes," *Progress in Surface Science*, vol. 83, no. 7-9, pp. 337–437, 2008.
- [7] T. Ando, N. Kodera, E. Takai, D. Maruyama, K. Saito, and A. Toda, "A high-speed atomic force microscope for studying biological macromolecules," *Proceedings of the National Academy of Sciences*, vol. 98, no. 22, pp. 12468–12472, 2001.
- [8] D. Abramovitch, "Low latency demodulation for atomic force microscopes, part I efficient real-time integration," in *American Control Conference*, June 2011, pp. 2252–2257.
- [9] A. Raman, S. Trigueros, A. Cartagena, A. P. Z. Stevenson, M. Susilo, E. Nauman, and S. Antoranz Contera, "Mapping nanomechanical properties of live cells using multi-harmonic atomic force microscopy," *Nature Nanotechnology*, vol. 6, no. 12, pp. 809–814, 2011.
- [10] D. Forchheimer, R. Forchheimer, and D. B. Haviland, "Improving image contrast and material discrimination with nonlinear response in bimodal atomic force microscopy," *Nature communications*, vol. 6, no. 6270, pp. 1–5, 2015.
- [11] K. S. Karvinen and S. O. R. Moheimani, "A high-bandwidth amplitude estimation technique for dynamic mode atomic force microscopy," *Rev. Sci. Instrum.*, vol. 85, no. 2, p. 023707, 2014.
- [12] M. G. Ruppert, K. S. Karvinen, S. L. Wiggins, and S. O. Reza Moheimani, "A kalman filter for amplitude estimation in high-speed dynamic mode atomic force microscopy," *IEEE Trans. Contr. Syst. Technol.*, vol. 24, no. 1, pp. 276–284, Jan 2016.
- [13] A. Girgis and T. Hwang, "Optimal estimation of voltage phasors and frequency deviation using linear and non-linear kalman filtering: Theory and limitations," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-103, no. 10, pp. 2943–2951, Oct 1984.
- [14] R. G. Brown and P. Y. C. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*. John Wiley & Sons Inc., 1997.
- [15] L. Markus and H. Yamabe, "Global stability criteria for differential systems," *Osaka Math. J.*, vol. 12, no. 2, pp. 305–317, 1960.
- [16] A. Varga, P. Van Dooren *et al.*, "Computational methods for periodic systems-an overview," in *Proc. of IFAC Workshop on Periodic Control Systems, Como, Italy*, 2001, pp. 171–176.
- [17] M. G. Ruppert and S. O. R. Moheimani, "Multimode Q control in tapping-mode AFM: Enabling imaging on higher flexural eigenmodes," *IEEE Trans. Contr. Syst. Technol.*, vol. 24, no. 3, pp. 1–11, 2016.
- [18] J. Tamayo and R. García, "Effects of elastic and inelastic interactions on phase contrast images in tapping-mode scanning force microscopy," *Appl. Phys. Lett.*, vol. 71, no. 16, pp. 2394–2396, 1997.