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Discrete modelling of soil–inclusion problems

K. Thoeni^{1,a *}, A. Effeindzourou^{1,b}, B. Chareyre^{2,c} and A. Giacomini^{1,d}

¹Centre for Geotechnical and Materials Modelling,
The University of Newcastle, Callaghan, NSW 2308, Australia

²Laboratoire 3SR
Université Grenoble-Alpes, F-38000 Grenoble, France

^aklaus.thoeni@newcastle.edu.au, ^bmakouala.effeindzourou@uon.edu.au,

^cbruno.chareyre@3sr-grenoble.fr, ^danna.giacomini@newcastle.edu.au

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Abstract. A generalised approach for the modelling of arbitrary shaped deformable structures in the framework of the discrete element method is presented. Minkowski sums of polytopes and spheres are used to describe the geometry of rounded cylinders and particle facets. In the current formulation, these new elements can be deformable. Their deformation is defined by the set of positions and orientations of their nodes. The elements can be connected to form arbitrary structures, such as grids and membranes. The constitutive behaviour of such connections is defined via an elastic perfectly plastic beam model. Contacts between other not connected structures or particles are detected based on three simple primitives: spheres, cylinders and thick rounded facets. The introduction of a virtual sphere at the contact point not only allows for straightforward contact handling but as well for the use of standard contact models based on sphere–sphere interactions. Hence, there is no need for developing new contact models. The approach is implemented into the open-source framework YADE. The capability of the newly developed approach for the modelling of soil–inclusion problems is presented.

Introduction

The modelling of the mechanical behaviour of soil–inclusion problems is very challenging since both the geometry and the governing mechanisms are strongly discontinuous. The discrete element method (DEM) is particularly well suited to solve such problems due to its robustness in tracking contacts in multibody systems and its ability in considering large displacements resulting from sliding at the interface between different objects. The classical DEM considers the interaction between rigid spheres only and rather simple contact laws are used to calculate the contact forces [1]. Generally, not only the soil is represented by spherical particles but also the inclusion. Thereby, spheres are either clumped or bonded together [2]. In the first case, the aggregate is rigid and the total force acting on the clump is calculated as the sum of all forces acting on the spheres representing the clump. In the second case, spheres belonging to the same aggregate interact via a bond where attractive forces are allowed and bonds can break. Nevertheless, both approaches introduce an artificial numerical roughness at the interface between the inclusion and the surrounding granular material. This can cause unrealistic artefacts and some limitations [3].

In this paper, a general method for the modelling of soil–inclusion problems with the DEM, which eliminates the artificial numerical roughness at the interface, is presented. In order to achieve this, the inclusions are represented by deformable cylinders and deformable thick facets which are defined via Minkowski sums. The concept of Minkowski sums to represent non-spherical particles in DEM was first introduced by Pournin and Liebling [4] and then further extended by Alonso-Marroquín [5] and Galindo-Torres et al. [6]. Only recently the approach has been extended for the modelling of general deformable structures by Bourrier et al. [7] and Effeindzourou et al. [8]. This

approach is implemented into the open-source framework YADE [9] and used to solve the soil-inclusion problem presented in this paper.

General Formulation

The formulation is based on three primitives (Fig. 1): spheres, cylinders and thick rounded facets (pfacets). Spheres are modelled as in the classical DEM [1] and are rigid. Cylinders are represented by a Minkowski sum of a line segment and a sphere (Fig. 1b) whereas pfacets are represented by a Minkowski sum of a facet (triangular element) and a sphere (Fig. 1c). Both, cylinders and pfacets are deformable. The deformation of a cylinder or pfacet is defined by the deformation of its nodes or vertices and linearly interpolated in-between. The mass of a cylinder or pfacet is lumped into its nodes or vertices.

Fig. 1: Three primitives and concept of a Minkowski sum: (a) sphere, (b) cylinder, and (c) pfacet.

Fig. 2: Possible contacts: (a) sphere-sphere, (b) cylinder-sphere, (c) cylinder-cylinder, (d) sphere-pfacet, (e) cylinder-pfacet, and (f) pfacet-pfacet.

Fig. 2 shows all possible interactions between the three primitives. The concept of a virtual sphere is introduced to handle all possible contacts. A virtual sphere with the same properties of its primitive is created at each contact point (Fig. 3b). Then in the last instance each contact is handled as a sphere-sphere interaction which allows the basic contact laws of the classical DEM formulation to be used to calculate the contact forces. The mathematical formulation for contact detection can be found in Effeindzourou et al. [8]. In the following, the contact laws used in this work are summarised.

Fig. 3: Concept of overlap and contact force between: (a) two spheres and (b) a sphere and a virtual sphere in a cylinder or pfacet.

The contact law relates the relative displacement and relative rotation to the contact force and contact moment respectively. The normal contact force $\mathbf{F}_n$ is a function of the normal contact stiffnesses k_n and the normal displacement $\mathbf{u}_n$ (overlap) between the two spheres in contact (Fig. 3). It is calculated as:

$$\mathbf{F}_n = k_n \mathbf{u}_n \quad \text{with} \quad k_n = \frac{2E_1 R_1 E_2 R_2}{E_1 R_1 + E_2 R_2} \quad (1)$$

where E_1 and E_2 correspond to the Young's modulus and R_1 and R_2 the radius of the two spheres in contact. The incremental shear force $d\mathbf{F}_s$ is related to the relative shear velocity $\dot{\mathbf{u}}_s$ and the tangential contact stiffness k_s by:

$$d\mathbf{F}_s = k_s \dot{\mathbf{u}}_s dt \quad \text{with} \quad k_s = \frac{2E_1 R_1 \nu_1 E_2 R_2 \nu_2}{E_1 R_1 \nu_1 + E_2 R_2 \nu_2} \quad (2)$$

where dt is the time step and ν_1 and ν_2 correspond to the Poisson's ratio associated to the two spheres in contact. The contact moment is separated into a twisting moment $\mathbf{M}_t$ and a bending moment $\mathbf{M}_b$. By adopting a vector representation $\boldsymbol{\Omega}_{12}$ of the relative rotation, the moments can be calculated as:

$$\mathbf{M}_t = k_t \boldsymbol{\Omega}_{12}^t \quad (3)$$

$$\mathbf{M}_b = k_b \boldsymbol{\Omega}_{12}^b \quad (4)$$

where k_b and k_t are the contact stiffnesses associated to bending and twisting respectively, and $\boldsymbol{\Omega}_{12}^b$ and $\boldsymbol{\Omega}_{12}^t$ are the bending and twisting components of the relative rotation associated to the two spheres.

The following elastic limits can be imposed to account for plastic deformation:

$$\|\mathbf{F}_n\| \leq \sigma_n^l A \quad (5)$$

$$\|\mathbf{F}_s\| \leq F_n \tan \varphi + \sigma_s^l A \quad (6)$$

$$M_b \leq \frac{\sigma_n^l I_b}{R} \quad (7)$$

$$M_t \leq \frac{\sigma_s^l I_t}{R} \quad (8)$$

where σ_n^l and σ_s^l are the tensile and shear strength respectively, $\varphi = \min(\varphi_1, \varphi_2)$ is the friction angle, $A = \pi R^2$ is the reference surface area or cross-section area, $I_t = \pi R^4/4$ and $I_b = \pi R^4/8$ are the reference polar and bending moments of inertia respectively, and $R = \min(R_1, R_2)$ is the reference radius of the contact.

Connected Cylinders and Deformable Structures

A single cylinder element behaves as a nearly rigid object whose deformation is reflected by the compliance of the interactions between the cylinder itself and other objects. Disregarding this compliance, the cylinder can be seen as strictly rigid but deformable in its longitudinal direction. The stiffness k_n considered in the longitudinal direction is:

$$k_n = \frac{EA}{L} \quad (9)$$

where E and L are the Young's modulus and the initial length of the cylinder respectively. The stiffness in the tangential direction for frictional contacts is defined as:

$$k_s = \frac{12E_b I_b}{L^3} \quad (10)$$

with E_b being the bending modulus of the cylinder.

Cylinder elements can be connected to form beams [7] and grids [8]. The classical beam theory is adapted to define the bending and twisting stiffness:

$$k_t = \frac{G_t I_t}{L} \quad (11)$$

$$k_b = \frac{E_b I_b}{L} \quad (12)$$

where G_t is the shear modulus associated with the twisting moment. Interconnected cylinders can be used to simulate beam and truss structures with large displacements. Note that the behaviour can be changed by setting specific contact stiffnesses to zero and by applying the elastic limits defined in Eqs. 5–8.

Pfacets can be combined in a similar way. A pfacet element consists of three vertices (nodes) and three connections (cylinders). The constitutive behaviour of a pfacet is defined by the three cylinders comprising the pfacet. Hence, connected pfacets act like connected cylinders with the only difference that they have two additional contact areas (one facet at the top and one at the bottom).

Fig. 4: Deformable structures: (a) four connected cylinders, (b) simple grid (1 cylinder/rib and 2 cylinders/rib in horizontal and vertical direction respectively), (c) grid structure, and (d) deformable sphere.

Application

A simple pull-out test is presented in order to show the capabilities of the model to solve soil-inclusion problems. No normal stress is applied on top of the sample and only gravitational forces are acting during the pull-out. The inclusions are pulled out completely in order to highlight the full capabilities of the presented approach.

The soil container consists of a cube with side lengths of 100 mm. The soil specimen consists of two equally high sphere assemblies with approximately 9500 spheres with a mean diameter of 4 mm and a standard deviation of 0.2 mm. The assemblies are generated by gravity deposition where a gap is introduced in between them in order to place the inclusion. The latter are generated from Minkowski sums with a sphere diameter of 1.5 mm. The material parameters used in the simulations are $E = E_b = 5e7$ Pa, $\nu = 0.3$, $\varphi = 20^\circ$ and density $\rho = 2650$ kg/m³. In order to limit the number of parameters they are the same for soil and inclusion except that no moments are considered for all contacts with spheres (i.e., $k_b = k_t = 0$). Two examples are shown, one with a grid-like inclusion with mesh openings of 10 mm $\times$ 10 mm, where each rib consists of two cylinders, and another with a membrane-like inclusion where each mesh opening of the grid is closed with four pfacets. The inclusions are square with side lengths of 10 cm. The grid-like inclusion consists of 440 cylinder elements whereas the membrane-like inclusion consists of 400 pfacet elements. The pull-out velocity is 0.05 m/s and a numerical damping of 0.2 is applied.

Fig. 5: Pull-out of (a) grid-like and (b) membrane-like inclusion at different stages ($u = 0$ cm, $u = 5$ cm and $u = 10$ cm).

Fig. 5 shows screen-shots of the simulations at three different stages. The first one represents the initial stage before the pull-out, the second one shows an intermediate stage with pull-out displacement of $u = 5$ cm and the last one corresponds to $u = 10$ cm. In Fig. 5a a clear rearrangement of the spheres is observed. White particles are pulled into the black layers of particle and the quantity of spheres towards the edge of the box in the pull-out direction increases. This is not the case for the membrane-like inclusion where the particle arrangement in the pull-out direction remains almost unchanged throughout the simulation (Fig. 5b). In the case of the membrane-like inclusion the spheres are rolling or sliding on the surface of the inclusion. In the case of the grid-like inclusion some spheres are interlocked within the grid openings and the pull-out can only be achievable when some of these sphere move up or down. Nevertheless, settlements of the surface of the assembly during the pull-out test can still be seen for the membrane-like inclusion, which is connected to particle rearrangements in the vertical direction.

Conclusions

The paper presents a general method for the efficient simulation of soil–inclusion problems based on the DEM. The inclusions are represented by deformable cylinders and deformable thick facets which are defined via Minkowski sums. In contrast to the classical approach of either clumping or bounding spherical particle, this new approach does not introduce any artificial numerical roughness at the interface. In addition, contact detection is rather simple and there is no need for the development of new contact models since the concept of a virtual sphere is introduced at the contact points. Hence, in the last instance each contact is handled as a sphere–sphere interaction. The full capabilities of the model are shown by performing a pull-out test where the inclusions are completely pulled out from the soil specimen. The results indicate that the model is capable of capturing large deformations and to efficiently simulate complex deformable shapes and soil–inclusion problems. It allows the representation of arbitrary deformable structures by using the standard contact models for spheres and can easily be implemented into a standard DEM code.

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