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The Lattice Boltzmann Method and the Problem of Turbulence

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Abstract. This paper reports a brief review of numerical simulations of homogeneous isotropic turbulence (HIT) using the lattice Boltzmann method (LBM). The LBM results shows that the details of HIT are well captured and in agreement with existing data. This clearly indicates that the LBM is as good as current Navier-Stokes solvers and is very much adequate for investigating the problem of turbulence.

Keywords: Direct numerical simulation, lattice Boltzmann method, turbulence

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INTRODUCTION

Since the work of Kim et al. [1], Spalart [2], direct numerical simulation (DNS) has become an effective tool for studying turbulent flows. In particular, DNS has greatly helping shedding some light in some fundamental aspects of turbulence. While DNSs of turbulent flows are still largely carried out by solving the Navier-Stokes equations, a relatively numerical approach is gaining momentum. The method, the lattice Boltzmann method (hereafter denoted LBM), started to be considered as a serious numerical tool after Frisch et al., [3] shows that Navier-Stokes equations can be recovered from the lattice gas automata (LGA) that predated the LBM. Since then, the LBM proved to be as effective as the Navier-Stokes solvers (based either on spectral methods or finite differences). However, despite the vast amount of research using the LBM, there are relatively few numerical studies using the LBM to investigate turbulent flows. There are even fewer LBM simulations dedicated to fundamental studies of the problem of turbulence.

Usually, the phenomenon of turbulence is investigated in homogeneous isotropic turbulence (HIT) because HIT is an adequate idealisation of turbulence which lends itself to theoretical analyses. Most DNSs of HIT have been carried out using Navier-Stokes solvers. Only relatively recently, LBM simulations of HIT have been performed successfully. In the present paper, we first briefly introduce the LBM and then present its application to investigate HIT.

THE LATTICE BOLTZMANN METHOD

Rather then solving the governing fluid equations (Navier-Stokes equations), the LBM solves the Boltzmann equation on a lattice. The basic idea of the LBM is to construct a simplified kinetic model that incorporates the essential physics of microscopic average properties, which obey the desired (macroscopic) Navier-Stokes equations ([3]). With a sufficient amount of symmetry of the lattice, the LBM implicitly solves these latter equations with second-order accuracy. An example of a lattice "cell" is shown in Figure 1. The lattice represented in Fig 1 is a three dimensional lattice with 18 moving particles and a rest particle (lattice model 3DQ19), which is the one used our LBM simulations.

FIGURE 1. D3Q19 square lattice
The Boltzmann equation is discretized on the cubic lattice (e.g. D3Q19) and results in the standard lattice Boltzmann equation (LBE) which, when combined with the Bhatnagar-Gross-Krook (BGK) approximation, governs the time and space variations of the single-particle distribution $f_i(x,t)$ at the lattice site $x$ and is expressed as:

$$f_i(x + e_i \Delta t, t + \Delta t) - f_i(x,t) = -\frac{1}{\tau} \left( f_i(x,t) - f_i^{eq}(x,t) \right), \quad i = 0, 1, \ldots, 8 \quad (1)$$

where $\tau$ is the relaxation time, $\Delta t$ the time step, $e_i (= \Delta x/\Delta t)$ is the velocity of the particle $i$ in the $i$-direction and $f_i^{eq}$ is the equilibrium single-particle distribution:

$$f_i^{eq} = \rho \omega_i (1 + 3(e_i \cdot u) + \frac{9}{2}(e_i \cdot u)^2 - \frac{3}{2}u^2), \quad (2)$$

where $\rho (= \sum_i f_i)$ is the fluid density, $u (\rho u = \sum_i f_i e_i)$ is the local fluid velocity and $\omega_i$ are the corresponding weights ($\omega_0 = 1/3$ for $i = 0$, $1/18$ for $i = 1$ to $6$ and $1/36$ for $i = 7$ to $18$; $i = 0$ corresponds to the rest particle at the centre of the lattice, $i = 1, \ldots, 6$ correspond to the particles on the axis aligned with $x, y$ and $z$, and $i = 7, \ldots, 18$ are related to the particles on the diagonal directions). The relaxation time is related to the kinematic viscosity of the fluid via the relation

$$\nu = \frac{2\tau - 1}{6} \quad (3)$$

In equ. (1), $\Delta x = \Delta y = \Delta z = 1 (x$ is the longitudinal direction and $y$ and $z$ the lateral directions). With this choice, $|e_i| = 1$, for $i = 1$ to $6$, and $\sqrt{2}$ for $i = 7$ to $18$; note that $|e_0| = 0$, and the variables such as , $x, t, u, \rho$ and $\nu$ are expressed in lattice units; $\nu$ is estimated once the Reynolds number is fixed ($\nu = U_0 M / RM$).

The left-hand side of (1) is the so-called streaming operation, which means that the particles moves to their nearest neighbours along the direction of their velocity (i.e. they radiate outwardly from the centre of the cubic lattice). The right-hand side is the collision term, here modelled by the BGK collision operator, which describes the redistribution of the particles at each node (for more details on the LBM see, for example, Chen & Doolen, ([4]) or Succi, ([5]). Thus, (1) is solved according to these two operations: collision and streaming. The collision step is described by

$$f_i^{new}(x,t) = f_i(x,t) - \frac{\Delta t}{\tau} (f_i(x,t) - f_i^{eq}(x,t)), \quad (4)$$

where $f_i^{eq}$ are calculated using (2.2). The streaming process is described by

$$f_i(x + e_i \Delta t, t + \Delta t) = f_i^{new}(x,t). \quad (5)$$

The collisions are entirely local, making the LBM efficiently parallelized. At time $t$, the particle distributions are updated based on (4); then, at time $t + \Delta t$, the particles propagate according to (5). Note that The above LBM is based on the so-called single relaxation time (SRT) model. Another LBM model called multi relaxation time (MRT) is also commonly used ([e.g. 6]). Here we concentrate only on the SRT-LBM.

**EXAMPLES OF LBM SIMULATIONS OF TURBULENT FLOWS**

This section presents two examples of LBM simulations of turbulent flows to illustrate the suitability of the LBM for investigating fundamental aspects of turbulence. These turbulent flows are those which are commonly used to investigate HIT.

**Three Dimensional Periodic Box Turbulence**

Before DNS of grid turbulence were carried out, HIT was simulated in a periodic three-dimensional box turbulence, mostly based on the Navier-Stokes solvers. There are now HIT box turbulence simulations based on the LBM ([e.g. 15, 16, 17, 18]). Yu et al. [16] clearly showed that the LBM captures well important features of the HIT, such as the power law decay, $k \sim r^n$, where $k$ is the turbulent kinetic energy and with $n$ well within the range obtained in classic DNS ([13]). Burattini et al. ([15]) found that the energy spectra and the value of the velocity derivative skewness in the initial period of decay were very close to those measured in grid turbulence. They also found that the power law decay, the growth of the Taylor microscale to be in good agreement with existing numerical and simulated data.
Grid Turbulence

It is common to experimentally investigate homogeneous and isotropic turbulence (HIT) by examining grid turbulence, generated by passing a uniform flow through a grid, usually made of vertical and horizontal bars, and placed normal to the main flow. It is generally accepted that grid-generated turbulence is the closest practical approximation to HIT, considered to be the simplest type of three-dimensional turbulence, which explains the considerable amount of work dedicated to this flow (see Ertunc et al., ([7]), for a brief but recent review on grid turbulence).

Using LBM, Djenidi ([8]) performed the first numerical simulation of grid turbulence with the actual grid in the computational domain. He showed that LBM provided results in agreement with experimental data and thus demonstrated the LBM is as effective as the classical Navier-Stokes solvers. This work was followed by several studies ([9], [10], [11], [12]) to investigate both small and large scale motions. For example, it was shown for the first time that ergodicity is well satisfied in grid turbulence (see Figure 2 which shows the instantaneous isocontour for the enstrophy, $\omega^2$, the vorticity variance), in planes perpendicular to the mean flow ([10]), demonstrating unequivocally the equivalence between temporal and spatial averages. In doing so, it definitely confirmed that hot-wire measurements in grid turbulence are appropriate for testing theoretical results based on spatially averaged statistics. An important result shown through these LBM simulations ([12]) is the Breakdown of Kolmogrov’s first similarity hypothesis (i.e. $E(k) = (u_K^2 \eta) K(k \eta)$; $E(k)$ is the 1D or 3D velocity spectrum, $u_K$ and $\eta$ the Kolmogorov velocity and length scales and $F(k)$ a universal function). This is well illustrated in Figure 2 ([12]) which shows the velocity spectra at Taylor microscale Reynolds numbers, $R_\lambda$, of about 20.

The first similarity hypothesis states that all spectra should collapse onto a universal function in the high wavenumber range when normalized by $u_K$ and $\eta$. Clearly, figure 3 does not support this hypothesis. The Kolmogorov
normalized spectra do not follow the spectrum of Comte-Bellot & Corssin ([14]) in the dissipative range for both the 1D and 3D spectra. It was shown ([12]) that the breakdown of they hypothesis occurs because the energy transfer is too small at all scales when $R_\lambda$ is below 20.

CONCLUDING REMARKS

The above relatively brief overview of numerical simulation of turbulence using the LBM clearly shows that this numerical method is well suited for carrying out fundamental studies of turbulence. The LBM is as good as current DNS solvers, with an advantage in terms of performing parallel computations, because of its local nature, which avoid solving the Poisson equation for the pressure.

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REFERENCES