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Liu, Xiaoli; Wang, Sijing; Wang, Shanyong; Wang, Enzhi "Fluid-driven fractures in granular materials" Published in Bulletin of Engineering Geology and the Environment Vol. 74, Issue 2, p. 621-636

Available from: <http://dx.doi.org/10.1007/s10064-014-0712-7>

The final publication is available at Springer via <http://dx.doi.org/10.1007/s10064-014-0712-7>

Accessed from: <http://hdl.handle.net/1959.13/1332960>

Fluid-driven fractures in granular materials

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Abstract: The fluid-driven fracture initiation and propagation process in granular materials is inherently a coupled hydro-mechanical problem. The bonded-particle method was utilised to simulate the hydraulic fracturing process in granular materials, and different failure mechanisms were evaluated by analysing microcrack formation. Hydraulic conductivity was controlled by pore size and connectivity in the direction of flow, and a strain-dependent formulation was preferred to highlight the inherent link between hydraulic conductivity and pore size. The results show that for consolidated formations, fluid-driven fracture initiation and propagation are dominated by tensile failure, while for unconsolidated formations, shear failure seems to be more important than tensile failure during the hydraulic fracturing process. In general, the results of the fluid-driven fracture simulations are in accordance with the experimental data.

1 Introduction

Fluid-driven fractures are encountered widely in the stimulation of petroleum, gas, and geothermal reservoirs, remediation of soil and groundwater aquifers, injection of wastes, sequestration of CO₂, and measurement of in-situ stresses by hydraulic fracturing, as well as in the transport of magma in geosystems [1-3]. The formation of fluid-driven fractures includes nucleation, growth, and termination in numerous rock types and stress regimes at scales ranging from several microns to many kilometres. In this process, fluid is pumped into the formation at a rate greater than can be absorbed by the rock. The pore fluid pressure in the formation rises, causing the rock to fail, either by developing shear slippage along planes of weakness or by propagating tensile hydraulic fractures. Given the broad application and importance of hydraulic fracturing technology, it is not a surprise that the mechanisms and modelling of the hydraulic fracturing process have been continuously studied since the 1950s, especially in relation to hydraulic fracturing treatment in the petroleum industry. These investigations have addressed various aspects of the problem both analytically and numerically [4-8]. Most research focuses on laboratory experiments, field tests, and theoretical analysis. Only recently has there been a rigorous effort to study the parametric dependence of fluid-driven fracture and the corresponding different limiting regimes. Numerical modelling is required due to the more complex, expensive and time-consuming characteristics of field tests and laboratory experiments. Obtaining proper results by hydraulic fracturing (in petroleum engineering in particular) depends highly on the geometric configuration of the fracture and the stress regime in the subsurface. This is why hydraulic fracturing has been a subject of active research, to clarify the mechanisms of fluid-rock interaction. Three types of fluid-driven fractures are encountered in engineering, as shown in Fig. 1. The geometry of hydraulic fractures is affected by the mechanical, thermal, and chemical conditions of the surrounding host rock.

Mathematical modelling of the hydraulic fracturing process is performed to predict the fracture response to both reservoir and fluid properties, in situ stresses, and the pumping rate. Corresponding models of such fractures may vary in their level of complexity and detailing of the physical processes involved. The

propagation of a plane-strain fracture under conditions of negligible fracturing fluid viscosity has been previously considered [9]. A self-similar solution was developed under the assumption of the dominance of inertial fluid forces for a particular injection. Other investigators [10-11] have assumed that the inertial fluid effects on either fracture propagation or fluid flow in the fracture are negligible and that fluid flow can be modelled by the lubrication theory. A solution to the problem of a penny-shaped hydraulic fracture propagating in an impermeable elastic solid was presented [4]. The solution is restricted to the so-called viscosity-dominated regime, where it can be assumed that the solid has zero toughness. Significant efforts have been oriented towards a more rigorous theoretical analysis of the hydraulic fracturing process. However, the complexity of the problem usually constrains analytical work to idealised or simplified models, which are either one-dimensional or axisymmetric (radial), as shown in Figure 2(a). The numerical simulation of fluid-driven fractures remains a particularly challenging computational problem, despite the significant progress made since the first algorithms were developed in the 1970s [12-13]. The challenge encountered in devising stable and robust algorithms stems from three distinct issues [14] that arise from the particular nature of this problem. Currently, there are many numerical methods used for hydraulic fracturing analysis, among which the finite element method (FEM) is the most widely used. A number of reliable hydraulic fracturing simulators are currently available, including 2D simulators (PROP, Chevron, etc.), pseudo-3D simulators (MFRAC-II, STIMPLAN, ENERFRAC, TRIFRAC, FracproPT, etc.), and 3D simulators (FRACPRO, GOHFER, FRACPAC, TerraFeac, HYFRAC3D, etc.). Recently, the extended finite element method (XFEM) was used to simulate fluid-driven fractures [6]. All of the simulators are based on continuum theory and linear elastic fracture mechanics. Except for the above-mentioned one-dimensional or axisymmetry hypotheses, only single fracture has been analysed, and predictions of the fracture width and length vary remarkably. Challenging difficulties in modelling arise from the non-local character of the fracture response and the highly non-linear coupling between the equations governing fluid flow in the fracture and those that govern rock deformation produced by fluid pressure. There is no existing reliable method to accurately measure fracture geometry during and after the fracturing process due to the complexity of the fracture profile, as shown in Figure 2(b).

Hydraulic fracturing consists of injecting a viscous fluid into a well under high pressure to initiate and propagate a fracture. Granular materials subjected to fluid flow may experience fracture formation and fluid flow location. Despite the many applications of hydraulic fracturing in poorly consolidated granular materials [15], the risk of failure is still high due to the currently limited understanding of this phenomenon. Our review of the literature has shown that no fundamental and systematic research has been conducted to provide physical models of hydraulic fracturing mechanisms, especially in poorly consolidated granular materials where there are no fracture initiation and propagation criteria targeting hydraulic fracturing. Generally, fluid-driven fracture initiation and propagation in brittle solid geomaterials are dominated by tensile failure and are based on conventional fracture mechanics, which assumes non-zero tensile strength exhibited by the material at the fracture tip. However, this may not be the case for fluid-driven fractures in weakly consolidated sediments. Contrary to strongly consolidated formations, poorly consolidated granular materials, both fine and coarse-grained, are incompact at the particle scale, and their strength is effective stress-dependent. Given their friable nature, poorly consolidated granular materials exhibit very low values of Young's modulus and uniaxial compression strength (UCS) and can carry little tensile stress. Poorly consolidated rocks are particularly prone to exhibit problems such as wellbore stability, surface subsidence, sand production, fluid losses, and casing damage. Low mechanical strength, high pore pressures and large leakoff of fluid often make stimulation operations more difficult and less efficient. The problems of reservoir compaction and the associated bedding plane slip have caused severe damage to hundreds of wells around the world [16-17]. Additionally, the created fractures are not always planar, and their geometry and

dimensions may become very difficult to predict, as shown in Fig. 2(b).

It is hypothesised that fluid-driven fracture criteria in granular materials consider tensile failure, shear failure, or a combined tensile-shear failure mode [15, 18-19]. As a result, the fractures show complicated geometrical features such as multiple segments. Segmentation is indeed one of the most often encountered features contributing to the complexity of hydraulic fractures. Recent observations based on geological evidence and laboratory experiments confirm that the occurrence of multisegmented hydraulic fractures is a common phenomenon. However, current hydraulic fracture models presume a single fracture or neglect interactions between multiple fractures that dramatically change hydraulic fracturing parameters such as fracture aperture, length, and net pressure. A new physical model that is applicable to weakly consolidated granular materials must be developed.

Fig. 1: Fluid-driven fracture models

Fig 2: Sketch of a plane-strain fluid-driven fracture

The goal of this study was to develop a bonded particle simulator for fluid-driven fracturing analysis in weakly consolidated granular materials. In this paper, a discrete element model using PFC3D was built and

calibrated to reproduce the mechanical and hydraulic responses of poorly consolidated granular materials. The hydraulic fracturing process was simulated, and the combined tensile-shear failure criterion was obtained and evaluated. It is concluded that in the case of weakly consolidated granular materials, shear failure seems to be more important than tensile failure during the hydraulic fracturing process, which is not compatible with what has been traditionally accepted in the oil and gas industry, i.e., that tensile failure occurs during the hydraulic fracturing process.

2 Bonded-particle method and hydro-mechanical coupling

2.1 Bonded-particle model

The bonded-particle model (BPM) was proposed by Potyondy and Cundall [20] for modelling rock behaviour numerically. In the model, rock is represented by a dense packing of non-uniformly sized circular or spherical particles that are bonded together at their contact points, and their mechanical behaviour is simulated by the distinct element method using the two- and three-dimensional discontinuum programs PFC2D and PFC3D. The microproperties consist of stiffness and strength parameters for the particles and the bonds. Damage is represented explicitly as broken bonds, which form and coalesce into macroscopic fractures when a load is applied. The BPM provides both a scientific tool to investigate the micromechanisms that combine to produce complex macroscopic behaviours and an engineering tool to predict these macroscopic behaviours. The micromechanisms that occur in rock or soil, controlled by the microstructure, are complex and difficult to characterise within the framework of current existing continuum theories. The BPM approximates rocks as cemented granular materials with an inherent length scale that is related to the grain size and provides a synthetic material that can be used to test hypotheses about how the microstructure affects the macroscopic behaviour. It does not impose theoretical assumptions and limitations on material behaviour as do most indirect models, which idealise the rock as an elastic continuum with many elliptical cracks. In the BPM, microfractures interact and coalesce into macroscopic fractures automatically.

In the DEM (PFC2D and PFC3D), the interaction between the particles is treated as a dynamic process, with states of equilibrium developing whenever the internal forces balance. The calculation cycle is a time-stepping algorithm that requires the repeated evaluation of the law of motion, Newton's second law, for each particle, a force-displacement law for each contact, and a permanent updating of the position of every particle. A more detailed description can be found in the software manual [21]. Material is represented by an assembly of particles that are joined together by breakable parallel bonds. A parallel bond approximates the mechanical behaviour of brittle elastic cement joining two bonded particles. Parallel bonds establish an elastic interaction between these particles that acts in parallel with the grain-based portion of the force-displacement behaviour. The bonds can transmit both forces and moments between particles. If the maximum stress exceeds the set strength, tensile or shear, then the parallel bond breaks and it is removed from the model along with its accompanying force, moment and stiffnesses. Thus, the damage occurs by bond breakages, and the studied material transforms from being solid to granular. The BPM mimics the mechanical behaviour of an assembly of grains joined by cement.

A parallel bond is defined by the mechanical behaviour of a finite-sized piece of cementing material deposited between the spheres. This type of contact is capable of transmitting both force and moments. A parallel bond may be visualised as a set of elastic springs with constant normal and shear stiffnesses. These springs are homogeneously distributed over a circular disk located on the contact plane between the particles and centred at the contact point. Parallel bonds are defined by five parameters: normal and shear stiffness [stress/displacement], normal and shear strength [stress], and bond radius. This contact acts in parallel to the contact bonds described in the DEM; thus, both parallel and contact bonds may coexist at the

same contact plane. A graphical representation of a parallel contact bond is shown in Fig. 3.

Fig. 3: Grain-cement system

The total force, $\bar{F}_i$, and moment, $\bar{M}_i$, transmitted through the parallel bond may be defined by the following expressions:

$$\begin{cases} \bar{F}_i = \bar{F}_i^n + \bar{F}_i^s \\ \bar{M}_i = \bar{M}_i^n + \bar{M}_i^s \end{cases} \quad (1)$$

where the superscripts n and s represent the normal and shear components of the force and moment vectors, respectively. Immediately after the parallel bond is formed, PFC3D resets the values of $\bar{F}_i$ and $\bar{M}_i$ to zero; hence, the subsequent force and momentum increments caused by the movement of the particles are calculated by

$$\begin{cases} \Delta \bar{F}_i^n = (-\bar{k}^n A \Delta U^n) n_i \\ \Delta \bar{F}_i^s = (-\bar{k}^s A \Delta U^s) \\ \Delta \bar{M}_i^n = (-\bar{k}^s J \Delta \theta^n) n_i \\ \Delta \bar{M}_i^s = (-\bar{k}^n I \Delta \theta^s) \end{cases} \quad (2)$$

where, A , J , and I denote the area, polar moment of inertia and moment of inertia of the parallel bond cross-section, respectively. These quantities are given by

$$A = \begin{cases} 2\bar{R}, & \text{for PFC2D} \\ \pi \bar{R}^2, & \text{for PFC3D} \end{cases}$$

$$J = \begin{cases} NA, & \text{for PFC2D} \\ \frac{1}{2}\pi\bar{R}^4, & \text{for PFC3D} \end{cases}$$

$$I = \begin{cases} \frac{2}{3}\bar{R}^3, & \text{for PFC2D} \\ \frac{1}{4}\pi\bar{R}^4, & \text{for PFC3D} \end{cases}$$

The new force and moment vectors are calculated as the summation of the old values of the vectors at the beginning of the time step and the vector increments. Thus, the new force and moment vectors are given as follows:

$$\begin{cases} \bar{F}_i^n \leftarrow \bar{F}_i^n n_i + \Delta\bar{F}_i^n \\ \bar{F}_i^s \leftarrow \{\bar{F}_i^s\}_{rot,2} + \Delta\bar{F}_i^s \\ \bar{M}_i^n \leftarrow \bar{M}_i^n n_i + \Delta\bar{M}_i^n \\ \bar{M}_i^s \leftarrow \{\bar{M}_i^s\}_{rot,2} + \Delta\bar{M}_i^s \end{cases} \quad (3)$$

The maximum tensile and shear stresses acting on the bond boundaries are derived from the beam theory and are calculated as follows:

$$\begin{aligned} \sigma_{\max} &= \frac{-\bar{F}^n}{A} + \frac{|\bar{M}_i^s|}{I} \bar{R} \\ \tau_{\max} &= \frac{|\bar{F}_i^s|}{A} + \frac{\bar{M}_i^n}{J} \bar{R} \end{aligned} \quad (4)$$

A detailed description of the PFC2D and PFC3D models for rock is given by [Potyondy & Cundall \[20\]](#).

2.2 Hydro-mechanical coupling

As mentioned in Section 1, fluid-driven fracturing is a hydro-mechanical coupling process. In PFC2D and PFC3D, fluid flow through porous media is modelled as a flow occurring between “pipes” connecting neighbouring pores. These pores are the void spaces inside a “domain”. A domain is created by every three (PFC2D) or four (PFC3D) neighbouring particles, such that each one of them is the vertex of a triangle (PFC2D) or tetrahedron (PFC3D) [22]. In the case of poorly consolidated granular materials, the rock particles may be acceptably described as spheres in PFC3D. By the same token, the rock pores may be represented as the void space between the spheres. Each pipe between two contiguous domains has a small space surrounded by three neighbouring particles. PFC3D models these pipes as cylindrical tubes with length l and aperture a [21]. Thus, the flow rate may be defined by the following equation:

$$q = \frac{\pi a^4 \Delta p}{128 \mu l} = \frac{k a^3 \Delta p}{l} \quad (5)$$

where q denotes pipe flow rate, μ denotes the fluid viscosity, $\Delta p = P_2 - P_1$ is the pressure difference between two adjacent domains, l and a denote the distance between the centres of two

domains and pipe aperture, respectively, and $k = \pi a / 16 \mu$ is the conductivity factor.

The pipe aperture is a function of the load applied to the two adjacent domains. It varies between a_0 when particles comprising the pipe are bonded but take no load and 0 as the force tends to infinity. Thus, the following empirical equation is used in PFC3D to calculate the magnitude of the pipe aperture as a function of the compressive load:

$$a = \frac{a_0 F_0}{F + F_0} \quad (6)$$

where F_0 is the value of the normal compressive force, F , at which the aperture decreases to $a_0/2$.

For the case of normal tensile force, the aperture is computed as the sum of the residual aperture and the normal distance between the surfaces of the two particles, g :

$$a = a_0 + mg \quad (7)$$

where g is the normal distance or gap between the surfaces of the two particles, and m is a dimensionless scaling multiplier. By default, $m = 1$. For calibration purposes, m can be set to different values.

Every time step, the flow rate received or produced by every domain, Σq , is calculated, and the consequent pressure disturbance Δp is calculated as

$$\Delta p = \frac{K_f}{V_d} (\Sigma q \Delta t - \Delta V_d) \quad (8)$$

where K_f represents the fluid bulk modulus, V_d is the apparent volume of the domain. The solution scheme alternates between applying the flow equation to all pipes and applying the pressure equation to all domains.

Consider a pressure perturbation Δp_p in a single domain; the flow into or out of that domain may be computed as:

$$q = N \frac{ka^3 \Delta p_p}{\bar{l}} \quad (9)$$

where N is the number of pipes connected to the domain, and $\bar{l}$ is the average distance between the centres of the domain being evaluated and all of its immediate neighbours. This fluid flow triggers a pressure response, Δp_r , defined as

$$\Delta p_r = \frac{K_f}{V_d} (\Sigma q \Delta t - \Delta V_d) = \frac{K_f q t}{V_d} \quad (10)$$

For stability, the pressure response must be less than the original pressure perturbation (i.e., $\Delta p_r \leq \Delta p_p$).

Hence, from Eq. (9) and Eq. (10)

$$\Delta t \leq \frac{\bar{l}V_d}{NK_fka^3}$$

Three forms of coupling between the fluid and the solid particles are involved in the above-mentioned process. First, changes in the aperture are controlled by contact opening and closing or changes in contact force. Second, mechanical changes in domain volumes cause changes in domain fluid pressures. Third, the domain pressure exerts tractions on the enclosing particles, and the force vector on a typical particle is described as

$$F_i = pn_i s \quad (11)$$

where n_i is the unit normal vector of the line joining the centres of a domain and a particle, and s is the projected area of a particle determined by the three contacting points between the particle and three neighbouring particles, which comprise a domain.

3 Bonded-particle modelling of fluid-driven fractures

In this section, the BPM was utilised for modelling fluid-driven fractures. The models were built to describe both the mechanical and the hydraulic properties of poorly consolidated granular materials. For continuum models, the input properties (such as modulus and strength) can be derived directly from measurements performed on laboratory specimens. For the BPM, which synthesises macroscale material behaviour from the interactions between microscale components, the input properties of the components are usually not known. Generally, the validity of the BPM is determined by a calibration process that involves comparing model behaviour with measured and observed responses of a real material at both laboratory and field scales. In this study, parameters such as the elastic modulus, Poisson's ratio, strength, and the shape of the stress-strain curve were duplicated to validate the results produced during the simulation runs. Using this method, the appropriate microproperties were chosen to simulate fluid-driven fractures.

To understand hydraulic fracturing in low-cohesive or cohesionless particulate materials, a series of laboratory experiments were recently conducted by Chang [15]. In this study, Chang's tests, in which silica flour was selected for the hydraulic fracturing experiments, were referred to in building the models and analysing the fluid-driven fracturing process. A schematic of the experimental setup and sketch of the experimental steps are shown in Fig. 4. The grain size distribution curve of the silica flour is shown in Fig. 5. Direct shear tests were conducted to determine the mechanical properties of the granular material made of silica flour. The experimental process was duplicated numerically using the BPM; therefore, the microproperties of the specimen were determined to be the input parameters for bonded-particle modelling. The input parameters are listed in Table 1. Sketches of the testing apparatus and the relationship between shear stress and shear (horizontal) displacement are shown in Fig. 6 and Fig. 7, respectively.

Fig. 6: Sketch of direct shear test apparatus (from Reference [15])

Fig. 7: Relationship of shear stress versus horizontal displacement for silica flour granular material

Table 1. Model microproperties determined from the specimen of Chang's test

Parameters	Values
Density of grain [kg/m3]	1680
Grain maximum diameter/Grain minimum diameter [m]	0.025/0.015
Young's modulus of grains [GPa]	4.8
Young's modulus of cement [GPa]	5.6
Normal stiffness of grains [GPa]	175
Shear stiffness of grains [GPa]	70
Normal stiffness of grains [GPa]	128
Shear stiffness of grains [GPa]	51.5
Grain friction coefficient	0.5
Tensile strength of cement [MPa]	0.57
Shear strength of cement [MPa]	0.62

Two types of models were built to describe the mechanical and hydraulic properties described in this section, as shown in Fig. 8. Using these models, the Darcy's seepage test was conducted numerically to validate the "pipe" and "domain" structures (shown in Fig. 9) in the model. Fig. 10 shows the pressure distribution of fluid for the numerical Darcy's test; the fluid enters from the bottom boundary and flows out from the top boundary. The fluid-driven fracturing process based on Chang's test [15] was simulated using these models. The fluid pressure of the injection point and the fluid-driven microfractures are shown in Fig. 11, and the injection pressure curves for different injection volumes were obtained and compared with the experimental curves, as shown in Fig. 12. The numerical analysis results agree well with the experimental results. The number of fluid-driven microfractures with different failure criteria was also calculated in the model, as shown in Fig. 13. This made it clear that tensile and shear combined microfractures occur when

hydraulic fracturing process is conducted in weakly consolidated granular materials. Moreover, it seems that the shear microfractures are dominant.

Fig.8: Bonded-particle models

Fig. 9: Pipe and domains

Fig. 10: Pressure distribution in Darcy’s numerical test

Fig. 11: Fluid-driven microfractures

Fig. 12: Injection pressure curves for different injection volumes: (a) 200 ml and (b) 350 ml

Fig. 13: Number of fluid-driven microfractures over time (injection fluid rate = constant)

Fig. 14: Fluid-driven microfractures over time (injection pressure = constant)

Additional numerical tests were conducted to study the effect of the particle size distribution on fluid-driven fractures. By using the same shape parameters (maximum particle size and minimum particle size) and microproperty parameters (both mechanical and hydraulic parameters) but different particle size distributions, the fluid-driven microfractures were calculated, and the results are shown in Fig. 14. It is concluded that the particle size distribution has a remarkable effect on the incidence of fluid-driven fractures. In this case, the number of fluid-driven fractures with a normal distribution of particle size in BPM is obviously larger than that with a uniform distribution of particle size.

Fig. 15: Injection pressure distribution over time determined by field hydraulic fracturing stress measurement and BPM numerical simulation

4 Conclusions

The fluid-driven fracture initiation and propagation process in granular materials is inherently a coupled hydro-mechanical problem. In this paper, the bonded-particle method was utilised to simulate the hydraulic fracturing process in granular materials. Other researcher's test results were used to obtain the microproperties or the input parameters in the BPM. The injection pressure distribution over time was computed and compared with the experimental data; the computed results were observed to agree well with the experimental results. The number of fluid-driven fractures was calculated by numerical hydraulic fracturing tests. This showed that the shear-tension combined failure mode occurred during hydraulic fracturing and that the failure mechanism dominating the hydraulic fracturing process in weakly consolidated granular materials is shear rather than tension. The hydraulic fracture fronts created in weakly consolidated granular materials have different characteristics compared to those in brittle solids. Moreover, the particle size distribution has a remarkable effect on the incidence of fluid-driven fractures. The number of fluid-driven fractures with normal distributions of particle size in BPM is clearly greater than the number of fractures with uniform distributions of particle size.

Based on the numerical hydraulic fracturing tests discussed in this paper, hydraulic fracturing in weakly consolidated granular materials was observed to be possible if fluid leak-off was minimised. The BPM approximates the mechanical behaviour of rock by representing it as a cemented granular material. The model was used to predict the incidence of fluid-driven fractures and formations adjacent to the hydraulic fracturing process in this study. Additionally, field hydraulic fracturing has recently been widely used to measure the in situ stress [23]. In this study, the BPM was also utilised to simulate fluid-driven fractures in brittle rock, as shown in Fig. 15. In this case, field hydraulic fracturing measurements were conducted in Yimeng District, Shandong Province of China. We chose the injection pressure curve, which was measured

between 437.97 m and 438.62 m beneath the ground surface, to be the figure of merit. Both linear elastic fracture mechanics (LEFM) and BPM were adopted to simulate the hydraulic fracturing process. The LEFM was used to invert the different formation macroparameters and was applied to study the microproperties of the BPM. Fig. 15 shows the associated calculated injection pressure data and field test curve. The discrete calculated data are in accordance with the field test data.

References

- [1] Berumen, S., D. Tiab, and F. Rodriguez. 2000. Constant rate solutions for a fractured well with an asymmetric fracture. *J. Petrol. Sci. Eng.* 25: 49-58.
- [2] Murdoch, L.C., and W.W. Slack. 2002. Forms of hydraulic fractures in shallow fine grained formations. *J. Geotech. Geoenviron.* 128: 479-487.
- [3] Raaen, A.M., E. Skomedal, H. Kjørholt, P. Markestad, and D. Okland. 2001. Stress determination from hydraulic fracturing tests: The system stiffness approach. *Int. J. Rock Mech. Min. Sci.* 38: 529-541.
- [4] Savitski, A.A., E. Detournay. 2002. Propagation of a fluid-driven penny-shaped fracture in an impermeable rock: Asymptotic solutions. *Int. J. Solids Struct.* 39: 6311-6337.
- [5] Adachi, J., and E. Detournay. 2008. Plane strain propagation of a hydraulic fracture in a permeable rock. *Eng. Fract. Mech.* 75: 4666-4694.
- [6] Ren, Q.W., Y.W. Dong, and T.T. YU. 2009. Numerical modeling of concrete hydraulic fracturing with extended finite element method, *Sci China Ser E-Tech Sci.* 52: 559-565.
- [7] Adachia, J., E. Siebritsb, A. Peircec, and J. Desroches. 2007. Computer simulation of hydraulic fractures. *Int. J. Rock Mech. Min. Sci.* 44:739-757.
- [8] Shin, H., and J.C. Santamarina. 2010. Fluid-driven fractures in uncemented sediments: Underlying particle-level processes. *Earth and Planetary Science Letters.* 299: 180-189.
- [9] Huang, N, A. Szewczyk, and Y. Li. 1990. Self-similar solution in problems of hydraulic fracturing. *ASME J Appl Mech.* 57:877-81.
- [10] Nilson, R.H. 1981. Gas driven fracture propagation. *ASME J Appl Mech.* 48:757-62.
- [11] Spence, D.A., P.W. Sharp. 1985. Self-similar solution for elastohydrodynamic cavity flow. *Proc Roy Soc London, Ser A.* 400:289-313.
- [12] Clifton, R.J., and A.S. Abou-Sayed. 1979. On the computation of the three dimensional geometry of hydraulic fractures, in: *Proceeding of SPE Symposium. On Low Permeability Gas Reservoirs*, eds. Richardson, T.X. ,307-313. Denver, Society of Petroleum Engineers (SPE 7943).
- [13] Clifton, R.J. 1989. Three-dimensional fracture-propagation models, in *proceeding of Recent Advances in Hydraulic Fracturing - SPE Monographs*, vol. 12, eds. J.L. Gidley, S.A. Holditch, D.E. Nierode, R.W. ,95-108, Veatch Society of Petroleum Engineers.
- [14] Lecampion, B., E. Detournay. 2007. An implicit algorithm for the propagation of a hydraulic fracture with a fluid lag. *Comput. Methods Appl. Mech. Engrg.* 196: 4863-4880.
- [15] Chang, H. 2004. Hydraulic fracturing in particulate materials, Ph.D. thesis, 267 pp., Georgia Institute of Technology, GA.
- [16] Bruno, M.S. 2002. Geomechanical and Decision Analyses for Mitigating Compaction-Related Casing Damage. *SPE Drilling & Completion*, 17: 179-188.
- [17] Zhang, X, N. Koutsabeloulis, and K. Heffer. 2007. Hydromechanical modeling of critically stressed and faulted reservoirs. *AAPG Bulletin.* 91: 31-50.
- [18] Soga, K., K.O. Gafar, M.Y.A. Ng, and S.K.A. Au. 2006. Macro and micro behaviour of soil fracturing. *Proceedings of the International Symposium on Geomechanics and Geotechnics of Particulate Media.* 421-427, Taylor and Francis, Balkema, Ube, Yamaguchi, Japan.

- [19] Alfaro, M.C., R.C.K. Wong. 2001. Laboratory studies on fracturing of low-permeability soils. *Can. Geotech. J.* 38: 303-315.
- [20] Potyondy, D.O., and P.A. Cundall. 2004. A bonded-particle model for rock. *Int.J.Rock Mech.Min.Sci.* 41: 1329-1364.
- [21] Itasca Consulting Group. 2004. PFC3D manual, Documentation provided with PFC3D V. 3.00.
- [22] Li, L., and R.M. Holt. 2002. Development of discrete particle modeling towards a numerical laboratory. In: *Numerical Modeling in Micromechanics via Particle Methods*. ed., H. Konietzky. 19-27. A.A. Balkema Publishers.
- [23] Raaen, A.M., E. Skomedal, H. Kjørholt, P. Markestad, and D. Okland. 2001. Stress determination from hydraulic fracturing tests: The system stiffness approach. *Int. J. Rock Mech. Min. Sci.* 38: 529-541.