Failure analysis of an infinite unsaturated soil slope

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Increases in rainfall and groundwater level rising can cause more frequent failures of unsaturated soil slopes, resulting in large-scale landslides. The aim of this paper is to propose a methodology for prediction of the failure of an infinite soil slope subject to steady unsaturated flow conditions. Probabilistic models for soil porosity, friction angle, matric suction and saturation are developed. The probability of slope failure for different groundwater levels and infiltration (or evaporation) intensities is investigated. Numerical results show that, for a slope with deep groundwater, the slope failure is mainly controlled by the rainfall infiltration, whereas for shallow groundwater, the slope failure is governed by the location of the groundwater level. A merit of the proposed methodology is that the fluctuation of the soil porosity along the depth of a slope is modelled as a Gaussian random field. The paper concludes that the proposed methodology can be used for prediction of slope failures under steady unsaturated flow conditions. Accurate prediction of slope failures can prevent catastrophic consequences of landslides.

Notation

c	cohesion at saturated state
e	void ratio
$f(y)$	safety indicator for plane at specific location y
G	limit state function
G_s	specific gravity of soil
h	numerical integration element size/element size of random field
h_w	hydraulic head
K_r	relative hydraulic conductivity
K_s	soil's saturated hydraulic conductivity
K_{ψ}	unsaturated hydraulic conductivity
K_0	saturated hydraulic conductivity of soil at surface
k	fitting parameter for shear strength criterion (Fredlund <i>et al.</i> , 1996)
l	depth of phreatic surface
m	fitting parameter for Gardner's equation
N	total number of samples for Monte Carlo simulations
n	soil porosity
$\bar{n}$	trend function for soil porosity
n_0	porosity of soil at ground surface
P_f	probability of slope failure
q_w	infiltration intensity of rainfall in a steady state
S	degree of saturation
$\mathbf{x}$	vector consisting of all the variables
y	depth of plane
y_w	parameter related to depth of weathering zone
y^*	thickness of soil layer
α	parameter in the van Genuchten (VG) model

β	parameter in VG model
γ	bulk unit weight of soil
γ_w	unit weight of water
δ_y	scale of fluctuation
η_w	normalised infiltration intensity ($= q_w/K_0$)
θ	slope angle
λ	power reduction equation
μ	mean value for variables involved in Monte Carlo simulations
μ_n	mean value for porosity profile
ρ_{ij}	correlation function for porosity fluctuation
σ	standard deviation for variables involved in Monte Carlo simulations
σ_n	standard deviation for porosity profile
σ_v	vertical overburden stress
τ	shear stress at the plane
$\bar{\tau}$	shear strength at the plane
ϕ	friction angle
ϕ_0	friction angle of soil at ground surface
φ	parameter in equation proposed by Gallipoli <i>et al.</i> (2003)
ω_n	pattern of fluctuation of soil porosity along the depth

1. Introduction

Rainfall infiltration and rise of groundwater level can cause the decrease of matric suction (ψ) and increase of degree of saturation (S), which can result in failures of unsaturated soil slopes. This is because the suction loss can severely weaken the

shear strength of unsaturated soils and the increase of degree of saturation can change the shear stress distribution along the depth of a slope owing to the increase of the average bulk unit weight (γ) of soil. Moreover, the shear strength of the soil is affected by the location of the phreatic surface and the rainfall intensity.

Various attempts have been made to estimate the safety of a slope subject to unsaturated flows, such as Iverson (2000); Cho and Lee (2002); Lu and Godt (2008); Ray *et al.* (2010); Travis *et al.* (2010); Li *et al.* (2013); Zhu *et al.* (2013) and so on. In general, most methods used for slope stability analyses with unsaturated steady seepages are deterministic, with the soil's mechanical and hydraulic properties treated as constants for a given soil layer. For example, Lu and Godt (2008) presented a framework for stability analysis of infinite slopes under steady unsaturated seepage conditions, considering a varying groundwater level and the variation of soil suction and moisture content above the groundwater level under steady infiltration conditions. It has been recently recognised that predicting the safety of slopes considering rainfall and groundwater level change inevitably involves uncertainties. For example, all the mechanical and hydraulic parameters of soils are determined by regression processes, based on experimental results, the variability of which cannot be ignored. Also, soil in the field cannot be ideally homogeneous along the depth, even if there is only one soil type. In other words, soil properties also exhibit spatial variability. For example, the porosity (n) of the surface soil is usually greater than that of the undisturbed soil buried deeper than the groundwater level. Studies have shown that the porosity of weathered soil typically varies from 0.45 to 0.30 in the upper 10 m of the soil profile (Cornforth, 2005; Selby, 1993). As such, the friction angle of the deep soil is usually greater than that of the surface soil (Cornforth, 2005; Lu and Godt, 2008; Rowe, 1969). Experimental evidence (e.g. Cornforth, 2005; Rowe, 1969) shows that the friction angle for a given sandy soil can vary from 30° to 55° and is inversely linearly proportional to its porosity. Mitchell *et al.* (1972) also showed the inversely linearly proportional relationship between the friction angle and its porosity for a silty fine sand (a lunar soil simulant). Lu and Godt (2008) adopted such an inversely linearly proportional relationship between the friction angle and its porosity for their study on infinite slope safety. In their study, the friction angle for sandy silt, silt and loess varies from 33° to 48°, owing to porosity change. In addition, according to previous investigation (Phoon and Kulhawy, 1999; Vanmarcke, 1977), soil properties do not change monotonically along the depth but can be fluctuant owing to the localised non-homogeneity.

Because of the uncertainty involved in predicting the slope safety, probabilistic methods for prediction of slope failure in the context of unsaturated conditions have been highlighted. The random finite-element method (RFEM) proposed by Griffiths and Fenton (1993) can generally be used to consider

the uncertainty involved in all aspects of geotechnical engineering. It has been successfully applied to slope stability assessment, bearing capacity estimation and retaining wall design (Fenton and Griffiths, 2008). However, in the case of unsaturated conditions, RFEM can be very complicated because it involves implementation of hydro-mechanical coupled constitutive models for unsaturated soils, which requires complicated numerical techniques (Sheng, 2011) with inevitable low computational efficiency. It can also be very time-consuming, especially if the RFEM has to be combined with the Monte Carlo method to estimate the probability of slope failure.

Uncertainty can also be incorporated into analytical and semi-analytical models to avoid the employment of finite-element analysis in predicting the probability of failure for slopes subject to unsaturated flows. For example, a number of probabilistic stability analyses (Cho, 2014; Santoso *et al.*, 2011; Zhang *et al.*, 2014) have been conducted for infinite slopes under rainfall infiltration considering the spatial variability of soil's saturated hydraulic conductivity (K_s). These analyses focus on the slope safety varying with the time under rainfall. However, few attempts have been made to study the joint effects of rainfall and varying groundwater level. In addition, in terms of soil properties, most probabilistic stability analyses only take into account the spatial variability of K_s , but not soil porosity. Although K_s plays an important role in calculating profiles of suction and saturation in an unsaturated slope, soil porosity is more fundamental. This is because all key factors in unsaturated slope analysis, such as hydraulic conductivity (K_s), water retention behaviour ($S-\psi$) and friction angle (ϕ), can be a function of soil porosity for a given soil type.

In this paper an attempt is made to propose a methodology for prediction of the reliability of an infinite soil slope subject to steady unsaturated flow conditions. In this methodology the fluctuation of the soil porosity (n) along the depth (y) is modelled as a Gaussian random field. The unsaturated Darcy's law, where the saturated hydraulic conductivity (K_s) is assumed to be a function of soil porosity (Carrier, 2003), is employed to calculate the suction distribution profile due to steady unsaturated flows related to rainfall and groundwater. A porosity-dependent water retention equation (Gallipoli *et al.*, 2003; Zhou *et al.*, 2012) is applied to calculate the saturation profile. Based on a comparative study in the paper by Sheng *et al.* (2011), the shear strength criterion proposed by Fredlund *et al.* (1996) is adopted to estimate the shear strength for unsaturated soils, in which the friction angle is also a function of soil porosity (Lu and Godt, 2008). Finally, the Monte Carlo method is used to estimate the probability of slope failure and the most unsafe location for an infinite unsaturated slope.

2. Formulation of slope failure

Consider an infinite slope with angle θ (see Figure 1). According to Iverson (2000) and Travis *et al.* (2010), all forces within an infinite slope can be assumed to vary only in the

Figure 1. An infinite slope subject to steady unsaturated flows

direction perpendicular to the slope surface and the phreatic surface can be assumed to lie parallel to the slope surface at a known depth (l). As shown in Figure 1, q_w is the infiltration intensity of rainfall in a steady state, which is perpendicular to the slope surface. The value of q_w is set to be negative for steady wetting (rainfall infiltration) and positive for steady drying (evaporation).

For an infinite slope, as shown in Figure 1, the failure criterion for an arbitrary plane (y) is that the shear stress at the plane exceeds the shear strength at the plane. In the risk analysis of slope failure, this criterion can be expressed in the form of a limit state function (G) as follows

$$1. \quad G(\mathbf{x}, y) = \bar{\tau}(\mathbf{x}, y) - \tau(\mathbf{x}, y) \quad (y \in [0, y^*])$$

where $\bar{\tau}$ is the shear strength at the plane, τ is the shear stress at the plane, y is depth of the plane and the thickness of soil layer is y^* . In this research, y^* is set to be 10 m as a demonstration. $\mathbf{x}$ is the vector consisting of all the variables involved in calculating $\bar{\tau}$ and τ . In the failure analysis, some or all of the variables ($\mathbf{x}$) can be assumed as random variables that follow

some probability distributions, such as normal distribution. The variables in vector $\mathbf{x}$ and their values will be discussed later in Section 4.

The shear stress (τ) at an arbitrary plane is caused by the vertical overburden stress (σ_v) induced by the self-weight of soil mass and pore water. The shear stress for an arbitrary plane at depth y can be written as

$$2. \quad \tau(y) = \sigma_v(y) \sin \theta \cos \theta$$

where the total vertical overburden stress (σ_v) can be calculated by integrating bulk unit weight (γ) from the plane to the ground surface as follows

$$3. \quad \sigma_v(y) = \int_0^y \gamma dy = \gamma_w \int_0^y [G_s - G_s n(y) + S(y)n(y)] dy$$

where γ_w is the unit weight of water ($=9.8 \text{ kN/m}^3$), G_s is the specific gravity that is usually a constant for soils ($2.65 \sim 2.7$ for different soil types), n is the soil porosity and S is the degree of saturation. Both soil porosity (n) and degree of saturation (S) are functions of the depth (y). Therefore, $n(y)$ and $S(y)$ are employed to represent the porosity profile and saturation profile along the depth, respectively. Owing to the high non-linearity of $n(y)$ and $S(y)$, Equation 3 usually cannot be integrated analytically. Here, the midpoint method is employed to conduct the numerical integration for Equation 3. The local truncation error for the midpoint method is in the order of $O(h^3)$, where h is the numerical integration element size. In this paper, h is set to be 10^{-3} . The midpoint method is also known as the modified Euler method, the details of which can be found in *Numerical Methods for Engineers* by Griffiths and Smith (1991).

For the shear strength ($\bar{\tau}$) of unsaturated soils, a number of criteria have been proposed to describe the shear strength variation due to suction change as well as saturation change. Sheng *et al.* (2011) undertook a comparative study on eight widely used shear strength criteria for unsaturated soils and concluded that the simple criterion proposed by Fredlund *et al.* (1996) predicts unsaturated soil strength reasonably well. Therefore, the strength criterion proposed by Fredlund *et al.* (1996) is employed here to determine the shear strength for an arbitrary plane in the infinite slope

$$4. \quad \bar{\tau}(y) = \left\{ \sigma_v(y) \cos^2 \theta + [S(y)]^k \psi(y) \right\} \tan \phi(y) + c$$

where k is the fitting parameter for the shear strength criterion, ψ is the matric suction, ϕ is the friction angle and c is soil cohesion at the fully saturated state (assumed to be zero here). $\psi(y)$ and $\phi(y)$ are profiles of matric suction and friction angle along the depth, respectively. According to Sheng *et al.* (2011),

parameter k depends on the soil type, the value of which is usually greater than one. For example, for compacted kaolin clay (Thu *et al.*, 2007), k is calibrated to 1.3 and for Madrid Gray clay (Escario and Juca, 1989), k is calibrated to 2.8.

Therefore, the limit state function of an arbitrary plane (y) can be specified by combining Equations 2–4 as follows

$$5. \quad G(\mathbf{x}, y) = \gamma_w (\cos^2 \theta \tan \phi(y) - \sin \theta \cos \theta) \\ \left\{ \int_0^y [G_s - G_s n(y) + S(y)n(y)] dy \right\} \\ + [S(y)]^k \psi(y) \tan \phi(y)$$

From Equation 5, it can be seen that the limit state function for an arbitrary plane in the soil slope involves four depth-dependent profiles, namely, the profile of soil porosity, $n(y)$, the profile of friction angle, $\phi(y)$, the profile of matric suction, $\psi(y)$, and the profile of degree of saturation, $S(y)$, all of which will be discussed in Section 3.

For a specific plane at the depth of y ($y \in [0, y^*]$), the safety of this plane can be defined using the limit state function of Equation 1

$$6. \quad f(y) = \begin{cases} 0 & \text{if } G(\mathbf{x}, y) \geq 0 \\ 1 & \text{if } G(\mathbf{x}, y) < 0 \end{cases}, \quad \text{for a given } y$$

$f(y)$ is the safety indicator for the plane at a specific location y . $f(y) = 1$ stands for the failure of plane y and it is safe when $f(y) = 0$. A failure of any plane in the slope leads to the failure of slope. The probability of slope failure of a plane at y depth (i.e. $P_f(y)$) can also be calculated by the Monte Carlo method

$$7. \quad P_f(y) = p[f(y) = 1] \approx \frac{1}{N} \sum_{i=1}^N f_i(y)$$

where N is the total number of samples (i.e. the number of Monte Carlo simulations), $f_i(y)$ represents the value of f (see Equation 6) at y depth when the i th sample of $\mathbf{x}$ is used for a simulation. $P_f(y)$ is function of y which varies between 0 and y^* . Therefore, the most unsafe location ($\tilde{y}$) can be defined as the plane with the maximum value of $P_f(y)$ along the depth, that is, $\tilde{y} = y|_{P_f=\max}$.

3. Models of soil properties

3.1 Profile of soil porosity, $n(y)$

Soil that forms slopes is typically the result of chemical weathering, and mechanical transport and consolidation. In one aspect, the chemical weathering that results in the reduction of soil becomes more distinct with the decrease in depth because

soil has more contact with destructive agents, such as the atmosphere, rainfall and temperature variation. In the other aspect, the consolidation pressure caused by soil self-weight is gradually decreasing with the decreasing depth. Consequently, for soils with the same chemical compositions, soil porosity at the ground surface is typically greater than that at deeper locations (Lu and Godt, 2008). Studies have shown that the porosity of weathered soil typically varies from 0.45 to 0.30 in the upper 10 m of the soil profile (Cornforth, 2005; Selby, 1993). Lu and Godt (2008) suggested a power reduction equation (λ) to model the reduction of porosity along the depth

$$8. \quad n(y) = n_0 - \lambda \Delta n, \quad \lambda = [1 + (y_w/y)]^{-1}$$

where n_0 is the porosity of soil at the ground surface, λ represents the power reduction equation, Δn is the total change of soil porosity when y changes from 0 to ∞ , and y_w is a parameter related to the depth of the weathering zone. In the weathering zone, the porosity change of the soil along the depth is more prominent. The purpose of including a parameter for the weathering zone (i.e. y_w) is to consider the non-linear change of soil porosity along the depth. For a given total change of porosity (e.g. Δn), different y_w values lead to different patterns of soil porosity profile. Equation 8 can be applied to any slope analysis model if the non-linear decrease of soil porosity along the depth needs to be considered. Besides, it can be replaced by either a linear equation or more complex non-linear equations, depending on the borehole testing results and the precision requirement of the curve-fitting. Equation 8 is adopted in this research, but revised by considering the thickness of the soil layer to be y^* rather than infinite, and then employed here as the trend function for soil porosity ($\tilde{n}$). As shown in Figure 2(a), the soil porosity profile along the depth considering spatial variation can be expressed as

$$9. \quad \begin{cases} n(y) = \tilde{n}(y) + \omega_n(y) \\ \tilde{n}(y) = n_0 - \lambda \Delta n, \quad \lambda = \frac{1 + (y_w/y^*)}{1 + (y_w/y)} \end{cases}$$

where Δn is redefined as the total change of soil porosity between the ground surface and depth y^* and $\omega_n(y)$ is the pattern of fluctuation of the soil porosity along the depth, which can be modelled by a Gaussian random field through setting a mean ($\mu_n = 0$), a standard deviation (σ_n) and a proper correlation function (ρ_{ij}). The pattern of fluctuation, that is, $\omega_n(y)$, is assumed to be unchanged for all samples of Monte Carlo simulation. In this research, the covariance matrix decomposition method (Cholesky decomposition) is employed to generate a random field (Fenton and Griffiths, 2008) and the correlation function, following Griffiths *et al.* (2011), is assumed as

$$10. \quad \rho_{ij} = \exp\left(-2 \frac{|y_i - y_j|}{\delta_y}\right)$$

Figure 2. Modelling spatial variability of soil porosity through random fields: (a) schematic diagram; (b) pseudo-program

in which y_i and y_j are depths of two points within depth y^* (i.e. $0 \leq y_i \leq y^*$, $0 \leq y_j \leq y^*$), and δ_y is the scale of fluctuation. Within the scale of fluctuation, values of porosity at two points show correlation, the tendency of which decreases with the distance between them. Porosities at any two points separated by distances beyond the scale of fluctuation have little correlation and behave independently. $h = y_{i+1} - y_i$ is the element size of the random field, which can be set to be identical to the numerical integration element size ($= 10^{-3}$). The pseudo-program for modelling the profile of soil porosity by a Gaussian random field is also presented in Figure 2(b).

Figure 3(a) shows the calculated results of Equation 9 with different values of scale of fluctuation ($\delta_y = 1.0, 10$ and 100 m). As shown in Figure 3(a), with a constant σ_n value ($= 0.006$ in this case), the predicted distribution of soil porosity fluctuates more if the scale of fluctuation decreases. The fluctuation pattern is also changed when the scale of fluctuation is different. When the scale of fluctuation is set to infinite, the predicted distribution curve is identical to the curve calculated by the trend function (see the dashed curve in Figure 3(a)). Figure 3(b) shows five different soil porosity profiles (i.e. profiles I, II, III, IV and V). Profile I shows constant porosity ($n_0 = 0.6$, $\Delta n = 0.0$) and profile II is the porosity profile represented by the trend function ($n_0 = 0.6$, $\Delta n = 0.1$, $y_w = 2$ m) with zero fluctuation ($\sigma_n = 0$). Profiles III, IV and V show the simulation results of Equation 9 with different values of σ_n ($= 0.006, 0.018$ and 0.03) at the same scale of fluctuation ($\delta_y = 1.0$ m) and trend function ($n_0 = 0.6$, $\Delta n = 0.1$, $y_w = 2$ m). Comparing the three different curves with different values of

σ_n indicates that greater σ_n leads to a larger amplitude of fluctuation.

3.2 Profile of friction angle, $\phi(y)$

Experimental evidence (Cornforth, 2005; Rowe, 1969) shows that the friction angle (ϕ) for a given soil can vary from 30° to 55° and is inversely linearly proportional to its porosity (n). Lu and Godt (2008) suggested the following linear relationship between the friction angle (ϕ) and porosity (n), which will be adopted directly in this paper

$$11. \quad \phi(y) = \phi_0 - \frac{\Delta\phi}{\Delta n} [n(y) - n_0]$$

where ϕ_0 is the friction angle of soil at the ground surface (i.e. $n = n_0$). $\Delta\phi$ stands for the change of friction angle when the porosity change is Δn . The friction angle is independent of porosity if $\Delta\phi = 0$.

3.3 Profile of matric suction, $\psi(y)$

According to Iverson (2000), the suction profile is the same as the distribution of the negative pore water pressure when $y \geq l$ (i.e. beneath the groundwater table)

$$12. \quad \psi(y) = -\gamma_w(y - l) \cos \theta \quad (y \geq l)$$

When $y < l$, the suction profile can be calculated by way of the extended Darcy's law, which can be applied to the steady-state

Figure 3. Soil porosity profiles simulated by way of random fields:
(a) simulated porosity profile with different scales of fluctuation;
(b) simulated porosity profile with different standard deviations

one-dimensional unsaturated conditions (Lu and Godt, 2008; Lu and Griffiths, 2004; Travis *et al.*, 2010). The extended Darcy's law can be written as

$$13. \quad K_\psi \frac{dh_w}{dy} = q_w$$

where h_w is the hydraulic head. According to Iverson (2000), $h_w = -\psi/\gamma_w - y \cos \theta$. q_w is the infiltration intensity of rainfall that is perpendicular to the slope surface. The value of q_w is set to be negative for infiltration and positive for evaporation. K_ψ is the unsaturated hydraulic conductivity at depth y . $K_\psi = K_s K_r$, in which K_s is the hydraulic conductivity for saturated soils and it is a function of soil porosity (n)

$$14. \quad K_s \propto \frac{e^3}{1+e} \Leftrightarrow K_s = K_0 \frac{n^3}{(1-n)^2} \frac{(1-n_0)^2}{n_0^3}$$

where K_0 is the hydraulic conductivity when soil porosity is equal to n_0 . K_r is relative hydraulic conductivity, which can be a function of either degree of saturation (S) or matric suction (ψ). A number of equations have been proposed to specify the relative hydraulic conductivity function for unsaturated soils (Fredlund and Xing, 1994; Gardner, 1956; Mualem, 1976; van Genuchten, 1980). Following Lu and Griffiths (2004), the well-known relative hydraulic conductivity equation proposed by Gardner (1956) is employed here

$$15. \quad K_r = \exp(-m\psi)$$

where m is a fitting parameter for Gardner's equation. Substituting Equations 14 and 15 into Equation 13 produces

$$16. \quad \begin{cases} \psi' = \frac{d\psi}{dy} = -\frac{q_w \gamma_w}{K_\psi} - \gamma_w \cos \theta & (y \leq l) \\ \psi = 0 & (y = l) \end{cases}$$

Equation 16 can be solved by using the finite-differential method, assuming $\psi' \approx (\psi_{i+1} - \psi_i)/h$. $h (= y_{i+1} - y_i)$ is the grid size of the finite-difference interval, which can be set to be equal to the element size of the random field ($= 10^{-3}$). Equation 16 can be rewritten as

$$17. \quad \begin{cases} \psi_{i+1} = \psi_i - h \frac{\eta_w \gamma_w (1-n_i)^2 n_0^3}{n_i^3 (1-n_0)^2 \exp(-m\psi_i)} - h \gamma_w \cos \theta & (y \leq l) \\ \psi_i = 0, y_i = l, \text{ when } i = 1 \end{cases}$$

where η_w is the normalised infiltration intensity ($= q_w/K_0$). If $K_s = K_0$ (i.e. porosity-independent), Equation 16 can be solved analytically as

$$18. \quad \psi(y) = -\frac{1}{m} \ln \left\{ \left(1 + \frac{\eta_w}{\cos \theta} \right) \exp[m \gamma_w (y-l) \cos \theta] - \frac{\eta_w}{\cos \theta} \right\}$$

Equation 18 can only be valid between the boundaries for evaporation and infiltration, that is

$$19. \quad \frac{\cos \theta}{\exp(m \gamma_w l \cos \theta) - 1} > \eta_w \geq -\cos \theta$$

Since $K_s \leq K_0$, the valid value of η_w must be inside the evaporation/infiltration boundaries represented by Equation 19. The analytic solution from Equation 18 can also be used to estimate the grid size adopted for Equation 17. As shown in Figure 4, the results obtained from Equation 17 match the analytic solution from Equation 18 very well when grid size is set to be 10^{-3} .

Figure 5(a) shows the calculated suction profile for an infinite slope subject to different steady flow conditions ($\eta_w = -0.8, 0,$

Figure 4. Determining the grid size for numerical integration

0.8) when the groundwater level is equal to 5 m. Three different soil porosity profiles are involved (porosity profiles I, III and V, as shown in Figure 3(b)). As shown in Figure 5(a), the suction distribution is independent of the porosity profile when $\eta_w = 0$. In other words, the suction profile is the same for soil porosity profiles I, III and IV when $\eta_w = 0$. When heavy evaporation occurs (e.g. $\eta_w = 0.8$) in the drought season, if K_s is a function of soil porosity that decreases along the depth in a fluctuating manner (see the calculated curves corresponding to porosity profiles III and V), the predicted suction is much greater than the suction calculated by assuming a constant soil porosity (see the calculated curve corresponding to porosity profile I). Figure 5(a) also shows the suction distribution along the depth for soil profiles with different standard deviations. As shown in Figure 3(b), the standard deviation (σ_n) of soil porosity is 0.006 and 0.03 for porosity profiles III and V, respectively. As shown in Figure 5(a), the fluctuation of the suction profile becomes more distinct with increasing amplitude of fluctuation of soil porosity.

3.4 Profile of degree of saturation, $S(y)$

For a given suction, the degree of saturation can be estimated by traditional water retention models, such as the BC model (Brooks and Corey, 1964), the VG model (van Genuchten,

Figure 5. Simulated (a) suction profile and (b) saturation profile in the slope

Basic random variables	Mean value, μ	Standard deviation, σ
Porosity at surface, n_0	0.6	0.06
Friction angle for surface soil, ϕ_0 : degrees	30.0	3
Total porosity change, Δn	0.1	0.01
Total friction angle change, $\Delta\phi$: degrees	25.0	2.5
Weathered depth, y_w : m	2.0	0.2
Fitting parameter in Gardner's model, m : kPa ⁻¹	0.01	0.001
Fitting parameter in VG model, α : kPa ⁻¹	0.01	0.001
Fitting parameter in VG model, β	2.0	0.2
Fitting parameter in Fredlund's model, k	1.5	0.15
Groundwater level, l : m	0~8	0.2
Normalised infiltration intensity, η_w	-0.8~0.8	0.08

Table 1. Random variables in limit state function

1980) and the FX model (Fredlund and Xing, 1994). With respect to the traditional water retention models, degree of saturation is uniquely determined by the suction for a given soil and independent of all the other soil variables. However, recent research (Gallipoli *et al.*, 2003; Sheng and Zhou, 2011; Sun *et al.*, 2007; Tarantino, 2009; Zhou *et al.*, 2012) showed that soil-water retention behaviour is also dependent on the porosity. Therefore, the traditional water retention equations can only be valid for a given soil with a specific porosity. A simple void-ratio-dependent water retention equation was proposed by Gallipoli *et al.* (2003) based on the VG model, which can be expressed as

$$20. \quad S = \left[\frac{1}{1 + [\alpha(e/e_0)^\varphi \psi]^\beta} \right]^{1-(1/\beta)}$$

where e is void ratio ($=n/(1-n)$), α and β are two parameters in the VG model which are calibrated when the void ratio is set to e_0 , and φ is a parameter introduced to quantify the effect of the void ratio on the soil-water retention behaviour. If the value of φ is set to zero, the term related to void ratio is removed and Equation 20 degenerates to the VG model. Zhou *et al.* (2012) suggested that φ can be set to $1/(\beta-1)$, based on the intrinsic constraints for a liquid-gas-solid mixture. Therefore, the profile of degree of saturation can be written as

$$21. \quad S(y) = \left\{ 1 + \left[\frac{n(y)}{n_0} \frac{1-n_0}{1-n(y)} \right]^{\beta/(\beta-1)} [\alpha\psi(y)]^\beta \right\}^{(1/\beta)-1}$$

where $n(y)$ and $\psi(y)$ can be calculated from Equations 9 and 16, respectively. Figure 5(b) shows different saturation profiles obtained using Equation 21 according to different profiles for soil porosity (i.e. porosity profiles I, III and V, as shown in Figure 3(b)), when $l=5$ m and $\eta_w=0.8$. As shown in Figure 5(b), for the evaporation condition, neglecting the

porosity dependency will overestimate the degree of saturation in the soil. In addition, the fluctuation of soil porosity will result in a fluctuant profile of degree of saturation. Furthermore, a greater value of standard deviation of soil porosity will make the fluctuation of the saturation profile more distinct.

4. Failure analysis of slope

The probability of slope failure will be estimated by Monte Carlo method using Equation 1 as the limit state function. In Equation 1, some of the variables included in vector x are treated as basic random variables, as shown in Table 1. These basic random variables are assumed to be of Gaussian distribution with known means and standard deviations. With the Monte Carlo method, a random sample is taken from the probability distribution function of each variable. Then, the sample is substituted into the limited state function G . If $G < 0$ for a given depth y in a slope, $f(y)=1$ (as shown in Equation 6). This process is repeated N times, known as the sample size, from which $P_f(y)$ can be determined by Equation 7. Meanwhile, the most unsafe location ($\bar{y}$) can be determined by finding the plane with a maximum probability of failure, namely the peak of the P_f - y curve. The fluctuation of soil porosity, which is simulated as a random field, will be included in the failure analysis. The fluctuation of soil porosity is controlled by values of σ_n and δ_y . In this section, δ_y is set to be 1 m. σ_n is set to be 0 and 0.006, which correspond to soil porosity profiles II and III (see Figure 3(b)), respectively. The fluctuation of soil porosity can be removed by setting $\sigma_n=0$. In this section, N is set to be 5000 to ensure the accuracy of simulations.

4.1 Scenario 1: Deep groundwater level ($l=4.0$ m)

In the first scenario, the groundwater level is set to be 4.0 m. The normalised infiltration ratio (η_w) is set to be from -0.8 (rainfall) to 0.8 (evaporation). Figure 6 shows the probability of failure $P_f(y)$ with respect to the location of the failure plane. As shown in Figure 6, for a slope with a deep groundwater

Figure 6. Probabilities of failure for a slope with a deep groundwater level

Figure 7. Probabilities of failure for a slope with a medium groundwater level

level, the slope failure is mainly controlled by the rainfall. For the equilibrium condition (i.e. $\eta_w = 0$), the probability of failure for the slope is almost equal to zero. The probability of failure of the slope markedly increases with the increase of the magnitude of the infiltration ratio for rainy conditions ($\eta_w < 0$). The most unsafe locations for rainy conditions ($\eta_w < 0$) are all less than 1 m, which is much higher than the groundwater level. Therefore, the failure caused by rainfall occurs above the groundwater level. The fluctuation of soil porosity will cause the fluctuation of probability of failure along the depth. However, the difference of the probability of failure caused by the monotonic and fluctuating soil porosities is relatively small.

4.2 Scenario 2: Medium groundwater level ($l = 2.0$ m)

The groundwater level is set to be 2.0 m in the second scenario. Similarly to the first scenario, the normalised infiltration ratio (η_w) is set to be from -0.8 (rainfall) to 0.8 (evaporation). The value of σ_m is set to be 0 and 0.006 for monotonic and fluctuating changes of soil porosity along the depth, respectively. Figure 7 shows the probability of failure $P_f(y)$ with respect to the location of the failure plane. The probability of failure decreases from around 0.8 to 0.7 for a heavy rainfall ($\eta_w = -0.8$) when the groundwater level rises from 4.0 m (the first scenario) to 2.0 m (the second scenario). For the equilibrium and drier conditions ($\eta_w \geq 0.0$), the probability of failure increases from around 0.0 to 0.1 due to the groundwater level rise. For a light rainfall ($\eta_w = -0.2$), the probability of failure almost remains the same (~ 0.1) for both scenarios. However, the most unsafe location is different for the two scenarios. In the first scenario, the most unsafe location is shallow and above the groundwater level for a light rainfall. For the second scenario, the most unsafe location is deep and beneath the

Figure 8. Probabilities of failure for a slope with a shallow groundwater level

groundwater level when $\eta_w = -0.2$. The fluctuation of soil porosity will cause the fluctuation of probability of failure along the depth, especially when the failure plane is beneath the groundwater level.

4.3 Scenario 3: Shallow groundwater level ($l = 0.5$ m)

Figure 8 shows the probability of failure $P_f(y)$ with respect to the location of the failure plane when the groundwater level is set to be 0.5 m. The normalised infiltration ratio (η_w) is set to be from -0.8 (rainfall) to 0.8 (evaporation). The value of σ_m is set to be 0 and 0.006 for monotonic and fluctuating changes of soil porosity along the depth, respectively. As shown in

Figure 8, although the increase of rainfall intensity will increase the probability of failure above the groundwater level, the probability of failure is governed by the location of the groundwater level. The most unsafe location is always beneath the groundwater level for all the normalised infiltration ratios. Compared with the first and second scenarios, the fluctuation of soil porosity will cause a distinct fluctuation of probability of failure along the depth for this case.

5. Conclusions

A methodology has been proposed in this paper for prediction of the failure of an infinite soil slope subject to steady unsaturated flow conditions. An inversely linearly proportional relationship between the friction angle and its porosity was adopted in the analysis. Probabilistic models for soil properties, namely, soil porosity, friction angle, matric suction and saturation, have been developed. The probability of slope failure for different groundwater levels and infiltration (or evaporation) intensities has been investigated thoroughly. It has been found from numerical results that, for a slope with a deep groundwater level, the slope failure is mainly controlled by the rainfall infiltration, whereas, for a shallow groundwater level, the slope failure is governed by the location of the groundwater level. It has also been found that, for a slope with deep groundwater, the most unsafe location is shallow and above the groundwater level; for medium groundwater, the most unsafe location is deep and beneath the groundwater level; and for shallow groundwater, the most unsafe location is always beneath the groundwater level. A merit of the proposed methodology is that the fluctuation of the soil porosity along the depth has been taken into account and modelled as a Gaussian random field. It can be concluded that the proposed methodology can be used for prediction of slope failures under steady unsaturated flow conditions.

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