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The Indentation Rolling Resistance of Spherically Profiled Idler Rolls

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ABSTRACT

The energy loss due to indentation rolling resistance in a conveyor belt system can account for up to 60% of the total power usage. It arises due to an asymmetric pressure distribution as a viscoelastic belt cover is indented by a rigid idler roll. This causes a retarding moment on the idler roll, dependent on the load, speed, idler diameter and the properties of the conveyor belt cover. As pouch conveyor systems become more prevalent in industry, the supporting structure, and thus the indentation rolling resistance of such systems is of importance. These designs typically employ curved belt profiles, or spherical idlers in order to aid belt tracking and closing. From this, a spherical indentation into a generalised Maxwell backing is modelled, and compared with experimental values. In addition to this, the strain dependency across the contact is investigated.

1. INTRODUCTION

Indentation rolling resistance is a drag force that arises due to the viscoelastic nature of the belt cover. Typically, due to the speed of the belt, and the diameter of the idler roll, the indentation of the cover occurs much quicker than the relaxation at the trailing edge of the contact. This induces a skewed load on the idler, resulting in a retarding moment [16]. Based on this definition, this hysteretic behaviour is less prominent in slower conveyors, or conveyors utilising large diameter idler rolls, as there is less discrepancy between the indentation and relaxation time.

Hager and Hintz [1] postulated this loss can contribute up to 60% of the total power usage. Given the magnitude of this loss mechanism, several authors [3, 4, 13, 15, 16] have proposed predictive models to estimate the extent of this loss in conventional conveying systems. Each model possesses different benefits, each with their own validity. These models apply to cylindrical idler rolls indented into a flat conveyor belt, and as such need modification to apply to the complex support systems existent in pouch conveyors. Nuttall [7] modified the theory presented by [4] to predict the losses within the Enerka-Becker pouch conveyor system. However as stated earlier, modern pouch conveyors utilise varying support configurations. Thus, a model needs to be developed that can simulate convex, flat and concave belt profiles, indented by flat and spherical (convex) idler rolls.

It is common practice to model the indentation of the idler roll into the bottom cover of the conveyor by utilising a one-dimensional stress/strain analysis. In this model, inter-fibre relations are omitted, thus each fibre makes an independent contribution to the overall stress, as shear between the fibres is neglected. This simplification is done (opposed to a two-dimensional model) in an attempt to minimise computational power, while still achieving accurate results.

1.1 MODELLING THE BELT USING A GENERALISED MAXWELL MODEL

The viscoelastic nature of the cover material induces a load, temperature and strain rate dependence on the indentation response. To predict this, [4] utilised a 3-parameter Maxwell model, which has been shown to provide reasonable predictions for viscoelastic materials. As belt manufacturers aim to improve the bottom cover material composition, in an attempt to minimise indentation rolling resistance, many polymers used are extremely temperature and strain rate dependent, and thus require a highly sensitive model. As will be shown, the 3-parameter model yields a limited approximation, especially given the frequency of indentation on a belt conveyor may exceed 1000 Hz.
The Generalised Maxwell Model, or Maxwell-Wiechert model, utilises a series of springs and dampers in parallel, as seen in Figure 1. This model, a generic form of Maxwell’s 3-parameter model, allows the loading contributions from varying polymer lengths in the material to be accounted for. This is particularly important for polymer blends, which are increasingly common in conveyor belts.

![Figure 1 Generalised Maxwell model.](image)

2. **Indentation Rolling Resistance**

It is becoming increasingly common for pouch conveyor systems to contain curved running surfaces in contact with the idler rolls. Curved surfaces allow the predictability of wear concentration, and the use of sacrificial parts, to minimise maintenance costs and downtime. As such, conventional methods used to calculate rolling resistance need to be modified.

Multiple models have been developed to predict this friction, and the model presented below builds on the work of several authors [7, 13]. Each model involves portraying the viscoelastic belt as a one-dimensional Winkler foundation, indented by a rigid idler roll. In addition to these models, curved belts and spherical idler rolls are investigated, as depicted in Figure 2. Previous tests have indicated that current bottom cover materials display a high dependence on temperature and load rate. This is illustrated below in Figure 3. To accommodate this, a high order Maxwell model is used in the approximation.
For small indentations \((z<<h)\), it is possible to approximate the indentation depth as a parabolic function [4]. Assuming that \(Z_0\) is the peak indentation, occurring at the centreline of the idler roll, the surrounding indentation can be estimated according to:
\[ w(x, y) = Z_0 - \frac{x^2}{2R_1} - \frac{y^2}{2R_2} + \frac{y^2}{2R_3} \]  

(1)

In the above equation, it should be noted that the radius \( R_3 \) is positive for a concave running surface, and negative for a convex surface. Similarly, this equation can be converted to a standard troughed configuration by setting \( R_2 \) and \( R_3 \) to infinity.

From the generalised Maxwell model, it is evident that the total stress in the system can be written as the sum of the stress in each individual strand:

\[ \sigma = \sigma_0 + \sum \sigma_i \]  

(2)

\[ \sigma_i = \sigma_{Ei} + \sigma_{\eta i} \]  

(3)

\[ \sigma_0 = E_0 \varepsilon \]  

(4)

\[ \sigma_{Ei} = E_i \varepsilon_E \]  

(5)

\[ \sigma_{\eta i} = \eta_i \frac{d\varepsilon_{\eta i}}{dt} \]  

(6)

where \( \varepsilon_E \) and \( \varepsilon_\eta \) correspond to the strains in the individual springs and dashpots respectively.

By taking the time derivatives of the individual strains:

\[ \dot{\varepsilon}_i = \varepsilon'_E + \varepsilon'_\eta \]  

(7)

\[ \varepsilon'_E = \frac{\sigma_{Ei}}{E_i} \]  

(8)

\[ \varepsilon'_\eta = \frac{\sigma_{\eta i}}{\eta_i} \]  

(9)

\[ \dot{\sigma}_i + \frac{E_i}{\eta_i} \sigma_i = E_i \dot{\varepsilon} \]  

(10)

The resulting ordinary differential equation relates the stress and strain in an individual element. Solving this, noting that \( \varepsilon = \frac{w(x, y)}{h} \), the stress distribution over the contact region can be determined:

\[ \sigma(x, y) = \frac{a^2 E_0}{2R_1 h} \left( 1 - \frac{x}{a} \right) \left( 1 + \frac{x}{a} \right) + \frac{a^2}{R_1 h} \sum_{i=1}^{N} E_i \varepsilon_E i \left( 1 + \frac{\nu E_i}{\alpha} \right) \left( 1 - e^{-\frac{a}{\nu h} \left( 1 - \frac{x}{a} \right)} - \left( 1 - \frac{x}{a} \right) \right) \]  

(11)

In the above equation, \( a \) is a function of \( y \), and is determined from equation (1) by setting the indentation to zero, with \( x \) equal to \( a \):
\[ a = \sqrt{2R_1Z_0 + R_1y^2\left(\frac{1}{R_3} - \frac{1}{R_2}\right)} \]  
\hspace{1cm} (12)

From this, we can determine the overall normal load applied by the belt, the opposing moment, and thus the drag force.

\[ F_N = 2 \int \int \sigma(x,y)dx\,dy \]  
\hspace{1cm} (13)

\[ M = 2 \int \int x\sigma(x,y)dx\,dy \]  
\hspace{1cm} (14)

\[ F_{drag} = \frac{M}{R_1} \]  
\hspace{1cm} (15)

From the above equations, it is possible to determine an explicit solution for a given \( y \) value, by expressing the ratio \( \varsigma = \frac{b}{a} \) [13]. From this, and equation (11), \( b \) can be determined. For simplicity however, a computational approach was used.

### 1.2 Experimental Setup

In order to validate the theoretical results, experimental tests were conducted on a laboratory test facility, specifically designed to accurately measure the various drag forces associated with idler roll motion [6]. This facility has the ability to investigate various idler rolls of different diameters and materials, at different loads, speeds and temperatures.

The measurement principle of the test facility is shown in Figure 4. The measurement idler roll is supported by four vertical load cells that are attached via virtual pivots at the top and bottom. The measurement roll is free to move horizontally as the belt moves over the idler roll. The horizontal resistance to motion is due to the rotating resistance of the idler roll (known as rim drag), the belt flexure resistance, and the indentation rolling resistance. The measurement roll is prevented from moving horizontally by the horizontal load cell that measures the combined resistances of rim drag, belt flexure resistance and indentation rolling resistance. The rim drag is also measured separately by a load cell that measures the torque to prevent the shaft of the measurement roll from rotating, while the belt flexure resistance is subtracted from the overall measurement as detailed in [6]. The normal load, \( F_N \), applied to the roll is achieved by deflecting the belt over the roll by hold-down rolls either side of the measurement roll. The normal load is varied by adjusting the belt tension, \( T \), using a hydraulic take-up system.

![Figure 4 Detail of the indentation rolling resistance measurement apparatus [6]](image)
In order to test situations indicated in Figure 2, profiled idlers were manufactured to represent case (c). This was achieved by machining a range of idler rolls with corrugated shells to simulate spherical idlers, see Figure 5(a). The profiled idler rolls were then installed in the indentation rolling resistance test facility as shown in Figure 5(b).

![Profiled idler roll.](image1)

![Profiled idler roll installed on the test facility.](image2)

Figure 5 Experimental test facility

In order to investigate how diameter and cross-radius (R2: Figure 3) affected measurements, 6 idler rolls were tested, according to Table 1.

**Table 1 Description of test idler rolls.**

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Cross-radius (mm)</th>
<th>Number of profiles (total)</th>
<th>Number of profiles in contact with belt</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>20</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>125</td>
<td>10</td>
<td>40</td>
<td>28</td>
</tr>
<tr>
<td>125</td>
<td>15</td>
<td>30</td>
<td>21</td>
</tr>
<tr>
<td>125</td>
<td>20</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>125</td>
<td>25</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>175</td>
<td>20</td>
<td>24</td>
<td>17</td>
</tr>
</tbody>
</table>

The aforementioned idlers were tested at a variety of loads and velocities, as specified below:

**Table 2 Test schedule.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Alterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>2, 4, 6, 8, 10 m/s</td>
</tr>
<tr>
<td>Load*</td>
<td>900, 1250, 1750, 2250 N</td>
</tr>
<tr>
<td>Temperature</td>
<td>30 °C</td>
</tr>
<tr>
<td>Belt Sag</td>
<td>1%</td>
</tr>
</tbody>
</table>

* The load values shown relate to certain positions of the hydraulic cylinder used to tension the belt, and thus increase the load. Due to temperature fluctuations, creep and material inconsistencies, the actual load varied with each test, as shown in the results.

As noted by [15], as an idler roll indents a steel cord belt, the pressure created is not limited to the bottom cover, but instead enters the insulation rubber surrounding the cords. From this, [15] predicted that the apparent bottom cover thickness is equal to the actual bottom cover thickness, plus half of the steel cord diameter. The belt used during these experiments was a steel cord belt; with diameter 5 mm cords and a 6 mm bottom cover. From this, the apparent thickness of the bottom cover is 8.5 mm. This is the value used in the simulations.
1.3 Findings

The range of experiments and simulations conducted enabled a comprehensive analysis to be performed of spherical idlers indenting a flat belt. Below, an analysis of how the indentation rolling resistance is affected by velocity, load, roll diameter and cross radius will be discussed, as well as the accuracy of the theoretical modelling techniques.

1.3.1 Dependence on Idler Diameter

For conventional troughed systems, idler diameters are typically within the range of 125 mm to 225 mm. This is based on a compromise between the indentation rolling resistance, manufacturing costs and the rotational inertia of the idlers. As can be seen in Figure 6, larger diameter idlers exhibit lower rolling resistance due to the increased contact area between the belt and idler. This distributes the load over a larger area resulting in lower stresses, and allows a more symmetrical pressure distribution. The corresponding cross sectional area (csa) of the contact was determined during the simulation, based on regions with stress greater than zero. This is also depicted in Figure 6. This is formulated based on the stress profile presented in equation (11).

![Figure 6](image)

Figure 6 The theoretical influence of normal force on the Indentation Rolling Resistance (left) and contact area (right) for various idler diameters.
1.3.2 Dependence on Cross Radius

The cross radius of an idler bears little impact on the overall drag of an idler roll compared to the idler roll diameter. Instead, it is more of a design preference to suit the supporting structure of the conveyor. Its theoretical impact on the rolling resistance is presented in Figure 7.

![Figure 7 The theoretical influence of normal force on the Indentation Rolling Resistance (left) and contact area (right) for various cross radii.](image)

As can be seen from Figure 7, the resistance is proportional to $F^4$, with the cross radius having little impact. In theory, this is not unexpected. Contrary to the primary radius, the cross radius bears little effect on the indentation or relaxation time of the backing material. This is due to the indentation stresses between relative cross-radii. As seen in Figure 7, the larger the cross radius, the larger the contact area between the belt and idler surface, resulting in a more even stress distribution, of lower magnitude.

1.3.3 Comparison between Experimental and Simulated Results

For belt conveying systems, the ability to accurately predict the indentation rolling resistance is critical to accurately predict the belt tensions and size associated components. As discussed earlier, many theories have been formulated, each with their own attributes, and unfortunately, downfalls. The following graph depicts a comparison between the theory outlined above, and the results achieved experimentally. These simulations were performed using a 20 element Maxwell model (41 parameters).

The results below depict data sets corresponding to a 2000, 3000, 4000 and 5000 N/m load across the idler roll, which in turn was converted to a normal force on each individual profile. Figure 8 portrays the results for the idler rolls of diameter 125 mm. The 4 plots depict the influence of the cross radius on the rolling resistance of the roll. Subsequently, Figure 9 outlines the results for the Ø75 mm and Ø175 mm idlers with a 20 mm cross radius. Figure 8(c) also belongs to this set as it defines the results belonging to the Ø125xR20 idler roll.
Figure 8 Comparison of experimental and predicted results for a Ø125 mm idler with (a) R10, (b) R15, (c) R20, (d) R25 mm cross radius.

Figure 9 Comparison of experimental and predicted results for (a) Ø75xR20 and (b) Ø175xR20.

As can be seen, there is a consistent shortfall across the results. The simulations of the Ø75xR20 idler perform the worst, with an average shortfall of approximately 40%. The 125mm diameter results improved. The error for this idler was 34%, 31%, 38% and 36%, corresponding to the cross radii of 10, 15, 20 and 25 mm respectively. The largest idler was an improvement again. This 175mm diameter idler had an average deviation of 23% from the experimental results.

These findings are evident in most one-dimensional Winkler foundation models attempting to simulate this behaviour. In addition to this:

- The use of a one-dimensional model, as stated earlier, does not account for shear between the polymer chains. The models produced by [2, 5] treats the cover material as a two-dimensional material. The existence of shear between polymers would encourage a non-uniform strain distribution throughout the cover material. If this were the case, depending on the applied load and the thickness of the bottom cover, the strain profile throughout the cover would taper off at a certain depth. This would introduce another ‘apparent cover thickness’, over which the strain is distributed. Rudolphi et al [14] states the inaccuracies of a one-dimensional model are approximately 50% of the predicted value (or a third of the experimental value). This corresponds with the recorded error of 23-40%.

- Friction between the idler roll and belt is very difficult to include in a theoretical model without developing a full FEM simulation. As the idler roll creates an increase in tension along the belt due to its drag force, this
will create regions of stick and slip in the contact region. It is evident that the belt is indented quicker than it relaxes. This will result in the majority of the slip occurring next to the trailing edge, thus opposing the motion.

- The DMA tests to obtain the rubber properties are performed at a constant strain setting. Typically between 0.1% and 2% values. This is discussed in section 1.3.5.

### 1.3.4 Effect of Maxwell Elements

The Maxwell parameters used to describe the viscoelastic behaviour of rubber, are obtained through fitting a Prony series to a master curve of the data. The accuracy of this fit is dictated by the number of Maxwell elements used. In order to test the effect of this fit on the simulated results, data was determined that corresponded to a 3, 5, 10, 15 and 20 element Maxwell model (7, 11, 21, 31 and 41 parameters respectively). The simulation results are shown below in Figure 10. The results shown correspond to a Ø75xR20 idler, with a distributed load of 4000 N/m across the idler.

![Figure 10 Influence of material data on indentation rolling resistance accuracy.](image)

As can be seen above, the accuracy, although still different to the experimental data, is greatly improved with a more accurate predictive model. It can also be seen that the lower order models appear to have difficulty simulating data at low belt speeds.

### 1.3.5 Strain Dependency

As mentioned in Section 1.3.3, several factors contribute to the inaccuracy of this model. One of which, is the inability of the model to accommodate variations in strain. A DMA test is performed at a constant strain, however in reality, most situations involve varying strain levels in a material. For instance, an idler roll being indented into a belt cover will experience low strain at the edges, and a maximum at the centreline of the idler. To account for this, a strain dependency is added to the model. A strain dependent DMA test is conducted, which determines the influence of the strain level, on the corresponding storage and loss moduli. By adjusting the storage and loss moduli accordingly, and refitting the master curve and Prony series, it is possible to determine a more accurate estimation of the stresses in that element. This is done at every strain level within the contact.

The strain sweep for the bottom cover material used in these calculations is shown below. As can be seen, the original test data was performed at 0.05% strain (corresponding to multiplier factors of 1).
Figure 11 Effect of strain on storage and loss modulus.

Using this data, the computational package is adjusted to calculate the strain at every point in the indentation profile. From this, an individual set of Maxwell parameters is determined, corresponding to each point, and used to determine the stress in that particular element. The graphs depicted below present Non-Strain Dependent (NSD), Strain Dependent (SD) and Experimental (Exp) results.

Similar to the presentation of initial results above in Figures 8-9, the results below depict data sets corresponding to a 2000, 3000, 4000 and 5000 N/m load across the idler roll. Figure 12 portrays the results for the idler rolls of diameter 125 mm. The 4 plots depict the influence of the cross radius on the rolling resistance of the roll. Subsequently, Figure 13 outlines the results for the Ø75 mm and Ø 175 mm idlers with a 20 mm cross radius.

Figure 12 Comparison of normal, strain dependent and experimental results for a Ø125 mm idler with (a) R10, (b) R15, (c) R20, (d) R25 mm cross radius.
Figure 13 Comparison of normal, strain dependent and experimental results for (a) Ø75xR20 and (b) Ø175xR20.

As can be seen from the above results, there is a consistent improvement through the addition of strain dependency to the material data. These changes are summarised in Table 3, where an average improvement of 5.7% occurs, with the inclusion of strain dependent material properties, in the theoretical model.

Table 3 Difference between Idler results with and without strain dependency.

<table>
<thead>
<tr>
<th>Idler</th>
<th>Error without strain dependency</th>
<th>Error with Strain Dependency</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>D75xR20</td>
<td>40.4%</td>
<td>34.8%</td>
<td>5.6%</td>
</tr>
<tr>
<td>D125xR10</td>
<td>34%</td>
<td>28.3%</td>
<td>5.7%</td>
</tr>
<tr>
<td>D125xR15</td>
<td>30.8%</td>
<td>25.5%</td>
<td>5.3%</td>
</tr>
<tr>
<td>D125xR20</td>
<td>37.9%</td>
<td>32.8%</td>
<td>5.1%</td>
</tr>
<tr>
<td>D125xR25</td>
<td>35.5%</td>
<td>30.4%</td>
<td>5.1%</td>
</tr>
<tr>
<td>D175xR20</td>
<td>23.2%</td>
<td>16%</td>
<td>7.2%</td>
</tr>
<tr>
<td>Average Improvement</td>
<td>5.7%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.4 DISCUSSION

The model presented predicts the indentation rolling resistance drag force, due to an asymmetric pressure distribution, which occurs as a moving belt is indented by a rigid idler roll. This function is solely dependent on the parameters of the belt cover material, the geometry of the indenting idler roll as well as the indenting load and belt velocity. The results of the experimental indentation rolling resistance tests showed a consistent shortfall in the theoretical prediction of the drag force. This shortfall ranged between 23% and 40%.

It was shown that part of the deficiency arises due to the inability of the model to accommodate strain variance. When included in this model, an average improvement of 5.7% was recorded, reducing the average deviation to 28.3% of the experimental value.

The remaining error is the result of approximation techniques, and the limitations of the model. As mentioned earlier, the respective Maxwell parameters are formed through the fitting of a Prony series to the master curve data, produced from the raw DMA results. The construction of the mastercurve is, in itself, an approximation, as is the Prony series that is subsequently fitted. These techniques are subject to varying minimisation techniques, and thus accumulate error in the process. Some recent authors [11] utilise the raw DMA data directly into the simulation. Furthermore, this displays the inelasticity of a one-dimensional model. Shear between the polymer chains within the belt cover would exacerbate the impact stress during loading, increasing the resultant drag force on the idler roll.
Friction between the idler roll surface and belt cover would also result in energy loss in the system. As discussed in section 1.3.3, when a tractive load is placed across a viscoelastic indentation, regions of stick and slip develop within the contact area. The stick regions ensure static contact remains between the belt and idler roll (provided the tractive load is less than the friction limit), whereas the slip zones allow creep to occur due to the tension gradient across the contact. These slip zones typically exist at locations where the normal pressure is not sufficient to maintain a frictive contact. This relative slip dissipates energy.

The relative error in each simulation requires discussion. From observation, it can be seen that the magnitude of the error increases with load, whereas minimal change is noticeable with speed. This is not unexpected, as the indentation rolling resistance bears little susceptibility to velocity. As belt speed increases, the indentation depth of the idler roll decreases due to the viscoelastic properties of the belt backing material. This results in a smaller contact area, and thus smaller slip zones within the contact.

From Table 3, it can be seen that the inclusion of strain dependency improved the results across all simulations. In particular, for the Ø125 mm idler roll, the improvement reduced with the dimension of the cross radius. For a given load, this is due to the smaller idler profiles indenting the belt further, exacerbating the error due to strain.

Using this theory to predict indentation rolling resistance losses of cylindrical idlers, [9] found similar conclusions. O’Shea [9] compared various rubber compounds, under similar experimental conditions, to determine the validity of numerous models. The models investigated were Jonkers [3], Spaans [15], QC-N [11] and Lodewijks [4]. In addition to these, O’Shea also compared the theory presented herein (for cylindrical idler rolls), based on the model of Rudolphi & Reicks [13].

The results depicted above also show velocity only has a minor influence on indentation rolling resistance. The experimental results depict an increase of approximately 10% from the data taken at 1 m/s, compared to 5 m/s. This is contrary to the load dependence, which shows an increase in resistance, approximately proportional to the increase in load.

1.5 CONCLUSIONS

An in-depth investigation was conducted, on the appropriateness of using a one-dimensional Winkler foundation model to predict the indentation rolling resistance that occurs in complex idler roll configurations and geometries, existent in current pouch conveying designs. The model proved to provide reasonably accurate predictions with the inclusion of strain dependency in the model with an average improvement of 5.7% seen across the results. This reduced the initial error observed from 33.6% to 27.9%.

REFERENCES