NOVA

University of Newcastle Research Online

nova.newcastle.edu.au

Teo, Yik R.; Fleming, Andrew J. "Optimal integral force feedback for active vibration control". Originally published in Journal of Sound and Vibration Vol. 356, p. 20-33 (2015)

Available from: <http://dx.doi.org/10.1016/j.jsv.2015.06.046>

© 2015. This manuscript version is made available under the CC-BY-NC-ND 4.0 license
<http://creativecommons.org/licenses/by-nc-nd/4.0>

Accessed from: <http://hdl.handle.net/1959.13/1326105>

Optimal Integral Force Feedback for Active Vibration Control

Yik R. Teo^{a,*}, Andrew J. Fleming^a

^a *Precision Mechatronics Lab, School of Electrical Engineering and Computer Science,
Faculty of Engineering and Built Environment, The University of Newcastle,
Callaghan, New South Wales, 2308, Australia.*

Abstract

This article proposes an improvement to Integral Force Feedback (IFF), which is a popular method for active vibration control for structures and mechanical systems. Benefits of IFF includes robustness, guaranteed stability and simplicity. However, the maximum damping performance is dependent on the stiffness of the system; hence, some systems cannot be adequately controlled. In this article, an improvement to the classical force feedback control scheme is proposed. The improved method achieves arbitrary damping for any mechanical system by introducing a feed-through term. The proposed improvement is experimentally demonstrated by actively damping an objective lens assembly for a high-speed confocal microscope.

Keywords: Force Feedback, Damping Control, Vibration Control

1. Introduction

The presence of undesirable vibrations is known to degrade the performance of structural and mechanical systems that may lead to system failures and malfunctions [1]. Vibrations appear due to the unwanted excitation of system resonances. A common method for reducing vibration is to artificially increase the viscous damping in the system. Damping control can be classified into passive or active. Examples of traditional passive approaches include viscoelastic damping and tuned-mass absorbers [2]. These approaches are commonly integrated into the structural or mechanical design and eliminate the need of additional controlled hardware and sensors. However, passive damping methods can be sensitive to changes in resonance frequencies, may be bulky, and may not perform well at low frequencies. On the other hand, active damping control has the potential to overcome the performance limitations of passive damping methods [3, 4]. Active control typically requires the integration of sensors and a feedback control system to artificially increase the damping ratio of a system.

*Principal corresponding author

Email addresses: yik.teo@newcastle.edu.au (Yik R. Teo),
andrew.fleming@newcastle.edu.au (Andrew J. Fleming)

Active damping control can be found in scientific and industrial applications where the speed and accuracy are key performance criteria, for example, scanning probe microscopy [5–8], nano-fabrication [9, 10], precision optics [11, 12], robotics [13], medical [14] and aerospace systems [15]. In addition, active damping control can also be found in defence applications where the system lifetime is important [16]. A number of techniques for damping control have been demonstrated successfully in the literature, these include Positive Position Feedback (PPF) [17], polynomial based control [18], acceleration feedback [19], shunt control [20, 21], resonant control [22], Force Feedback [23–26], and Integral Resonance Control (IRC) [27, 28]. Among these techniques, PPF controllers, velocity feedback controllers, force feedback controllers, and IRC controllers have been shown to guarantee stability when the plant is strictly negative imaginary [29]. Controllers with automatic synthesis have also been successfully applied to vibration control applications. Examples include robust $\mathcal{H}_\infty$ controllers [30–32] and LMI based controllers [33]. However, such methods are rarely used in practice because of their implementation complexity.

In references [23–26] Integral Force Feedback (IFF) is described for vibration control. This technique utilizes a force sensor and integral controller to directly augment the damping of a mechanical system. The major advantages of IFF are the simplicity of the controller, guaranteed stability, excellent performance and robustness to variation of resonance frequency. However, the maximum damping achievable with IFF is a function of the system properties, in particular the system’s stiffness relative to the actuator stiffness. This means that some systems can be critically damped using IFF while other systems exhibit insufficient damping.

In this work, an improvement to integral force feedback is described which allows an arbitrary damping ratio to be achieved for a mechanical system. The modification amounts to replacing the integral controller with a first-order low-pass filter. Although the additional complexity is negligible, the damping performance is significantly improved. This result allows integral force feedback control to be applied to systems that were not previously suited.

2. Classical Integral Force Feedback (CIFF)

Integral force feedback control has been widely employed for augmenting the damping of flexible structures. The feedback law is simple to implement and under common circumstances provides excellent damping performance with guaranteed stability. Figure 1 illustrates a structure G equipped with a piezoelectric actuator that produces a force F_a with internal stiffness K_a . A force sensor is collocated with the piezoelectric actuator and measures the axial force F_s acting on the system G . The variable d represents the mechanical displacement.

The classical integral force feedback controller has a block diagram representation illustrated in Figure 2. The transfer function between the unconstrained piezo expansion δ to the sensor force F_s is adapted from [23]

$$G_{F_s\delta}(s) = \frac{F_s}{\delta} = K_a \left\{ 1 - \sum_{i=1}^n \frac{v_i}{1 + s^2/\omega_i^2} \right\}, \quad (1)$$

Figure 1: Structure G with a piezoelectric transducer.

Figure 2: Block diagram representation of Classical Integral Force Feedback.

where ω_i is the natural frequency of the system and v_i is the fraction of modal strain energy for the i^{th} mode. The modal zeros z_i is given as

$$z_i^2 = \omega_i^2(1 - v_i). \quad (2)$$

The integral force feedback controller is

$$C_{d1}(s) = \frac{K_{d1}}{K_a s}, \quad (3)$$

where K_{d1} is the gain of controller. The maximum modal damping is [23]

$$\zeta_i^{\max} = \frac{\omega_i - z_i}{2\omega_i}, \quad (4)$$

and the corresponding controller gain is

$$K_{d1} = \omega_i \sqrt{\frac{\omega_i}{z_i}}. \quad (5)$$

The proofs of (4) and (5) are given in Appendix A. The root locus plot corresponding to CIFF is shown in Figure 3(a). The main limitation of the classical method is that the maximum modal damping (4) depends on the distance between the system poles ω_i and zeros z_i . If the distance between the pole and zero is small, the maximum modal damping achievable with CIFF is reduced.

Figure 3: Typical root locus plots

Figure 4: Block diagram representations of Optimal Integral Force Feedback.

3. Optimal Integral Force Feedback (OIFF)

This article proposes an improvement of the classical integral force feedback methodology. A new feed-through term β is introduced into the system as shown in Figure 4(a). The new system in Figure 4(a) is cast into the CIFF structure via a change of variable. This allows a straightforward comparison between the CIFF and OIFF systems and to make use of previous results reported for CIFF. In particular, the expressions for maximum modal damping (4) and controller gain (5). This is done by

equating the systems in Figure 4(a) and Figure 4(b), that is

$$\begin{aligned} K_a \left\{ 1 - \sum_{i=1}^n \frac{v_i}{1 + s^2/\omega_i^2} \right\} + \beta &= \hat{K}_a \left\{ 1 - \sum_{i=1}^n \frac{\hat{v}_i}{1 + s^2/\omega_i^2} \right\}, \\ K_a + \beta - \sum_{i=1}^n \frac{K_a v_i}{1 + s^2/\omega_i^2} &= \hat{K}_a - \sum_{i=1}^n \frac{\hat{K}_a \hat{v}_i}{1 + s^2/\omega_i^2}. \end{aligned} \quad (6)$$

From (6) we obtain the new expressions via a change of variable

$$\hat{K}_a = K_a + \beta, \quad (7)$$

and

$$\hat{v}_i = \frac{K_a v_i}{\hat{K}_a} = \frac{K_a v_i}{K_a + \beta}. \quad (8)$$

Note that the change of variables (7) and (8) are not unique, other options are possible.

The transfer function from $\hat{\delta}$ to $\hat{F}_s$ is

$$\hat{G}_{F_s \delta}(s) = \hat{K}_a \left\{ 1 - \sum_{i=1}^n \frac{\hat{v}_i}{1 + s^2/\omega_i^2} \right\}. \quad (9)$$

The modal zeros are now a function of β

$$\hat{z}_i(\beta) = \sqrt{\omega_i^2 \left(1 - \frac{K_a}{K_a + \beta} v_i \right)}. \quad (10)$$

This results in an extra degree of freedom that allows the position of the zeros to be modified. As β decreases, the zeros move closer to the real axis, under the condition that $K_a(v_i - 1) < \beta < 0$ is satisfied. The integral force feedback controller is

$$C_{d2}(s) = \frac{K_{d2}}{\hat{K}_a s}, \quad (11)$$

The root locus plot for a typical OIFF system is shown in Fig. 3(b). Notice that the location of the zero changes with respect to β .

The new maximum modal damping is

$$\hat{\zeta}_i^{max} = \frac{\omega_i - \hat{z}_i(\beta)}{2\hat{z}_i(\beta)}, \quad (12)$$

which corresponds to the new optimal gain

$$K_{d2} = \omega_i \sqrt{\frac{\omega_i}{\hat{z}_i(\beta)}}. \quad (13)$$

The derivations of (12) and (13) are discussed in Appendix A.

On the other hand, given a desired modal damping ζ_d , the value of β required is

$$\beta = -K_a + \frac{K_a v_i (2\zeta_d + 1)^2}{4\zeta_d(1 + \zeta_d)}. \quad (14)$$

Figure 5: Mechanical diagram of a second-order mechanical system where F_s is the measured force acting between the actuator and the mass in the vertical direction.

3.1. Comparison between Integral Resonance Control and Integral Force Feedback

Integral Resonance Control is a well-known damping technique that is based on displacement feedback while Integral Force Feedback is based upon force feedback [27]. Aside from the difference feedback variables, the major difference is the open-loop structure of the systems. In IRC, the open-loop transfer function is represented by [27]

$$G_{IRC}(s) = \sum_{i=1}^n \frac{\alpha_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}, \quad (15)$$

where $\alpha_i > 0 \forall i$ and n represents the number of modes that sufficiently describe the properties of the structure. For IFF, the open-loop transfer function is

$$G_{IFF}(s) = K_a \left\{ 1 - \sum_{i=1}^n \frac{v_i}{1 + s^2/\omega_i^2} \right\}, \quad (16)$$

where ω_i is the natural frequency of the system and v_i is the fraction of modal strain energy for the i^{th} mode. In classical IFF, the only control variable is the feedback gain, while in the case of IRC, the feedback gain and feed-through variable are both variables. In the proposed method, an additional feed-through term is added to the classic IFF structure, which provides a structure similar to that found in IRC control. However, although both damping techniques introduces an additional feed-through term, the optimal values and closed-loop response are different for each case due to the different open-loop systems.

4. Case Study

In this section, the performance of CIFF and OIFF are demonstrated on a simple mechanical system.

4.1. Mechanical Dynamics and System Properties

The mechanics of a second order system are illustrated in Figure 5 with parameters listed in Table 1. The equation of motion for this system is

Table 1: Parameters of a simple mechanical system.

Parameter	Variable	Value
Mass	M_p	250g
Flexure Stiffness	k_f	300N/ μ m
Actuator Stiffness	K_a	100N/ μ m
Flexure Damping	c_f	10N/ms ⁻¹

$$M_p \ddot{d} + c_f \dot{d} + (K_a + k_f)d = F_a, \quad (17)$$

where M_p is the mass of the platform and the stiffness and damping coefficient of the flexures are denoted by k_f and c_f respectively. The force of the actuator is F_a and the stiffness is K_a . A force sensor is collocated with the actuator and measures the load force F_s . The configuration of the system is such that the actuator and flexure appear mechanically in parallel, hence, the stiffness coefficients can be grouped together,

$$k = K_a + k_f \quad (18)$$

This simplifies the equation of motion (17) to

$$M_p \ddot{d} + c_f \dot{d} + k = F_a. \quad (19)$$

The transfer function from actuator force F_a to the displacement of the mass d is

$$G_{dF_a}(s) = \frac{d}{F_a} = \frac{1}{M_p s^2 + c_f s + k} \quad (20)$$

The sensor force F_s can be written as

$$\begin{aligned} F_s &= F_a - dK_a, \\ &= F_a - K_a F_a G_{dF_a}(s), \\ &= F_a (1 - K_a G_{dF_a}(s)). \end{aligned} \quad (21)$$

The transfer function between the applied force F_a and measured force F_s is found by rearranging (21).

$$G_{F_s F_a}(s) = \frac{F_s}{F_a} = 1 - K_a G_{dF_a}(s). \quad (22)$$

The force developed by the actuator F_a is

$$F_a = K_a \delta. \quad (23)$$

Recall that δ is the unconstrained piezo expansion.

Substituting (23) into (22), we obtain the transfer function from the unconstrained piezo expansion δ to the force of the sensor F_s

$$\begin{aligned} G_{F_s \delta} &= \frac{F_s}{\delta}, \\ &= K_a \frac{F_s}{F_a}, \\ &= K_a (1 - K_a G_{dF_a}(s)). \end{aligned} \quad (24)$$

A valid assumption is that the effect of the damping in the flexure c_f is small and thus negligible. Hence, equating (1) and (24) results in an expression for v_i and ω_i

$$\begin{aligned} K_a \left\{ 1 - \sum_{i=0}^n \frac{v_i}{1 + s^2/\omega_i^2} \right\} &= K_a (1 - K_a G_{dF_a}(s)) \\ &= K_a \left(1 - \frac{K_a}{M_p s^2 + k} \right) \\ &= K_a \left(1 - \frac{K_a/k}{s^2/\frac{k}{M_p} + 1} \right) \end{aligned} \quad (25)$$

where

$$v_i = \frac{K_a}{k} \quad \omega_i^2 = \frac{k}{M_p}. \quad (26)$$

The frequency of the open-loop poles of (22) are

$$\omega_1 = \sqrt{\frac{k}{M_p}} = \sqrt{\frac{k_a + k_f}{M_p}}. \quad (27)$$

The corresponding open-loop zeros are

$$\begin{aligned} z_1 &= \sqrt{\omega_i^2(1 - v_i)}, \\ &= \sqrt{\frac{K_a + k_f}{M_p} \left(1 - \frac{K_a}{k} \right)}, \\ &= \sqrt{\frac{k_f}{M_p}}. \end{aligned} \quad (28)$$

which for the system are $\omega_1 = 6.4$ kHz and $z_1 = 5.5$ kHz.

4.2. Control Design

Classical Integral Force Feedback.

The open-loop frequency response of $G_{F_s F_a}(s)$ is shown in Figure 6. A key observation is that its phase response lies between 0 and 180°. This is a general property of flexible structures with inputs and outputs proportional to the applied and measured forces. The integral controller (3) has a constant phase lag of 90° so the loop-gain of the system lies between -90 and 90°. As a result, the closed-loop system has an infinite gain margin and phase margin of 90°.

The optimal gain and maximum damping ratio for the example system using CIFF is $K_{d1} = 4.3 \times 10^4$ and $\zeta_1^{max} = 0.077$. These values can be validated by the numerical root-locus plot shown in Figure 9. The numerically obtained optimal gain is 4.57×10^4 and the corresponding damping ratio is 0.077. This correlates closely with the predicted values which supports the accuracy of the assumptions made in deriving the optimal gain.

Figure 6: Frequency response of $G_{F_s F_d}(s)$.

Figure 7: Block diagram of the mechanical system using OIFF.

Optimal Integral Force Feedback.

The CIFF method can be extended to include the feed-through term β as illustrated in Figure 4(a). For practical implementation, the equivalent controller $\hat{C}_d(s)$ presented to the plant, i.e the transfer function from F_s to δ , is found as

$$\hat{C}_d(s) = \frac{C_{d2}(s)}{1 + C_{d2}(s)\beta}. \quad (29)$$

The additional “feed-through term β is effectively added into the controller. Hence, no physical modification of piezoelectric actuator or mechanical system is required.

The relationship between β and ζ described in (14) for the case study is plotted in Figure 8. The maximum modal damping with CIFF is 0.077; however, with OIFF, the maximum modal damping ranges from 0.077 to 1 with different values of β .

The root locus of the system is shown in Figure 9. The optimal feedback gain, maximum damping ratio and corresponding value of β is given in Table 2. These values are validated by the numerical root-locus plot in Figure 9 which is also given

Figure 8: The relationship between β and ζ for OIFF

Table 2: Comparison between analytic and numerically obtained damping ratio and feedback gain.

β	Analytic		Numerical	
	$\hat{\zeta}^{max}$	K_{d2}	$\hat{\zeta}^{max}$	K_{d2}
-6.67×10^7	0.500	5.65×10^4	0.501	5.57×10^4
-6.85×10^7	0.600	5.93×10^4	0.601	5.94×10^4
-6.98×10^7	0.707	6.21×10^4	0.708	6.23×10^4
-7.07×10^7	0.800	6.45×10^4	0.801	6.49×10^4
-7.13×10^7	0.900	6.69×10^4	0.902	6.70×10^4

Figure 9: Root locus comparison between CIFF and OIFF.

in Table 2. These values correlates closely with the predicted values which supports the accuracy of the assumptions made in deriving the optimal gain. The closed-loop transfer function between the reference r to the sensor force F_s is

$$G_{F_s r}(s) = \frac{F_s}{r} = \frac{\hat{C}_d G_{F_s \delta}}{1 + \hat{C}_d G_{F_s \delta}}. \quad (30)$$

when $s = 0$

$$G_{F_s r}(0) = \frac{\hat{C}_d G_{F_s \delta}(0)}{1 + \hat{C}_d G_{F_s \delta}(0)} = \frac{G_{F_s \delta}(0)}{G_{F_s \delta}(0) + \beta}. \quad (31)$$

This shows that the DC gain of the closed-loop increases as β is decreased. Recall that the maximum damping ratio of the closed-loop system increase as β is decreased.

Effects of external force disturbance

We examine the effect of external force disturbances to the system. Consider the block diagram shown in Figure 10, where an external force disturbance ω acts on the platform of the system in addition to the actuator force F_a . The transfer function from the disturbance ω to the displacement of the platform d is

$$G_{d\omega}(s) = \frac{G_{dF_a}(1 + \hat{C}_d K_a)}{1 + \hat{C}_d K_a(1 - K_a G_{dF_a})} \quad (32)$$

Figure 10: Block diagram of the CIFF system for analysis.

Figure 11: Frequency response of $G_{d\omega}(s)$, where d is scaled to μm .

Figure 11 shows the frequency response of $G_{d\omega}(s)$ for both CIFF and OIFF with $\zeta_d=0.707$. From the frequency response, we can conclude that the improvement in damping results in a system that is more sensitive to external force disturbance.

5. Experimental Results

The experiment was conducted on a Queensgate OSM-Z-100B objective lens positioner and Olympus 100 x objective lens as shown in Figure 12. This is single-axis lens positioner has a range of $100 \mu\text{m}$ and a static stiffness of 1.5 N m^{-1} . The weight of the objective lens is 88.8 g . The damping controller is implemented using analog electronics. The block diagram of the experimental set-up is shown in Figure 13. The open-loop frequency response of the stage was measured from the input of the voltage amplifier u_2 which is proportional to the input force F_a to the force sensor F_s and position sensor output d . The frequency responses were measured using a HP 89410A vector signal analyser with an excitation of $100 \text{ mV}_{\text{pp}}$ random noise signal.

Figure 12: Queensgate OSM-Z-100B objective lens positioner and Olympus 100 x objective lens.

Figure 13: Block diagram of the experimental set-up.

The open-loop frequency responses are shown in Figure 14. The open-loop resonance frequency is around 383 Hz. The frequency responses in Figure 14 reveal an extremely high modal density. The first two modes are relative close in frequency. The root locus of the system using CIFF corresponds to $\beta = 0$ in Figure 15. Here, the optimal gain and maximum damping ratio are numerically obtained from the root locus plot.

$$K_{d1} = 1700 \quad \zeta^{max} = 0.33 \quad (33)$$

Figure 15 also includes the root locus plots of the system using OIFF at different values of β . The relationship between β and ζ (shown in Figure 16) was numerically found from the root locus plot. These values are summarised in Table 3. The maximum damping ratio can be increased from 0.33 to 0.85 by adjusting the value of the

Table 3: Experimental results: Numerically obtained data.

β	$\hat{\zeta}^{max}$	K_{d2}	β	$\hat{\zeta}^{max}$	K_{d2}
0.0	0.33	1700	-0.4	0.49	2350
-0.1	0.36	1840	-0.5	0.57	2500
-0.2	0.39	2060	-0.6	0.68	2840
-0.3	0.44	2160	-0.7	0.85	3300

(a) Open-loop frequency response measured from the input of the voltage amplifier u_2 to the output of the force sensor F_s and closed-loop frequency response measured from u_1 to the output of the force sensor F_s for CIFF and OIFF.

(b) Open-loop frequency response measured from the input of the voltage amplifier u_2 to the output of the position sensor d and closed-loop frequency response measured from u_1 to the output of the position sensor d for CIFF and OIFF.

Figure 14: Experimental results: Open and closed-loop frequency responses using OIFF and OIFF

Figure 15: Experimental results: Root locus using OIFF.

Figure 16: Experimental relationship between ζ and β using OIFF.

Figure 17: Step response of the system using CIFF and OIFF with a reference of $1\mu\text{m}$.

feed through term β . Figure 14 also shows the closed-loop frequency responses of the system using CIFF and OIFF with $\beta = -0.6$. The closed-loop frequency responses are measured from u_1 to the force sensor F_s and position sensor output d . The closed-loop response shows that with both methods the first mode has been effectively damped. However, the damping ratio and settling time improvements cannot be directly observed from the frequency plots because of the difference in DC gain.

The most direct method for experimentally observing damping ratio and settling time is the step response of the system which is plotted in Figure 17. The overshoot of the system with CIFF is 18% as compared to 1% for the system with OIFF. This shows the significant improvement in the system's damping. Furthermore, the 0.1% settling time of the system with CIFF controller is 0.173 compared to 0.102 for the system with OIFF, an improvement of over 40%. The additional oscillation in both step responses is due to the uncontrolled higher-order dynamics of the system.

6. Conclusion

This article describes an extension to integral force feedback damping control that allows arbitrary mechanical damping to be achieved for any mechanical system. An additional feed-through term is added to the system to provide an extra degree of freedom that can be used to arbitrarily manipulate the system zeros and maximum damping. Simulation results on a second-order system demonstrate an increase in the maximum achievable damping from 0.077 to 1 using the proposed extension. This result will allow high-performance mechanical systems to be critically damped with a first-order

Figure A.18: Root locus plot of a system.

control law. The experimental results on a objective lens positioner demonstrates an increase in the maximum damping from 0.33 to 0.68. The closed-loop frequency responses shows that the dominant first resonance mode have been attenuated by 20 dB and an improvement in the step response. Furthermore, the step response of the system with OIFF shows an significant improvement in both the overshoot and settling time as compared to the system with CIFF. Future work includes modelling the system in a negative imaginary framework to facilitate system with multiple sensors and actuators.

Appendix A. Derivation of maximum modal damping and controller gain.

In a CIFF system, the closed-loop poles with open-loop system (1) and controller (3) is

$$s(s^2 + \omega_i^2) + K_{d1}(s^2 + z_i^2) = 0, \quad (\text{A.1})$$

where the pole position

$$s = \rho(K_{d1})(-\zeta + i\sqrt{1 - \zeta^2}). \quad (\text{A.2})$$

is a function of the integral force feedback gain K_{d1} .

The expression of the maximum modal damping (4) and controller gain (5) can be found analytically from the root locus plot shown in Figure A.18. The solutions were computed using Maple, a computer algebra software. The procedures are listed below.

1. Replace (A.2) into (A.1),

$$\begin{aligned} \rho(K_{d1}) \left(-\zeta + i\sqrt{-\zeta^2 + 1} \right) \left(\omega^2 + (\rho(K_{d1}))^2 \left(-\zeta + i\sqrt{-\zeta^2 + 1} \right)^2 \right) \\ + K_{d1} \left(\omega^2 + (\rho(K_{d1}))^2 \left(-\zeta + i\sqrt{-\zeta^2 + 1} \right)^2 \right) = 0 \end{aligned} \quad (\text{A.3})$$

2. Separate (A.3) into real and imaginary components.

$$\text{Real: } -4 (\rho (K_{d1}))^3 \zeta^3 + 2 (\rho (K_{d1}))^2 K_{d1} \zeta^2 + 3 \zeta (\rho (K_{d1}))^3 - \rho (K_{d1}) \omega^2 \zeta - (\rho (K_{d1}))^2 K_{d1} + K_{d1} z^2 \quad (\text{A.4})$$

$$\text{Imaginary: } 4 (\rho (K_{d1}))^3 \zeta^2 \sqrt{-\zeta^2 + 1} - 2 \sqrt{-\zeta^2 + 1} (\rho (K_{d1}))^2 K_{d1} \zeta - \sqrt{-\zeta^2 + 1} (\rho (K_{d1}))^3 + \rho (K_{d1}) \sqrt{-\zeta^2 + 1} \omega^2 \quad (\text{A.5})$$

3. The pole position that corresponds to the maximum damping is the point when a line tangent from the origin intersects with the root locus. This can be found analytically by differentiating (A.4) and (A.5) with respect to K_{d1} , i.e. $\frac{d}{dK_{d1}}$ (A.4) and $\frac{d}{dK_{d1}}$ (A.5).
4. Replace the variables $\rho(K_{d1}) = c_p$ and $\frac{d}{dK_{d1}} \rho(K_{d1}) = c_{dp}$, where c_p and c_{dp} are constants, into (A.4), (A.5), $\frac{d}{dK_{d1}}$ (A.4) and $\frac{d}{dK_{d1}}$ (A.5).
5. Solve the four equations in Step 4 simultaneously to find solutions for the unknowns ζ, ρ, dro, K_{d1} .

There are eight possible solutions from Step 5, however, only one of the solutions is non-trivial and feasible. The solutions includes

- Two trivial solution correspond to $c_p = c_p$. These solutions are neglected as both the controller gain $K_{d1} = \frac{(\omega^2 + c_p^2)c_p}{c_p^2 + z^2}$ and $K_{d1} = -\frac{(\omega^2 + c_p^2)c_p}{c_p^2 + z^2}$ requires c_p which is still an unknown variable.
- Two solutions correspond to an imaginary valued controller gain $K_{d1} = \frac{\omega^2}{\sqrt{-\omega z}}, \omega z > 0$. These solutions are neglected.
- Two solutions correspond to an imaginary valued $c_p, \omega z > 0$. These solutions are neglected because the absolute value of a complex number has to be real valued.
- One solution correspond to a negative valued damping ratio where $\zeta = -\frac{\omega - z}{2z}, \omega > z$ which is neglected.
- The remaining solution is

$$\zeta_i^{\max} = \frac{\omega_i - z_i}{2\omega_i}, K_{d1} = \omega_i \sqrt{\frac{\omega_i}{z_i}}. \quad (\text{A.6})$$

which corresponds to the maximum damping (4) and controller gain (5) expressions.

In a OIFF system, the same proof procedure can be done to obtain the maximum modal damping (12) and controller gain (13). However, this is not required as the OIFF system can be cast into a CIFF system via a change of variables. Hence, the expressions in (4) and (5) can be re-written to incorporate the change of variables

$$\hat{\zeta}_i^{max} = \frac{\omega_i - \hat{z}_i(\beta)}{2\hat{z}_i(\beta)}, K_{d2} = \omega_i \sqrt{\frac{\omega_i}{\hat{z}_i(\beta)}}. \quad (\text{A.7})$$

Acknowledgement

The authors would like to thank Prof. Andr Preumont from the Active Structures Laboratory, Universit libre de Bruxelles, Belgium for providing the solution to the maximum modal damping and controller gain for Classical Integral Force Feedback.

References

- [1] A. Preumont, Vibration control of active structures, Kluwer Academic Publishers, 2002.
- [2] E. I. Rivin, Passive vibration isolation, ASME, 2003.
- [3] D. J. Inman, Vibration with control, John Wiley and Sons Ltd., Chichester, England, 2006.
- [4] M. Moshrefi-Torbati, J. Forrester, A. Forrester, A. Keane, M. Brennan, S. Elliott, Novel active and passive anti-vibration mountings, Journal of Sound and Vibration 331 (7) (2012) 1532 – 1541.
- [5] N. Jalili, K. Laxminarayana, A review of atomic force microscopy imaging systems: application to molecular metrology and biological sciences, Mechatronics 14 (8) (2004) 907–945.
- [6] A. J. Fleming, B. J. Kenton, K. K. Leang, Bridging the gap between conventional and video-speed scanning probe microscopes, Ultramicroscopy 110 (9) (2010) 1205–1214. doi:10.1016/j.ultramic.2010.04.016.
- [7] Y. K. Yong, S. S. Aphale, S. O. R. Moheimani, Design, identification, and control of a flexure-based xy stage for fast nanoscale positioning, IEEE Transactions on Nanotechnology 8 (1) (2009) 46.
- [8] Yin Zhang, H. Zhao, L. Zuo, Contact dynamics of tapping mode atomic force microscopy, Journal of Sound and Vibration 331 (23) (2012) 5141 – 5152.
- [9] A. Ferreira, C. Mavroidis, Virtual reality and haptics for nanorobotics, IEEE Robotics and Automation Magazine 13 (3) (2006) 78–92.
- [10] A. A. Tseng, S. Jou, A. Notargiacomo, T. P. Chen, Recent developments in tip-based nanofabrication and its roadmap, Journal of Nanoscience and Nanotechnology 8 (5) (2008) 2167–2186.

- [11] L. Vaillon, C. Philippe, Passive and active microvibration control for very high pointing accuracy space systems, *Smart Materials and Structures* 8 (6) (1999) 719–728.
- [12] A. J. Bronowicki, N. S. Abhyankar, S. F. Griffin, Active vibration control of large optical space structures, *Smart Materials and Structures* 8 (6) (1999) 740–752.
- [13] J. Lin, Z. Huang, P. Huang, An active damping control of robot manipulators with oscillatory bases by singular perturbation approach, *Journal of Sound and Vibration* 304 (1–2) (2007) 345 – 360.
- [14] M. Jaensch, M. U. Lampérth, Development of a multi-degree-of-freedom micropositioning, vibration isolation and vibration suppression system, *Smart Materials and Structures* 16 (2) (2007) 409–417.
- [15] M. D. Rao, Recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes, *Journal of Sound and Vibration* 262 (3) (2003) 457 – 474.
- [16] S.-y. Wu, T. L. Turner, S. A. Rizzi, Piezoelectric shunt vibration damping of an f-15 panel under high-acoustic excitation, in: *SPIE’s 7th Annual International Symposium on Smart Structures and Materials*, International Society for Optics and Photonics, 2000, pp. 276–287.
- [17] J. L. Fanson, T. K. Caughey, Positive position feedback control for large space structures, *AIAA Journal* 28 (4) (1990) 717–724.
- [18] S. S. Aphale, B. Bhikkaji, S. O. R. Moheimani, Minimizing scanning errors in piezoelectric stack-actuated nan positioning platforms, *IEEE Transactions on Nanotechnology* 7 (1) (2008) 79–90.
- [19] M. Ahmadian, S. N. Mahmoodi, Modified acceleration feedback for active vibration control of aerospace structures, *Smart Materials and Structures* 19 (6) (2010) 65015–65024.
- [20] A. J. Fleming, S. O. R. Moheimani, Sensorless vibration suppression and scan compensation for piezoelectric tube nan positioners, *IEEE Transactions on Control Systems Technology* 14 (1) (2006) 33–44.
- [21] A. J. Fleming, S. Behrens, S. O. R. Moheimani, Optimization and implementation of multi-mode piezoelectric shunt damping systems, *IEEE/ASME Transactions on Mechatronics* 7 (1) (2002) 87–94.
- [22] H. R. Pota, S. O. R. Moheimani, M. Smith, Resonant controllers for smart structures, in: *Proc. IEEE Conference on Decision and Control*, Phoenix, Arizona, 1999, pp. 631–636.
- [23] A. Preumont, B. De Marneffe, A. Deraemaeker, F. Bossens, The damping of a truss structure with a piezoelectric transducer, *Computers & structures* 86 (3) (2008) 227–239.

- [24] A. Preumont, *Mechatronics, Dynamics of electromechanical and piezoelectric systems*, Springer, Dordrecht, The Netherlands, 2006.
- [25] A. J. Fleming, Nanopositioning system with force feedback for high-performance tracking and vibration control, *IEEE Transactions on Mechatronics* 15 (3) (2010) 433–447.
- [26] A. J. Fleming, K. K. Leang, Integrated strain and force feedback for high performance control of piezoelectric actuators, *Sensors and Actuators A* 161 (1-2) (2010) 256–265. doi:10.1016/j.sna.2010.04.008.
- [27] S. S. Aphale, A. J. Fleming, S. O. R. Moheimani, Integral resonant control of collocated smart structures, *IOP Smart materials and Structures* 16 (2007) 439–446. doi:10.1088/0964-1726/16/2/023.
- [28] A. J. Fleming, S. S. Aphale, S. O. R. Moheimani, A new method for robust damping and tracking control of scanning probe microscope positioning stages, *IEEE Transactions on Nanotechnology* 9 (4) (2010) 438–448. doi:10.1109/TNANO.2009.2032418.
- [29] I. R. Petersen, A. Lanzon, Feedback control of negative-imaginary systems, *Control Systems, IEEE* 30 (5) (2010) 54–72.
- [30] Bao-Lin Zhang, G.-Y. Tang, Active vibration control of offshore steel jacket platforms using delayed feedback, *Journal of Sound and Vibration* 332 (22) (2013) 5662 – 5677.
- [31] N. Debnath, S. Deb, A. Dutta, Frequency band-wise passive control of linear time invariant structural systems with h optimization, *Journal of Sound and Vibration* 332 (23) (2013) 6044 – 6062.
- [32] Y. Cheung, W. Wong, H-infinity optimization of a variant design of the dynamic vibration absorber—revisited and new results, *Journal of sound and vibration* 330 (16) (2011) 3901–3912.
- [33] C. Lee, S. M. Salapaka, Fast robust nanopositioning: A linear-matrix-inequalities-based optimal control approach, *IEEE/ASME Transactions on Mechatronics* 14 (4) (2009) 414–422.