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Design of a Two Degree of Freedom Resonant Miniature Robotic Leg

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Abstract—This article identifies the design considerations for a two degree of freedom (DoF) miniature robotic leg utilizing piezoelectric bimorph actuators with a specific focus on the resonance modes of the system. An analytical model was developed using three independent lumped mass models with superposition for tuning the resonance frequencies and optimizing the performance of the leg. The model was verified both experimentally and using FEA.

I. INTRODUCTION

There are many advantages to the miniaturization of autonomous robots. The reduction in size allows the robot access to restricted locations such as, inside water pipes [1], through rubble [2], [3] and even inside the human body [4]. Another advantage is the potential reduction in cost and the possibility for disposable robots. Additionally, due to the small power requirements, it is possible to power a miniature robot from ambient energy sources such as light, electric fields, magnetic fields or vibration [5].

There are several challenges associated with designing a miniature robot. The reduced size of the robot increases the complexity of the power supply, mechanical design, sensors and control of the robot. To combat this, aspects of the design are simplified to reduce the complexity. One common method of reducing the complexity is to reduce the number of actuators or degrees of freedom (DoF) the robot has.

There has been an increased interest in the development of miniature robots over the past decade [6]. For example, Wood et al have developed several miniature robots utilizing piezoelectric bender actuators for locomotion [7]–[11]. One such example is a 1.7 g hexapod configuration autonomous robot that was able to achieve a forward travel speed of 0.9 body lengths per second [12]. This robot was approximately 4.8 cm long and consisted of three leg pairs where each pair of legs were driven by three piezoelectric benders arranged to amplify the displacement and produce a tripod gait.

The use of resonant vibration in miniature robotics was reported in [13]. Their work outlines the development of a range of robots capable of traveling over both land and water. These robots used forced vibration of continua to achieve two axis locomotion utilizing a single piezoelec-

tric actuator. This actuator was driven with frequencies between 0.1 to 100 kHz.

This article investigates the design of a two DoF miniature robot leg. In order to generate locomotion two piezoelectric benders are used. The displacement of the benders are amplified by the mechanical structure of the leg and by driving the two benders at the resonances of the system, a high speed gait can be achieved. The main focus of this work is to identify key design parameters and insights that dictate the resonant frequencies of the miniature robot leg and their associated motion. To achieve this, three lumped mass models are derived and combined using superposition. The model is then compared to FEA and real world experimental results.

The following section will outline the overall leg design, including the physical layout and electrical configuration. Next the parameters for the lumped mass model of the leg are derived and compared to experimental results. The paper will be concluded with a discussion on the key parameters effecting the resonant frequencies of the leg and an outline of the future direction.

II. LEG CONFIGURATION

The size and shape of the leg is intended to be used on a hexapod style miniature robot similar in design to the one shown in Fig. 1. Each leg consists of two parallel piezoelectric bimorph benders joined at the tip by an aluminum end-effector comprised of a flexure part and a leg part as shown in Fig. 2. Unlike most other miniature robot legs [14], the piezoelectric actuators are integrated into the mechanical structure.

Fig. 1. 3-D representation of a hexapod robot using this leg design

The benders are driven using the biased bipolar electrical configuration [15] which utilizes both the positive and negative electric fields for maximum deflection and force.

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Fig. 2. Prototype two degree of freedom piezoelectric leg

By driving each bender with a sinusoidal waveform and varying the phase between the two benders a wide range of end effector motion paths can be achieved. Of most interest, when driven 90° out of phase a circular motion at the tip of the end effector can be produced. A miniature robot comprised of several legs can be controlled by varying the phase and magnitude of each leg.

The unique design of the leg results in two resonant modes that are reasonably close together and by altering a small set of variables these resonant modes can be made to overlap. In general the first resonant mode is dominated by rotation about the z-axis and is referred to as the lifting mode. The second resonant mode is characterized by rotation about the x-axis and is referred to as the swinging mode. By forcing these modes to overlap the resonant motion of the leg will be a combination of lifting and swinging.

III. MODELING

The modal superposition technique is used to develop a 3-DoF lumped mass model for the previously discussed miniature robot leg. This method was chosen over numerical methods as a lumped mass model can be used to provide insight into the design parameters of the system. This technique uses superposition of multiple single DoF lumped mass systems to create a complete model of the system. Each individual system is comprised of either a mass, stiffness and force, or a torque, rotational stiffness and rotational inertia as per Fig. 3, where M is the mass, I is the rotational inertia, K is the stiffness, F is the force and τ is the torque.

Each lumped mass system will be related a point that is along the centroid of the flexure and midway between the two piezoelectric benders, shown in Fig. 2. The three axis to be determined are: displacement along the y-axis δ_y , rotation about the x-axis θ_x and rotation about the z-axis θ_z . In order to solve the dynamic equation and find the resonant frequencies, the stiffnesses, mass and Inertias of the individual systems must be determined. The damping factor of the system is ignored to simplify this analysis.

Fig. 3. Single DoF equivalent lumped mass model

Additionally, the force and torques must be found to solve for the static displacements such that,

$$\begin{bmatrix} F_y \\ \tau_x \\ \tau_z \end{bmatrix} = \begin{bmatrix} K_y & 0 & 0 \\ 0 & J_x & 0 \\ 0 & 0 & J_z \end{bmatrix} \begin{bmatrix} \delta_y \\ \theta_x \\ \theta_z \end{bmatrix}. \quad (1)$$

A. Linear DoF δ_y

In order to determine the stiffness and mass parameters for the y-axis an appropriate model for the system must be developed. Fig. 4 shows the mass and stiffness model used for the y-axis lumped mass model. This model was used in combination with the Rayleigh method of equating the maximum potential energy to the maximum kinetic energy of the system, $PE_{\max} = KE_{\max}$. In order to solve this equation the relative displacements of each mass were found such that,

$$\delta_y = \delta_{\max}, \quad (2)$$

$$\delta_p = \delta_{\max} A, \quad (3)$$

$$A = \frac{K_f}{K_f + K_p}, \quad (4)$$

where δ_y is the displacement of the leg mass, $\delta_{\max}$ is the maximum displacement, δ_p is the displacement of the bender mass and K_p and K_f are the stiffnesses of the bender and flexure.

Using Eq. (2) to (4) the maximum potential energy of the system is,

$$PE_{\max} = \frac{1}{2} [K_f (\delta_{\max} - \delta_{\max} A)^2 + K_p (\delta_{\max} A)^2], \quad (5)$$

$$PE_{\max} = \frac{1}{2} \delta_{\max}^2 [K_f (1 - A)^2 + K_p A^2]. \quad (6)$$

Similarly the maximum kinetic energy is given by,

$$KE_{\max} = \frac{1}{2} \left[(M_p + M_f) \omega^2 \delta_{\max}^2 A^2 + \frac{M_l}{2} \omega^2 \delta_{\max}^2 \right], \quad (7)$$

$$KE_{\max} = \frac{1}{2} \omega^2 \delta_{\max}^2 \left[(M_p + M_f) A^2 + \frac{M_l}{2} \right], \quad (8)$$

where M_y , M_p , M_l and M_f respectively are the lumped masses of the y-axis DoF, bender, leg and flexure respectively. Lastly, the lumped mass and stiffness can be found by equating the maximum potential and kinetic energy

such that,

$$\omega = \sqrt{\frac{K_f}{(M_p + M_f)A^2 + \frac{M_l}{2}}}, \quad (9)$$

$$K_y = K_f, \quad (10)$$

$$M_y = (M_p + M_f)A^2 + \frac{M_l}{2}, \quad (11)$$

where M_y is the equivalent mass of the y-axis DoF and K_y is the equivalent stiffness of the y-axis DoF.

This model can also be used to determine the static displacements of the leg end-effector by finding the effective stiffness and force while ignoring the effect of the masses such that,

$$K_{y,static} = \frac{K_f K_p}{K_f + K_p}, \quad (12)$$

$$F_y = \frac{(F_1 + F_2)K_f}{2(K_p + K_f)}, \quad (13)$$

F_y , F_1 and F_2 are the forces of the y-axis DoF and the piezoelectric benders respectively and $K_{y,static}$ is the equivalent static stiffness of the leg.

Fig. 4. Lumped mass model for y-axis DoF.

The stiffness of the piezoelectric bender, K_p , in the y-axis is found using the boundary conditions for a cantilever beam [16],

$$K_p = \frac{3Dw_p}{2l_p^3}, \quad (14)$$

where K_p is the stiffness of the beam, w_p is the width of the beam, l_p is the length of the beam and D is the flexural stiffness of a triple layer beam, given by $D = \int E(y)y^2 dy$ [15]. Similarly, the bending stiffness of the flexure is

$$K_f = \frac{36E_f I_f}{P^3}, \quad (15)$$

where K_f is the stiffness of the flexure, E_f is the Young's modulus of the flexure, I_f is the second moment of area of the flexure and P is the distance between the piezoelectric actuators.

The lumped mass, M_p , of the actuator was determined by looking at the kinetic energy of an element of the actuator, $dT = \frac{1}{2}m(x)v(x)^2$ and using a velocity of $v(x) = v_{max}\frac{x}{l_p}$ and the mass per unit length of $m(x) = \rho_p A_p dx$ where v_{max} is the velocity at the tip of the actuator, l_p is the free length of the actuator, ρ_p is the density of the

actuator and A_p is the area. By substituting the velocity and mass equations

$$M_p = m_p \frac{1}{3}, \quad (16)$$

$$M_f = m_f \frac{w_p}{l_f}, \quad (17)$$

where $m_p = \rho_p A_p l_p$ and $m_f = \rho_f A_f l_f$. The mass of the free flexure is ignored and the leg mass, $M_l = \int \rho_l A_l(y)l_l dy$ is assumed to be completely lumped at the reference point.

B. Rotational DoF θ_x

The lumped mass model for the rotational DoF about the x-axis is shown in Fig. 5. The reference axis for this degree of freedom is located through the centroid of the flexure with respect to the y-z plane. Using this model the lumped parameters are,

$$K_{\theta x} = K_{fx} + 2\left(\frac{P}{2}\right)^2 K_p, \quad (18)$$

$$I_x = 2I_{px} + I_{fx} + I_l, \quad (19)$$

$$\tau_x = \frac{P(F_1 - F_2)}{2}, \quad (20)$$

where $K_{\theta x}$ is the equivalent rotational stiffness for the x-axis DoF, K_{fx} is the rotational stiffness of the flexure, I_x , I_{px} , I_{fx} and I_l are the rotational inertias of the equivalent system, actuator, flexure and leg respectively and τ_x is the torque of the x-axis DoF.

Fig. 5. Lumped mass model for θ_x DoF

Because of the symmetrical construction of the leg, the rotational stiffness J_{fx} was found by analyzing half of the flexure and assuming a guided - simply supported boundary condition beam [16], shown in Fig. 6. The angular displacement was found at the simply supported end and Hooke's law was used to determine the rotational stiffness,

$$K_{fx} = \frac{9E_f I_f}{P}. \quad (21)$$

The rotational inertia of a volume can also be found by performing the integral $I = \int_V \rho(r)r^2 dV$ for the volume. In the case of the end-effector a compact cross section is

Fig. 6. Boundary conditions for flexure

assumed, reducing the integral to be evaluated over the length of the leg. The lumped inertias of the flexure and actuators are found by assuming a concentrated mass at a distance r from the center of rotation and ignoring the mass of the free flexure such that,

$$I = mr^2, \quad (22)$$

$$I_{fx} = M_f \left(\frac{P}{2} \right)^2, \quad (23)$$

$$I_{px} = M_p \left(\frac{P}{2} \right)^2. \quad (24)$$

C. Rotation DoF θ_z

The lumped mass model for the z-axis rotational DoF can be seen in Fig. 7. The reference axis for this DoF is located through the centroid of the flexure with respect to the x-y plane. The lumped inertia and rotational stiffness are found using the Rayleigh method described in the y-axis DoF section such that,

$$K_{\theta z} = K_{fz}, \quad (25)$$

$$I_z = (I_{pz} + I_{fz})B^2 + \frac{I_{Iz}}{2}, \quad (26)$$

$$B = \frac{K_{fz}}{K_{fz} + K_{pz}}, \quad (27)$$

where $K_{\theta z}$ is the equivalent rotational stiffness for the z-axis DoF, K_{fz} is the rotational stiffness of the flexure, I_z , I_{pz} and I_{fz} are the rotational inertias of the equivalent system, actuator and flexure respectively.

Fig. 7. Lumped mass model for the θ_z DoF

Similar to the y-axis, the static equivalent stiffness and

torque are,

$$K_{\theta z, static} = \frac{2K_{pz}K_{fz}}{K_{pz} + K_{fz}}, \quad (28)$$

$$\tau_z = \frac{(\tau_1 + \tau_2)K_{fz}}{2(K_{pz} + K_{fz})}. \quad (29)$$

where $K_{\theta z, static}$ is the equivalent stiffness of the z-axis DoF and τ_z , τ_1 and τ_2 are the torques of the z-axis DoF and the piezoelectric benders respectively.

The rotational stiffness for the flexure can be found by analyzing the torsional stiffness of the flexure, given by the equation,

$$K = \frac{JG}{l}, \quad (30)$$

where K is the flexural stiffness, J is the torsion constant based on the cross-sectional area, G is the shear modulus and l is the length of the element. For the flexure,

$$J = \frac{1}{3} w_f t_f^3, \quad (31)$$

$$K_{fz} = \frac{3G_f J}{P}. \quad (32)$$

The rotational stiffness of the piezoelectric bender at the tip can be found using Hooke's law and the equations for motion of the piezoelectric bender from [15] and [17], using this,

$$K_{pz} = \frac{Dw_p}{l_p}. \quad (33)$$

The rotational inertia of the flexure is calculated using the same method as the x-axis inertia except that the rotational axis is changed and therefore the cross section is changed such that,

$$I_{fz} = \frac{m_f(w_f^2 + t_f^2)}{12}. \quad (34)$$

The rotational inertia of the leg can be assumed to be identical to that of the x-axis inertia.

The method used to determine rotational inertia of the actuator is slightly more complicated due to the loading conditions of the piezoelectric bender. To simplify the calculation the actuator is assumed to have a concentrated cross section such that the integral will be only in one direction, the x-axis. To determine the rotational inertia, first the angular velocity and linear momentum must be found with respect to the rotational axis,

$$dL_p(x) = \rho_p w_p t_p v_p(x)(N - x)dx, \quad (35)$$

$$\Omega_p(x) = \frac{v_p(x)}{x}, \quad (36)$$

where $v_p(x)$ is the velocity of the actuator with respect to x and $N - x$ is the distance to the rotational axis. Using these equations and integrating along the length of the

beam, the rotational inertia is,

$$dI_{pz} = \frac{dL_p(x)}{\Omega_p(x)}, \quad (37)$$

$$dI_{pz} = \frac{dL_p(x)}{\Omega_p(x)}, \quad (38)$$

$$dI_{pz} = \rho_p w_p t_p N \left(x - \frac{x^2}{N} \right) dx, \quad (39)$$

$$I_{pz} = m_p l_p N \left(\frac{1}{2} - \frac{L}{3N} \right). \quad (40)$$

D. Finite element analysis

The leg was also analyzed using the Finite Element Analysis (FEA) tool ANSYS to determine the range of motion and resonance modes of the leg. This analysis showed that the first two resonance modes were dominated by rotational motion about the x and z axis. The first two resonant modes can be seen in Fig. 8. The third resonance was significantly higher than the first two and features significant displacement along the y -axis, corresponding to the δ_y DoF.

Fig. 8. A) First resonant mode shape, b) Second resonant mode shape

E. First resonant modes

The resonance frequency for a single DoF system is given by $f = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$ or $f = \frac{1}{2\pi} \sqrt{\frac{K}{I}}$. Table I lists the resonance frequencies of the first modes in the δ_y , θ_x and θ_z degrees of freedom for the lumped mass, FEA and experimental results. The analytical model, FEA and experimental results are generally in agreement.

By analyzing the equations for stiffness, mass and inertia, insights into the properties that effect the resonant frequencies can be determined.

This work has highlighted several parameters that have a significant effect over particular DoF compared to others. The length of the bender has a moderate impact on the δ_y DoF resonant frequency and a significant effect on the resonant frequency for the θ_x DoF while having very little effect on the θ_z DoF resonance. This feature can be exploited to change the θ_x resonant frequency with a marginal alteration to the free length of the actuators. Conversely the separation distance P has a minimal impact on the θ_x DoF but a significant impact on the δ_y and θ_z DoFs. Care must be taken when tuning the resonant frequencies using these two methods to not effect the maximum displacements.

An alternative method of tuning the resonant frequencies is to alter the lumped masses of the leg. If done correctly, adding or removing masses will have little to no effect on the stiffness and therefore maximum displacements of the beam. The simplest method of tuning the mass of the leg is to change the shape of the end effector. Increasing mass at the base of the leg will have a minimal impact on the rotational DoFs as this mass is located very near to the axis of rotation. Conversely, mass added or removed from the tip will significantly alter the resonant frequencies of the rotational DoFs.

IV. EXPERIMENTAL RESULTS

An experimental analysis was conducted using a prototype leg identical in design to the leg shown in Fig. 2 and Fig. 9. The purpose of this analysis was to determine the frequency responses and the static displacements of the leg so as to verify the analytical model. The control signals for the piezoelectric benders were generated using an Agilent 33500B dual channel signal generator connected to two Pdm200 voltage amplifier from PiezoDrive to generate the high voltage driving signals. The leg end effector velocities were measured using a PDV 100 laser vibrometer from Polytec and the low frequency displacements were measured with an LAT61 laser distance sensor from di-soric.

The leg was constructed using two custom made piezoelectric benders measuring 4.5 mm wide, 0.4 mm thick by 15 mm in length. The actuators were glued to a 3-D printed, ABS plastic base 6 mm apart with two part epoxy and with a free length of approximately 12.8 mm. The leg end effector and flexure was machined out of a single piece of aluminum and glued across the tips of the actuators.

TABLE I
FIRST RESONANT MODES FOR EACH DOF

DoF	Lumped Mass	FEA	Experimental
δ_y	1030	946 Hz	720 Hz
θ_x	391 Hz	427 Hz	360 Hz
θ_z	334 Hz	410 Hz	270 Hz

Fig. 9. Miniature robot leg end effector.

Fig. 10. Experimental end-effector displacement with actuators driven 90° out of phase at 5 Hz.

The end effector x, y and z axis displacements were measured by driving the benders 90° out of phase at a frequency of 5 Hz. All displacement measurements were taken at the tip of the end-effector. A summary of the displacements achieved can be seen in Fig. 10 and Table II.

The frequency response shown in Fig. 11, identifies a resonance at approximately 270 Hz, 360 Hz and 720 Hz for the X, Z and Y axis respectively. This response was generated by driving both benders with identical noise with a peak to peak voltage of 10 V and a bandwidth of 1 kHz. This result compares reasonably well with both the FEA and analytical analysis shown in Table. I. The coupling shown in Fig. 11 between the different axis is

TABLE II
MAXIMUM TIP DISPLACEMENTS OF THE LEG IN Y AND Z AXIS

Axis	Analytical	FEA	Experimental
X	478 μ m	374.9 μ m	462.2 μ m
Y	144.3 μ m	192.6 μ m	250.1 μ m
Z	441 μ m	460.1 μ m	552.1 μ m

Fig. 11. Frequency response of miniature robotic leg in x, y and z axis.

primarily due to the alignment of the vibrometer with respect to the leg, and the alignment of the components of the leg during the assembly process.

Discrepancies between the results are contributed to the imperfect mounting and assembly of the leg structure and the non-rigid nature of the leg support structure, overall resulting in a less stiff structure. Additionally damping is present in the real world results and the lumped mass model ignores twisting and shear stress of the piezoelectric actuators.

V. CONCLUSION

The experimental results show that a miniature robotic leg produced in this way can provide enough force and motion to achieve significant motion of the end-effector. The main shortcomings of this design is the repeatability of the leg construction. In order to improve this the manufacturing process must be simplified to attain a uniform and repeatable leg. Another approach to this problem is to fine tune the resonant modes after construction such that the resonant frequencies of all the robots legs are equal. Ideally a combination of these improvements will be investigated

The next step will be to analyze the response of the leg during intermittent ground contact. Preliminary results have shown that an intermittent ground contact will have a damping effect of the frequency response of the leg, however this is greatly dependent on a wide range of variables and more research is required. The final goal of this research is to develop an untethered, autonomous hexapod miniature robot.

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