Scale-by-scale energy budgets which account for the coherent motion

Thiesset¹, F. & Danaila¹, L. & Antonia², R.A & Zhou³, T.
¹ CORIA CNRS UMR 6614, University of Rouen, France
² School of Mechanical Engineering, University of Newcastle, Australia
³ Civil and Resource Engineering, University of Western Australia, Australia
E-mail: danaila@coria.fr

Abstract. Scale-by-scale energy budget equations are written for flows where coherent structures may be prominent. Both general and locally isotropic formulations are provided. In particular, the contribution to the production, diffusion and energy transfer terms associated with the coherent motion is highlighted. Preliminary results are presented in the intermediate wake of a circular cylinder for phase-averaged second-and third-order structure functions. The experimental data provide adequate support for the scale-by-scale budgets.

1. Introduction

Characterizing and understanding turbulent flows are necessary steps prior to modelling and predicting their statistics as well as the dynamical behaviour which underpins these statistics. Since the pioneering work of Townsend (1976) numerous studies have confirmed the persistence of the organized or coherent motion (CM) in a wide variety of turbulent shear flows. Since CM plays an important role in the context of turbulence dynamics, a lot of effort has been devoted to the extraction of coherent structures, with the aim of learning more about their dynamical nature and their contribution to the total statistics. The meaningful extraction of CM entails establishing clear criteria for defining it and hence distinguishing it from the mean flow and the random motion (RM). Since the major challenge is to predict turbulent flow statistics, a first (quite important) step would be to predict the interactions between CM and RM. The process of turbulence birth starts with mean velocity gradients (stagnation points or regions), followed by instabilities which lead to CM and are followed by the onset of RM. For decaying flows such as wakes, there is now abundant experimental and numerical support of the persisting influence of the CM far downstream of the turbulent energy injection (Cimbala et al. (1988), Bisset et al. (1990)).

In turbulent flows, the energy is generally distributed over a wide range of scales. A first feature which distinguishes CM from RM is the scale(s) at which these two motions exist. CM is generally characterized by a single scale, or a very restricted range of scales (a narrow peak in the energy spectrum) whereas RM is represented by a wide range of scales. Kolmogorov (Kolmogorov (1941)) stipulated that there is a scale beyond which the influence of the anisotropic/coherent large scales is not perceptible anymore and, as a consequence, the velocity statistics become locally isotropic. Presumably for this reason, the influence of CM on RM was not considered.
at the level of an arbitrary scale, implicitly supposing that the effect of CM is confined to the level of its characteristic scale. Alternately, both the nonlocality of the cascade and the finite Reynolds number (FRN) effects lead to the fact that CM might play a role on RM at a given, arbitrary scale.

The focus of this study is to understand the relationship between the three main ingredients of the flow (mean velocity, CM and RM) at a given scale (small, intermediate and large). The variables of the problem are the scale \( r \) (separation between two spatial locations) and the CM phase \( \phi \) (also interpreted as the transit period of the CM).

The separation between the CM and RM is done according to a triple decomposition of the instantaneous velocity field proposed by Reynolds & Hussain (1972), viz.

\[ U_i = \overline{U}_i + \tilde{u}_i + u'_i \]

where \( \overline{U}_i, \tilde{u}_i, u'_i \) respectively stand for the mean, coherent and random velocities along the \( i \)-th direction (\( \pi \) indicates time averaging).

Since CM is characterized by a time/scale periodicity, it is useful to introduce the operation of phase-averaging (i.e. averaging over the CM period), noted as \( \langle \cdot \rangle \). The resulting statistics will therefore depend on the scale \( r \) and on the phase \( \phi \). The physical quantities are the energy of the RM and CM for a given set of parameters \( (r, \phi) \), which are second-order structure functions calculated with velocity increments at a scale \( \vec{r} \), defined as

\[
\langle \Delta q'^2 \rangle (\vec{r}, \phi) = \langle (u'_i(\vec{x} + \vec{r}) - u'_i(\vec{x}))^2 \rangle, \quad \Delta \tilde{q}^2 (\vec{r}, \phi) = \langle (\tilde{u}_i(\vec{x} + \vec{r}) - \tilde{u}_i(\vec{x}))^2 \rangle,
\]

where \( \langle \cdot \rangle \) indicates phase averaging (see Reynolds & Hussain (1972) for the procedure), \( q \) indicates the total kinetic energy and double indices indicate summation. As a first step, local isotropy will be supposed so that the dependence is on \( r \) (the modulus of \( \vec{r} \)) only, which corresponds to considering (for instance) the central region of a cylinder wake.

The aim of this study is twofold.

- First, we derive scale-by-scale energy budgets for both CM and RM which mathematically emphasize their interaction at a vectorial scale \( \vec{r} \) separating two spatial points. We provide transport equations of \( \langle \Delta q'^2 \rangle \) and \( \Delta \tilde{q}^2 \), the time-averaged values of \( \langle \Delta q'^2 \rangle (\vec{r}, \phi) \) and \( \Delta \tilde{q}^2 (\vec{r}, \phi) \), General (Section 2.1), as well as locally isotropic formulations (Section 2.2), are provided. These equations evidence additional terms corresponding to the transport, production, diffusion and forcing of RM by CM.

- Second, we turn our attention to the particular case of the wake behind a cylinder (Section 3), in which the scale-phase second-and third-order structure functions are calculated from hot-wire measurements with phase reference. We have chosen this flow because it is characterized by a persisting Bénard-Von Kármán street, even in the so-called 'far field' (Cimbala et al. (1988), Bisset et al. (1990)). Specifically, we identify the energy distributions of the CM and RM at a given \( (r, \phi) \) in the intermediate wake of a circular cylinder. We finally show that the scale-by-scale budget of the random field is well supported by experimental data on the wake centreline.

2. Analytical development

2.1. General formulation

The starting point is the triple decomposition given by Eq. 1, and the objective is to derive transport equations for quantities such as time averages of \( \langle \Delta q'^2 \rangle (\vec{r}, \phi) \) and \( \langle \Delta \tilde{q}^2 \rangle (\vec{r}, \phi) \).

As an example, Reynolds & Hussain (1972) described analytically the interaction between turbulence and an organized wave. These authors derived dynamical and one-point energy
Then, considering the gradient with respect to the mid-point \( \vec{x} \) equations of the organized and random velocity increment, \( \Delta \tilde{u} \) and \( \Delta u' \), respectively, viz. (2004), (3) and (4) are each written at the two points \( \vec{x} \) and \( \vec{x}^+ = \vec{x} + \vec{r} \), where \( \vec{r} \) is the separation vector between the two points, and the superscript \( ' + \) hereafter denotes quantities associated with coherent motion, one step before the temporal averaging. These authors (Reynolds & Hussain (1972)) obtained the dynamical equations for both the random and the coherent motion, viz.

\[
\frac{\partial \tilde{u}_i}{\partial t} + U_j \frac{\partial \tilde{u}_i}{\partial x_j} + \tilde{u}_j \frac{\partial U_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \tilde{u}_i \tilde{u}_j - u_i u_j \right) + \frac{\partial}{\partial x_j} \left( \langle u'_i u'_j \rangle - \tilde{u}'_i \tilde{u}'_j \right) = - \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} .
\]

(3)

\[
\frac{\partial u'_i}{\partial t} + U_j \frac{\partial u'_i}{\partial x_j} + u_j \frac{\partial U_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( u'_i \tilde{u}_j - u'_i u_j \right) = - \frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_j^2} .
\]

(4)

In (3) and (4), \( \nu \) is the kinematic viscosity, \( p \) is the kinematic pressure, and repeated indices signify summation. Following the procedure established by Antonia et al. (1997) and Danaila et al. (2004), (3) and (4) are each written at the two points \( \vec{x} \) and \( \vec{x}^+ \) = \( \vec{x} + \vec{r} \), where \( \vec{r} \) is the separation vector between the two points, and the superscript \( '+' \) hereafter denotes quantities considered at the point \( \vec{x} + \vec{r} \). The substraction of one from the other yields the transport equations of the organized and random velocity increment, \( \Delta \tilde{u}_i = u'_i - \tilde{u}_i \) and \( \Delta u'_i = u'_i + u'_i \) respectively, viz.

\[
\frac{\partial \Delta \tilde{u}_i}{\partial t} + \Delta \left( U_j \frac{\partial \tilde{u}_i}{\partial x_j} \right) + \Delta \left( \tilde{u}_j \frac{\partial U_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \langle u'_i u'_j \rangle - u'_i u'_j \right) = - \left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \Delta \tilde{p} + \nu \left( \frac{\partial^2}{\partial x_j \partial x_j} + \frac{\partial^2}{\partial x_j^+ \partial x_j^+} \right) \Delta \tilde{u}_i ;
\]

(5)

and

\[
\frac{\partial \Delta u'_i}{\partial t} + \Delta \left( U_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left( u'_j \frac{\partial U_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( u'_i \tilde{u}_j - u'_i u_j \right) = - \left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_i^+} \right) \Delta p' + \nu \left( \frac{\partial^2}{\partial x_j \partial x_j} + \frac{\partial^2}{\partial x_j^+ \partial x_j^+} \right) \Delta u'_i .
\]

(6)

At this stage, the statistics at the two points \( \vec{x} \) and \( \vec{x}^+ \) are considered as being independent. Then, considering the gradient with respect to the mid-point \( \bar{X} = \frac{1}{2} (\vec{x} + \vec{x}^+) \) (Hill (2002), Danaila et al. (2004))

\[
\frac{\partial}{\partial x_j} = - \frac{\partial}{\partial r_j} + \frac{1}{2} \frac{\partial}{\partial X_j} ; \quad \frac{\partial}{\partial x_j^+} = \frac{\partial}{\partial r_j} + \frac{1}{2} \frac{\partial}{\partial X_j} .
\]

(7)

by multiplying (5) and (6) with \( 2 \Delta \tilde{u}_i \) and \( 2 \Delta u'_i \) respectively, applying phase followed by a time averaging, and finally noting that

\[
\langle \Delta u'_i \Delta u'_i \rangle = \langle \Delta u_i \Delta u_j \rangle - \Delta \tilde{u}_i \Delta \tilde{u}_j
\]

(8)

\[
\langle \Delta u_j \Delta q^2 \rangle = \Delta \tilde{u}_j \Delta q^2 + \Delta \tilde{u}_j \langle \Delta q^2 \rangle + \langle \Delta u'_i \Delta q^2 \rangle + 2 \Delta \tilde{u}_i \langle \Delta u'_i \Delta u'_i \rangle .
\]

(9)
we obtain the energy budgets for the organized and random motions. These are

\[
\frac{\partial}{\partial t} \Delta q^2 + U_j \frac{\partial \Delta q^2}{\partial x_j} + \frac{1}{2} \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^*} \right) \left[ \left( \tilde{u}_j + \tilde{u}_j^* \right) \Delta q^2 + 2 \left( u_j^* + u_j^{**} \right) \Delta u_j^* \Delta \tilde{u}_j \right] \\
+ 2 \Delta \tilde{u}_j \Delta \tilde{u}_j \frac{\partial U_j}{\partial x_j} - \left\{ \left( u_j^* + u_j^{**} \right) \Delta u_j^* \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^*} \right) \Delta \tilde{u}_j + 2 \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^*} \right) \Delta \tilde{u}_j \Delta \tilde{p} \right\} \\
+ \frac{\partial}{\partial r_j} \Delta \tilde{u}_j \Delta q^2 - 2 \Delta \tilde{u}_j \frac{\partial}{\partial r_j} \left\{ \Delta u_j^* \Delta \tilde{u}_j \right\} \\
+ \nu \left[ - \left( 2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial x_j^2} \right) \Delta q^2 - 2 \left( \frac{\partial \tilde{u}_j}{\partial x_j} \frac{\partial \tilde{u}_j}{\partial x_j} + \frac{\partial \tilde{u}_j^*}{\partial x_j} \frac{\partial \tilde{u}_j^*}{\partial x_j} \right) \right] + 2 \left( \tilde{\epsilon} + \tilde{\epsilon}^+ \right) = 0; \quad (10)
\]

and

\[
\frac{\partial}{\partial t} \Delta q^2 + U_j \frac{\partial \Delta q^2}{\partial x_j} + \frac{1}{2} \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^*} \right) \left[ \left( \tilde{u}_j + \tilde{u}_j^* \right) \Delta q^2 + \left( \tilde{u}_j^* + \tilde{u}_j^{**} \right) \left( \Delta q^2 \right) \right] \\
+ 2 \Delta \tilde{u}_j \Delta \tilde{u}_j \frac{\partial U_j}{\partial x_j} + \left\{ \left( u_j^* + u_j^{**} \right) \Delta u_j^* \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^*} \right) \Delta \tilde{u}_j + 2 \left( \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j^*} \right) \Delta \tilde{u}_j \Delta \tilde{p} \right\} \\
+ \frac{\partial}{\partial r_j} \Delta \tilde{u}_j \Delta q^2 - 2 \Delta \tilde{u}_j \frac{\partial}{\partial r_j} \left\{ \Delta u_j^* \Delta \tilde{u}_j \right\} \\
+ \nu \left[ - \left( 2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial x_j^2} \right) \Delta q^2 - 2 \left( \frac{\partial \tilde{u}_j}{\partial x_j} \frac{\partial \tilde{u}_j}{\partial x_j} + \frac{\partial \tilde{u}_j^*}{\partial x_j} \frac{\partial \tilde{u}_j^*}{\partial x_j} \right) \right] + 2 \left( \epsilon' + \epsilon'^+ \right) = 0, \quad (11)
\]

where \( \Delta q^2 = \Delta \tilde{u}_j \Delta \tilde{u}_j \) and \( \Delta q^2 = \Delta u_j^* \Delta u_j^* \) are respectively the CM and RM kinetic energies at a given scale. The quantities \( \tilde{\epsilon} = \nu \left( \frac{\partial \tilde{u}_j}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_j^*} \right)^2 \) and \( \epsilon' = \nu \left( \frac{\partial u_j^*}{\partial x_j} + \frac{\partial u_j^*}{\partial x_j^*} \right)^2 \) are the mean energy dissipation rates of the coherent and the random motions, respectively.

For the sake of simplicity, (10) and (11) can be formally written as

\[
I_c + A_{cm} + D_{cc} + D_{rc}^1 + P_{cm} - P_{rc} + T_c + F_c + \mathcal{V} + 2 \left( \tilde{\epsilon} + \tilde{\epsilon}^+ \right) = 0; \quad (12)
\]

\[
I_r + A_{rr} + D_{rr}^2 + P_{cm} + P_{rc} + D_{cp} + T - F_c + \mathcal{V} + 2 \left( \epsilon' + \epsilon'^+ \right) = 0, \quad (13)
\]

where \( I, A, D, P, T, F \) et \( \mathcal{V} \) denote respectively the non stationarity, advection, diffusion, production, transfer, forcing and viscous terms. The subscripts \( m, c, r \) correspond to the mean, coherent and random motions, and \( D_p \) indicates the pressure diffusion.
Equations (10) and (11) are the general formulations of the scale-by-scale budgets which account for the coherent motion in which each term depends on the separation vector $\vec{r}$. For homogeneous flows and in the limit of large separations, the scale-by-scale budgets (10) and (11) are fully consistent with one-point energy budget provided by Reynolds & Hussain (1972) (Eqs. (3.2b) and (3.2c) p. 266 in Reynolds & Hussain (1972)). Each term can be evaluated by DNS (Direct Numerical Simulations), without any other assumption. Equations (10) and (11) provide a general basic framework which ought to allow the physics of the interaction between coherent and random fields to be unravelled.

By comparison with Danaila et al. (2004), there are additional terms which emerge in the present equations, e.g. the terms $P_{rc}$, $T_c$ and $F_c$ which can be identified as the production of random fluctuations by the coherent motion, the coherent kinetic energy transfer and the forcing associated by the presence of a coherent motion are emphasized. All three are present in equations (10) and (11), but with opposite sign. This means that what represents a loss of energy for CM (10), constitutes a gain of energy for RM (11). Further, we can shed some light on the transport of random statistical quantities by the organized motion $D^1_{rc}$ and $D^2_{rc}$.

2.2. The locally homogeneous and isotropic context

We now turn our attention to the derivation of (10) and (11) in a locally homogeneous and isotropic context. This assumption yields considerable simplifications of the analytical development and allows an easier comparison with simpler quantities, which could be therefore inferred from measurements (Danaila et al. (1999), Danaila et al. (2001)). Considering first a locally homogeneous turbulent flow, the viscous term simplifies such as

$$
\nu \left[ - \left( 2 \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial X_j^2} \right) \Delta q^2 - 2 \left( \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} + \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} \right) \right] = 2\nu \frac{\partial^2}{\partial r_j^2} \Delta q^2, \quad (14)
$$

since $\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} = 0$ and $\frac{\partial^2}{\partial X_j^2} = 0$, Hill (1997). The same simplification holds for the coherent motion. Then, in the context of local isotropy, the divergence and the Laplacian operators are expressed as:

$$
\frac{\partial}{\partial r_j} = \frac{2}{r} r + \frac{\partial}{\partial r}, \quad \frac{\partial^2}{\partial r_j^2} = \left( \frac{2}{r} + \frac{\partial}{\partial r} \right) \frac{\partial}{\partial r}, \quad (15)
$$

By further using (15), after multiplying (10) and (11) with $r^2 = r_j r_j$, integrating with respect to $r$ and dividing by $r^2$, we obtain

$$
\frac{1}{r^2} \int_0^r s^2 \left( A_{cm} + D_{rc} + D^1_{rc} + P_{cm} + P_{rc} + D_{cp} \right) ds + \Delta \bar{u}_l \Delta \bar{q}_l^2 - \frac{2}{r^2} \int_0^r \Delta \bar{u}_l \frac{\partial}{\partial s} s \left\langle \Delta u'_l \Delta u'_l \right\rangle ds - 2\nu \frac{\partial}{\partial r} \Delta q^2 + \frac{4}{3} \epsilon = 0; \quad (16)
$$

$$
\frac{1}{r^2} \int_0^r s^2 \left( A_{cm} + D_{rr} + D^2_{rc} + P_{rm} - P_{rc} + D_{cp} \right) ds + \left\langle \Delta u_l \Delta q^2 \right\rangle - \Delta \bar{u}_l \Delta \bar{q}_l^2 - \frac{2}{r^2} \int_0^r \Delta \bar{u}_l \frac{\partial}{\partial s} s \left\langle \Delta u'_l \Delta u'_l \right\rangle ds - 2\nu \frac{\partial}{\partial r} \Delta q^2 + \frac{4}{3} \epsilon = 0. \quad (17)
$$

Equations (16) and (17) are the scale-by-scale energy budgets of the organized and random motions respectively, in a locally homogeneous and isotropic context. Here, $s$ is a dummy...
variable and the subscript $\parallel$ denotes the direction parallel to the separation vector. When the spatial separation is inferred by invoking Taylor hypothesis, this direction coincides with the direction of the mean flow.

The first line of Eqs. (16) and (17) represents the energy contribution of the largest scales (Danaila et al. (2004)). The main difference with respect to the extended form of Kolmogorov equation Antonia et al. (1997) equation, consists in several extra terms due to the presence of CM. The effective energy transfer of the random velocity component is explicit and is thus constituted of the total energy transfer $\langle \Delta u_\parallel \Delta q^2 \rangle$ (including the coherent and random contributions), from which is subtracted the coherent energy transfer $\Delta u_\parallel \Delta q^2$ and the forcing term $\frac{2}{\kappa} \int_0^\infty \Delta u_\parallel \frac{\partial}{\partial s} s^2 \langle \Delta u'_{\parallel} \Delta u'_s \rangle ds$.

3. Results in the wake of a circular cylinder

3.1. Measurements

The analytical considerations previously developed are now used to assess the essential physics and particularly the dynamical nature associated with the presence of the organized motion. Hot-wire measurements previously made at the University of Newcastle (Zhou et al. (1992)) are used to calculate two-point statistics of CM and RM. The measurements were conducted in an open-circuit wind tunnel with a square working section of $0.35 \times 0.35m$ and $2.4m$ long. The cylinder of diameter $d = 12.7mm$ was placed horizontally to generate the wake flow. The three vorticity components were measured simultaneously by means of a four-X-wire probe, tested in grid turbulence by Antonia et al. (1998). The downstream location investigated here is $40d$, sufficiently far from the energy injection to expect the local isotropy to be verified and close enough to accurately extract the organized motion. The free stream velocity $U_0$ is $3m.s^{-1}$ corresponding to a Reynolds number, based on the cylinder diameter and upstream velocity of $Re_d = 2525$, and a Taylor micro-scale Reynolds number of about 70 on the wake centerline.

When making the hot wire probe, great effort has been attempted to keep $\Delta x = \Delta y = \Delta z$, which are about $6\eta$, where $\eta$ is the Kolmogorov length scale, and can be obtained using $\eta \equiv (\nu^3/\epsilon)^{1/4}$. This spatial separation leads to an attenuation of the measured velocity gradients, which should have been corrected by using the spectral method proposed by Zhu & Antonia (1996). However, the estimation based on isotropic assumption, i.e. $\epsilon'_{iso} = 15\nu (\frac{du}{dz})^2$, can approximate the full energy dissipation rate properly, around the wake centerline, in terms of the mean value (Zhu & Antonia (1996)). The spatial separation $\Delta x$ is calculated by means of the Taylor’s hypothesis, $\Delta x = -U_0 \Delta t$, where $U_c$ is the mean velocity at the center of the vortices and is equal to $U_c = 0.92U_0$ at $40d$ downstream the cylinder (Zhou & Antonia (1992)).

To calculate phase averaged statistics, the transverse velocity signal $v$ is bandpass filtered at a frequency centered on Strouhal frequency. The Hilbert transform $h$ of the filtered signal $v_f$ is calculated and the phase is inferred from $\phi(t) = \tan^{-1} \left( \frac{h(t)}{v_f(t)} \right)$. Finally, the phase is divided into 32 segments and phase averaged statistics are calculated for each class. Statistics were calculated over 750 integral time-scales, convergence of the statistics was checked and found to be satisfactory.

3.2. Phase-scale second-and third-order structure functions, on the wake centerline

The essential difference between our approach and the classical energy budget equations is the phase averaging operation which allows us to assess the temporal dynamics associated with the CM, one step before time averaging. Second-and third-order structure functions are as usually functions of $r$, but specific to our methodology, they are also functions of the phase $\phi$ of the CM, before being time-integrated.
Fig1(a) and 1(b) represent the phase-scale distributions of the kinetic energy of the organized and random motions respectively. In Fig1(a), one can note a strong temporal periodicity of period $\phi = \pi$ of the coherent motion kinetic energy. This periodicity might also be observable on the $r$ axis, at scales characteristic of the organized motion (not shown). This emphasizes the spatio-temporal periodicity of the Von Kármán street. Concerning the energy distribution of random fluctuations (Fig1(b)), the influence of the CM is less perceptible. Its shape in scale $r$ is very similar to that of the 'classical' time-integrated second-order structure function.

Figure 1. (a) $\log_{10}(\Delta q^2)$ function of $r$ and $\phi$ on the wake center line at $x = 40d$, $r$ is normalized by $L_v = 4.2d$, the streamwise distance between two consecutive vortices. (b) $\log_{10}(\langle \Delta q^2 \rangle)$ function of $r$ and $\phi$ on the wake center line at $x = 40d$. (c) Transfer term $-\langle \Delta u \parallel \Delta q^2 \rangle / \epsilon r$ function of $r$ and $\phi$ on the wake center line at $x = 40d$. (d) Phase averaging of the spanwise vorticity component $\tilde{\omega}_z$ normalized by $d/U_0$ in the plan $(\phi, y)$.

The non linear transfer term $-\langle \Delta u \parallel \Delta q^2 \rangle$ divided by $\epsilon r$ is displayed on Fig1(c). The temporal periodicity is strongly discernible, and reveals two maxima at a phase location $\phi = \pm \pi/2$ and a scale $r \approx L_v/10 \approx \lambda$, where $\lambda$ is the Taylor micro-scale. Furthermore, at $\phi = 0$ this term reveals some negative values, which might a priori be associated with a local, inverse cascade. Noticeable are also maximum values larger than $4/3$, presumably signifying very accelerate cascade.

In Fig1(d), it is represented the phase-averaged vorticity spanwise component. This is a
pertinent criterium of the Von Kármán street, which has a negative sign vortex centered at \( \phi = -\pi/2, y = 0.75d \). Its partner of positive sign, not visible on the figure, is located at \( \phi = \pi/2, y = -0.75d \). Therefore, the two maxima of the non-linear transfer term coincide with the vortex centers. There is evidence that the organized motion induces a strong temporal effect on the kinetic energy transfer at a scale \( r \).

### 3.3. Scale-by-scale budget

We now turn our attention to time-averaged structure functions. Fig. 2 contains the total non-linear transfer \(-\langle \Delta u||\Delta q^2 \rangle\), the additional coherent transfer and forcing due to the coherent motion \( \Delta \tilde{u}||\Delta q^2 + \frac{2}{r^2} \int_0^r \Delta \tilde{u} \frac{\partial}{\partial s} s^2 \langle \Delta u'||\Delta u'_i \rangle ds \) and the effective transfer inferred from their sum, in function of the separation \( r/L_v \).

![Figure 2](image_url)

**Figure 2.** Non linear transfer term divided by \( \epsilon r \). —— total transfer term \(-\langle \Delta u||\Delta q^2 \rangle\), - - - - coherent transfer and forcing term \( \Delta \tilde{u}||\Delta q^2 + \frac{2}{r^2} \int_0^r \Delta \tilde{u} \frac{\partial}{\partial s} s^2 \langle \Delta u'||\Delta u'_i \rangle ds \), · · · · difference \(-\langle \Delta u||\Delta q^2 \rangle + \Delta \tilde{u}||\Delta q^2 + \frac{2}{r^2} \int_0^r \Delta \tilde{u} \frac{\partial}{\partial s} s^2 \langle \Delta u'||\Delta u'_i \rangle ds \).

For weakly turbulent flows the non linear transfer term is smaller than \( \frac{4}{3} \epsilon r \), because of the cross-over between viscous and large-scale effects (Danaila et al. (1999), Danaila et al. (2004)). Here, \(-\langle \Delta u||\Delta q^2 \rangle/\epsilon r \approx 0.93\). The additional energy transfer associated with the coherent motion is negative, its value is quite small, but nonnegligible. Its contribution is non zero for all separation with a maximum contribution located at about \( 2\lambda \). Finally, the maximum effective transfer of the random motion is smaller than the total transfer by about 12%.

On the wake centerline, the isotropic scale-by-scale budget of the random motion is:

\[
-\frac{1}{r^2} \int_0^r s^2 A_{\text{rm}} ds - \langle \Delta u||\Delta q^2 \rangle
\]

\[+ \Delta \tilde{u}||\Delta q^2 + \frac{2}{r^2} \int_0^r \Delta \tilde{u} \frac{\partial}{\partial s} s^2 \langle \Delta u'||\Delta u'_i \rangle ds + 2\nu \frac{\partial}{\partial r} \Delta q^2 = \frac{4}{3} \epsilon' r, \tag{18}
\]
which means that in the limit of large scales, the advection term is almost entirely compensated by the energy dissipation rate. All terms of the scale-by-scale budget (18) are shown in Fig. 3.

\[ \frac{r}{L} \]

Figure 3. Term in equation (18) divided by \( \epsilon' \).

- \( \frac{4}{3} r \)
- \( \frac{1}{r^2} \int_0^r s^2 A_{rm} ds \)
- \( -\langle \Delta u_\parallel \Delta q^2 \rangle \)
- \( -\Delta \bar{u}_\parallel \Delta q^2 \)
- \( \frac{2}{r^2} \int_0^r \Delta \bar{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u'_\parallel \Delta u'_i \rangle ds \)
- \( 2\nu \frac{\partial}{\partial r} \Delta q^2 \)
- Left hand side of (18).

The balance between the right-and left-hand sides of (18) is reasonably satisfied at all scales. The accuracy with which (18) is validated appears to reflect that local isotropy is achieved. Several local isotropy tests were made (to be presented elsewhere) and compared with experimental data in the far-field of a wake Browne et al. (1987); overall, these tests seem to indicate that the use of \( \epsilon_{iso} \) is justifiable even though individual components of the full energy dissipation rate exhibit departures from isotropy. Therefore, the experimental investigation in the cylinder intermediate wake supports the analytical considerations provided in Section 1.

4. Concluding remarks

Scale-by-scale budgets of the kinetic energy associated with the organized and random fields are derived using both general and isotropic frameworks. They reveal additional diffusion, production, transport terms as well as extra energy transfer and forcing terms associated with the presence of the coherent motion. These considerations put explicitly in evidence the fact that the effective energy transfer of the random motion consists of the difference between the total energy transfer and the energy transfer and forcing by the organized structures. Hot wire measurements in the intermediate wake of a circular cylinder are subsequently used to highlight the temporal dynamics due to the organized motion. It is shown that the energy transfer is clearly influenced by the coherent motion, revealing two maxima corresponding to the locations of the vortex centers. The time-averaged scale-by-scale budget (18) is also reasonably well supported by the experimental data.
References


