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Best, D. J.; Rayner, J. C. W.; Allingham, David "A note on formulae for CMH statistics for JAR data" Food Quality and Preference Vol. 31, Issue 1, p. 19-21 (2014)

Available from: <http://dx.doi.org/10.1016/j.foodqual.2013.07.006>

Accessed from: <http://hdl.handle.net/1959.13/1055682>

A Note on Formulae for CMH Statistics for JAR Data

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Summary. Cochran-Mantel-Haenszel (CMH) test statistics can be used to compare products rated on JAR ‘just about right’ scales and for similar categorical data. In this note we consider the case of paired or related data. The simplified statistics we give allow ‘by hand’ calculation when the data set is small. We also give an R package for larger data sets or for when the user is not interested in ‘by hand’ calculation.

Key Words: McNemar statistic; nominal data; ordinal data; Stuart statistic.

1. Introduction

Analysis of three-category (flavour too weak, flavour just right, flavour too strong) JAR ‘just about right’ data is an example of data to which the methods of this note can be applied. Lawless and Heymann (2010, p.336), for example, use categorical methods to analyse two-product randomised block categorical data. On page 338 they say that “[f]or more than two products the Cochran-Mantel-Haenszel (‘CMH’) method can be used, but it is computationally intensive and requires statistical software.” Fritz (2009) in Appendix G, p.30, of the ASTM manual on JAR scales says, in similar vein, “there are no simple formulae for hand calculation”. This note aims to alleviate this lack of simple formulae. Section 2 gives some simplified formulae and section 3 considers some data of Gacula et al. (2009). Section 4 gives a simple method of calculating the CMH ‘mean scores’ statistic for JAR data.

2. Simple Formulae

2.1 Square tables

For the comparison of two products the CMH test statistic for JAR data is the statistic of Stuart (1955) for the analysis of square $k \times k$ contingency tables where the data are paired.

Suppose the frequencies for the $k \times k$ table are n_{ij} , for $i, j = 1, 2, \dots, k$, and also that we use the usual dot notation $n_{i\cdot} = \sum_{j=1}^k n_{ij}$, $n_{\cdot j} = \sum_{i=1}^k n_{ij}$ and $n_{..} = \sum_{i=1}^k \sum_{j=1}^k n_{ij}$. Stuart’s test statistic is

$$S = d^T V^{-1} d$$

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in which $d^T = (d_1, \dots, d_{k-1}) = (n_{1\cdot} - n_{\cdot 1}, \dots, n_{(k-1)\cdot} - n_{\cdot (k-1)})$ and V has elements

$$v_{ii} = n_{i\cdot} + n_{\cdot i} - 2n_{ii}$$

$$v_{ij} = -(n_{ij} + n_{ji}) \text{ for } i \neq j.$$

S has asymptotic distribution χ^2_{k-1} .

For the case of a 2×2 table, $S = d_1^2 / (n_{1\cdot} + n_{\cdot 1} - 2n_{11}) = (n_{12} - n_{21})^2 / (n_{12} + n_{21})$ which is the statistic of McNemar (1947).

For 3×3 tables

$$S = (v_{11}d_2^2 + v_{22}d_1^2 - 2v_{12}d_1d_2) / D \text{ in which } D = \det V = v_{11}v_{22} - v_{12}^2.$$

For 4×4 tables,

$$S = (Ad_1^2 + Ed_2^2 + Kd_3^2 + 2Bd_1d_2 + 2Cd_1d_3 + 2Fd_2d_3) / D$$

in which

$$D = \det V = v_{11}v_{22}v_{33} + 2v_{12}v_{13}v_{23} - v_{11}v_{23}^2 - v_{22}v_{13}^2 - v_{33}v_{12}^2,$$

$$A = v_{22}v_{33} - v_{23}^2, B = -(v_{12}v_{33} - v_{13}v_{23}), C = v_{12}v_{23} - v_{13}v_{22},$$

$$E = v_{11}v_{33} - v_{13}^2, F = -(v_{11}v_{23} - v_{12}v_{13}) \text{ and } K = v_{11}v_{22} - v_{12}^2.$$

As is clear, this formula avoids having to invert a 3×3 matrix.

2.2 More General Formula for S

A formula for S for more than two products can be given if raw categorical (categories labelled $1, 2, \dots, k$) data for each of the judges (labelled $1, 2, \dots, c$) is available for each of the products (labelled $1, 2, \dots, t$). Section 2.1 applied when $t = 2$. First put $X_{uvw} = 1$ if product u is assessed by judge v as being in category w and $X_{uvw} = 0$ otherwise. Then the more general formula is

$$S = \frac{t-1}{t} \sum_{u=1}^t q_u^T V^{-1} q_u$$

in which the vector q_u has w th element $X_{u\cdot w} - X_{\cdot\cdot w} / t$ for $w = 1, 2, \dots, k-1$ and the (w, z) th element of the matrix V is, for $w, z = 1, \dots, (k-1)$,

$$v_{wz} = - \sum_{v=1}^c X_{\cdot vw} X_{\cdot vz} / t^2 \text{ for } w \neq z, \text{ and } v_{ww} = X_{\cdot\cdot w} / t - \sum_{v=1}^c X_{\cdot vw}^2 / t^2 \text{ for } w = z.$$

S can be shown to have an approximate $\chi^2_{(t-1)(k-1)}$ distribution. This general S statistic is the CMH generalized association statistic simplified for JAR data. We illustrate use of the formula for S in the next section.

3. Cosmetics example

Suppose we consider the JAR moisture scale data of Gacula et al. (2009, p.686) with $t = 3$ cosmetics A, B and C which are categorized by $c = 6$ judges. The data are shown in Table 1 where the $k = 3$ categories are denoted by 1, 2 and 3. From Table 1 we can sum over judges to obtain the cosmetics $\times$ categories matrix of counts shown in Table 2. Each count in Table 2 is a $X_{u\cdot w}$ value. Thus, for example, $X_{2\cdot 2} = 4$. Table 2 is not the usual type of two-way contingency table that would have no underlying block or repeated measures structure. This implies it is not valid to check the homogeneity of count distributions for each product in Table 2 using the standard Pearson χ^2 (chi-squared) test. Observe that the data layout in Table 1 is a randomised block design. It is possible to use ANOVA for these data but with only three different data values this analysis may be suspect. Lawless and Heymann (2010, p.339) say “[i]f there are only a few response categories the data should be treated as categorical or ordinal.” We emphasise that the numbers 1, 2 and 3 in Table 1 are just codes for a JAR scale, where 1 means ‘a little moist’, 2 means ‘just about right’, and 3 means ‘a little dry’.

Table 1. Categorizations made by six judges on the products A, B and C

	Judge					
Products	1	2	3	4	5	6
A (1)	3	3	3	2	3	3
B (2)	2	2	2	1	3	2
C (3)	1	1	1	1	2	1

Table 2. Products $\times$ categories counts

	Category		
Products	1	2	3
A (1)	0	1	5
B (2)	1	4	1
C (3)	5	1	0

Table 3. JAR codes for the cosmetics data and X s

Judge (v)	A	B	C	$X_{\cdot v1}$	$X_{\cdot v2}$	$X_{\cdot v1}X_{\cdot v2}$	$X_{\cdot v1}^2$	$X_{\cdot v2}^2$
1	3	2	1	1	1	1	1	1
2	3	2	1	1	1	1	1	1
3	3	2	1	1	1	1	1	1
4	2	1	1	2	1	2	4	1
5	3	3	2	0	1	0	0	1
6	3	2	1	1	1	1	1	1
Sum				6	6	6	8	6

Table 3 repeats the coded JAR values of Table 1 and also gives the X values for the cosmetics data derived from the JAR values. We defined the X_{uvw} in the previous section. Recall that the simplified CMH general association statistic is $S = \{(t-1)/t\} \sum_{u=1}^t q_u^T V^{-1} q_u$. Notice in Table 3 that the $X_{\cdot vw}$ values are summed over products so that, for example, $X_{\cdot 51} = 0$. Also note that $X_{\cdot \cdot 2} = 6$, the sum over judges of the $X_{\cdot v2}$ values. Asymptotically S has a $\chi_{(t-1)(k-1)}^2$ distribution. Table 2 gives the $X_{u\cdot w}$ values.

We find $q_1^T = (-6, -3)/3$, $q_2^T = (-3, 6)/3$, $q_3^T = (9, -3)/3$,

$$V = \frac{1}{9} \begin{pmatrix} 10 & -6 \\ -6 & 12 \end{pmatrix}, \quad V^{-1} = \frac{9}{84} \begin{pmatrix} 12 & 6 \\ 6 & 10 \end{pmatrix},$$

and substitution yields $S = 13.71$ with a χ^2_4 p-value of 0.008. The small p-value verifies an obvious pre-analysis observation from Table 2, that there are differences in distribution for the moisture ratings of the three cosmetics.

4. The CMH ‘mean’ statistic

The statistic S is valid for both nominal and ordinal data, but if the data are ordinal then further analysis is possible. To compare product means of ordinal scores Desu and Raghavarao (2004, p.284) suggest using the statistic M defined as

$$M = \frac{c(t-1)F}{c-1+F}$$

where F is the F value for products in a two-way ANOVA without interaction. M is the CMH ‘mean scores’ statistic simplified for JAR or similar data.

For the cosmetics moisture data $F = 41.67$ and so $M = 10.71$. As M has an approximate χ^2_{t-1} distribution, an approximate p-value is 0.00. It appears the population mean scores differ. Note that both M and F use the codes 1, 2 and 3 as scores. This is an arbitrary choice that the statistic S does not use. The statistic $S - M = 3.0$ and using a χ^2_2 approximation this is not significant. Thus the significant S value for these data is due to differences in mean scores.

The authors have prepared an R (R Development Core Team, 2012) package called ‘crblocks’ to calculate the statistics S and M and their p-values using the χ^2 approximation and permutation methods. See Bi (2009) for a discussion of permutation methods. Executable code to run in the command window of a PC is also available.

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Appendix: R Package

The R package 'crblocks' can be installed from a CRAN repository using the following command in R:

```
> install.packages('crblocks')
```

Thereafter, the package is accessed by entering:

```
> library('crblocks')
```

To perform the analysis, for example on data in a file called 'gacula.txt' and using 1,000 simulations, use the function 'catrandpvaluepermute'; the output is below.

```
> catrandpvaluepermute('gacula.txt',1000)
Statistic    dof    data value    chi^2 p-value    Simulated p-value
S              4      13.71          0.008          0.002
M              2      10.71          0.000          0.000
L^2            1      10.71          0.000          0.000
```

The file 'gacula.txt' is

```
# One line per judge
# Products A    B    C
3          2    1
3          2    1
3          2    1
2          1    1
3          3    2
3          2    1
```