Scaling range of velocity and passive scalar spectra in grid turbulence

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Isotropic velocity and scalar fluctuations are closely approximated by slightly stretching a heated grid flow through a short (1.36:1) contraction. The heating is such that temperature serves as a passive scalar, and the velocity/scalar time scale ratio is about one. At small values of Taylor microscale Reynolds number ($10 < R_\lambda < 10^2$), the spectrum of the temperature fluctuations has a more discernible scaling range than the spectrum of the velocity fluctuations. The scaling-range exponent for the thermal spectrum, m_θ , exhibits a power-law function of R_λ and tends to the Kolmogorov value of 5/3 more rapidly than that for the velocity spectrum, m_u . Both m_θ and m_u are closer to the Kolmogorov value with the contraction than with no contraction. The trends for the present measurements supplemented with previously published data for larger R_λ ($> 10^2$) indicate that, to obtain a 5/3 scaling range, R_λ must exceed 10^3 . The ratio $(5/3 + m_u)/m_\theta$ is approximately 2, in close conformity with the proposal of Danaila and Antonia [“Spectrum of a passive scalar in moderate Reynolds number homogeneous isotropic turbulence,” *Phys. Fluids* **21**, 111702 (2009)]. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4731295>]

I. INTRODUCTION

The Kolmogorov^{1,2} or “K41” theory postulates that, for sufficiently large Reynolds numbers, the small scale motion (SSM) is statistically independent of the large scales. It follows that the SSM should be statistically universal, locally isotropic, and described by parameters such as ν , the kinematic viscosity, and $\langle \varepsilon \rangle$, the mean energy dissipation rate.^{1,2} When a scalar is introduced in a turbulent flow so that it can be treated as “passive” (i.e., it does not affect the dynamics of the flow), the rate at which it is mixed, so that a uniform concentration is eventually reached, is primarily governed by $\langle \chi \rangle$, the mean scalar dissipation rate; one therefore expects that passive scalars (e.g., temperature, humidity, and dye) should observe the same K41 postulates.^{3,4} If the notion that the SSM is universal were strictly valid then there would be a realistic hope for a successful statistical theory. However, growing evidence from experiments^{5–8} and simulations^{9,10} suggests that the SSM is affected by the large-scale anisotropy, which propagates down to the small scales. The SSM includes both the dissipative and inertial scales; the inertial scales will be more affected by finite Reynolds number effects than the dissipative scales. The attention paid to the SSM seems justified for the reasons outlined previously, but also in the context of developing a theory/model to predict the dispersion/mixing of a passive scalar.^{11–16} In this regard, the present study is carried out to further understand the mixing of velocity and scalar fluctuations in turbulence that is more closely isotropic than in earlier studies.

For isotropic turbulence at large Taylor microscale Reynolds numbers, the spectrum of u is expected to exhibit a “5/3” power-law behaviour in the inertial subrange,^{1,3} viz.,

$$\phi_u(k_1) = C_u \langle \varepsilon \rangle^{2/3} k_1^{-5/3}, \quad (1)$$

where $\phi_u(k_1)$ is the one-dimensional spectrum $\int_0^\infty \phi_u(k_1) dk_1 = u'^2$, k_1 is the one-dimensional wavenumber; the Taylor microscale Reynolds number is defined as $R_\lambda = \lambda u'/\nu$, where λ is the

Taylor microscale and u is the streamwise-velocity fluctuation (a prime denotes the root-mean-square value). According to Sreenivasan's¹⁷ survey, the Kolmogorov constant C_u in Eq. (1) may be considered quasi-universal, at least for $R_\lambda \gtrsim 50$, with a magnitude of ~ 0.5 . For a passive scalar introduced in a turbulent flow, the scalar spectrum also exhibits a 5/3 scaling range in the inertial-convective subrange,^{3,4}

$$\phi_\theta(k_1) = C_\theta \langle \varepsilon \rangle^{-1/3} \langle \chi \rangle k_1^{-5/3}, \quad (2)$$

where $\int_0^\infty \phi_\theta(k_1) dk_1 = \theta'^2$ and θ is the scalar fluctuation. Sreenivasan¹⁸ has indicated that, for shear flows at $R_\lambda < 10^3$, the scalar spectrum is less steep than that given by Eq. (2) and so, the Obukhov-Corrsin constant C_θ is only defined for $R_\lambda \gtrsim 10^3$. He also indicated that for grid turbulence, where the large scales are expected not to depart significantly from isotropy, the anticipated 5/3 scaling range is observed for R_λ smaller than 10^2 . On the basis that the available sheared-turbulence and grid-turbulence data for C_θ seem to be scattered between 0.3 and 0.5, Sreenivasan¹⁸ recommended an average, R_λ independent, value of ~ 0.4 .

Since the present study focuses on grid turbulence, we shall briefly discuss the findings of Sreenivasan^{17,18} in this context. Grid turbulence is nominally the least complex form of turbulence in view of the absence of mean shear and hence, the absence of forcing by the large scales on scales that fall in the “inertial” (or more rigorously the “scaling”) and dissipative ranges. We first note that the recommendations by Sreenivasan^{17,18} for quasi-universal magnitudes of C_u and C_θ , while supposedly of some empirical value, may not accurately reflect the available information on the relative behaviour of the velocity and scalar fluctuations in grid turbulence and sheared turbulence. For example, the distinction made by Sreenivasan¹⁸ regarding the difference in behaviour of $\phi_\theta(k_1)$ between grid turbulence and sheared turbulence could equally have been made for $\phi_u(k_1)$. In fact, Antonia and Burattini¹⁹ showed that, for grid turbulence or more generally decaying-type turbulence such as on the centreline of jets and wakes, the asymptotic “4/5” law of Kolmogorov²

$$-\langle (\delta u)^3 \rangle = (4/5) \langle \varepsilon \rangle r \quad (3)$$

($\delta u = u(x+r) - u(x)$ is the velocity increment between two points separated by a distance r) is approached very slowly as R_λ is increased. Values of R_λ in excess of 10^4 may be required before an inertial subrange is actually established, i.e., the effect of “finite Reynolds number” (FRN) persists up to at least $R_\lambda \approx 10^4$. A much smaller value of R_λ , about 10^3 , appears sufficient in forced turbulence.¹⁹ The obvious implication is that Eq. (1) should be satisfied more rapidly in sheared turbulence than in grid turbulence. However, even in sheared turbulence, it is unlikely that C_u can be estimated unambiguously when $R_\lambda \lesssim 10^3$.

The observations of Sreenivasan¹⁸ indicate that, for the case of $\phi_\theta(k)$, the trends are reversed, i.e., Eq. (2) is attained more quickly for grid turbulence than for sheared turbulence. However, the experimental grid-turbulence data and the numerical box-turbulence data for $-\langle \delta u (\delta \theta)^2 \rangle$ examined by Danaila *et al.*²⁰ suggest that R_λ would need to exceed 10^3 for the maximum value of $-\langle \delta u (\delta \theta)^2 \rangle / (\langle \chi \rangle r)$ to reach the asymptotic “4/3” law of Yaglom²¹

$$-\langle \delta u (\delta \theta)^2 \rangle = (4/3) \langle \chi \rangle r \quad (4)$$

($\delta \theta = \theta(x+r) - \theta(x)$ is the temperature increment between two points separated by a distance r). It is therefore unlikely, contrary to the claim by Sreenivasan¹⁸ that Eq. (2) holds for grid turbulence at $R_\lambda < 10^3$. This is supported by the spectral measurements of Mydlarski and Warhaft^{5,6} for passive-grid and active-grid flows; they have shown that, for both $\phi_u(k_1)$ and $\phi_\theta(k_1)$, there is a scaling range, where

$$\phi_u(k_1) \sim k_1^{-m_u}, \quad (5)$$

$$\phi_\theta(k_1) \sim k_1^{-m_\theta}, \quad (6)$$

with the magnitudes of the scaling-range exponents m_u and m_θ increasing monotonically towards the asymptotic value of 5/3 as R_λ approaches 10^4 . By using Sreenivasan's²² modified form

of Eq. (1), i.e.,

$$\phi_u(k_1) = C_u^+ \langle \varepsilon \rangle^{2/3} k_1^{-5/3} (k_1 \eta)^{5/3-m_u} \quad (7)$$

(η is the Kolmogorov length scale), Mydlarski and Warhaft⁵ found that, after extrapolation, the exponent can be approximated by $m_u = 5/3 - a_u R_\lambda^{-2/3}$ with $a_u = 5.25$. The second term on the right side (i.e., $a_u R_\lambda^{-2/3}$) represents the effect of FRN; this term is negligible when R_λ approaches 10^4 , which is consistent with the previously mentioned FRN effect on $-(\delta u)^3/(\langle \varepsilon \rangle r)$. From their analysis, Mydlarski and Warhaft⁵ showed that the scaling-range coefficient for the velocity spectrum, $C_u^+ (= 4.5 - 2.4m_u)$, approaches the Kolmogorov constant of 0.5 ($= C_u$ in Eq. (1)) as m_u approaches $5/3$ for increasing R_λ . For the scalar, $\phi_\theta(k_1)$ may be written in the form analogous to Eq. (7), viz.,

$$\phi_\theta(k_1) = C_\theta^+ \langle \varepsilon \rangle^{-1/3} \langle \chi \rangle k_1^{-5/3} (k_1 \eta)^{5/3-m_\theta}, \quad (8)$$

where the scaling-range exponent m_θ is closer to $5/3$ than m_u , and that the scaling-range coefficient for the scalar spectrum, C_θ^+ , approaches the Obukhov-Corrsin constant, C_θ (in Eq. (2)), as m_θ approaches $5/3$ for increasing R_λ . Although a trend for C_θ^+ as a function of m_θ has yet to be established, a least-squares curve fit to Mydlarski and Warhaft's⁶ data (their Figure 7) shows $m_\theta = 5/3 - a_\theta R_\lambda^{-2/3}$ with $a_\theta \approx a_u/2 = 5.25/2$. This is consistent with Figure 9 of Mydlarski and Warhaft,⁶ where the difference between the scaling-range exponents, $m_\theta - m_u \approx (a_u/2) R_\lambda^{-2/3}$, is a decreasing function of R_λ . The magnitude of a_θ ($\approx a_u/2$) implies that the departure from $5/3$ is about half as small for $\phi_\theta(k_1)$ than for $\phi_u(k_1)$. Danaila and Antonia²³ (see also Danaila *et al.*²⁰) provided an explanation for the difference between m_u and m_θ , or equivalently between a_u and a_θ , by adopting the nonlinear characteristic time $\tau(k)$ of eddies of size k^{-1} (k is the magnitude of the wavenumber vector $\vec{k}$), first proposed by Batchelor²⁴ and later used by Kraichnan,²⁵ to study energy transfer in the inertial range of isotropic turbulence, viz.,

$$\tau(k) \sim \left[\int_0^k p^2 E(p) dp \right]^{-1/2}, \quad (9)$$

where $E(p)$ is the three-dimensional energy spectrum of the wavenumber “ p .” This characteristic time $\tau(k)$ essentially represents the effect of the cumulative strain imposed on a structure of size k^{-1} by all larger structures. Danaila and Antonia²³ have used this characteristic time to formulate a closure hypothesis for the nonlinear energy-transfer term in the transport equation of $E_\theta(k)$. This, together with the inclusion of a term reflecting the FRN effect on $E(k)$ and $E_\theta(k)$, has allowed them to obtain an expression for $E_\theta(k)$ and to compare it with $E(k)$. The analytical expression is

$$E_\theta(k) = \frac{\langle \chi \rangle \tau(k)}{k} \sim \frac{1}{k} \left[\int_0^k p^2 E(p) dp \right]^{-1/2}. \quad (10)$$

Danaila and Antonia²³ showed that, if $E(k)$ and $E_\theta(k)$ assume power laws similar to those described by Eqs. (5) and (6) (note that for isotropic turbulence, the three-dimensional spectra E and E_θ should have the same slopes in the scaling range as the one-dimensional spectra ϕ_u and ϕ_θ), then

$$m_\theta = (5/3 + m_u)/2 \quad (11)$$

after substituting the scaling-range expressions for $E(k)$ and $E_\theta(k)$ in Eq. (10), which take into account the FRN effect. Equation (11) implies that, although m_θ is closer to $5/3$ than m_u , it will reach its asymptotic value of $5/3$ when m_u reaches $5/3$.

To provide support for Eq. (11), Danaila and Antonia²³ have used data obtained from both passive and active grids.^{5,6} For the majority of these data, the small scales may be considered to be only approximately isotropic, as shown, for example, by the three-component vorticity measurements of Antonia *et al.*²⁶ with the exception of Mydlarski and Warhaft⁶—the anisotropy of the small-scale temperature fluctuations is enhanced by the presence of a mean-temperature gradient. The large scales have non-negligible departures from isotropy. The ratio of streamwise to cross-stream velocity fluctuations u'^2/v'^2 or u'^2/w'^2 in the central core region of the flow falls between 1.1 and 1.8, and may vary slightly due to different grids.^{5,27–29}

To improve isotropy at the large scales, an effective method is to introduce a short secondary contraction downstream of the grid.^{27–32} The overall effect is to alter the ratio of streamwise to cross-stream velocity (or vorticity) fluctuations so that it approximates 1 more closely. Comte-Bellot and Corrsin,²⁷ Lavoie,²⁸ and Lavoie *et al.*,²⁹ have showed that, by using a contraction with an area ratio of 1.27 and 1.36, this reduced the anisotropy ratio u^2/v^2 or u^2/w^2 to a value somewhere between 0.9 and 1.2. Notably, with the contraction, and for a round-bar grid of 44% solidity (known as “Rd44w”) with a wire helically wrapped around each bar to reduce vortex shedding, Lavoie²⁸ found that $u^2/w^2 \approx 0.99$. In Sec. II, we discuss in more detail the anisotropy ratio (u^2/w^2) for the grids (including Rd44w) selected for the present study.

When a grid flow is immediately heated with a mesh of fine wires (known as a “mandoline,” see Sec. II for further description) and is then slightly stretched by a short 1.36:1 contraction, we observed that the skewnesses for the scalar fluctuation and its derivative ($S_\theta \approx 0.05$ and $-S_{\partial\theta/\partial x} \approx 0.09$, respectively) are quite small.³³ For a similar grid, with no contraction and/or with no wire wrapped on each bar, the magnitudes of S_θ and $-S_{\partial\theta/\partial x}$ are nearly twice as large. For the kurtosis of the scalar fluctuation and its derivative, the magnitudes of K_θ and $K_{\partial\theta/\partial x}$ are larger when the (1.36:1) contraction is not used.³³ Expectantly, the skewness is zero if the grid turbulence is isotropic and the kurtosis (subtracted by 3) is zero if the distribution of the passive scalar in isotropic grid turbulence is Gaussian. The implication is that the secondary contraction improves the isotropy of the scalar fluctuations since, with the contraction, the magnitudes for both the skewness and the kurtosis are closer to zero.

For the present study, the aim is to examine the relative behaviour of the spectra $\phi_u(k_1)$ and $\phi_\theta(k_1)$ in grid turbulence with a smooth (1.36:1) contraction for which the large scales (and the small scales) are more nearly isotropic than for grid turbulence with no contraction. Of particular interest are the relative magnitudes of the spectral scaling-range exponents m_u and m_θ , and how closely Eq. (11) is satisfied. Also of interest are the consequent relative magnitudes of the scaling-range coefficients C_u^+ and C_θ^+ , and the manner in which they approach C_u (the Kolmogorov constant) and C_θ (the Obukhov-Corrsin constant), respectively. Although the maximum value of R_λ covered by the present experiments is small (no more than 10^2), the measurements are supplemented with available large- R_λ ($\approx 10^2 \rightarrow 10^3$) experimental data from literature and should provide a useful indication of how the asymptotic states for the velocity and temperature fields, as represented by Eqs. (1) and (2), are approached for isotropic turbulence.

II. APPARATUS

Figure 1 shows a schematic diagram of the wind-tunnel test section used for the present investigation. The test section includes a short (1.36:1) secondary contraction that is located at $11M$ downstream of the grid. The design of the contraction, based on a linear theory of turbulence^{30,34} and the empirical data of Comte-Bellot and Corrsin,²⁷ was carefully undertaken by Lavoie²⁸ and Lavoie *et al.*²⁹ to improve the isotropy of the large scales and to reduce the effect of grid geometry.

FIG. 1. The test section of the wind tunnel. The 1.36:1 contraction is located $\approx 11M$ downstream of the grid (scale: $l = 0.1 \text{ m} \approx 4M$).

FIG. 2. The biplanar grid; $M = 24.76$ mm. Mesh solidity is $\sigma = d/M(2 - d/M)$; x -coordinate axis lies on the centreline of the duct.

Three different biplanar grids (Figure 2) previously used by Lavoie²⁸ and co-workers^{29,31,33} are used here. The first grid (Sq35) consists of square bars with a solidity ratio (σ) of 0.35, the second (Rd35) has round bars ($\sigma = 0.35$), and the third (Rd44w) is made of round bars with a thin wire wrapped around each bar ($\sigma = 0.44$). For each grid, the mesh size is $M = 24.76$ mm. The flow entering the grid is taken from an open-circuit (9:1 contraction) wind tunnel, and is driven by a centrifugal blower that is controlled by a variable-cycle (from 0 to 1500 rpm) power supply. Further details of the wind tunnel are provided in Refs. 28, 29, and 33. For the majority of new data presented here, the grid-mesh Reynolds number ($R_M = MU_o/\nu$) is $\approx 10^4$, where U_o (≈ 6 m/s) is the mean free-stream velocity entering the grid.

In Figure 3(a), the velocity on the centreline of the wind tunnel is measured using a Pitot-static tube and a (100 Pa) micro-manometer. The velocity through the contraction part of the tunnel is not constant, and the time required for turbulence to be convected from the grid (at say location $s = 0$) to a downstream position (at $s = x$) is defined by^{27–29}

$$t = \int_0^x \langle U(s) \rangle^{-1} ds. \quad (12)$$

Equation (12) simplifies to $t = x/U_o$ if no contraction is used. The non-dimensional time scale, tU_o/M , allows direct comparison between results obtained downstream of the contraction and those obtained with no contraction.^{27–29} With the exception of a slight stretching of the initial grid flow in the contraction, for the most part of the duct ($\approx 80M$ in length), there is no mean shear, no production of turbulent kinetic energy, and the turbulence simply decays. For the present results, the measurement range $20 \lesssim x/M \lesssim 100$ (or $22 \lesssim tU_o/M \lesssim 110$ using Eq. (12)) is sufficiently far downstream of the regions of initially developing turbulence and accelerated flow in the contraction, and avoids possible effects of the duct exit (measurements furthest from the grid are $\approx 12M$ short of the exit plane of the duct). In the range $20 \lesssim x/M \lesssim 100$ shown in Figure 3(a), the time-averaged

FIG. 3. (a) Centreline distribution of the wind-tunnel velocity, and Lavoie's²⁸ measurements of (b) the velocity profile just downstream of the (1.36:1) secondary contraction and (c) the flow anisotropy on the centreline of the test section (see also Lavoie *et al.*²⁹).

velocity ($U_{cl}/U_o \approx 1 \pm 0.01$) is independent of streamwise position, and the variation in wall pressure is no more than 1% of the dynamic pressure in the duct. For the same wind tunnel, Lavoie's²⁸ cross-wire measurements in the range $16 \lesssim tU_o/M \lesssim 25$ have established that the grid flow is approximately homogeneous with spanwise velocity distribution $U(y)/U_{cl} \approx 1 \pm 0.02$ and turbulent kinetic energy $q(y)^2/q_{cl}^2 \approx 1 \pm 0.05$ (see examples in Figure 3(b); the subscript "cl" denotes the centreline value).

For clarity, we show in Figure 3(c) Lavoie's²⁸ measurements of the anisotropy ratio u^2/w^2 in the duct. The improvement in isotropy with the contraction is obvious when compared with the same grid flows with no contraction. Although the effect of grid geometry persists, the difference in the anisotropy ratio, especially between the square-bar grid and the round-bar grids, is smaller. With no contraction, the anisotropy ratio is more sensitive to the geometry of the grid and, for the round bars (Rd35 and Rd44w), u^2/w^2 is dependent on x . Figure 3(c) shows that, with the 1.36:1 contraction, the dependence of u^2/w^2 on x for the three grids is rather weak. The anisotropy increases slightly with x for both Sq35 and Rd35; this possibly reflects the persistence of large scales produced by these grids. For Rd44w with each bar helically wrapped by a wire to disrupt the large scales, turbulence appears to relax very close to isotropy. Overall, by using the contraction to reduce the flow anisotropy, Lavoie *et al.*²⁹ observed that the decay rate of turbulent kinetic energy is less dependent on grid geometry; with no contraction, the large-scale anisotropy tends to increase the magnitude of the decay rate. The decay rate and the choice of the optimum virtual origin (t_o) are further discussed in Sec. IV.

To obtain a passive scalar, the grid flow is heated with a mesh array of 0.5-mm-diameter Chromel-A wires located just ($1.5M$) downstream of the grid (see Figure 1). This method of heating is the same as that used by Zhou *et al.*^{35,36} and Antonia *et al.*³⁷ (see also Danaila and Antonia²³) to introduce the scalar in the Sq35 grid flow; it is based on the mandoline technique developed by Warhaft and Lumley.³⁸ The horizontal and vertical wires of the mandoline heater are separated by a gap of $0.6M$ and have the same mesh size as the turbulence-generating grid. The grid flow is slightly warmer than the ambient air; a fixed temperature difference, $\Delta T \approx 2^\circ\text{C}$, is maintained by adjusting a variable-voltage (from 0 to 275 V) power supply. The maximum voltage limits the grid-mesh Reynolds number (R_M) to no more than 3×10^4 ; the maximum flow speed (U_o) obtained with the present grids is about 16 m/s. For the present measurement range of flow speeds (≈ 6 –16 m/s), the pressure change across the (1.36:1) contraction is too small to produce any noticeable compressibility effects³⁹ on the grid turbulence. The Prandtl number (Pr) is fixed at ≈ 0.71 . The Taylor microscale Reynolds number is in the range $10 < R_\lambda < 10^2$.

III. MEASUREMENT TECHNIQUE

For the present results, the streamwise velocity (u) and the temperature (θ) fluctuations are measured simultaneously using hot and cold wires. Both wires are etched from a fine Wollaston (Pt-10%Rh). The hot wire (diameter $d_{hot} \approx 2.50 \mu\text{m}$; length $l_{hot} \approx 200d_{hot}$) is operated at constant temperature with an overheat ratio of 1.5. To minimise contamination by velocity, the cold wire ($d_{cold} \approx 0.63 \mu\text{m}$; $l_{cold} \approx 1000d_{cold}$) is operated at a constant current of 0.1 mA with a sensitivity of $1 \Omega/^\circ\text{C}$. To avoid interference of the measurement signals, the wires are parallel with a spanwise separation of 1 mm and the cold wire is located just slightly upstream of the hot wire. For the present range of measurements, the distance between the wires is 1.5 – 3.0η ; the wire lengths are in the range $0.6 \lesssim \eta/l_{hot} \lesssim 1.5$ and $0.5 \lesssim \eta/l_{cold} \lesssim 1.1$. Overall, the wire length-to-diameter (l/d) ratios are chosen to minimise attenuation at high frequencies while maintaining η/l as close to 1 as practicable. For $\eta/l \gtrsim 0.5$, the effect of attenuation⁴⁰ underestimates u'^2 by no more than $\pm 4\%$. For a cold wire of $l/d \approx 1000$, the error⁴¹ in measuring θ'^2 is $\pm 5\%$. The uncertainty in $\langle \varepsilon \rangle$ and $\langle \chi \rangle$, estimated by the method of propagation of errors,⁴² is no more than $\pm 10\%$.

The hot-wire and cold-wire signals are digitised with a 12 bit (200 kHz) A/D converter and are low-pass filtered at half the sampling rate. To ensure that the records are of sufficient duration, we have checked that the signal fluctuations and their derivatives have indeed converged with the sample size (1×10^6 for each experimental run); the sampling period ($t_s U_o/\eta$) is in the order of 10^6 . The power spectrum of the signal is estimated by using the Welch⁴³ FFT method. The cutoff frequency is determined by the response time of the cold wire provided by Antonia *et al.*⁴⁴ It is close

FIG. 4. Examples of curve fit and extrapolation (dashed lines) for the Kolmogorov non-dimensionalised spectrum of velocity and temperature (solid lines) (Rd44w with the contraction; $tU_o/M \approx 70$; $R_M \approx 10^4$; $R_\lambda \approx 40$). The DNS data ($\square$) of Abe *et al.*⁴⁷ and Antonia and Abe⁴⁸ are for the centreline of a fully developed turbulent channel flow ($h^+ = 1020$; $R_\lambda \approx 90$).

to (but usually smaller than) the Kolmogorov frequency, i.e., $0.8 \lesssim f_K(2\pi\eta/U_o) < 1$. While the temporal resolution is adequate, the measured spectrum is usually contaminated by electronic noise and imperfect spatial resolution of the hot and cold wires. To remove the contamination, we corrected each u spectrum based on the assumption that the dissipative range of the spectrum scales on the Kolmogorov velocity and length scales. Djenidi and Antonia⁴⁵ indicated that this assumption holds even when the Reynolds number is small, in contrast to the asymptotic requirement of Kolmogorov^{1,2} provided inhomogeneity effects are small. For convenience of curve fitting the spectrum in the dissipative range, we adopted a “model spectrum” outlined in Pope,⁴⁶ and the model constants are calibrated against available data from experiments and direct numerical simulations (DNS) (e.g., see Djenidi and Antonia⁴⁵). It should be noted that, for the purpose of extrapolation, the method of curve fit is of secondary importance provided that the experimental or DNS spectrum is sufficiently well resolved. For the passive scalar, we extended the same treatment to curve fit and extrapolate the dissipative range of the Kolmogorov-normalised θ spectrum. To check that the extrapolation provides adequate closure at the high wavenumbers, each spectrum is weighted by the wavenumber and is compared with previously published experimental and modelling data. In the examples shown in Figure 4, we included the (DNS) data of Abe *et al.*⁴⁷ and Antonia and Abe⁴⁸ for comparison with the present measurements. Review of other published data from experiments³⁶ and modelling⁴⁹ (not shown in Figure 4) indicated a similar trend at the high wavenumbers ($k_1\eta \gtrsim 0.8$). The reasonable agreement between the present results and previously published data reflects the adequacy of the Kolmogorov scaling and helps to validate the extrapolation technique. The uncertainty in estimating u'^2 and θ'^2 by using the spectral method, i.e., $\int_0^\infty \phi_u(k_1)dk_1 = u'^2$ and $\int_0^\infty \phi_\theta(k_1)dk_1 = \theta'^2$, is no more than $\pm 5\%$. Truncation of the curve fits at $k_1\eta = 1$ underestimates the derivative variances, i.e., $\int_0^\infty k_1^2 \phi_\psi(k_1)dk_1 = (\partial\psi/\partial x)^2$ with $\psi = u, \theta$, by no more than 10%.

For the measurements shown in Figure 4 and in the next few pages, the spectra are non-dimensionalised by using the Kolmogorov length $\eta = (\nu^3/\langle\varepsilon\rangle)^{1/4}$, velocity $u_K = (\nu\langle\varepsilon\rangle)^{1/4}$, and temperature $\theta_K = \langle\chi\rangle^{1/2}(\nu/\langle\varepsilon\rangle)^{1/4} = (\langle\chi\rangle\eta/u_K)^{1/2}$. The methods for calculating $\langle\varepsilon\rangle$ and $\langle\chi\rangle$ are described in Sec. IV and the spectral results are discussed in Sec. V.

IV. THE MEAN DISSIPATION RATES

For grid turbulence, Zhou *et al.*^{35,36} have shown that the mean dissipation rates $\langle\varepsilon\rangle$ and $\langle\chi\rangle$ are well approximated by the streamwise decay rate of the turbulent kinetic energy, defined here as $q'^2 = u'^2 + v'^2 + w'^2$, and the temperature variance, θ'^2 , respectively. From the same wind tunnel, Lavoie *et al.*²⁹ have measured the ratio $r_{uw} = u'^2/w'^2$ and we have made use of the axial symmetry of the flow $v'^2 = w'^2$ to determine the turbulent kinetic energy, i.e., $q'^2 = u'^2 + 2w'^2$. For the present grids with the contraction, Lavoie *et al.*²⁹ have established that the turbulence is nearly isotropic and that both $q'^2 \sim (t - t_0)^{n_q}$ and $u'^2 \sim (t - t_0)^{n_u}$ exhibit a power-law rate of decay, where their decay rates are about the same, i.e., $n_q \approx n_u$.

For the present study, the decay rates for u'^2 and θ'^2 are determined from a total of 9 data points obtained in the range $22 \lesssim tU_o/M \lesssim 110$. Figures 5 and 6 show that both u'^2 and θ'^2 follow a power-law decay, where $u'^2 \sim (t - t_o)^{n_u}$ and $\theta'^2 \sim (t - t_o)^{n_\theta}$ with the power-law exponents $n_u, n_\theta < 0$. The mean dissipation rates are defined as

$$\langle \varepsilon_d \rangle = -\frac{1}{2} \frac{dq'^2}{dt} = -\frac{1}{2} \left(1 + \frac{2}{r_{uw}} \right) \frac{du'^2}{dt}, \quad (13)$$

$$\langle \chi_d \rangle = -\frac{1}{2} \frac{d\theta'^2}{dt}, \quad (14)$$

where each data point is calculated by using the 3-point centre-difference scheme and then averaged over its two closest points. To avoid end effects due to the scheme, the data point on each end of a batch of 9 points is removed. Taking the time derivative of the power laws for Eqs. (13) and (14) gives the relations

$$\langle \varepsilon_d \rangle \sim -\frac{1}{2} \left(1 + \frac{2}{r_{uw}} \right) n_u (t - t_o)^{n_u-1}, \quad (15)$$

$$\langle \chi_d \rangle \sim -\frac{1}{2} n_\theta (t - t_o)^{n_\theta-1}. \quad (16)$$

The virtual origin, t_o , is selected so that both n_u and n_θ are independent of the measurement range $22 \lesssim tU_o/M \lesssim 110$ (using Eq. (12)) and should account for the effect of the secondary contraction. If no contraction is used, the correction for the virtual origin is $t - t_o = (x - x_o)/U_o$ by assuming Taylor's hypothesis. Since the mandoline (source of heat) is introduced immediately downstream of the grid (source of turbulence), t_o should be about the same for both the velocity and passive-temperature fields. The optimum value for t_o is determined by the method described by Lavoie *et al.*²⁹ and Antonia *et al.*³⁷ which involves the Taylor microscale (λ) and the Corrsin microscale (λ_θ). The microscales are defined as

$$\lambda^2 \sim \nu(q'^2/\langle \varepsilon_d \rangle) \sim (-\nu/n_u)(t - t_o), \quad (17)$$

$$\lambda_\theta^2 \sim \kappa(\theta'^2/\langle \chi_d \rangle) \sim (-\kappa/n_\theta)(t - t_o), \quad (18)$$

where $\lambda^2/(t - t_o) \sim (-\nu/n_u)$ and $\lambda_\theta^2/(t - t_o) \sim (-\kappa/n_\theta)$ should be constant in the power-law decay range.^{29,37} For each grid flow shown in Figures 5 and 6, we find that the same optimum value of t_o satisfies both the velocity and temperature measurements. Varying $t_o U_o/M$ by ± 1.0 changes n_u and n_θ by no more than ± 0.03 . For the curve fits in Figures 5 and 6, the 90% confidence limit for n_u and

FIG. 5. The power-law decay of turbulent kinetic energy, $q'^2 = u'^2 + 2w'^2$, and its mean dissipation rate, $\langle \varepsilon \rangle$, downstream of each grid with adjustment for optimum virtual origin $t_o U_o/M$. $u'^2/w'^2 \approx 1.11$ (Sq35), ≈ 1.07 (Rd35) and ≈ 0.99 (Rd44w). n_u is the decay exponent; $R_M \approx 10^4$.

FIG. 6. The power-law decay of temperature variance, θ'^2 , and its mean dissipation rate, $\langle\chi\rangle$, downstream of each grid with adjustment for optimum virtual origin $t_o U_o / M$. n_θ is the decay exponent; $R_M \approx 10^4$.

n_θ is ± 0.03 . In Figure 7(a), the ratio between the microscales,

$$\left(\frac{\lambda}{\lambda_\theta}\right)^2 \sim Pr \frac{q'^2 / \langle\varepsilon_d\rangle}{\theta'^2 / \langle\chi_d\rangle} \sim Pr \frac{\tau}{\tau_\theta}, \quad (19)$$

is proportional to the velocity/scalar time scale ratio τ/τ_θ . For each grid, τ/τ_θ ($\approx 1 \pm 0.1$) is reasonably independent of the range of the measurements. Since both q'^2 and θ'^2 decay according to a power law, we have checked that the time scale ratio indeed closely matches the ratio between the temperature and the velocity power-law exponents, i.e., $\tau/\tau_\theta \approx n_\theta/n_u$. Inspection of Table I shows that, for the three grids, the ratio n_θ/n_u is from ≈ 1.03 to 1.04 .

Since decaying grid turbulence is an approximation to isotropic turbulence, comparison between the “decay” dissipation rates and the “isotropic” dissipation rates should serve as a measure of the flow anisotropy. For homogeneous isotropic turbulence, the mean dissipation rates are defined by the wavenumber-weighted spectrum of u and θ , viz.,

$$\langle\varepsilon_{\text{iso}}\rangle = 15\nu \int_0^\infty k_1^2 \phi_u(k_1) dk_1, \quad (20)$$

$$\langle\chi_{\text{iso}}\rangle = 3\kappa \int_0^\infty k_1^2 \phi_\theta(k_1) dk_1. \quad (21)$$

Figures 7(b) and 7(c) show that, for each grid, the flow is very nearly isotropic, where the decay/isotropic ratios for $\langle\varepsilon_d\rangle/\langle\varepsilon_{\text{iso}}\rangle$ and $\langle\chi_d\rangle/\langle\chi_{\text{iso}}\rangle$ are $\approx 1 \pm 0.1$. The decay/isotropic ratios for the Kolmogorov length η_d/η_{iso} , velocity $u_{\text{kd}}/u_{\text{kiso}}$, and temperature $\theta_{\text{kd}}/\theta_{\text{kiso}}$ are $\approx 1 \pm 0.03$. The close

FIG. 7. (a) The time scale ratio, τ/τ_θ , and the anisotropy of the estimated mean dissipation rates (b) $\langle\varepsilon_d\rangle/\langle\varepsilon_{\text{iso}}\rangle$, and (c) $\langle\chi_d\rangle/\langle\chi_{\text{iso}}\rangle$ from the data in Figures 5 and 6. The present data are with the contraction; the data with no contraction are from Zhou *et al.*³⁵ and Antonia *et al.*³⁷

TABLE I. Review of decay exponents (n_u and n_θ) of different grid flows from the same wind tunnel with and without the (secondary) contraction. The grid is at $tU_o/M = 0$; $M = 24.76$ mm. The “microscale” method is used to determine the optimum virtual origin ($t_o U_o/M$) for measurements in the range $22 < tU_o/M < 110$; $r_{uw} = u'^2/w'^2$ is the anisotropy ratio. For the measurements of n_θ , the “mandoline” heating method is used; $\Delta T \approx 2^\circ\text{C}$. For the present measurements, the 90% confidence limit for both n_u and n_θ is ± 0.03 . $R_M \approx 10^4$.

Reference	Grid	$t_o U_o/M$	$-n_u$	$-n_\theta$	r_{uw}	Initial condition
Antonia <i>et al.</i> ³⁷	Sq35	0	1.33	1.37	/	No contraction
Lavoie ²⁸	Sq35	7.0	1.05	/	1.45	No contraction
	Rd35	6.0	1.19	/	1.27	
	Rd44w	6.0	1.18	/	1.30	
Lavoie <i>et al.</i> ²⁹	Sq35	5.0	1.18	/	1.11	1.36:1 contraction
	Rd35	5.0	1.22	/	1.07	
	Rd44w	7.0	1.10	/	0.99	
Present measurements	Sq35	5.0	1.18	1.21	/	1.36:1 contraction
	Rd35	6.0	1.23	1.27	/	
	Rd44w	6.0	1.14	1.18	/	

agreement between the “decay” and “isotropic” measurements lends support to the extrapolation technique discussed in Sec. III.

A review summary of results in Table I shows that, when isotropy is improved with the contraction (i.e., $r_{uw} = u'^2/w'^2$ is closer to 1), the magnitude of n_u is less different between each grid. For an optimum virtual origin, however, the magnitude of n_θ is slightly larger than that of n_u ; for for Rd44w that produces the most isotropic flow, n_θ/n_u is not exactly equal to 1. This evidence may suggest that the passive-scalar field, although mixed and advected by a very nearly isotropic flow, does not behave exactly as the velocity field. In Sec. V, the differences between the two fields, as identified in their power spectra, show that their characteristics are indeed not the same.

V. THE POWER SPECTRUM

Figure 8(a) shows typical spectra of velocity (u) and temperature (θ) for a grid flow (Sq35) stretched by the secondary (1.36:1) contraction. The other grid flows (Rd35 and Rd44w) stretched by the contraction produce very similar spectra, i.e., the grid geometry has negligible effect on the spectral scaling range. However, for each grid flow, there is a significant scaling range for ϕ_θ but not for ϕ_u . This difference is observed in Figures 8(a) and 8(b) with and without the contraction, respectively; it is a feature of grid turbulence at low Reynolds numbers $R_\lambda \lesssim 10^2$ (e.g., Sepri⁵⁰ and Rey and Boudjemaa⁵¹). Mydlarski and Warhaft^{5,6} showed that, by increasing R_λ , this widens

FIG. 8. Effect of inlet conditions on the Kolmogorov non-dimensionalised velocity (u) and temperature (θ) spectra; (a, b) grid Sq35 with and without the contraction; $tU_o/M \approx 30 \rightarrow 40$ (present measurements, Zhou *et al.*⁵⁵ and Lavoie²⁸); (c) active grid with no contraction; $tU_o/M \approx 70$ (Mydlarski and Warhaft^{5,6}).

FIG. 9. (a, b) Examples of compensated spectrum (present data, Rd44w with the 1.36:1 contraction; $tU_o/M \approx 70$; $R_\lambda \approx 35$). (c) The coefficients for optimum compensation ($10 < R_\lambda < 10^3$); the measurements ($u \triangle$, $\theta \blacktriangle$) are for Rd44w with the contraction. For the Sq35 data ($u \square$, $\theta \blacksquare$) and the data of Mydlarski and Warhaft⁵ ($u \nabla$) and Yeh and Van Atta⁵² ($u \triangleright$, $\theta \blacktriangleright$), the grid flows are not stretched by a contraction. The velocity data points $\diamond$, $\circ$, and $\times$ are obtained from Figure 10(b). The temperature data point $\blacktriangledown$ is obtained from Figure 10(c).

the scaling range and, for $R_\lambda > 10^2$, both ϕ_u and ϕ_θ exhibit nearly the same wide scaling range (Figure 8(c)).

Figures 9(a) and 9(b) show examples of u and θ spectra, which are “optimally” compensated:

$$C_u^+(k_1) = \langle \varepsilon \rangle^{-2/3} k_1^{5/3} (k_1 \eta)^{-5/3+m_u} \phi_u(k_1), \quad (22)$$

$$C_\theta^+(k_1) = \langle \varepsilon \rangle^{1/3} \langle \chi \rangle^{-1} k_1^{5/3} (k_1 \eta)^{-5/3+m_\theta} \phi_\theta(k_1), \quad (23)$$

compared with those which are “ $m_u = m_\theta = 5/3$ ” compensated:

$$C_u(k_1) = \langle \varepsilon \rangle^{-2/3} k_1^{5/3} \phi_u(k_1), \quad (24)$$

$$C_\theta(k_1) = \langle \varepsilon \rangle^{1/3} \langle \chi \rangle^{-1} k_1^{5/3} \phi_\theta(k_1). \quad (25)$$

For the optimally compensated spectra, the scaling-range exponents, i.e., $0 \ll (m_u, m_\theta) < 5/3$ and $m_u < m_\theta$, are selected by a method of trial-and-error to produce the largest plateaux in the spectra. The results are shown in Figure 9(c), where the scaling-range coefficients C_u^+ and C_θ^+ are the maximum values corresponding to their respective, optimally compensated, power spectra. Changing the scaling-range exponents m_u and m_θ by $\pm 5\%$ generally alters the respective coefficients C_u^+ and C_θ^+ by no more than $\pm 10\%$. The uncertainty in the estimates of the plateaux C_u^+ and C_θ^+ is about $\pm 10\%$.

To supplement the present data, we have included in Figure 9(c) the measurements of C_u^+ obtained from Yeh and Van Atta⁵² (their Figures 2 and 3; $x/M = 35$) and those reported by Mydlarski and Warhaft⁵ (their Figure 11). Further examples of C_u^+ and C_θ^+ are determined from the spectra shown in Figure 10; the optimally compensated spectra are recalculated from the $5/3$ compensated spectra reported by Schedvin *et al.*⁵³ (their Figure 8 includes the data of Kistler and Vrebalovich⁵⁴ and Comte-Bellot and Corrsin⁵⁵) and Mydlarski and Warhaft⁶ (their Figure 6).

Inspection of Figures 9(a), 9(b), and 10 clearly shows that the scaling ranges for both the velocity and the temperature spectra do not exhibit a $5/3$ slope (for $R_\lambda < 10^3$). Instead, the scaling ranges (represented by the largest spectral plateaux) indicate that the slopes are less steep, i.e., $5/3 - m_u$ and $5/3 - m_\theta$. This is consistent with Eqs. (7) and (8), where the scaling-range coefficients C_u^+ and C_θ^+ are not expected to match the constants C_u and C_θ .

Figure 9(c) shows that C_u^+ and C_θ^+ increase approximately linearly with $5/3 - m_u$ and $5/3 - m_\theta$, respectively. Although both the velocity and temperature scaling ranges are sensitive to the contraction, they do not behave the same. In terms of the magnitude and the rate-of-change, the contraction tends to affect C_u^+ more than C_θ^+ . For the same magnitude ($m_u = m_\theta$) of the optimum scaling-range exponents, $C_u^+ < C_\theta^+$.

FIG. 10. Review of spectral data with the effect of compensation; (a, b) velocity; with contraction $\diamond R_\lambda \sim 40 \rightarrow 50$ (Comte-Bellot and Corrsin⁵⁵); no contraction $\circ R_\lambda = 280$ (Schedvin *et al.*⁵³); no contraction $\times R_\lambda = 670$ (Kistler and Vrebalovich⁵⁴); (c) temperature; no contraction $R_\lambda = 582$ (Mydlarski and Warhaft⁶). The spectra (from bottom to top) are vertically offset by 0, 1, and 2, respectively.

In Figure 11, the scaling-range exponents are plotted in terms of the Reynolds number R_λ . For each grid (Sq35, Rd35, and Rd44w) with the 1.36:1 contraction, the error bars indicate small variations obtained for a fixed speed ($R_M \approx 10^4$) over the region $22 \lesssim tU_o/M \lesssim 110$. Figure 11 shows that, with this contraction, the scaling-range exponents are about the same for all three grids. This evidence lends support to Lavoie's²⁸ deduction that the effect of grid geometry is rather small with the present contraction.

Since Rd44w produces the most isotropic turbulence (Figure 3(c)) and the decay/isotropic ratios $\langle \varepsilon_d \rangle / \langle \varepsilon_{iso} \rangle$ and $\langle \chi_d \rangle / \langle \chi_{iso} \rangle$ are $\approx 1 \pm 0.1$ (see Sec. IV), we decided to test different flow speeds (i.e., ≈ 6 –16 m/s; see Sec. II) for a fixed downstream distance ($tU_o/M \approx 70$) from this grid to obtain a wider range of Reynolds number ($30 < R_\lambda < 60$). These measurements (for Rd44w) are shown as triangular symbols ($\triangle$, $\blacktriangle$) in Figures 9(c) and 11–14; the Rd44w spectra for u and θ are given in Figure 15(a).

Figures 11 and 12 show that the scaling-range exponents vary with R_λ ; the scaling range widens as R_λ is increased (Figure 15(a)). According to Mydlarski and Warhaft's⁵ results, the Reynolds-number variation reasonably follows a “ $-2/3$ ” power law (i.e., $m_u \propto R_\lambda^{-2/3}$) and that a $5/3$ scaling range is not observed until $R_\lambda \gtrsim 10^4$. The formulas used to describe the scaling-range exponents, after Danaila and Antonia,²³ may be written as

$$m_u = 5/3 - a_u R_\lambda^{-2/3}, \quad (26)$$

$$m_\theta = (5/3 + m_u)(a_\theta/a_u) = (5/3)(2a_\theta/a_u) - a_\theta R_\lambda^{-2/3}. \quad (27)$$

FIG. 11. The scaling-range exponents m_u and m_θ as functions of R_λ . The present measurements are mostly for Rd44w with the contraction ($u \triangle$, $\theta \blacktriangle$); for each grid flow (Sq35, Rd35, and Rd44w), the error bars represent measurements over the range $tU_o/M \approx 22 \rightarrow 110$ at a fixed $R_M (\approx 10^4)$. For the data of Danaila and Antonia²³ ($u \square$, $\theta \blacksquare$), Mydlarski and Warhaft⁵ ($u \nabla$, $\theta \blacktriangledown$), and Yeh and Van Atta⁵² ($u \triangleright$, $\theta \blacktriangleright$), the grid flow is not stretched by a contraction. See Table II for a summary of the curve fits.

FIG. 12. The difference between the scaling-range exponents, $m_\theta - m_u$, as a function of R_λ . The present measurements with the contraction are shown as $\blacktriangle$ (Rd44w). With no contraction, the data are shown as $\square$ (Danaila and Antonia²³), ∇ (Mydlarski and Warhaft⁵), and $\triangleright$ (Yeh and Van Atta⁵²).

Given that the scaling-range coefficients in Figure 9(c) have a linear dependence on the scaling-range exponents, we have adopted the following expressions, after Mydlarski and Warhaft:⁵

$$C_u^+ = C_u + b_u (5/3 - m_u), \quad (28)$$

$$C_\theta^+ = C_\theta + b_\theta (5/3 - m_\theta). \quad (29)$$

Using Eqs. (26) and (27), the scaling-range coefficients are expressed in terms of R_λ :

$$C_u^+ = C_u + a_u b_u R_\lambda^{-2/3}, \quad (30)$$

$$C_\theta^+ = C_\theta + 5/3 (1 - 2a_\theta/a_u) b_\theta + a_\theta b_\theta R_\lambda^{-2/3}, \quad (31)$$

where the empirical coefficients (a_u , a_θ , b_u , b_θ , C_u , and C_θ) are determined from the available experimental data.

In Figure 11, the least-squares curve fits to the data using Eqs. (26) and (27) yield $a_u/a_\theta \approx 2.0 \pm 0.02$, where the rms difference is no more than $\pm 5\%$. From this, it naturally follows that

$$m_\theta - m_u \approx (a_u/2) R_\lambda^{-2/3}. \quad (32)$$

Figure 12 shows that, for Reynolds number in the range $R_\lambda \lesssim 10^2$, the difference between the scaling-range exponents ($m_\theta - m_u$) is non-negligible; both the scatter in the data and the magnitude of a_u ($=4.25$) with the 1.36:1 contraction appear slightly smaller than those with no contraction. Overall, least-squares curve fits of the form given by Eq. (32) are broadly consistent with the trends of the available data, and the rms difference is no more than $\pm 20\%$. Note that, for the measurements obtained with no contraction, the value $a_u = 5.25$ matches that reported by Mydlarski and Warhaft.⁵ According to Figure 12, as $m_\theta - m_u$ tends to zero for $R_\lambda > 10^2$, any effects due to different initial conditions are likely to diminish and/or are not expected to be easily distinguishable.

Figure 13 shows the scaling-range coefficients (C_u^+ and C_θ^+) plotted in terms of R_λ . Since R_λ is small with the available measurements (especially for C_θ^+), we have adopted the “universal” constants $C_u \sim 0.5$ and $C_\theta \sim 0.4$ recommended by Sreenivasan^{17,18} to provide the initial curve fits using Eqs. (28)–(31). The coefficients (b_u , b_θ , C_u , and C_θ) are then reiterated, by trial-and-error, to minimise the rms difference. For each curve fit shown in Figures 9(c) and 13, the rms difference is no more than $\pm 6\%$. Figure 13 shows that the optimised curve fit for C_u^+ with no contraction is in good agreement with the measurements of Schedvin *et al.*,⁵³ Kistler and Vrebalovich,⁵⁴ and Saddoughi and Veeravalli,⁵⁶ obtained for large Reynolds numbers in the range $10^2 < R_\lambda < 10^3$ (indicated by the error bars in Figure 13).

From Eqs. (30) and (31), and with $a_u/a_\theta \approx 2.0$ (from the curve fits in Figures 11 and 12), it follows that

$$C_\theta^+ - C_u^+ \approx C_\theta - C_u + (a_\theta b_\theta - a_u b_u) R_\lambda^{-2/3}. \quad (33)$$

FIG. 13. The scaling-range coefficients C_u^+ and C_θ^+ as functions of R_λ . The symbols are as defined in Figure 11. For the curve fits (solid and dashed lines), the empirical coefficients are summarised in Table II. The error bars indicate the approximate loci of other velocity measurements obtained with a slight (1.27:1) contraction (Comte-Bellot and Corrsin⁵⁵) and with no contraction (Kistler and Vrebalovich,⁵⁴ Schedvin *et al.*,⁵³ Saddoughi and Veeravalli⁵⁶).

Although there are rather few data points in Figure 14, it is evident that the magnitude of $C_\theta^+ - C_u^+$ is systematically smaller with the 1.36:1 contraction than with no contraction. The least-squares fit of the form given by Eq. (33) confirms this trend; for each curve fit, the rms difference is $\sim \pm 20\%$. The uncertainty in measuring $C_\theta^+ - C_u^+$ from the spectra is about $\pm 20\%$ and the apparent difference between the trends in Figure 14 cannot be attributed to scatter. Attempts to improve the curve fits would require more experimental data points at higher R_λ ($\gtrsim 10^3$) than there are currently available; such (future) experiments possibly require the use of active grids.⁵⁻⁷ Nevertheless, it should be clear that, from observing the trends in Figure 14, to reach the 5/3 scaling range, where the constants are approximately universal, i.e., $C_\theta^+ - C_u^+ \approx C_\theta - C_u$, the value of R_λ needs to far exceed 10^3 .

VI. DISCUSSION

According to Figure 11, the secondary contraction plays a dominant role in controlling the scaling-range exponents (m_u and m_θ) since the effect of grid geometry is reduced by the contraction.^{28,29} To gain some understanding of the effects of larger contractions, we reviewed the spectra reported by Uberoi³⁰ (Figure 15(b)). To conform with linear theory, Uberoi plotted his velocity spectrum as a function ck_1 , where $c \gg 1$ is the contraction ratio. Uberoi's measurements show that, by increasing c , this tends to shift the turbulent energy to the higher wavenumbers and increase the magnitude of the scaling-range exponent, i.e., $m_u \propto c$ (see also Figures 8(a) and 8(b)). Since m_u and a_u are related according to $m_u = 5/3 - a_u R_\lambda^{-2/3}$ (Eq. (26)), we therefore suspect that, for a fixed Reynolds number ($R_\lambda < 10^2$), the magnitude of a_u should decrease with increasing contraction ratio, i.e., $a_u \propto c^{-1}$.

FIG. 14. The difference between the scaling-range coefficients, $C_\theta^+ - C_u^+$, as a function of R_λ . The present measurements for Rd44w with the contraction are shown as $\blacktriangle$. With no contraction, the Sq35 measurements are shown as $\square$ and the data point of Yeh and Van Atta⁵² is shown as $\triangleright$. Each curve fit is given by Eq. (33). The asymptotic value $C_\theta - C_u \sim -0.1$ follows the recommendation of Sreenivasan.^{17,18}

FIG. 15. (a) Present u and θ measurements for grid Rd44w with contraction (ratio $c = 1.36$; location $x_c/M = 11$) and mandoline (location $x_m/M = 1.5$) showing the effect of increasing Reynolds number ($30 < R_\lambda < 60$) at $tU_o/M \approx 70$. (b) Uberoi's³⁰ measurements for $R_M = 3710$ showing the effect of increasing contraction ratio; the pre-contraction ($x_c/M = 21$) data are shown as $\circ$; the post-contraction ($c = 4, 9$, and 16) data are shown as $\bullet$; vertical offset is 2 decades. (c) Warhaft's⁵⁹ measurements for grid Sq34 with contraction ($c = 4$; $x_c/M = 60$) at $R_M = 9700$ showing the effect of increasing streamwise distance ($x_m/M = 5, 10$, and 15) between the mandoline and the turbulence-generating grid; the dotted-dashed lines are for $x/M = 53$ (upstream of contraction) and the solid lines (with slope m_θ) are for $x/M = 69$ (downstream of contraction); vertical offset is arbitrary.

From the experimental results summarised in Table II, we find that, while the magnitude of a_u may change with different initial conditions (in this case, it is mainly the secondary contraction), the ratio $a_u/a_\theta \approx 2.0$ is generally unaffected. This finding seems consistent with that of Jayesh *et al.*⁵⁷ From their measurements over the Reynolds-number range $30 \lesssim R_\lambda \lesssim 130$ using different grids and methods of heating, Jayesh *et al.*⁵⁷ obtained $m_u = 1.34$ and $m_\theta = 1.58$, where the ratio $a_u/a_\theta = (5/3 + m_u)/m_\theta \approx 1.9$ is independent of R_λ . Their result is in close agreement with both the present finding for a_u/a_θ (Table II) and the proposal of Danaïla and Antonia²³ (Eq. (11)).

The method of heating is of interest as it affects the turbulent mixing. With the “mandoline” technique, the resulting temperature field is sufficiently homogeneous,^{38,58} and the velocity/scalar time scale ratio, which is an important parameter for modelling,^{13,16} tends to be closer to unity than that obtained by heating the flow with the turbulence-generating grid³⁸ or with a plenum “toaster.”⁵⁸ With the mandoline, the initial scale of the thermal fluctuations can be controlled by adjusting the distance (x_m) between the mandoline and the grid. Figure 15(c) shows that, for a small value of x_m ($\lesssim 10M$), the spectra for θ obtained at both the inlet and exit planes of a 4:1 secondary contraction

TABLE II. Summary of parameters for the curve fits shown in Figures 9(c) and 11–14. The measurements marked “*” are the same as those first reported by Mydlarski and Warhaft.⁵ Those marked “ $\oplus$ ” are in conformity with the proposal of Danaïla and Antonia.²³ The constants marked “ $\otimes$ ” are selected to minimise the rms difference between the curve fit and the experimental data, and are consistent with the recommendations of Sreenivasan,^{17,18} i.e., $C_u \sim 0.5$ and $C_\theta \sim 0.4$.

Eq.	Symbol	Contraction ratio		Description	Note
		$c = 1$	$c = 1.36$		
(26)	a_u	5.25*	4.25	The rate-of-change in the scaling-range exponent $m_u = f(R_\lambda)$.	$a_u \propto c^{-1}$; $c \geq 1$
(27)	a_u/a_θ	2.0 $\oplus$	2.0 $\oplus$	The ratio $(5/3 + m_u)/m_\theta$ (the uncertainty is ± 0.02).	a_u/a_θ is constant
(28)	b_u	2.4*	1.6	The rate-of-change in the scaling-range coefficients for	$b_u < b_\theta$
(29)	b_θ	7.5	5.8	optimally compensated velocity and scalar spectra.	
(28)	C_u	0.51 $\otimes$	0.51 $\otimes$	The Kolmogorov constant and the Obukhov-Corrsin	C_u and C_θ are
(29)	C_θ	0.44 $\otimes$	0.44 $\otimes$	constant for a 5/3 power-law scaling range.	constant; $C_u \gtrsim C_\theta$

are nearly the same. According to Warhaft's⁵⁹ measurements, increasing x_m tends to shift the post-contraction (θ) spectrum to a higher frequency and slightly increases the magnitude of m_θ ; this has an overall effect of increasing the time scale ratio⁵⁹ (i.e., $\tau/\tau_\theta \approx n_\theta/n_u$) from 1.5 (at $x_m = 5M$) to 2.3 (at $x_m = 15M$).

The velocity and the temperature scaling-range coefficients (C_u^+ and C_θ^+) tend towards the Kolmogorov and the Obukhov-Corrsin constants (C_u and C_θ) at different rates. Figure 9(c) shows that the rate-of-change in the scaling-range coefficient, with respect to the scaling-range exponent, is slower for the velocity than for the passive scalar, i.e., $b_u < b_\theta$. A small reduction in the magnitudes for both b_u and b_θ is obtained by using the 1.36:1 contraction (Table II). How b_u and b_θ behave for more extreme contractions (i.e., $c \gg 1$) is a topic for future investigation.

At low Reynolds numbers $R_\lambda \lesssim 10^2$, the effect of the 1.36:1 contraction on the scaling-range coefficients C_u^+ and C_θ^+ is evident. A comparison of the present data in Figure 13 with the data of Comte-Bellot and Corrsin⁵⁵ shows that, with the present contraction, $C_u^+ \approx 1.0$. With no contraction, the magnitude of the velocity scaling-range coefficient ($C_u^+ \approx 1.5$) tends to be larger. Figures 13 and 14 suggest that, as the Reynolds number is increased ($R_\lambda > 10^2$), effects due to different initial conditions should diminish and the difference between C_u^+ and C_θ^+ becomes negligible. From the present curve fits (Table II), the prediction for the asymptotic value $C_\theta - C_u \sim -0.1$ follows the recommendation of Sreenivasan.^{17,18}

In summary, the present findings provided some new insight into how the scaling-range coefficients (C_u^+ and C_θ^+) tend towards the universal constants for closely isotropic turbulence. Clearly, the manner in which C_u^+ and C_θ^+ approach universality (i.e., C_u and C_θ) is not unique and that this depends on initial conditions. Indeed, if R_λ is much less than 10^2 , the velocity and scalar fields behave differently, and the initial conditions (e.g., changes to the contraction size and/or the method of heating) will affect the morphology of the two fields. Such information is useful if an attempt is to be made to improve any predictive models for turbulent mixing.

VII. CONCLUSION

The use of a smooth (1.36:1) contraction improves the isotropy of the large scales for turbulent flows downstream of three different grid geometries. The introduction of a passive scalar by using a mandoline heater located just downstream of the grid allows comparison between the (spectral) characteristics of the velocity and the scalar (temperature) fields with and without the effect of the contraction.

The present grid turbulence closely follows a power-law rate of decay. Although the heating is carefully controlled such that the velocity/scalar time scale ratio is close to 1, the magnitude of the power-law decay exponent is consistently slightly larger for the scalar than for the velocity (i.e., $n_\theta/n_u \approx 1.03$ to 1.04), independently of the grid geometry.

For low Taylor microscale Reynolds numbers ($R_\lambda \lesssim 10^2$), the thermal spectrum has a more prominent scaling range than the velocity spectrum. The spectral scaling range follows a power law and the scaling-range exponent is larger for the scalar than for the velocity, i.e., $m_\theta > m_u$. The magnitudes for m_u and m_θ are closer to the Kolmogorov value of $5/3$ with the 1.36:1 contraction than with no contraction. As the Reynolds number is increased, m_θ approaches $5/3$ more rapidly than m_u , and the difference between m_θ and m_u diminishes according to the form $m_\theta - m_u \propto R_\lambda^{-2/3}$. The ratio $(5/3 + m_u)/m_\theta$ ($\approx 2.0 \pm 0.02$) tends to be unaffected by initial conditions and conforms closely with the findings of Danaïla and Antonia.²³

The velocity and temperature scaling-range coefficients of the (optimally) compensated spectra C_u^+ and C_θ^+ increase linearly with their respective scaling-range exponents $5/3 - m_u$ and $5/3 - m_\theta$. The magnitudes of the rate-of-change in C_u^+ and C_θ^+ with respect to their scaling-range exponents are smaller with the 1.36:1 contraction than with no contraction. For the same magnitude of m_u and m_θ or for a fixed R_λ ($\lesssim 10^2$), the scaling-range coefficient is generally smaller for the velocity spectrum than for the scalar spectrum, i.e., $C_u^+ < C_\theta^+$. The difference between the scalar and the velocity scaling-range coefficients reasonably follows the form $(C_\theta^+ - C_u^+) - (C_\theta - C_u) \propto R_\lambda^{-2/3}$. The scaling-range coefficients C_u^+ and C_θ^+ are unlikely to reach C_u (the Kolmogorov constant) and C_θ (the Obukhov-Corrsin constant) for the values of R_λ below 10^3 . The manner in which C_u and C_θ

are approached (from small R_λ) should be useful for the purpose of refining theory/model to predict turbulent mixing when the effect of finite Reynolds number cannot be neglected.

On a more general note, the present results suggest that the use of the contraction which improves the isotropy at the large scales leads to the parameters such as m_u and m_θ (Figure 11) being closer to the expected asymptotic values corresponding to very large R_λ . This in turn implies that it would be worthwhile to try and extend the present range of R_λ before reappraising the question, addressed, for example, by Antonia and Burattini¹⁹ based on imperfectly isotropic turbulence data, of how large should R_λ be in order to attain the “4/5” law of Kolmogorov.

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