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Sensor Fault Tolerant Direct Torque and Flux Control of Induction Motors

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Abstract: In this paper we propose the use of a multisensor switching control strategy for fault tolerant direct torque and flux control (DTFC) of induction motors. The proposed strategy combines three current sensors and associated observers that estimate the rotor flux. The estimates provided by the observers are compared at each sampling time by a switching mechanism which selects the sensors–observer pair with the smallest error between the estimated rotor flux magnitude and a desired flux reference. The measurements provided by the selected pair are used by a DTFC controller to implement the control law. Pre-checkable conditions are derived that guarantee fault tolerance under an abrupt fault of a current sensor. These conditions are such that measurements from faulty sensors are automatically avoided by the switching mechanism, thus maintaining good performance levels even in the presence of a faulty sensor. Simulation results under realistic conditions illustrate the effectiveness of the scheme.

1. INTRODUCTION

The use of induction motor drives in high performance servo systems, such as automated production or transportation systems, has become increasingly popular due to recent advances in motor control techniques (Leonhard, 1996). In such applications, faults in any of the components of the feedback loop (the motor itself, the driving circuits, sensors, actuators, etc.) can seriously deteriorate the overall system performance (Lee and Ryu, 2003). It is thus crucial, in order to avoid failure of the whole system, to design fault tolerant control strategies which are capable to maintain good performance levels even under the occurrence of component faults.

In this paper, we investigate the use of the multisensor switching strategy introduced by Martínez et al. (2006); Seron et al. (2008b) in a direct torque and flux control (DTFC) scheme (Takahashi and Noguchi, 1986; Depenbrock, 1988). The structure of the proposed fault tolerant control strategy is depicted in Figure 1. It consists of a bank of three rotor flux observers, an estimate switching mechanism and the DTFC controller. Each observer provides estimates of the rotor flux based on noisy measurements of two phase currents. At each sampling time, the switching mechanism selects the observer with the smallest error between the estimated rotor flux magnitude and a desired flux reference, and passes only the selected current measurements to the DTFC controller. The latter uses these currents to compute estimates of the stator flux and electric torque to implement the control law.

A standard approach to achieve fault tolerance is to endow the control system with explicit fault detection and compensation capabilities (see, for example, Bennett et al. (1996) and Lee and Ryu (2003) for the use of this approach in induction motor control systems). In contrast, our proposed strategy comes with pre-checkable conditions which guarantee that, when a current sensor fails, the observers that use measurements from the faulty sensor are automatically avoided by the switching mechanism, thus maintaining good performance levels even under sensor fault. Thus, our scheme achieves faulty sensor detection and isolation “implicitly” by guaranteeing that the switching cost avoids selecting faulty sensors. A similar fault tolerant scheme was successfully employed in Seron et al. (2008a) in combination with *field oriented control* of induction motors. The scheme proposed in the current paper, for *direct torque and flux control* (DTFC), makes use of a different switching selection criterion which considerably simplifies the analysis and allows to derive tighter fault tolerance guaranteeing conditions. An aspect of the proposed scheme that should be emphasised is the fact that it endows a well-established control technique for induction motors; namely, DTFC, with sensor fault tolerant capabilities. In that sense, it should be appreciated that no theoretical stability proofs exist, in general, for this control technique when implemented using stator flux estimates (rather than true values), save for the important fact that it works in numerous real life implementations in industry. Thus, a pervasive assumption of our approach here will be that the DTFC controller effectively stabilises the machine and that permanent regime is reached before the occurrence of any fault. Under these circumstances, as the analysis of this paper shows, the proposed scheme guarantees fault tolerance of the closed-loop system.

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Fig. 1. Proposed fault tolerant control scheme.

2. MODEL OF THE INDUCTION MOTOR

The induction motor can be represented in different frames depending on the control scheme to be implemented. For the purposes of the proposed fault tolerant switching strategy, it will be convenient to consider the model of the induction motor in a reference frame fixed with the stator, with components denoted by (a, b) (Krause et al., 1995). In this reference frame, the motor electromagnetic variables can be described by the following dynamic equation:³

$$\frac{dx}{dt} = A(\omega)x + Bu, \quad (1)$$

$$x = [i_{sa} \ i_{sb} \ \psi_{ra} \ \psi_{rb}]^T, \quad u = [u_{sa} \ u_{sb}]^T,$$

where the state x is composed by the stator current components i_{sa}, i_{sb} and the rotor flux components ψ_{ra}, ψ_{rb} , and the input u consists of the stator voltage components u_{sa}, u_{sb} . The matrices $A(\omega)$ and B in (1) have the form

$$A(\omega) = \begin{bmatrix} a_{r11}\mathcal{I} & a_{r12}\mathcal{I} & a_{i12}(\omega)\mathcal{J} \\ a_{r21}\mathcal{I} & a_{r22}\mathcal{I} & a_{i22}(\omega)\mathcal{J} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{\sigma L_s}\mathcal{I} \\ 0 \end{bmatrix}, \quad (2)$$

where $a_{r11} \triangleq -\frac{R_s}{\sigma L_s} - \frac{1-\sigma}{\sigma\tau_r}$, $a_{r12} \triangleq \frac{1}{c\tau_r}$, $a_{i12}(\omega) \triangleq -\frac{n_p\omega}{c}$, $a_{r21} \triangleq \frac{M}{\tau_r}$, $a_{r22} \triangleq -\frac{1}{\tau_r}$, $a_{i22}(\omega) \triangleq n_p\omega$,

$$\mathcal{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad (3)$$

and where n_p is the number of pole pairs of the induction machine, R_s, R_r, L_s, L_r , are the stator and rotor resistances and self-inductances, respectively, M is the mutual inductance, $\tau_r = L_r/R_r$, $\sigma = 1 - M^2/(L_s L_r)$ and $c = \sigma L_s L_r / M$. The rotor speed ω satisfies the dynamic equation

$$\frac{d\omega}{dt} = \frac{n_p M}{J L_r} [\psi_{ra} i_{sb} - \psi_{rb} i_{sa}] - \frac{\tau_L}{J}, \quad (4)$$

where J is the moment of inertia of the rotor and τ_L is the load torque. For future reference, we define the magnitude of the rotor flux as

$$\psi_{rd} = \sqrt{\psi_{ra}^2 + \psi_{rb}^2}. \quad (5)$$

We observe that the stator current components i_{sa}, i_{sb} in (1) are the two-phase projection on the (a, b) plane of the three-phase currents i_R, i_S and i_T . This projection is given by the transformation (Krause et al., 1995)

$$\begin{aligned} i_{sa} &= (1/3)(2i_R - i_S - i_T), \\ i_{sb} &= (\sqrt{3}/3)(i_S - i_T). \end{aligned} \quad (6)$$

³ The dependency of the variables on time t is omitted for simplicity.

We assume that the rotor speed w and the phase currents i_R, i_S and i_T are measured. The state variables i_{sa} and i_{sb} in (1) are then directly computed from (6), whereas ψ_{ra} and ψ_{rb} will be estimated by means of flux observers.

3. DTFC

DTFC is based on the use of hysteresis loops of both stator flux magnitude,

$$\psi_{sd} = \sqrt{\psi_{sa}^2 + \psi_{sb}^2}, \quad (7)$$

and electromagnetic torque,

$$\tau_e = \frac{3}{2} n_p (i_{sb} \psi_{sa} - i_{sa} \psi_{sb}), \quad (8)$$

to control the output voltage of an inverter through a lookup table. Indeed, the difference between (estimates of) ψ_{sd} and τ_e and their corresponding reference signals are passed through separate hysteresis based comparators to determine logic levels representing the desired action on flux and torque according to the current operating condition (for example, the desired action may be to increase flux magnitude and decrease torque). The lookup table then heuristically selects the best output voltage vector according to the position of the stator flux vector (ψ_{sa}, ψ_{sb}) in the (a, b) -plane and the desired action on electromagnetic torque and stator flux magnitude (Takahashi and Noguchi, 1986; Depenbrock, 1988).

The reference signal for flux magnitude, denoted as $\psi_{s,ref}$, is an external constant signal whereas the reference for electromagnetic torque, τ_{ref} , is an internal signal (it can also be given externally) provided by an additional proportional-integral (PI) controller loop (see Figure 1) that forces the rotor speed to follow an external speed reference signal ω_{ref} , that is, $\tau_{ref} = P_w(\omega_{ref} - \omega) + I_w \int_0^t (\omega_{ref} - \omega) dv$, where P_w and I_w are design parameters. The overall DTFC based controller can then be expressed as the following function of the rotor speed, the external reference signals ω_{ref} and $\psi_{s,ref}$, and stator currents and fluxes $i_{sa}, i_{sb}, \psi_{sa}, \psi_{sb}$ (required to compute flux magnitude (7), electromagnetic torque (8) and flux vector position):

$$u = \mathcal{K}_{DTFC}(\omega(\cdot), i_{sa}(\cdot), i_{sb}(\cdot), \psi_{sa}(\cdot), \psi_{sb}(\cdot), \omega_{ref}, \psi_{s,ref}). \quad (9)$$

4. CURRENT SENSORS AND FLUX OBSERVERS

The phase currents i_R, i_S and i_T satisfy the relation

$$i_R + i_S + i_T = 0. \quad (10)$$

Hence, one could measure two phase currents and calculate the third one from (10). However, we propose to employ three sensors, one per phase, having measurement equations of the form

$$i_{j,m} = i_j + \eta_j, \quad j = R, S, T, \quad (11)$$

(where η_R, η_S and η_T are bounded, uncorrelated zero-mean measurement noises) and take advantage of the redundancy provided by these three measurements.

For the fault tolerant switching strategy we estimate the rotor flux components independently of the control strategy. To this end, we employ three observers, each one based on measurements from two phases. Observer 1 uses

measurements (11) from phases R and S and computes, based on (10) and (6),

$$\begin{aligned} i_{T,1} &= -i_{R,m} - i_{S,m}, \\ i_{sa,1} &= (1/3)(2i_{R,m} - i_{S,m} - i_{T,1}), \\ i_{sb,1} &= (\sqrt{3}/3)(i_{S,m} - i_{T,1}). \end{aligned} \quad (12)$$

Then, the values of $i_{sa,1}$ and $i_{sb,1}$ obtained in (12) are used in the observer dynamic equation

$$\frac{d\hat{x}_1}{dt} = A(\omega)\hat{x}_1 + Bu + G(\omega) \begin{bmatrix} \hat{i}_{sa,1} - i_{sa,1} \\ \hat{i}_{sb,1} - i_{sb,1} \end{bmatrix}, \quad (13)$$

where $\hat{x}_1 = [\hat{i}_{sa,1} \hat{i}_{sb,1} \hat{\psi}_{ra,1} \hat{\psi}_{rb,1}]^T$ is the state estimate provided by observer 1, and $A(\omega)$, B and u are as in (1). The observer gain matrix $G(\omega)$ in (13) has the following form proposed by Kubota and Matsuse (1994):

$$G(\omega)^T = \begin{bmatrix} g_1 & g_2(\omega) & g_3 & g_4(\omega) \\ -g_2(\omega) & g_1 & -g_4(\omega) & g_3 \end{bmatrix}, \quad (14)$$

where $g_1 = (K-1)(a_{r11} + a_{r22})$, $g_2(\omega) = (K-1)a_{i22}(\omega)$, $g_3 = (K^2-1)(a_{r21} + a_{r11}c) - cg_1$, $g_4(\omega) = -cg_2(\omega)$, and all remaining symbols are as defined in (2)–(3). This gain matrix is such that for each fixed value of ω , the eigenvalues of the closed-loop matrix $A(\omega) + G(\omega)C$, with

$$C = [\mathcal{I} \ 0], \quad (15)$$

and $\mathcal{I}$ as in (3), have negative real part and are proportional to those of $A(\omega)$ by a factor of $K > 0$.

In a similar way, observer 2 uses measurements (11) from phases R and T to compute $i_{S,2} = -i_{R,m} - i_{T,m}$, and observer 3 uses measurements (11) from phases S and T to compute $i_{R,3} = -i_{S,m} - i_{T,m}$. Then, following the same procedure as with observer 1, we obtain analogous equations to (12)–(13) for observers 2 and 3.

Each observer outputs the variable

$$z_j = [i_{sa,j} \ i_{sb,j}]^T, \quad j = 1, 2, 3, \quad (16)$$

consisting of the calculated stator current components, and makes it available to the controller whenever the switching mechanism selects this observer, according to a criterion described in the following section.

5. ESTIMATE SWITCHING MECHANISM AND CONTROL IMPLEMENTATION

When the induction motor reaches a permanent regime, the squared magnitude of the rotor flux, $\psi_{rd}^2 \triangleq \psi_{ra}^2 + \psi_{rb}^2$ is (ideally) equal to its desired reference squared value $\psi_{r,ref}^2$. Thus, we could measure each observer's performance by considering the error signal $\hat{\psi}_{rd,j}^2 - \psi_{r,ref}^2$, where $\hat{\psi}_{rd,j}^2 \triangleq \hat{\psi}_{ra,j}^2 + \hat{\psi}_{rb,j}^2$, $j = 1, 2, 3$, is the estimated squared magnitude of the rotor flux. However, as is well known, the input voltages of an induction motor are PWM signals generated by the inverter that actuates the motor. These PWM signals have a fundamental component with appropriate amplitude and frequency that drives the motor to the permanent regime. The higher harmonic components, on the other hand, lead to the, so-called, *ripple* (or high frequency oscillation) of the electromagnetic variables, which is usually attenuated by the “low-pass” nature of the induction motor itself. Hence, the actual magnitude of the rotor flux can be seen as the sum of a mean value and a (small) high-frequency ripple. We thus propose to filter the

signals $\hat{\psi}_{rd,j}^2$ and $\psi_{r,ref}^2$ before computing the error signal in order to essentially compare their mean values. That is, for each observer we consider the following *error signal*:

$$\pi_j \triangleq |\overline{\hat{\psi}_{rd,j}^2} - \overline{\psi_{r,ref}^2}|, \quad j = 1, 2, 3, \quad (17)$$

which measures the deviation of the filtered estimated squared magnitude of the rotor flux, denoted as $\overline{\hat{\psi}_{rd,j}^2}$, from the filtered rotor flux squared reference signal, denoted as $\overline{\psi_{r,ref}^2}$. The filtered signals are obtained by passing each signal through a low-pass filter with transfer function

$$H(s) = \frac{1}{(T_H s + 1)^2}. \quad (18)$$

The design parameter $T_H > 0$ in (18) will be chosen so that all harmonic components of the signals are sufficiently attenuated and the resulting filtered signals essentially represent their mean values (see footnote 5 in Section 8).

Remark 1. We observe that other expressions for the error signal, instead of (17), can be used by the switching mechanism to perform the estimate selection. However, the choice (17) is convenient because of the independence of the rotor flux magnitude from the load torque which, in turn, reduces the sensitivity of the switching mechanism to changes in the load state of the induction motor. ♦

The error signals π_j are subsequently sampled with a sampling period T_s to obtain the discrete-time signals

$$\pi_j[k] = \pi_j(kT_s), \quad \text{for } j = 1, 2, 3 \text{ and } k = 0, 1, \dots \quad (19)$$

Finally, the sampled error signals $\pi_j[k]$ are compared at each time k according to the following switching criterion:

$$\ell_k = \operatorname{argmin}_j \{\pi_j[k] : j \in \{1, 2, 3\}\}. \quad (20)$$

At each sampling time k , then, the observer with index ℓ_k computed from (20) is selected by the switching mechanism and the selected currents $z_{\ell_k}(kT_s)$ (see (16)) passed on to the controller during $kT_s \leq t < (k+1)T_s$ for the implementation of the DTFC law (9). The selected currents will be employed to compute estimates of the stator flux components $\hat{\psi}_{sa}, \hat{\psi}_{sb}$ (for example, by means of a standard Luenberger observer). Thus, the controller is implemented in the following way:

$$u(t) = \mathcal{K}_{\text{DTFC}}(\omega(kT_s), z_{\ell_k}(kT_s), \hat{\psi}_{sa}(kT_s), \hat{\psi}_{sb}(kT_s), \omega_{ref}, \psi_{s,ref}) \text{ for } kT_s \leq t < (k+1)T_s. \quad (21)$$

When compared with standard DTFC strategies based on the use of a single observer, the proposed multi-observer switching strategy has similar performance under healthy operation of all current sensors and, more importantly, it has the advantage of preserving good performance levels under sensor outage. These properties will be analysed and illustrated in the following sections.

6. PERFORMANCE UNDER HEALTHY OPERATION

In this section we will analyse the performance of the scheme of Figure 1 in permanent regime and when all current sensors are operational.

6.1 Induction motor variables

In permanent regime the rotor speed ω and the flux magnitude ψ_{rd} in (5) reach the desired constant values $\omega = \omega_{ref}$ and $\psi_{rd} = \psi_{r,ref}$. Also, the rotor flux components

are given by $\psi_{ra}(t) = \psi_{r,ref} \cos(\omega_\rho t + \rho_0)$ and $\psi_{rb}(t) = \psi_{r,ref} \sin(\omega_\rho t + \rho_0)$, for some phase shift ρ_0 . Similarly, the stator current components are given by (Krause et al. (1995), see also Seron et al. (2008a)) $i_{sa}(t) = I_{ab} \sin(\omega_\rho t + \rho_0 + \rho_{ab})$ and $i_{sb}(t) = -I_{ab} \cos(\omega_\rho t + \rho_0 + \rho_{ab})$, where

$$I_{ab} = \sqrt{\frac{\psi_{r,ref}^2}{M^2} + \frac{\tau_L^2}{J^2 \mu^2 \psi_{r,ref}^2}}, \quad \omega_\rho = n_p \omega_{ref} + \frac{M \tau_L}{\tau_r J \mu (\psi_{r,ref})^2}$$

and $\cos \rho_{ab} = -\left[1 + \left(\frac{J \mu \psi_{r,ref}^2}{M \tau_L}\right)^2\right]^{-1/2}$. Finally, the phase currents $i_R(t)$, $i_S(t)$ and $i_T(t)$ are sinusoids with amplitude I_{ab} , frequency ω_ρ , and phases $\rho_R = \rho_0 + \rho_{ab}$, $\rho_S = \rho_0 + \rho_{ab} - \frac{2\pi}{3}$ and $\rho_T = \rho_0 + \rho_{ab} - \frac{4\pi}{3}$, respectively. The permanent regime values will be used in Section 7 to analyse the performance of the multisensor switching system under faulty operation and to derive fault tolerance conditions.

6.2 Observer variables

To analyse the observer variables in permanent regime, we define the observer estimation errors as

$$\tilde{x}_j = [\tilde{i}_{sa,j} \ \tilde{i}_{sb,j} \ \tilde{\psi}_{ra,j} \ \tilde{\psi}_{rb,j}]^T \triangleq x - \hat{x}_j, \quad j = 1, 2, 3, \quad (22)$$

where x is the state of the system (1) and $\hat{x}_j$ is the state estimate provided by observer j , for $j = 1, 2, 3$. It is easy to show using (1), (15) and the observer equations (12), (13) and the corresponding ones for observers 2 and 3 that, under healthy operation of all current sensors, the estimation errors (22) satisfy the dynamic equations

$$\dot{\tilde{x}}_j = [A(\omega) + G(\omega)C]\tilde{x}_j + G(\omega)\eta_j, \quad j = 1, 2, 3, \quad (23)$$

where, for each fixed ω , the matrix $A(\omega) + G(\omega)C$ is stable by design (see discussion after (14)) and $\eta_1 = [\eta_R \ \frac{\sqrt{3}}{3}(\eta_R + 2\eta_S)]^T$, $\eta_2 = [\eta_R \ -\frac{\sqrt{3}}{3}(\eta_R + 2\eta_T)]^T$ and $\eta_3 = [-\frac{\sqrt{3}}{3}(\eta_S + \eta_T) \ -\frac{\sqrt{3}}{3}(\eta_S + \eta_T)]^T$.

When $\omega = \omega_{ref}$ in permanent regime, since the noises η_j , $j = 1, 2, 3$ are bounded by assumption, the states of (23) will be ultimately bounded. This is shown in the following result, reproduced from Seron et al. (2008a).

Lemma 2. Let the noises be elementwise⁴ bounded as $|\eta_j| \leq \bar{\eta}_j$, $j = 1, 2, 3$, for some vectors $\bar{\eta}_j$ with positive elements. Let $V\Lambda V^{-1}$ be the Jordan decomposition of the matrix $A(\omega_{ref}) + G(\omega_{ref})C$. Then the flux estimation errors are elementwise ultimately bounded as

$$|\tilde{\psi}_{ra,j}| \leq \epsilon_{a,j}, \quad |\tilde{\psi}_{rb,j}| \leq \epsilon_{b,j}, \quad (24)$$

for $j = 1, 2, 3$, where

$$\begin{aligned} \epsilon_{a,j} &\triangleq [0 \ 0 \ 1 \ 0] |V| |\operatorname{Re}(\Lambda)^{-1}| |V^{-1}G(\omega_{ref})| \bar{\eta}_j, \\ \epsilon_{b,j} &\triangleq [0 \ 0 \ 0 \ 1] |V| |\operatorname{Re}(\Lambda)^{-1}| |V^{-1}G(\omega_{ref})| \bar{\eta}_j. \end{aligned} \quad (25)$$

◇

We next use Lemma 2 to obtain ultimate bounds for the error signals (17). First, using the fact that $\psi_{rd}^2 = \psi_{ra}^2 + \psi_{rb}^2$ in permanent regime, we can write

$$\begin{aligned} \pi_j &= |\hat{\psi}_{ra,j}^2 + \hat{\psi}_{rb,j}^2 - \psi_{r,ref}^2| \\ &= |(\psi_{ra} - \tilde{\psi}_{ra,j})^2 + (\psi_{rb} - \tilde{\psi}_{rb,j})^2 - \psi_{r,ref}^2| \\ &= |\hat{\psi}_{ra,j}^2 + \hat{\psi}_{rb,j}^2 - 2\psi_{ra}\tilde{\psi}_{ra,j} - 2\psi_{rb}\tilde{\psi}_{rb,j}|, \end{aligned} \quad (26)$$

for $j = 1, 2, 3$, where a bar over a signal denotes the signal filtered by the transfer function (18). Thus, using (24) and

⁴ $|M|$ and $\operatorname{Re}(M)$ indicate the elementwise magnitude and real part, respectively, of a (possibly complex) matrix (vector) M .

noting that the transfer function $H(s)$ in (18) has two real poles and is such that $H(0) = 1$ (and hence upper bounds on its output signal are less than or equal to upper bounds on its input signal), we have

$$\pi_j \leq \epsilon_{a,j}^2 + \epsilon_{b,j}^2 + 2\psi_{r,ref}\epsilon_{a,j} + 2\psi_{r,ref}\epsilon_{b,j}.$$

Finally, the sampled error signal $\pi_j[k]$, $j = 1, 2, 3$, defined in (19), satisfies the same bounds, that is,

$$\pi_j[k] \leq \bar{\pi}_j, \quad \bar{\pi}_j \triangleq \epsilon_{a,j}^2 + \epsilon_{b,j}^2 + 2\psi_{r,ref}(\epsilon_{a,j} + \epsilon_{b,j}), \quad (27)$$

where $\epsilon_{a,j}$ and $\epsilon_{b,j}$ are defined in (25).

If the bounds on the noises are small, then the bounds (27) on the sampled observer error signals under healthy operation will also be small. As we will show in Section 7 below, this is in stark contrast with the bounds that these observer error signals have when a current sensor associated with the corresponding observer fails. This difference in bounds between healthy and faulty operation is the key to achieve fault tolerance in the proposed approach.

7. PERFORMANCE UNDER CURRENT SENSOR FAULT AND FAULT TOLERANCE

In this section we consider the performance of the switching control scheme under abrupt faults of current sensors. We will model an abrupt fault as an instantaneous change in one of the sensor measurement equations, from (11) to

$$i_{j,m} = \eta_j^F, \quad j \in \{R, S, T\}, \quad (28)$$

where η_R^F , η_S^F , η_T^F are bounded measurement noises. We will assume that only one sensor can fail at the time.

In the following two subsections we will analyse the behaviour of the variables relevant to the switching scheme under the *working hypothesis* that, in the presence of a faulty sensor, only measurements from healthy sensors are selected by the switching controller. The analysis will finally allow us to derive conditions, in Subsection 7.3, that guarantee that the *working hypothesis* is satisfied.

7.1 Induction motor variables

Under the working hypothesis that only measurements from healthy sensors are used by the switching controller, the performance in permanent regime of the induction motor is not affected by a fault in a current sensor. Thus, all equations presented in Section 6.1 are still valid.

7.2 Observer variables

If a fault modelled by (28) occurs in the sensor that measures the phase current i_R , it follows from (12) in Section 4 that only observers 1 and 2 will be affected by this fault whereas observer 3 will remain unaffected. Similarly, a fault in the sensor that measures the phase current i_S (i_T) affects only observers 1 and 3 (2 and 3).

Substituting (28) in the equations of the observers affected by each fault we have, after some calculations, that the estimation errors in permanent regime ($\omega = \omega_{ref}$) change their dynamics from (23) to

$$\dot{\tilde{x}}_l^F = A_G \tilde{x}_l^F + G(\omega_{ref})[E_l^F i^F + \eta_l^F], \quad l \in \mathcal{L}^F, \quad (29)$$

where $\tilde{x}_l^F = [\tilde{i}_{sa,l}^F \ \tilde{i}_{sb,l}^F \ \tilde{\psi}_{ra,l}^F \ \tilde{\psi}_{rb,l}^F]^T$ denotes the “under-fault” estimation errors, $A_G \triangleq A(\omega_{ref}) + G(\omega_{ref})C$, $F \in$

$\{R, S, T\}$ denotes the type of fault, and $\mathcal{L}^F$ denotes the set of indices of the observers affected by the fault F . The set $\mathcal{L}^F$, the vectors E_l^F , for $l \in \mathcal{L}^F$, and the input i^F change according to the type of fault as described in the table:

Fault	$\mathcal{L}^F$	$E_l^F, l \in \mathcal{L}^F$	i^F
R	$\{1, 2\}$	$E_1^R = -\begin{bmatrix} 1 \\ \sqrt{3}/3 \end{bmatrix}, E_2^R = \begin{bmatrix} -1 \\ \sqrt{3}/3 \end{bmatrix}$	i_R
S	$\{1, 3\}$	$E_1^S = \begin{bmatrix} 0 \\ -2/\sqrt{3} \end{bmatrix}, E_3^S = \begin{bmatrix} 1 \\ -\sqrt{3}/3 \end{bmatrix}$	i_S
T	$\{2, 3\}$	$E_2^T = \begin{bmatrix} 0 \\ 2/\sqrt{3} \end{bmatrix}, E_3^T = \begin{bmatrix} 1 \\ \sqrt{3}/3 \end{bmatrix}$	i_T

The “under-fault” noise input η_l^F in (29) is a combination of measurement noises; for example, for a fault in the sensor that measures the phase current i_R , these inputs take the form $\eta_1^F = [\eta_R^F \ (\sqrt{3}/3)(\eta_R^F + 2\eta_S)]^T$ and $\eta_2^F = [\eta_R^F - (\sqrt{3}/3)(\eta_R^F + 2\eta_T)]^T$. Note that (29) is a stable linear system driven by two bounded external inputs: the phase current i^F and the “under-fault” noise η_l^F . Taking Laplace transforms in (29) we have that the components of the flux estimation errors in response to i^F satisfy

$$\begin{bmatrix} \tilde{\psi}_{ra,l}^F \\ \tilde{\psi}_{rb,l}^F \end{bmatrix}_{i^F} = \begin{bmatrix} \tilde{H}_{a,l}^F(s) \\ \tilde{H}_{b,l}^F(s) \end{bmatrix} i^F, \quad l \in \mathcal{L}^F, \quad (30)$$

where $\tilde{H}_{a,l}^F(s) = [0 \ 0 \ 1 \ 0] (sI - A_G)^{-1} G(\omega_{ref}) E_l^F$, and $\tilde{H}_{b,l}^F(s) = [0 \ 0 \ 0 \ 1] (sI - A_G)^{-1} G(\omega_{ref}) E_l^F$.

Since, in permanent regime, i^F is a sinusoid (see Section 6.1) then the flux estimation error components in (30) will also be sinusoids of the form

$$\begin{bmatrix} \tilde{\psi}_{ra,l}^F \\ \tilde{\psi}_{rb,l}^F \end{bmatrix}_{i^F} = \begin{bmatrix} \tilde{a}_l^F \sin(\omega_p t + \tilde{\rho}_{a,l}^F) \\ \tilde{b}_l^F \cos(\omega_p t + \tilde{\rho}_{b,l}^F) \end{bmatrix}, \quad l \in \mathcal{L}^F, \quad (31)$$

where $\tilde{a}_l^F = |\tilde{H}_{a,l}^F(j\omega_p)| I_{ab}$, $\tilde{b}_l^F = |\tilde{H}_{b,l}^F(j\omega_p)| I_{ab}$, $\tilde{\rho}_{a,l}^F = \arg \tilde{H}_{a,l}^F(j\omega_p) + \rho^F$, $\tilde{\rho}_{b,l}^F = \arg \tilde{H}_{b,l}^F(j\omega_p) + \rho^F - \pi/2$, and where ρ^F is equal to ρ_R , ρ_S or ρ_T according to the type of fault. Applying the principle of superposition to the system (29), assuming zero initial conditions, yields the following form for the flux estimation errors:

$$\begin{aligned} \tilde{\psi}_{ra,l}^F &= \tilde{a}_l^F \sin(\omega_p t + \tilde{\rho}_{a,l}^F) + \tilde{\eta}_{ra,l}^F, \\ \tilde{\psi}_{rb,l}^F &= \tilde{b}_l^F \cos(\omega_p t + \tilde{\rho}_{b,l}^F) + \tilde{\eta}_{rb,l}^F, \end{aligned} \quad (32)$$

where we have combined (31) with the components of the flux estimation errors in response to the noise η_l^F , which we denote $\tilde{\eta}_{a,l}^F$ and $\tilde{\eta}_{b,l}^F$.

Next, we have that the observer error signals (17) can be written, for $l \in \mathcal{L}^F$, as (see (26))

$$\pi_l^F = |(\tilde{\psi}_{ra,l}^F)^2 + (\tilde{\psi}_{rb,l}^F)^2 - 2\psi_{ra}\tilde{\psi}_{ra,l}^F - 2\psi_{rb}\tilde{\psi}_{rb,l}^F|, \quad (33)$$

where $F \in \{R, S, T\}$, according to the type of fault, and ψ_{ra} , ψ_{rb} are sinusoidal. Substituting (32) in (33) we obtain, after some trigonometric and algebraic manipulations and further taking mean values (recall the discussion after (18)), that π_l^F satisfies

$$\pi_l^F = |\sigma_l^F + (\tilde{\eta}_{a,l}^F)^2 + (\tilde{\eta}_{b,l}^F)^2|, \quad (34)$$

where we have defined, for $F \in \{R, S, T\}$, the quantity

$$\begin{aligned} \sigma_l^F &\triangleq \frac{(\tilde{a}_l^F)^2}{2} + \frac{(\tilde{b}_l^F)^2}{2} - \psi_{r,ref} \tilde{a}_l^F \sin(\tilde{\rho}_{a,l}^F - \rho_0) \\ &\quad + \psi_{r,ref} \tilde{b}_l^F \sin(\tilde{\rho}_{b,l}^F - \rho_0), \quad l \in \mathcal{L}^F, \end{aligned} \quad (35)$$

with ρ_0 as defined in Section 6.1. Thus, we conclude from (34) that, for $l \in \mathcal{L}^F$,

$$\pi_l^F \geq \bar{\pi}_l^F, \quad \bar{\pi}_l^F \triangleq \begin{cases} \sigma_l^F & \text{if } \sigma_l^F \geq 0 \\ |\sigma_l^F| - [(\epsilon_{a,l}^F)^2 + (\epsilon_{b,l}^F)^2] & \text{if } \sigma_l^F < 0, \end{cases} \quad (36)$$

where $\epsilon_{a,l}^F$ and $\epsilon_{b,l}^F$ are computed as in Lemma 2 but using the “under-fault” bounds for the noises. Since the sampling operation preserves bounds, then (36) also yields a lower bound for the sampled signals $\pi_l^F[k]$ (see (19)), that is,

$$\pi_l^F[k] \geq \bar{\pi}_l^F, \quad l \in \mathcal{L}^F. \quad (37)$$

7.3 Fault tolerance conditions

Under healthy operation, the sampled error signals $\pi_j[k]$, defined in (19), on which the switching strategy bases its decision (20) at each sampling time, are upper bounded as in (27). On the other hand, under a current sensor fault, the sampled error signals $\pi_l^F[k]$ associated with the two observers affected by the fault are lower bounded as in (37). Thus, the scheme with switching criterion (20) will be fault-tolerant under a fault of type F , where $F \in \{R, S, T\}$ indicates a fault in the sensor measuring the phase current i_R , i_S or i_T , if

$$\bar{\pi}_l^F > \bar{\pi}_j, \quad \text{for } l \in \mathcal{L}^F \text{ and } j \in \{1, 2, 3\} \setminus \mathcal{L}^F, \quad (38)$$

where $\bar{\pi}_l^F$ and $\bar{\pi}_j$ were defined in (36) and (27), respectively. Note that, if conditions (38) hold, then observer j , $j \in \{1, 2, 3\} \setminus \mathcal{L}^F$, will be chosen by the switching criterion (20) over observers l , $l \in \mathcal{L}^F$, which are the ones affected by the faulty sensor measurements. We observe from equations (23), (24), (25), (27), (30), (31) and (35)–(37), that the healthy and faulty bounds $\bar{\pi}_j$ and $\bar{\pi}_l^F$, respectively, not only depend on the sensor noise characteristics but also on the reference values, $\psi_{r,ref}$ and ω_{ref} , the load torque τ_L and the observer gain matrix $G(\omega_{ref})$ (which, in turn, depends on the observer parameter K , see (14) and subsequent discussion). Then, to guarantee fault tolerance, condition (38) should be fulfilled for all operating conditions of the induction motor. This verification can be made numerically specifying the operating region of the motor, for example, $\omega_{ref} \in [\omega_{min}, \omega_{max}]$, $\tau_L \in [\tau_{L,min}, \tau_{L,max}]$ and fixed $\psi_{r,ref}$, making an appropriate discretisation of this region and computing the healthy and faulty bounds at each point of the discretisation for a given value of the observer parameter K . If condition (38) is satisfied for all the points of the discretisation, we can anticipate that it would be valid for the whole operating region since the induction motor variables and the bound expressions are smooth enough to assume that (38) still holds between two points of the discretisation. In the following subsection we illustrate this approach by means of a particular example.

8. SIMULATION RESULTS

We consider an induction motor with the following parameters: $R_r = 0.39923\Omega$, $R_s = 1.165\Omega$, $J = 0.0812\text{Nm}$, $L_s = 0.13995\text{Hy}$, $L_r = 0.13995\text{Hy}$, $M = 0.13421\text{Hy}$ and $n_p = 2$. The desired reference value for the rotor flux amplitude is $\psi_{r,ref} = 0.888\text{Wb}$, and the operating region for the rotor speed and torque load is specified as $\omega_{ref} \in [125, 140]\text{rad/sec}$ and $\tau_L \in [10, 30]\text{Nm}$. Proceeding

Fig. 2. Switching sequence ℓ_k and observer (filtered and sampled) error signals $\pi_j[k]$ [Wb²], $j = 1, 2, 3$.

as indicated in Section 7.3, we can anticipate to achieve fault tolerance when the induction motor operates in the predefined region by selecting $K = 2.1$ as the observer gain parameter. The sensor noises in (11) are bounded as $|\eta_R| \leq 0.01A$, $|\eta_S| \leq 0.01A$ and $|\eta_T| \leq 0.01A$. The same bound of $0.01A$ is used for the “under-fault” noises in (28). The filter parameter⁵ in (18) and the switching sampling period in (19) are chosen as $T_H = 20ms$ and $T_s = 0.10ms$, respectively. The parameters for the PI controller of the DTFC strategy were chosen as $P_w = 1.8816$ and $I_w = 18.816$. The reference signal for rotor speed is a ramp that starts from zero at $t = 0s$ and reaches its desired constant value $\omega_{ref} = 130rad/s$ at $t = 2.6s$. The simulation scenario is as follows. At $t = 1s$ a load with $\tau_L = 10Nm$ is applied. At $t = 3.5s$ a fault in the sensor that measures the phase current i_R occurs, that is, its measurement equation changes from (11) to (28). The last 3 plots of Figure 2 show the (filtered and sampled) error signals $\pi_j[k]$, $j = 1, 2, 3$, defined in (19), corresponding to observers 1, 2 and 3, respectively. As analysed in Section 7.2, after the fault at $t = 3.5s$ the error signals for observers 1 and 2 quickly move to values noticeably away from zero, whereas the error signal for observer 3 maintains the same small values (near zero) as before the occurrence of the fault. The top plot of Figure 2 shows the switching signal ℓ_k resulting from the switching mechanism decision (20). Note that after the fault at $t = 3.5s$ the switching mechanism only selects observer 3, as guaranteed by the fault tolerance conditions. Figure 3 shows the response of the rotor speed ω (top) and the stator flux magnitude ψ_{sd} (bottom) under the fault tolerant control scheme. Note that the simulated fault at $t = 3.5s$ has no noticeable impact on these responses. Although not shown in this example, a similar situation in terms of the fault tolerance conditions (38) for faults in phases S and T holds true. Thus, the scheme correctly selects the appropriate

Fig. 3. Rotor speed ω [rad/sec] and stator flux magnitude ψ_{sd} [Wb] under the fault tolerant control scheme.

observer in the event of a fault in any of the phase current sensors provided only one sensor fails at a time.

9. CONCLUSIONS

We have proposed the use of a strategy based on multi-sensor switching for sensor fault tolerant DTFC control of induction motors. The strategy combines stator current sensors, rotor flux observers and a switching mechanism that selects the sensors-observer pair with the smallest error between the estimated flux magnitude and a desired flux reference. The estimates provided by the selected pair are used by a DTFC controller. We have provided pre-checkable conditions that guarantee fault tolerance under an abrupt fault of a current sensor. These conditions are such that measurements from the faulty sensor are automatically avoided by the switching mechanism, thus maintaining good performance levels even in the presence of a faulty sensor. Extensions of the strategy to speed-sensorless DTFC control is under current investigation.

REFERENCES

- S.M. Bennett, S. Patton, R.J. and Daley, and D.A. Newton. Torque and flux estimation for a rail traction system in the presence of intermittent sensor faults. In *UKACC International Conference on Control '96*, volume 1, pages 72–77, September 1996.
- M. Depenbrock. Direct Self-Control (DSC) of Inverter-Fed Induction Machine. *IEEE Trans. on Power Electronics*, 3:420–429, 1988.
- P.C. Krause, O. Wasynczuk, and S.D. Sudhoff. *Analysis of Electric Machinery*. IEEE press, New York, 1995.
- H. Kubota and K. Matsuse. Speed sensorless field-oriented control of induction motor with rotor resistance adaptation. *IEEE Trans. on Ind. Appl.*, 30(5):1219–1224, 1994.
- K.-S. Lee and J.-S. Ryu. Instrument fault detection and compensation scheme for direct torque controlled induction motor drives. *IEEE Proceedings. Control Theory and Applications*, 150(4), 2003.
- W. Leonhard. *Control of Electrical Drives*. Springer, Berlin, Germany, 1996.
- J.J. Martínez, X.W. Zhuo, J.A. De Doná, and M.M. Seron. Multi-sensor switching strategy for automotive longitudinal control. In *Proc. of the 2006 American Ctrl. Conf.*, Minneapolis, USA, 2006.
- M.M. Seron, M.E. Romero, and J.A. De Doná. Sensor fault tolerant control of induction motors. In *Proceedings of the 17th IFAC World Congress*, Seoul, July 2008a.
- M.M. Seron, X.W. Zhuo, J.A. De Doná, and J.J. Martínez. Multi-sensor switching control strategy with fault tolerance guarantees. *Automatica*, 44(1):88–97, 2008b.
- I. Takahashi and T. Noguchi. A New Quick response and High - Efficiency Control Strategy of an Induction Motor. *IEEE Transactions on Industry Applications*, 22:820–827, 1986.

⁵ The filter (18) is designed to have sufficient attenuation at $2\omega_p$, the frequency of the most significant components of the switching error signals in faulty conditions. Since $\omega_p = n_p \omega_{ref} + \frac{M\tau_L}{\tau_r J \mu(\psi_{r,ref})^2}$ (see Section 6.1), a lower bound on ω_p for the range $\tau_L \in [10, 30]Nm$ and $\omega_{ref} \in [125, 140]rad/sec$ is obtained by setting $\omega_{ref} = 125rad/sec$ and (for simplicity) $\tau_L = 0Nm$. This yields the bound $\omega_p > 250rad/sec$, and hence $1/(2\omega_p) < 2ms$. Thus, we choose $T_H = 20ms$ in (18), ten times larger than the computed upper bound on $1/(2\omega_p)$.