Spatial structure of ULF waves: Comparison of magnetometer and Super Dual Auroral Radar Network data

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Abstract. The spatial structure of ultralow frequency (ULF) waves is usually, though not exclusively, estimated from ground-based magnetometer measurements. This paper compares ULF wave spatial structure obtained from coincident ground magnetometer and HF radar measurements and addresses the interpretation of Pc5 azimuthal wave numbers. ULF spatial structures estimated from magnetometer and radar data were quite different for the October 23, 1994, event presented by *Ziesolleck et al.* [1998]. Azimuthal wave numbers (m) were 3–5 and 12 for the ground and ionosphere, respectively. We reexamine this event and attempt to explain why the spatial structure of the ULF wave in the ionosphere, seen by the Saskatoon Super Dual Auroral Radar Network (SuperDARN) radar, may differ from that deduced from the magnetometer data. The radar data are used to develop a two-dimensional (2-D) model of the spatial distribution of ULF amplitude and phase in the ionosphere. Our modeling shows that the differences between ground and ionosphere measurements may be explained by spatial integration. In general, m numbers deduced from ground measurements should be smaller than the ionospheric values, and they are strongly dependent on the ionospheric ULF amplitude and phase distribution in both latitude and longitude.

1. Introduction

Determining the spatial structure of magnetohydrodynamic (MHD), ultralow frequency (ULF) waves is important for understanding generation and propagation properties of these disturbances in the Earth's magnetosphere. The most common method of obtaining ULF wave spatial structure relies on remote sensing using ground-based magnetometers. During the 1960s it was found that the radial (or ground north-south) structure could provide estimates of magnetospheric plasma mass density through the field line resonance (FLR) phenomena [Obayashi and Jacobs, 1958; Gul'elmi, 1967; Kitamura and Jacobs, 1968]. This involved determining the amplitude and phase structure with latitude [e.g., Samson et al., 1971] or from magnetic conjugate locations [e.g., Sugiura et al., 1964]. Experimental investigations of longitudinal properties of ULF waves have concentrated on the phase structure or m number [Ol-

son and Rostoker, 1978]. Southwood [1975] has shown how azimuthal phase properties might be used to estimate the energy flux in the wave. More recently, Mann et al. [1999] have shown the importance of azimuthal wave numbers in the identification of reflection properties around the magnetopause.

ULF waves propagating from the magnetosphere must pass through the ionosphere to be detected on the ground. Several studies showed how the ULF signals detected on the ground are influenced by the anisotropic ionosphere conductivity [Hughes, 1974; Hughes and Southwood, 1976; Glassmeier, 1984]. In addition to a rotation of the wave disturbance vector, small-scale features with scale sizes less than the ionospheric E region altitude ($\simeq 100$ – 120 km) are considerably smoothed. A simpler model using empirical formulations for ionospheric currents and the Biot-Savart law, to obtain the ground magnetic signature, was described by Poulter and Allan [1985]. This study was concerned with damping properties of FLRs and concentrated on latitudinal spatial structure. In this paper we use a similar model to Poulter and Allan [1985] to investigate the two-dimensional (2-D) spatial structure of ULF waves.

Recently, it has become possible to compare the properties of ULF waves in the ionosphere with those obtained at ground level. The ionospheric data are ob-

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tained from HF and VHF radars, probing the ionosphere above a magnetometer array. Walker *et al.* [1979] showed a good agreement between the latitudinal structure and FLR properties using the Scandinavian Twin Auroral Radar Experiment (STARE) and ground magnetometer data. A similar study by Samson *et al.* [1991] used magnetometer data from the Canadian Auroral Network for the OPEN Unified Study (CANOPUS) and the Super Dual Auroral Radar Network (SuperDARN), which showed similar spectral properties with latitude. Following the observations of large- m -number signals in SuperDARN data [Fenrich *et al.*, 1995], a comparison of the latitudinal and azimuthal spatial structure of ULF waves obtained from CANOPUS and SuperDARN was reported by Ziesolleck *et al.* [1998]. They showed several events where the FLR amplitude and phase structure measured by the radar was "smeared" at the ground. This was explained in terms of spatial integration introduced by the ionosphere as modeled by Hughes and Southwood [1976] and Poulter and Allan [1985]. However, the discussion of the azimuthal structure focused on the phase where the m numbers obtained from the magnetometer data were found to be smaller than those obtained from the radar. The data for October 23, 1994, were presented as an example where the radar and magnetometer m numbers did not agree well. Ziesolleck *et al.* [1998] discussed several possible explanations for the observed discrepancy without favoring any particular mechanism.

There are several mechanisms which may be responsible for obtaining different m numbers from ground-based magnetometer compared with radar data. These mechanisms can be divided into two groups: (1) physical phenomena accompanying ULF wave energy propagating between the F region and the ground, and (2) effects arising from the way the measurement was obtained. Propagation through the ionosphere is known to cause a $\sim 90^\circ$ rotation of the polarization vector in the lower ionosphere [Hughes, 1974] and the ground signal becomes a spatially integrated combination of the disturbance in the ionosphere [Hughes, 1974; Poulter and Allan, 1985]. Differences due to the measurement methodology include (1) phase aliasing between the ground-based magnetometers [Ziesolleck *et al.*, 1998], (2) contributions from the latitudinal phase gradient when the magnetometer spacing on the ground is not aligned with the FLR contour [Ziesolleck *et al.*, 1998], (3) effects of the line-of-sight velocity component due to the different beam directions when using a single radar [Ziesolleck *et al.*, 1998], and (4) effect of grating lobes of the radar on the 2-D phase-amplitude distribution [Greenwald *et al.*, 1985].

The $\sim 90^\circ$ rotation of the wave polarization vector modifies all spatial wave components equally and can be accounted for. Phase aliasing and nonalignment of the ground magnetometers with a FLR contour can be easily recognized if the 2-D horizontal structure of the ULF amplitude and phase is restored from radar measure-

Figure 1. Sketch of spatial integration effect on ULF wave amplitude and phase due to the presence of the ionosphere.

ments. Effects from single line-of-sight measurements using one radar may be eliminated by using two radars with overlapping fields of view. This is not always possible in practice. The grating lobes affect the ULF wave pattern in a complicated manner and may produce additional amplitude maxima and phase distortions. They become important at the higher frequencies and require detailed consideration of the parameters of each radar when the frequency exceeds 16 MHz [Greenwald *et al.*, 1985]. Spatial integration of the amplitude and phase structure of ULF waves arises from "radiated" fields from ionospheric currents. This is well known within the context of FLRs and the latitudinal distribution of amplitude and phase [Hughes and Southwood, 1976]. In this paper we show that a similar process also occurs in the longitudinal distribution, and this can explain discrepancies in m numbers between the radar and magnetometer measurements.

Consider the interaction of a shear Alfvén mode ULF wave with the ionosphere as shown in Figure 1. The amplitude distribution of Hall currents generated by the ULF wave has a horizontal scale size Λ_i at the ionosphere height H . For the case shown, the ground magnetic field perturbation has a larger scale size $\Lambda_g \sim 2H + \Lambda_i$ due to the field from the integrated ionosphere current as seen on the ground, calculated from the Biot-Savart law. This also leads to the flattening of the ground phase gradient [Hughes, 1974]. When the ionospheric distribution of Hall currents also has a finite scale size in longitude, then m numbers measured by magnetometers on the ground will be different compared with the values in the ionosphere. To estimate the contribution from this mechanism, it is necessary to determine whether the longitudinal amplitude distribution has a small or large scale size with respect to the effective E layer height. This requires information about the spatial distribution of the ULF wave properties in the ionosphere, and these can be provided by radar measurements. Therefore 2-D maps of the ULF wave amplitude and phase are very important for distinguishing between different mechanisms that may cause different m numbers at the ground compared with the ionosphere.

In this paper we reexamine the data for October 23, 1994, reported in *Ziesolleck et al.*, [1998]. We show that the spatial integration, in addition to changing the meridional ULF wave structure on the ground, also affects the longitudinal characteristics when the amplitude distribution in the ionosphere has a finite azimuthal scale size. The modifications due to spatial integration include an increase in the amplitude scale size and a decrease of the m numbers at ground level, compared with the ionosphere.

2. Analysis of Data

The time series, power spectra for selected ranges, and phase data for 0232-0402 UT, October 23, 1994, obtained from the SuperDARN radars at Saskatoon and Kapuskasing and the CANOPUS magnetometers are presented in Figures 6 to 9 of *Ziesolleck et al.* [1998]. The magnetometer data showed a dominant spectral peak at 1.67 mHz. Unfortunately, we found that the Saskatoon and Kapuskasing merged radar data had insufficient spatial coverage to be useful for comparison with the magnetometer data in this case. Spatial coverage over the magnetometer array was sufficient for data from the Saskatoon radar. The single radar measures line-of-sight velocity data, and ULF wave polarization effects may affect the results [e.g., *Fenrich et al.*, 1995; *Ziesolleck et al.*, 1998]. This point is discussed further in the next section. We have taken the data set for the time interval 0235-0335 UT, October 23, 1994, from selected CANOPUS magnetometers at Rabbit Lake (RAB), Back (BAC), and Gillam (GIL), and the Saskatoon radar. The radar data were inspected and corrected for minor gaps. Each spatial map (range, beam) was converted to the same sampling time grid to provide continuous time series for each radar spa-

Figure 2. The spatial distribution of the spectral peak frequency in Doppler velocity spectra from the Saskatoon radar for 0235-0335 UT, October 23, 1994. The heavier shading indicates a higher frequency. The 1.67 mHz signal is examined in this paper.

Figure 3. (a) Spatial amplitude and (b) phase for the 1.67 mHz Doppler velocity data from Saskatoon for 0235-0335 UT, October 23, 1994. Normalized amplitude gradations in shading indicate 0.2 steps from high power (black) to low power (white). ULF phase variations are plotted in 30° contour steps. The lighter shading means a higher phase value, this is, phase decreases eastward. The westward FLR region is seen around 70° latitude.

tial cell. The radar frequency during this event was 10.3 MHz, well below 16 MHz, which allows us to exclude grating lobe effects.

The radar data provide a more detailed view of the two-dimensional ULF spatial structure. In order to appreciate the spatial distribution of wave power and the relationship to the observed 1.67 mHz peak in the magnetometer data, the distribution of the frequency of the spectral maxima from the radar range/beam cells is shown in Figure 2. The 1.67 mHz frequency dominates the spatial map, extending across the longitude field of view. Lower frequencies appear at higher latitudes. However, an interesting feature is the 1.39 mHz region over -38° to -45° in longitude and 67° to 69° in latitude dividing the two 1.67 mHz sections. *Ziesolleck et al.* [1998] analyzed data from beam 9 which passes

directly through this lower-frequency region. Hence the spectral maxima in the radar data vary spatially, and it is therefore possible for the spectral maxima obtained from radar and magnetometer data to be different. Also, note that coincident azimuthal measurements at ground level may only be obtained from the magnetometers at BAC and RAB.

The spatial variations of amplitude and phase for the 1.67 mHz spectral component are shown in Figures 3a and 3b, respectively. The larger amplitudes appear at the western edge of the radar field of view. These values include the effect of using single-radar data. However, as discussed in the next section, the finite scale size in both latitude and longitude is preserved when this is taken into account. The phase shows a gradient in both latitude and longitude. The longitudinal phase change is used to determine the m number and would usually be interpreted as a signature of tailward propagation of Kelvin-Helmholtz generated waves

Figure 4a. Latitude variation of amplitude and phase of the Saskatoon radar data over $3^\circ \times 3^\circ$ area across FLR structure.

Figure 4b. The longitudinal phase structure of the Saskatoon radar data at 68.4° latitude. This latitude corresponds to the CANOPUS magnetometer at BAC and is close to that at RAB.

at the magnetopause. Figure 4a shows the ULF amplitude and phase from the radar data over the area from -45° to -48° E and 68° to 71° N. The latitudinal variation is consistent with FLRs. Ziesolleck *et al.* [1998] used phase values at the resonance latitude, $\approx 70.2^\circ$ N, to estimate ionospheric m numbers, while their ground values were obtained from the line of stations situated at $\approx 68^\circ$ N, substantially equatorward of the FLR contour. To avoid this, we used ionospheric phase values from the same coordinates as the ground magnetometers. The longitudinal phase variation in the radar data at 68.4° N (magnetic latitude of BAC) is shown in Figure 4b. From this, the resulting m number in the ionosphere over RAB-BAC was calculated to be ~ 12 . The phase differences obtained from the three magnetometers are shown in Table 1. Positive phase indicates that the signal at RAB is leading, which agrees, in sign, with the radar data. However, the phase difference with longitude is ~ 3 times smaller for the magnetometer data, which gives an estimate of $m = 3 - 5$ for the RAB-BAC pair.

The detailed ULF wave structure from the radar data significantly enhances the information traditionally obtained from magnetometers. Figure 3b shows that the more spatially detailed radar data exclude the possibility of spatial aliasing in the longitudinal phase from the magnetometer spacing. Figure 4b shows that in the

Table 1. Phase Difference Values for the H and D Components of the Three CANOPUS Magnetometers

Stations ^a	H	D
RAB-BAC	42°	67°
RAB-GIL	6°	-47°

^aStations are Rabbit Lake (RAB), Back (BAC), and Gillam (GIL)

ionosphere the phase shift between RAB and BAC does not exceed 180° , so that the ground magnetometer data are not affected by spatial aliasing. The amplitude map in Figure 3a shows nonuniformity of the FLR amplitude with longitude in the line-of-sight data. This may decrease the ground m values via spatial integration of the ULF wave field. Therefore, for this particular event the major contributions to the observed discrepancy in m numbers are due to spatial integration effects and/or line-of-sight constraints in using single-radar measurements. In the next section we model both these effects in order to estimate each contribution to the observations.

3. Modeling of Data

The aim of modeling the data is to highlight changes in m that occur due to the interaction of ULF waves with the ionosphere during the event of October 23, 1994. It is not intended to provide a detailed mapping of the ULF structure from the magnetosphere to the ground. The modeling consists of two parts: (1) we consider the effect of using line-of-sight velocity measurements from a single radar and how this distorts the ULF wave structure, and (2) on the basis of the ionospheric distribution of the ULF wave amplitude and phase, we calculate the spatial distribution of amplitude and phase at the ground using the Biot-Savart law and investigate the effect of 2-D spatial integration on the m numbers.

3.1. Line-of-Sight Velocity Effect When Using Single Radar Data

The ULF amplitude measured by the Saskatoon radar shows spatial nonuniformity with both latitude and longitude (Figure 3a). While the finite meridional scale size may be explained by the FLR structure (Figure 4a), the limited longitudinal extent of the amplitude requires closer examination. Longitudinal scale sizes of ULF energy are usually considered to be quite large, up to 3000 km [e.g., *Greenwald and Walker*, 1980]. However, to our knowledge, radar-data-derived longitudinal scale sizes of ULF waves have not been published. The observed longitudinal nonuniformity in Figure 3a can arise from either the actual ULF wave amplitude distribution or an apparent one resulting from the geometrical effect when using single-radar measurements as described by *Fenrich et al.*, [1995]. This effect reflects the fact that a single radar measures the line-of-sight component of the plasma drift velocity $V_{\parallel} = V \cos(\mathbf{V} \cdot \mathbf{k})$, where $\mathbf{V}$ and $\mathbf{k}$ are drift velocity and radar wave vector, respectively. For simplicity, assume that the ULF wave in the ionosphere is linearly polarized with the electric field vector in the same direction with respect to the L shell geometry. For regions where $\mathbf{V} \perp \mathbf{k}$, the measured $V_{\parallel}$ would be zero. Looking at Figure 3a, one might identify such a region in the vicinity of the amplitude minimum near beam 9. This would occur even

Figure 5. The model ionospheric data with infinite longitudinal scale size ($\Lambda_i^{lon} = \infty$). Amplitude distribution is shown by a grey scale map with darker tones corresponding to larger values, and it was normalized to its value at -50°E , 70°N . Phase distribution is presented by labeled contours.

if the incident longitudinal amplitude distribution was uniform. To decide whether the longitudinal structure in Figure 3a is the real or apparent property of the ULF wave, we modeled the radar response to various ionospheric distributions. First, we used an ionospheric ULF field distribution with uniform amplitude in longitude and a linear azimuthal phase gradient with $\mathbf{V}$ orthogonal to beam 9. In the meridional plane we used a Gaussian-shaped amplitude profile with an effective half-width $\Lambda_i^{lat}/2 = 100$ km and arctangent phase to resemble FLR properties. These ionospheric 2-D distributions are shown in Figure 5.

The modeled response for single radar measurements is shown in Figure 6. This reproduces the main features of the experimental amplitude distribution (Figure 3a), including the large-amplitude region at the westward edge of the field of view and the strong amplitude decrease across beam 9. However, the modeled and measured phase distributions are quite different. The modeled phase (Figure 6b) shows a 180° flip across beam 9 due to the change of sign in $\cos(\mathbf{V} \cdot \mathbf{k})$ when the angle between $\mathbf{V}$ and $\mathbf{k}$ crosses 90° . This feature would be easily detected in the experimental data. Instead, the experimental phase decreases monotonically eastward. The absence of the phase flip in the experimental data shows that $\mathbf{V}$ has a component aligned with beam 9. The magnitude of this component is large enough to move the 180° phase flip outside the radar field of view. Consequently, there is a contribution from the longitudinal nonuniformity in the spatial amplitude distribution properties of the ULF wave in the ionosphere. To illustrate this, we repeated the modeling for a finite longitudinal amplitude distribution and $\mathbf{V}$ parallel to beam 9 (Figure 7). The longitudinal amplitude was characterized by $\Lambda_i^{lon}/2 = 190$ km with its maximum centered

Figure 6. Modeled radar response to ionospheric data in Figure 5 with drift velocity perpendicular to beam 9: (a) amplitude and phase data in the same format as Figure 5; (b) the modeled phase for comparison with Figure 3b.

at -50°E , 70°N and the same longitudinal phase and latitudinal parameters as Figure 5. The distribution modeled for the single radar located at Saskatoon is shown in Figure 8. For this orientation of $\vec{V}$ the geometric effect is negligible. In contrast to Figure 6, this modeled distribution (Figure 8) resembles the main features of the experimental data in both the amplitude and phase except for a lower-amplitude region at the southeast part of the radar field of view.

3.2. Ground Data Modeling

In modeling the ground signal, the equations of *Poulter and Allan* [1985] were used to calculate the 2-D structure on the ground through integration of the ionospheric Hall currents. The Hall currents were assumed to be proportional to the ULF electric field, obtained from the radar Doppler measurements. We also assumed that they are horizontal and have the same polarization at all points in the ionospheric plane. Integra-

Figure 7. The model ionospheric data with a limited latitudinal scale size ($\Lambda_i^{lon}/2 = 190$ km), which reproduce the main features of the data in Figure 3. Data format is the same as in Figure 5.

tion was performed based on the Gauss formula within ± 1300 km limits in both north-south and east-west directions. This value approximately corresponds to the horizon distance for the ionospheric E region. The effective ionosphere height was chosen as $H = 100$ km, and the skin depth of the signal into the Earth was also set to 100 km. Using the same ionospheric ULF wave parameters as in Figure 7 (i.e., finite Λ_i^{lon}), we obtained the corresponding ground distribution as shown in Figure 9. The phase structure on the ground looks very different from that in the ionosphere.

In order to better understand Figure 9, consider the effects in latitude and longitude separately as illustrated in Figure 10. We focus on the ionosphere and ground distributions of the ULF amplitude and phase along the magnetic latitude and longitude across the amplitude maximum at -50°E , 70°N . In addition to the ex-

Figure 8. Modeled radar response to ionospheric data in Figure 5 with drift velocity parallel to beam 9. Data format is the same as in Figure 5.

Figure 9. The expected amplitude and phase distribution seen at the ground due to the data in Figure 7 ($\Lambda_i^{lon}/2 = 190$ km). Both the amplitude and phase of the signal in the ionosphere contribute to the spatial distribution seen at ground level. Data format is the same as in Figure 5.

pected broadening of the amplitude, the phase shows a smaller slope just beneath the region of largest amplitude and has additional extrema near the edges. Detailed analysis of 1-D model curves such as these, obtained for different values of Λ_i , showed that distortions of the phase-amplitude distribution on the ground directly under the location of the largest amplitude decrease with increase of the ratio Λ_i/H . Consider two extreme cases: (1) When $\Lambda_i/H = 0$, we essentially have a point source in the ionosphere which illuminates an area on the ground of size $\Lambda_g \sim 2H$ and with constant phase corresponding to the phase of the source. (2) For $\Lambda_i/H = \infty$, the amplitude and phase distribution on the ground is almost equal to that in the ionosphere. For the intermediate case $0 < \Lambda_i/H < \infty$, the amplitude scale size on the ground increases by a doubled ionospheric height as $\Lambda_g \sim \Lambda_i + 2H$, and the ground phase gradient becomes smaller than in the ionosphere. Additional extrema of the phase occur at the edges of the amplitude distribution (Figure 10b). At large distances from the amplitude maximum, the phase returns to its value at the maximum of the amplitude distribution, that is, it is effectively a point source when looking from large distances. These results show that azimuthal wave number discrepancies between the ground and ionosphere in both the magnitude and sign of m may occur. In general, the magnitude of the phase gradient on the ground should be smaller than in the ionosphere, which leads to smaller m values. If the ground sensors lie in regions of reversed phase gradient, then even the sign of m may be altered.

It is clear from the 2-D structure shown in Figure 9 that the ground signal depends on the spatial structure of both the amplitude and phase in the ionosphere. The

amplitude functions have been "smeared" as shown by previous authors. However, in addition to this, there are two extrema in the phase, one to the northeast and the other to the southwest of the center of the amplitude maximum. Figures 7 and 9 show that while the azimuthal phase distribution (i.e., m number) in the ionosphere does not depend on longitude, its value on the ground does change with both latitude and longitude. Furthermore, the model shows that for the given ionospheric structure, the phase difference for the RAB-BAC magnetometer pair should be larger than for RAB-GIL, which is in qualitative agreement with the experimental data (Table 1). The phase distribution shown in Figure 9 allows the range of possible m values, as observed on the ground, to be determined, and its dependence on latitude can be shown. The longitudinal phase derivative at constant latitude was calculated, and the minimum and maximum values identify the minimum and maximum m numbers (at each latitude) that confine the shaded region in Figure 11. The dashed line in Figure 11 shows the ionosphere m number, which is constant and always larger in absolute value than those

Figure 10. Modeled 1-D ionospheric and ground amplitude-phase distributions along (a) magnetic latitude and (b) longitude, which were obtained from Figure 6 via its section across the amplitude maximum at -50°E , 70°N .

Figure 11. Modeled range of possible ground m numbers (shaded area) versus latitude. See text for comments.

at the ground. The shaded region shows the range of possible m numbers for any magnetometer spacing at a given latitude. Figure 11 shows that spatial integration also applies to finite scale sizes in the longitudinal direction, which leads to smaller m numbers measured on the ground and may even alter their sign.

4. Discussion and Conclusions

In order to explain differences in ground-based compared with ionospheric (radar) azimuthal m numbers, the horizontal structure of ULF wave energy observed on October 23, 1994, has been examined. To interpret these data, the single-radar response to the distribution of ULF wave amplitude and phase in the ionosphere was modeled. The 2-D spatial integration at ground level of the ionospheric currents was also modeled to obtain the amplitude and phase distributions on the ground. The presence of the ionosphere is known to modify the amplitude and phase structure of ULF waves when measured on the ground, and SuperDARN radars provide a unique, detailed spatial coverage for investigating the fields in the ionosphere. For example, Figures 3 and 4a illustrate the ability of the radar to identify the location of the resonant contour and its spatial scale size.

Modeling the October 23, 1994, event in this paper has shown how radar and magnetometer data can give different estimates of ULF spatial structure. It is important to recognize that the ionosphere itself does not change the ULF wave energy spatial scale size and phase gradient structure. Major changes in these parameters occur during propagation of ULF energy from the ionosphere through the neutral atmosphere to the ground. The process is made clear if we follow an incident shear Alfvén wave with a specific m number from the magnetosphere to the ground. In the lower ionosphere (where the conductivity is largest), the wave generates Hall currents, which produce an isotropic (in the neutral atmo-

sphere) propagating electromagnetic signal. The structure here has the same m as in the magnetosphere. The ground magnetometer signal is an integrated view over the sky, each spatial contribution weighted according to the Biot-Savart law. It is here that ground m values may differ from the magnetosphere and ionosphere due to the spatial integration. Consider the case where the incident wave energy consists of some narrow wave number spectrum (constant azimuthal phase gradient). If the longitudinal scale size of the ULF amplitude distribution is large in comparison with the height of the E region ($\Lambda_i^{lon} \gg H$), the azimuthal phase gradient and, consequently, the ground m numbers remain the same as in the ionosphere. In contrast, if $\Lambda_i^{lon} \sim H$, then on the ground the spectrum is converted to a band that contains both positive and negative wave numbers (Figure 11). The upper and lower bounds of this band depend on the height and scale size of the signal in the ionosphere. However, the absolute value does not exceed the ionospheric m number. Thus 2-D spatial integration can explain the difference in azimuthal wave numbers obtained from radars and ground magnetometers, when ground measurements provide estimates of m which are smaller than values in the ionosphere [Ziesolleck *et al.*, 1998].

Finally, the experimental data and modeled effects of ULF spatial structure presented in this paper show the following:

1. The spatial amplitude and phase structure of induced ionospheric currents affects the azimuthal characteristics of ULF waves estimated using ground magnetometer data. The latitudinal structure associated with FLRs is quite well understood. As more work is directed toward understanding ULF azimuthal structure, it is important to realize that similar constraints apply in the longitudinal direction as well. Two-dimensional spatial integration can be a major cause of the difference between azimuthal m values measured by radars compared with those obtained using magnetometers on the ground. When the ULF wave amplitude distribution has a finite scale size in longitude, the model calculations show that this always leads to smaller m values on the ground, in agreement with the experimental data.

2. When interpreting spatial maps of ULF amplitude and phase from a single radar, one should always consider possible contributions from the line-of-sight velocity measurement. This can introduce distortions of the original ionospheric signal. With reasonable assumptions, it has been shown that the data for October 23, 1994, must have a finite longitudinal scale size which affects the m value measured at the ground.

With respect to future work, the predictions of the model may be improved by obtaining estimates of the polarization properties of the electric field over the ionospheric spatial grid. This can be achieved by using data from two radars with overlapping fields of view, and such a study is in progress.

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