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Axial Line Analysis Revisited: Reconsidering its Value for Architecture

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Abstract: Developed in the late 1970s and refined over the following two decades, the Axial Line method is part of the Space Syntax suite of tools and associated theories that sought to apply mathematics to architectural and urban plans to uncover the relationship between social structure and spatial configuration. The aim of this paper is to consistently and critically demonstrate the Axial Line method. This paper is a response to repeated propositions that this approach is largely impenetrable to the average built environment professional. In response to these calls for clarity and consistency, this paper offers a series of worked examples, based on hypothetical architectural settings, of the three main stages in the process. The first stage involves the construction of an Axial Line map, the second applies a series of mathematical formulas to the data contained in the map, and the third suggests how the mathematical results might be interpreted and represented graphically. In presenting a critical interpretation of the Axial Line approach, the paper offers several original contributions to the method.

Keywords: Space Syntax, Axial Line Analysis, Architectural Analysis, Axial Line Map

Introduction

INSPIRED BY THE work of Alexander [1964: 1966], March and Steadman [1971] and Kruger [1979], in the 1980s a growing number of architectural scholars were drawn to use logical, mathematical or geometric models as tools to assist the design process or for the analysis of architecture [Stiny 1975; Mitchell 1990; Mitchell and McCullough 1995]. One of the most successful of these groups developed the Space Syntax set of analytical approaches [Hillier and Hanson 1984; Hillier 1996; Hanson 1998]. Over the following decades this group remained focused on the application and continual refinement of a set of tools. This focus both ensured the longevity of their ideas, but it also effectively marginalized them from the body of computational researchers in architecture. This marginalization occurred, in part, because Space Syntax researchers developed their own journals and conferences and began to focus on the serial refinement of sometimes-obscure dimensions of their own methods. As a direct result of this comparative isolation, within two decades, the group had become so hermetic that it was difficult for external scholars to develop a detailed understanding of their practices. This comparative isolation has led to several researchers, including Klarqvist [1993], Osman and Suliman [1994] and Dovey [1999] being critical of the lack of transparency in the methods developed and used by Space Syntax researchers. While Dovey [1999] even implies that this opacity may be deliberate – to avoid external academic scrutiny – a more likely explanation is that the approach has developed over time and there are relatively few papers that critically summarize the various methods taking into account recent developments. For example, Peponis and Wineman [2002] and Bafna [2003]

both offer substantial and informative overviews of Space Syntax, but in recent years only Ostwald [2011a; 2011b] has provided a critical, worked example, of a Space Syntax tool; the Justified Plan Graph. This realization is the catalyst for the present paper, which is focused on the second major analytical tool developed by Space Syntax researchers; the Axial Line map.

The primary purpose of the present paper is to provide a worked demonstration of the Axial Line method, including the production of an Axial Line map, the mathematical processing of data derived from this map and the interpretation of the results. For this reason, the present paper does not contain extensive original research, but rather its original contribution is its detailed and critical compilation and combination of past research along with several new refinements to the analytical approach. Finally, because like most analytical tools the Axial Line method is relative (that is, the results mean more in comparison) then, instead of producing a worked example of a real building, it is more informative to provide a series of hypothetical examples of similar buildings, but with different planning, to develop an understanding of what a comparative analysis entails.

Background

Space Syntax is the collective title given to a number of techniques and associated theories that seek to use mathematics to draw connections between spatial configurations and social effects. Bill Hillier and Julienne Hanson authored the seminal works of Space Syntax, first collectively and then later separately [Hillier and Hanson 1984; Hillier 1996; Hanson 1998] and these were swiftly supplemented with a range of refinements, many of which were proposed by Alan Penn [Penn *et al.* 1997; Penn 2003] or John Peponis [Peponis *et al.* 1998; Peponis and Wineman 2002]. The work of Peponis in particular has shaped the variation of the Axial Line method described in this paper.

Space Syntax promotes a conceptual shift in understanding architecture wherein “dimensional” thinking is rejected in favor of “relational” reasoning. This shift relies on the process of translating architecturally defined space into a series of graphs that may be mathematically analyzed (graph *analysis*) and then interpreted (graph *theory*) in terms of their two-dimensional spatial characteristics. This method is almost entirely concerned with spatial relations and the belief that such configurations are a direct reflection of the social structure that created it. Thus, Peponis and Wineman [2002] argue that the central supposition underlying Space Syntax research “is that it is possible to identify certain underlying structures of space that are linked to observable patterns of behavior and that these patterns, in turn, create social function, whether generative or reproductive” [272]. Bafna [2003] argues the central premise of Space Syntax research is that the structure of society “is inherently spatial, and inhabited space has a fundamentally social logic” [18]. In practice, Space Syntax tools have been used to predict how people will use space, insofar as some spaces have natural tendencies to promote concentrated use, while others are sufficiently isolated that they are unlikely to be natural spaces of social interaction [Bafna 2003].

While there is an extensive body of published data providing support for Space Syntax, papers which record results that do not match the Space Syntax predictions are rare, and such data, if it is reproduced at all, is typically regarded as anomalous. This is one of the potentially problematic dimensions of the Space Syntax discipline; balanced and thoughtful criticism can be rare. This is not to suggest that Space Syntax has hidden flaws, but rather

that ongoing robust testing and criticism is part of the natural cycle of theory development [Kuhn 1962; Latour 1987].

The Axial Line Method

One of the key Space Syntax methods relies on the generation of an Axial Line map. A technical definition of an Axial Line map – which is paraphrased in multiple papers – is that, it is the set of the *fewest* and *longest* straight lines required to cover *every* convex space in a plan, while ensuring that those lines that can be connected are connected and all non-trivial circulation loops are made. The two qualifiers, *convex space* and *circulation loops* are explained as follows:

- A convex space is one that can be viewed, in its totality, from every point within that space. If the space is concave (that is, part of the space is occluded from view), it must be partitioned or divided in such a way as to identify only convex spaces (fig. 1).

Fig. 1: A Convex Shape is One where any Point on the Perimeter may be Linked Directly to all other Points on the Perimeter (A-B) without Moving Outside the Perimeter (C-D). Convex Space (Left), Non-convex Space (Center), and Non-convex Space Partitioned into Two Convex Spaces (Right)

- A “circulation loop” is a path around a space. If the path is visually connected at all points (or has only minor visual obstructions like columns) then it is considered a “trivial” loop. If the obstruction is sufficient to disconnect the experience of a space, it is a “non-trivial” loop (fig 2).

Fig. 2: Trivial and Non-trivial Circulation Loops

An alternative explanation of the Axial Line process is that it identifies the simplest (fewest), most efficient (longest), network diagram (connected) that can represent a complete building plan (covers all convex spaces and completes all non-trivial loops). In practice the Axial Line map looks like a set of angled, referenced, lines, all of which intersect at least one other line. Because the Axial Line method relies on long sight-lines, it is most useful for the analysis of urban spaces, where open spaces tend to connect large urban areas into a network of vistas, most of which correlate to streets and parks (although connections via tunnels, and bridges can also be handled by the method). Indeed, several of the mathematical results arising from the analysis of urban Axial Plans suggest a correlation between various data and the volume of traffic on particular streets [Hiller *et al.* 1993], crime rates [Hillier and Shu 2000] and property values [Desyllas 2000]. The Axial Line method may also be usefully applied to the analysis of large buildings including galleries [Batty *et al.* 1998] and offices [Ermal and Peponis 2008].

Applications of the Axial Line method to domestic plans are relatively rare although Hanson [1998] provides several examples. One of the reasons for this is that the method generally requires larger sets of data to produce useful results. However, one often overlooked way of achieving a sufficient critical mass in the data is the comparative analysis of multiple houses. While other Space Syntax tools, including the Justified Plan Graph, have been used for the comparative analysis of houses – either by the same architect [Bafna 1999; Major and Sarris 1999; Ostwald 2011b] or in the same historic setting [Hanson 1998] – the Axial Line method has rarely been applied in this way.

Almost all of the research using Axial Line analysis from the last decade has been undertaken using one of a range of software tools. While highly efficient, these programs actually mask what is happening in the process and some have been found to contain mathematical errors in their programming [Ostwald 2011a]. For a person who has a detailed understanding of the method this is a reasonable compromise, but for a “non-expert” user, the “black-box” approach to developing results is more problematic. In response to this, the present paper develops and demonstrates a manual mapping method and a transparent mathematical process. This method involves several new rules, variations or processes that are demonstrated for the first time in this paper.

While the manual method might be slow, it does allow for a researcher’s intuition to at least partially shape the ultimate Axial Line map. While intuition can support the production of maps that more closely approximate the intentions of designers, they are also necessarily more subjective. Thus, the present method allows researchers to balance ease of production with objectivity to produce a stable and singular result. Furthermore, if the method is metic-

ulously followed, it will produce a completely objective result. There is an additional advantage to using the manual method; it allows the researcher time to immerse themselves in the spatial qualities of the plan prior to the mathematical results being calculated. This is useful because, without this level of understanding, any errors in the software calculations are unlikely to be immediately recognizable as such.

There are many ways of producing an Axial Line map and each version has its own strengths and weaknesses. Moreover, it is also possible to produce multiple different, but similarly “valid”, maps from the same architectural plan. Ideally, the method for producing an Axial Line map should limit both of these occurrences by producing only a single, optimal option. For this reason, the process adopted in the present paper for drawing the Axial Line map is based on two of the more stable and repeatable versions [Peponis *et al.* 1997; Turner *et al.* 2005] with some modifications to reflect the spirit of Hillier’s and Hanson’s [1984] original version. In contrast, the software-supported variations of these methods tend to use the computer to identify every possible line (an “all line map”) and then reduce that to a minimal set; a methodical, if somewhat inelegant, approach that prioritizes the computer’s mass-processing capacity ahead of its ability to apply logic [Turner *et al.* 2005]. An alternative would be to follow Batty and Rui [2003] and use isovists to generate the axial map.

Manual Process

The manual process for producing an Axial Line map commences with a partitioning of the plan into specific convex areas. This partitioning is done by extending lines – known as “s-lines” – from every reflex wall angle until they reach another wall surface. Once completed these s-lines will have defined all of the spaces in a building into concise zones (or “s-spaces”) [Peponis *et al.* 1997]. Note that it is not necessary to draw s-lines between convex wall angles and s-lines do not extend along the generating wall’s surface. S-lines represent a demarcation that must be crossed for an entire wall surface to become visible (fig. 3).

Fig. 3: An Example of S-lines Extended from Reflex Wall Angles

Once the s-lines have been drawn, the variation of the method developed in the present paper continues – returning to Hillier’s and Hanson’s [1984] original version – with the process of finding the longest line that can be drawn on the plan and which crosses at least one s-line. If the longest line has no equivalences elsewhere in the plan, it is called an “m-line” and drawn in. There are multiple different types of m-lines which have been defined by

space syntax researchers although in the present context it is useful to understand the variations, although these are not significant for the following analysis (fig. 4). It is possible that a number of potential m-lines will possess identical characteristics (length, connections and s-line intersections) and a decision must be made about which one to keep, and which to remove. Peponis *et al.* [1997] selected one of these at random, whereas Turner *et al.* [2005] notes that this may compromise the final plan by impacting on later drawn lines. The solution proposed in the present paper is to sketch in potential lines (or “p-lines”) noting that only one is needed, and the remainder may be erased once sufficient lines have been drawn to produce a complete map (fig. 5).

Fig. 4: M-line Types; based on Turner *et al.* [2005]

Fig. 5: Example of Three Potential Lines (P-lines 1a, 1b and 1c) that have Identical Length, Connections and s-line Intersections. Only One of these is Required for the Final Map and Analysis

Any s-lines crossed by an m-line or p-line are removed as a sign that this zone or partition in the plan has been adequately covered. Then, the next longest m-line intersecting at least one s-line is drawn and so on. This new process, which departs from Peponis’s version, is more closely aligned to the intentions of Hillier’s and Hanson’s original work. Thereafter, to ensure all non-trivial circulation loops are drawn, a modified version of Peponis *et al.*’s [1997] polygon check is used; see next section. Once all s-lines have been removed, the map

should be reviewed and any superfluous m-lines or p-lines removed. At the end of the mapping process, the only thing that remains of the plan is a set of angled, intersecting m-lines, which may be numbered for reference, and are called an Axial Line map.

Worked Example: Axial Plan Map Generation

Consider the floor plans of three hypothetical villas, each with the same floor area and external dimensions but with different internal spatial configurations. The three broad planning strategies they represent are “linear” (or enfilade), “branching” (or arborescent) and “circuitous” (or rhizomorphous). These three houses are respectively known as the Villa Epsilon ($V\epsilon$), the Villa Zeta ($V\zeta$) and the Villa Eta ($V\eta$). The following ten steps are typically required to generate the Axial Line maps for a floor plan. However, for reasons that will soon become apparent, not all of these stages are relevant for all of the Villas.

Step 1. The first step requires the production of plan drawings, stripped of any detail other than their base geometric properties (fig. 6).

Fig. 6: Building Floor Plans for Three Hypothetical Villas

Step 2. In the second stage, the set of s-lines in the plan are traced (fig. 7).

Fig. 7: S-Lines Marked on Floor Plans

Step 3. Once all of the s-lines have been traced, then the process of drawing m-lines can commence. It is recommended that as each m-line is generated it is given a unique identity label (typically a number). The first m-line is located by running a ruler over the plan, seeking the longest axis that intersects at least one s-line. If two m-lines of equal length are identified which intersect different s-lines, both should be drawn. After all lines are drawn one of these identical length lines may be identified as superfluous and removed. In the case

of the three villas, there are two clear starting m-lines (1 and 2). These lines are drawn simultaneously as they are identical length and intersect different sets of s-lines (fig. 8).

Fig. 8: The Longest Lines are Marked on the Floor Plans (in All Three Cases Two M-lines, Identical in Length and Intersecting Different S-lines are Generated)

Step 4. Once the first m-line(s) (or set of equivalent, p-lines) are drawn, then the second m-line line is identified. Before generating the second m-line, remove or reduce the opacity of any s-lines the first m-line intersected. The method for generating the second m-line is identical to the first line; locate the longest m-line in the system that intersects at least one (remaining) s-line. Again, if two lines of equal length intersect different s-lines then draw both, and if multiple lines of equal length intersect the same s-lines also sketch all annotate them appropriately as p-lines, and note that one must later be retained, the rest removed. In the case of the three villas, the Villa Epsilon has a clear second (actually third) m-line, whereas the other two villas each have a pair of identical possible m-lines (or two p-lines until one of each pair is deleted) (fig. 9)

Fig. 9: The ‘Second’ Longest Lines are Marked on the Floor Plans (in the Case of the Villas Zeta and Eta they are P-lines)

Step 5. The process of seeking the next longest m-line line, which intersects an s-line, is repeated until all s-lines have been intersected by m-lines or p-lines. This step ensures that every wall surface in the plan is surveilled by one of the axial lines (the final set of m/p-lines). For the three villas, seven iterations of this step are required to achieve sufficient lines intersecting all s-lines (fig. 10).

VILLA EPSILON

VILLA ZETA

VILLA ETA

VILLA EPSILON

VILLA ZETA

VILLA ETA

VILLA EPSILON

VILLA ZETA

VILLA ETA

VILLA EPSILON

VILLA ZETA

VILLA ETA

Fig. 10: Steps in the Process of Ensuring that all S-lines have been Crossed by m/P-Lines

Step 6. While the stages outlined so far may identify the m/p-lines that intersect every s-line, they do not guarantee that all non-trivial circulation loops have been completed. This situation occurs whenever it is possible to circulate completely around a wall or space; typically called an “island”. To check that all non-trivial circulation loops have been completed extend a polygon from the surfaces of an island until it meets another wall, island or m/p-line. If the polygon reaches a wall or island additional m-lines (Step 7) are required. This intuitive procedure provides the same outcome as Peponis *et al.*’s [1997] rigorous definition required for automation. This situation occurs in the Villa Eta where there are two, symmetrical looping circuits, through the space. However, because both polygon’s shaded in the Villa Eta do not meet a surface, the rule suggests that the existing m/p-lines are sufficient to comprehensively represent the spatial configuration (fig. 11).

Fig. 11: Checking for “Islands” in the Three Villas and using the Extended Polygon Method for the Two “Islands” in the Villa Eta

Step 7. Where polygon contact occurs additional m-lines are required. In this instance the goal is to seek the fewest in number and longest in length m-lines (prioritized in that order) that will prevent contact. Where the polygons of multiple islands make contact with other wall-sets, this process is conducted simultaneously for all polygons. The Villa Eta, in the previous step, displayed the simultaneous development of two polygons to test two cases but neither polygon needed additional m-lines.

Step 8. It is unlikely, but theoretically possible, that discrete, separated sets of lines can be generated. If this is the case the fewest and longest lines must be added that can singularly connect any previously unconnected lines. In the example villas all connections are made.

Step 9. In this step a decision is made regarding which p-lines to keep and whether there are any superfluous m-lines that need to be removed. This process involves a number of protocols as follows (listed hierarchically where lower protocols must respect higher ones), with procedures following:

Protocols:

1. The process begins with a consideration of the longest lines and progresses towards the shortest.
2. No line may be deleted if it will result in an associated point no longer being surveilled (see procedure 1).
3. No lines may be removed which will allow an island surface extension polygon to meet a wall-set or island.
4. Any line with connections which are a subset of the connections of another line is a possible candidate for deletion (see procedure 2).
5. If multiple lines make identical connections, the shorter line(s) may be removed.
6. If multiple lines are identical in all respects, for consistency retain the line running Northwest – Southeast (see procedure 3).

Procedures:

1. To check select points associated with a line, (point A is associated with lines 8 and 10, Point B with lines 3, 4, 5, 7 and 9) and draw check lines (c-line) from associated points to their vertices. If a c-line is intersected by only one m-line that m-line must

not be deleted. C-lines could be generated from an all line map, or by drawing all lines (shown in fig 4 and as demonstrated for points A and B in fig. 12).

Fig. 12: Example of Points Associated with M-lines. Lines not Relevant to Point a or b are Shown long-dashed, C-lines Shown Short-dashed

2. Using Turner *et al.*'s [2005] algorithmic process as the basis for line deletion, the connections of a line are determined thus (using Villa Epsilon Line 1 as an example): $C1 = \{1, 2, 5, 6a, 6b, 8, 9, 10\}$ Connections for line 8 in Villa Epsilon are $C8 = \{1, 2, 8, 10\}$ therefore connections of line 8 are a subset of line 1, and is a candidate for possible deletion.

3. If two lines are identical in all of their properties then a decision must be made to remove one. The nature of the decision does not affect the mathematical result, but it can lead to confusion when trying to replicate a method. For this reason the present paper proposes the practice of retaining the line which runs closest to 45° from the upper left corner to the lower right corner (or running Northwest to Southeast if geography is known); the other line(s) is removed. In the case of Villa Epsilon the lines removed are 2, 4 6a & 6b, 7, 9 10 (fig. 13).

Fig 13a

Fig 13b

Fig 13c

Fig 13d

Fig 13e

Fig 13f

Fig 13g

Fig 13h

Fig. 13: The Complete Set of Lines for the Three Villas are Tested and, Following the Protocol, Lines are Removed Until only an Ideal set Remains for Each Plan

Step 10. The plan now contains the set of the fewest and longest lines required to represent the spatial configuration. When the plan is finally removed, the Axial Line map is all that remains, and it is all that is required for the mathematical analysis to begin (fig. 14).

Fig. 14: Axial Line Maps of the Three Villas

Worked Example: Mathematical Analysis

Having arrived at an objective Axial map for each of the Villas, mathematical analysis can then be applied to this map to derive various qualities associated with integration, isolation and control [Ostwald 2011a]. In this worked example, the Villa Epsilon is used to demonstrate the eleven steps that potentially make up the method.

Step 1. As a starting point, the number of lines in the complete system (K), is determined. In the case

Step 1.	K
Number of lines.	4

Step 2. Depth is measured as the number of steps, or intersections, between a starting line and another line. Total Depth (TD), is the number of connections each line makes with all others, referenced to the relative depth of those lines. In the case of the Villa Epsilon, line 1 ‘connects’ to itself at depth 0, to lines 3 and 5 at a depth of 1, and to line 8 at a depth of 2. Similarly, line 3 connects to itself at depth 0, to line 5 at depth 1, to line 1 at depth 2 and line 8 at depth 3. TD is the sum of the number of connections between a particular line, and every other line in the set weighted by level (L). It is calculated by adding together, for each level of the justified graph, the number of nodes (n_x) at that level of depth multiplied by the L (0, 1, 2, 3, 4, ...). Thus:

$$TD = (0 \times n_x) + (1 \times n_x) + (2 \times n_x) + ... (X \times n_x)$$

In the case of the Villa Epsilon, the four lines are evenly divided between two TD values, 4 and 6.

Step 2.	Level (relative depth) of other lines				TD
	0	1	2	3	
Line 1	1	2	1	0	4
Line 3	1	1	1	1	6
Line 5	1	2	1	0	4
Line 8	1	1	1	1	6

Step 3. Calculating the Mean Depth (*MD*) in the Axial Line map is achieved by dividing the total depth of a line by one less than the total number of lines in the system, or:

$$MD = \frac{TD}{(K-1)}$$

For the Villa Epsilon, because the system is evenly divided between two *TD* values, it is also evenly divided between two *MD* values; 1.33 and 2.

Step 3.	<i>TD</i>	K-1	<i>MD</i>
Line 1	4	3	1.333
Line 3	6	3	2
Line 5	4	3	1.333
Line 8	6	3	2

Step 4. The next step involves the calculation of Relative Asymmetry (*RA*); a way of normalizing the range of possible results to between 0.0 and 1.0. This is important because it allows for a direct comparison to be made between different Axial Line maps. The *RA* for the system is calculated as follows:

$$RA = \frac{2(MD-1)}{K-2}$$

Step 4.	<i>MD-1</i>	<i>K-2</i>	<i>RA</i>
Line 1	0.333	2	0.333
Line 3	1	2	1
Line 5	0.333	2	0.333
Line 8	1	2	0.1

Step 5. In this stage the level of integration (i) of each line in the system is calculated. Being a function of relative asymmetry, it can also be used (and is used frequently) to compare different axial maps. The formula for i is as follows:

$$i = \frac{1}{RA}$$

Step 5.	i
Line 1	3
Line 3	1
Line 5	3
Line 8	1

Step 6. Real relative asymmetry (RRA) allows for a comparison of a line to a scaled, idealized benchmark configuration. RRA is calculated as follows:

$$RRA = \frac{RA}{D_k}$$

Where D is a value indexed in Hillier and Hanson [1984; 112]. For a system containing 4 axial lines D can also be calculated using Peponis's and Periklakis's [1985] formula. For the present paper, a D value of 0.357 has been used for the calculations.

Step 6	D_k	RRA
Line 1	0.357	0.933
Line 3	0.357	2.801
Line 5	0.357	0.933
Line 8	0.357	2.801

Step 7. Integration (i) is one of the most important results derived from the mathematical analysis of Axial Line maps. This is because a higher integration value typically correlates with a higher degree of pedestrian traffic. Integration is calculated as follows:

$$i = \frac{1}{RRA}$$

Step 7	<i>i</i>
Line 1	1.072
Line 3	0.357
Line 5	1.072
Line 8	0.357

Step 8. Control value (*CV*) is a measure of the number of axial lines that are accessible only through specific axial lines in the system. In order to calculate *CV*, the first step is to create an intersection matrix, which shows that lines that intersect with each other and calculates the number of connections (*NC n*) for each line. *CVe* values for each line can be calculated with the following formula;

$$CV_e = \frac{1}{NC_n}$$

The *CV* value for a line is produced by adding the *CVe* value(s) of each line it intersects. For example:

Line 1 intersects line 8 (*CVe* 0.5) and line 5 (*CVe* 1) to total *CV*=1.5.

Step 6.	<i>NC n</i>					<i>Cve</i>	<i>CV calc</i>	<i>CV</i>
	Line 1	Line 2	Line 3	Line 4	<i>NCn</i>	0.5	0.5 + 1	1.5
Line 1	0	0	1	1	2	1	1	0.5
Line 3	0	0	1	0	1	0.5	0.5 + 1	1.5
Line 5	1	1	0	0	2	1	1	0.5
Line 8	1	0	0	0	1	0.5	0.5 + 1	1.5

Step 9. The Difference factor (*H*) “quantifies the spread or degree of configurational differentiation among integration values” [Hanson 1998; 30]. *H* is calculated using:

$$H = - \sum \left[\frac{a}{t} \ln \left(\frac{a}{t} \right) \right] + \left[\frac{b}{t} \ln \left(\frac{b}{t} \right) \right] + \left[\frac{c}{t} \ln \left(\frac{c}{t} \right) \right]$$

Where, *a* = Max *RA*, *b* = Mean *RA*, *c* = Min *RA* and *t* = *a* + *b* + *c*

a	2.801
b	1.867
c	0.933
t	5.601

In the case of the Villa Epsilon, $H = 1.0113$

Step 10. The Relative Difference Factor (H^*) normalizes the unrelativized H result into a scale between $\ln 2$ and $\ln 3$ [Zako 2006] and is calculated as follows:

$$H^* = \frac{(H - \ln 2)}{(\ln 3 - \ln 2)}$$

For the Villa Epsilon, $H^* = 0.7846$

Step 11. The complete set of results may then be combined into a single matrix summarizing the most important information about the plan (Tables 1 – 3).

Table 1: Villa Epsilon—results

Line	<i>TD</i>	<i>MD</i>	<i>RA</i>	<i>i</i>	<i>RRA</i>	<i>CV</i>
<i>Line 1</i>	4	1.3333	0.3333	3	0.933	1.5
<i>Line 3</i>	6	2	1	1	2.801	0.5
<i>Line 5</i>	4	1.3333	0.3333	3	0.933	1.5
<i>Line 8</i>	6	2	1	1	2.801	0.5
<i>Minimum</i>	4	1.3333	0.3333	1	2.801	0.5
<i>Mean</i>	5	1.6666	0.6666	2	1.867	1
<i>Maximum</i>	6	2	1	3	0.933	1.5
<i>H</i>	1.0113					
<i>H*</i>	0.7846					

Table 2: Villa Zeta–results

Line	<i>TD</i>	<i>MD</i>	<i>RA</i>	<i>i</i>	<i>RRA</i>	<i>CV</i>
<i>Line 1</i>	9	2.25	0.8333	1.2	2.367	0.5
<i>Line 3</i>	6	1.5	0.3333	3	0.947	1.3333
<i>Line 4</i>	5	1.25	0.1666	6	0.474	2.5
<i>Line 8</i>	8	2	0.6666	1.5	1.894	0.3333
<i>Line 9</i>	8	2	0.6666	1.5	1.894	0.3333
<i>Minimum</i>	5	1.25	0.3333	1.2	0.474	0.3333
<i>Mean</i>	7.2	1.8	0.5333	2.64	1.515	1
<i>Maximum</i>	9	2.25	1.6666	6	2.367	2.5
<i>t</i>					4.356	
<i>H</i>	0.9401					
<i>H*</i>	0.6090					

Table 3: Villa Eta–results

Line	<i>TD</i>	<i>MD</i>	<i>RA</i>	<i>i</i>	<i>RRA</i>	<i>CV</i>
<i>Line 1</i>	5	1.25	0.16667	6	0.474	1.5
<i>Line 3</i>	5	1.25	0.16667	6	0.474	1.5
<i>Line 4</i>	6	1.5	0.33333	3	0.947	0.66667
<i>Line 8</i>	6	1.5	0.33333	3	0.947	0.66667
<i>Line 9</i>	6	1.5	0.33333	3	0.947	0.66667
<i>Minimum</i>	5	1.25	0.16667	3	0.474	0.66667
<i>Mean</i>	5.6	1.4	0.26667	4.2	0.758	1
<i>Maximum</i>	6	1.5	0.33333	6	0.947	1.5
<i>t</i>					2.178	
<i>H</i>	1.0613					
<i>H*</i>	0.9080					

Finally, completing the mathematical analysis allows for the generation of further diagrams coded (by color, shading or line thickness) to indicate specific integration hierarchies or, where there is more extensive data, to identify only certain levels of integration (say results in the top 25%). This visual representation of data can be useful to designers seeking to quickly understand a spatial configuration without reference to the summary matrix.

Fig. 15: Axial Maps Weighted for Integration. Thicker Lines Equal Higher Integration

Conclusion

Traditionally, the focus of Axial Line analysis has been the calculation of integration or conversely segregation values as these have been shown to correlate to a number of social phenomenon. One of the most widely published of these is the correlation between integration and volume of movement. This correlation can be informative at any scale, from building interior to urban fabric, although it can only be used to predict relative trends. In the case of the three villas in the present paper, it is possible to identify, for each villa, the paths through space that most rapidly provide visual cohesion to a plan. That is, the most efficient way for a visitor to develop an understanding of a set of spaces is to follow the path with the highest integration (*i*) values. Alternatively, the line with the highest integration value is also the most efficient path for a security patrol, or docent in a gallery, to follow to visually survey the maximum amount of space in the plan. For the Villa Epsilon, lines 1 and 5 are equal in integration potential, but they are only marginally higher than the values for lines 3 and 8. The Villa Eta results are also relatively undifferentiated. However, for the Villa Zeta, line 4 is the most significant, in terms of integration, in the entire system and that plan displays a wide range of potential results.

Because the example villas are relatively simple, it is possible to see in the Axial Maps how the plans of each differ. For example, the Villa Epsilon is a mostly linear plan where, with one exception, there is only one path that can be taken to experience the majority of the plan. The Villa Zeta, in contrast, has a planning pattern where every room opens from a single, central space and adjacent corridor, and the Villa Eta, has a complex, rhizomorphic plan, with many possible connecting paths. These qualities are readily apparent in a visual examination of the Axial Line maps for each villa and their implications for inhabitation are also reasonably straightforward. For example, a plan organized with a linear hierarchy suggests a significant degree of privacy is provided to inhabitants, with access to the deepest spaces only afforded to a select few. At the other end of the spectrum, a rhizomorphic plan suggests a high degree of adaptability allowing inhabitants to follow multiple paths through the configuration. Beyond the general observations above, the three examples are too limited to develop further conclusions. Indeed, for an analysis of architecture to have sufficient data to be statistically relevant, it must either focus on large buildings like prisons or museums [Hillier and Tzortzi, 2006] or it must consider sets of buildings that are linked in some way (by the same architect, same timeframe, similar location or program).

Finally, the purpose of this paper has not been to offer evidence for the efficacy of space syntax methods in general, or the Axial Line graph in particular. A considerable body of

evidence, albeit some of it contested, already exists. Instead, this paper has collated a body of knowledge relating to one of the key Space Syntax methods and presented a reasoned and replicable version of that method, drawing on a combination of past research and adding several new insights. Original methodological refinements in this paper include the amalgamation of Turner *et al.*'s [2005] line limitation routine into Peponis *et al.*'s [1997] line generation procedure, modified to prioritize line length as a key determinant in drawing sequence as Hillier and Hanson's [1984] original methodology intended. In addition, the introduction of p-lines eliminates a random variable from the map generation procedure and several new strategies provide enhanced graphic consistency.

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