Near-wall similarity between velocity and scalar fluctuations in a turbulent channel flow

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The degree of near-wall similarity between velocity and scalar fields is examined with the use of direct numerical simulation databases for a smooth wall turbulent channel flow with passive scalar transport. Particular attention is given to correlations between velocity and scalar fluctuations as well as those between their derivatives. In the vicinity of the wall, the largest correlation is between the wall-normal vorticity fluctuation and the spanwise scalar derivative, mainly due to the significantly reduced effect of the fluctuating pressure gradient. Other correlations peak at locations where the effect of the pressure fluctuation is smallest. The near-wall similarity between momentum and thermal streaks is not perfect because momentum streaks are often intensified or weakened by the fluctuating pressure gradient. © 2009 American Institute of Physics. [DOI: 10.1063/1.3081555]

I. INTRODUCTION

It is well established that there is a close similarity between $u_1$, $u_2$, and $u_3$ denote the streamwise, wall-normal, and spanwise velocity fluctuations, respectively, and $\theta$ (temperature fluctuation) in the near-wall region of turbulent shear flows as a result of the close analogy between the velocity and scalar fields (viz., the molecular Prandtl number is nearly unity and analogous boundary conditions, i.e., no velocity and temperature fluctuations at the wall, are used) (see, for example, Refs. 1–7). To a large degree, the similarity is reflected in the close correspondence between momentum and thermal streaks.

The near-wall similarity between $u_1$ and $\theta$ leads to a similarity between small-scale velocity and scalar fluctuations since the wall-normal derivatives ($u_{1,2}$ and $\theta_2$) and spanwise derivatives ($u_{1,3}$ and $\theta_3$) are concentrated at the edges of the streaks and their spanwise interfaces, respectively. This latter similarity can be readily studied using direct numerical simulation (DNS) data. DNS data for a turbulent channel flow were used in Ref. 8 to examine the correlation between the fluctuating vorticity vector $\omega_i$ ($i=1,2,3$ represent the streamwise, wall-normal, and spanwise directions, respectively) and the scalar gradient vector $\theta_i(=\partial \theta / \partial x_i)$ ($x_1$, $x_2$, and $x_3$ represent the streamwise, wall-normal, and spanwise directions, respectively; they are used interchangeably with $x$, $y$, and $z$). A close correspondence was found near the wall between different components of $\omega_i$ and $\theta_i$, specifically between $\omega_{1,2}(=-u_{1,2})$ and $\theta_2$, and between $\omega_{2,3}(=-u_{1,3})$ and $\theta_3$, despite the fact, noted by Corrsin\textsuperscript{9}, that $\omega_i$ is solenoidal ($\nabla \cdot \omega_i = 0$) while $\theta_j$ is lamellar ($\nabla \times \theta_j = 0$).

While the near-wall similarity between velocity and temperature fluctuations and also between their spatial derivatives has been examined,\textsuperscript{1–8} there remains some uncertainty regarding the relative magnitudes of the correlations between $u_1$ and $\theta$, $u_{1,2}$ and $\theta_2$, $u_{1,3}$ and $\theta_3$, and $\omega_2$ and $\theta_3$. The major objective of this paper is to clarify this issue, especially in the context of the role played by the pressure fluctuation. The influence of the Reynolds number is also addressed partially. It is hoped that the paper will provide new insight into the near-wall similarity between velocity and scalar fields and lead to the development of more reliable turbulence models for the scalar field (e.g., the development of the two-equation $k-e$ model\textsuperscript{10}).

The paper is organized as follows. The present DNS databases are described briefly in Sec. II. In Sec. III, the quality of the similarity between the velocity and scalar fields is investigated quantitatively. Reasons for a departure from similarity are also considered. In Sec. IV, an attempt is made to quantify the near-wall similarity with the use of Taylor series expansions.

II. DNS DATABASES

The present DNS databases are for a turbulent channel flow with passive scalar transport (Abe \textit{et al.}\textsuperscript{8}). Three values of $h^+ (=u_h \nu = 180$, 395, and 640; $u_+$ is the friction velocity, $\nu$ the kinematic viscosity and $h$ the channel half-width; the superscript denotes normalization by wall variables) are used. The molecular Prandtl number, $Pr(=\nu/\kappa)$, is 0.71 ($\kappa$ is the thermal diffusivity).

The fully developed turbulent channel flow is driven by a constant mean streamwise pressure gradient. A passive scalar (temperature) is introduced by imposing a constant time-averaged heat flux at each wall.\textsuperscript{8} The bulk and wall mean temperature increase linearly in $x_1$. The wall temperature fluctuation is assumed to be zero. In this context, an instantaneous temperature difference $\Theta$ defined by

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\[
T = \frac{\partial(T_w)}{\partial x} x - \Theta
\]

\[T \text{ and } T_w \text{ are the local temperature and wall temperature, respectively; the angular bracket represents integration over } z \text{ and } t \text{ (time) is used when time integrating the energy equation since } \langle T \rangle, \langle T_w \rangle, \text{ and } \langle T_m \rangle \text{ increase linearly in the } x \text{ direction for the present configuration } ((T_m) \text{ denotes the bulk mean temperature). The introduction of } \Theta \text{ enables the application of the nonslip boundary condition in the } y \text{ direction. For the other } (x \text{ and } z) \text{ directions, the periodic boundary condition is imposed. The governing equation for the scalar field can then be expressed as}
\]

\[
\frac{\partial \Theta^+}{\partial t^+} + U_j^+ \frac{\partial \Theta^+}{\partial x_j^+} = \frac{1}{h^+ \Pr} \frac{\partial^2 \Theta^+}{\partial x_j^+ \partial x_j^+} + U_1^+ \left( \frac{2}{\int_0^1 U dy} \right) \frac{\partial \Theta^+}{\partial y^+}
\]

where \(U_i\) is the instantaneous velocity in the \(i\)th direction and the superscript \# denotes the normalization by the channel half-width \(h\) (see also Kasagi et al.\(^4\)). The last term of Eq. (2) plays a role in keeping the time-averaged heat-flux constant across the channel. Note that the present thermal boundary condition differs somewhat from the internal heat source condition used by Kim and Moin.\(^3\) In the latter study, while the thermal boundary condition is zero, the time-averaged heat flux is not constant throughout the channel.

There may be other boundary conditions for the wall temperature fluctuation (e.g., an extremely thin wall) which could lead to an impairment in the near-wall similarity between velocity and scalar fields (see, for example, Refs. 5 and 6). Our choice for a zero wall temperature fluctuation was considered previously by Kasagi et al.\(^11\) who pointed out that the wall temperature fluctuation is extremely small for most materials at \(Pr=0.71\). The numerical methodology together with the validation of some important small-scale turbulence quantities was given by Abe et al.\(^7,8\) and is not repeated here. The domain size \((L_x \times L_y \times L_z)\), number of grid points \((N_x \times N_y \times N_z)\), and spatial resolution \((\Delta x, \Delta y, \Delta z)\) for the DNS databases used are given in Table I.

### III. NEAR-WALL SIMILARITY

The distributions of \(\bar{u}_1 \bar{\theta}/u_1' \bar{\theta}'\) (an overbar and a prime denote averaging in space and time and the root-mean-square value, respectively) are shown in Fig. 1(a). Also included are the data of Kasagi et al.\(^4\) for \(h^+=150\) and Abe et al.\(^1\) at \(h^+=1020\). The largest magnitude of the correlation occurs in the near-wall region, as inferred. However, the peak is not located at the wall but at \(y^+=7\). The Reynolds number is unlikely to affect the similarity between \(u_i\) and \(\theta\) except at \(h^+=180\), where the magnitudes are slightly larger due to the low Reynolds number effects.\(^12\)

There are several correlation coefficients between small-scale velocity and scalar fluctuations whose magnitudes are comparable to \(u_1 \bar{\theta}/u_1' \bar{\theta}'\). In Fig. 1(b), an expanded scale has been used to highlight differences between correlation coefficients corresponding to the products \(\omega_2 \theta_3, -\omega_2 \theta_3, u_{1,2} \theta_2, u_{1,3} \theta_3, \text{ and } u_1 \theta\) at \(h^+=640\). Close to the wall, \(\omega_2 \theta_3/\omega_2 \theta_3'\) remains almost constant and has a slightly larger value than the other four correlation coefficients (at \(y^+=0.15\), \(\omega_2 \theta_3/\omega_2 \theta_3'\) is 0.95, whereas the magnitude of the other four correlations is about 0.90). The difference, which is small, can be readily discerned in the joint probability density functions (pdfs) of Fig. 2. Indeed, the joint pdf between \(\omega_2\) and \(\theta_3\) is fully consistent with the larger correlation between these two quantities and exhibits significantly less spread than the joint pdf between \(u_i\) and \(\theta\). With increasing distance from the wall, the other four correlations exhibit peaks with almost the same magnitudes but at different \(y\) locations (see Table II).

The large magnitudes of near-wall correlation coeffi-

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**TABLE I. Domain size, grid points, and spatial resolution.**

<table>
<thead>
<tr>
<th></th>
<th>180</th>
<th>395</th>
<th>640</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_x \times L_y \times L_z)</td>
<td>12.8h \times 2h \times 6.4h</td>
<td>12.8h \times 2h \times 6.4h</td>
<td>12.8h \times 2h \times 6.4h</td>
</tr>
<tr>
<td>(L_x^+ \times L_y^+ \times L_z^+)</td>
<td>2304 \times 360 \times 1152</td>
<td>5056 \times 790 \times 2528</td>
<td>8192 \times 1280 \times 4096</td>
</tr>
<tr>
<td>(N_x \times N_y \times N_z)</td>
<td>768 \times 128 \times 384</td>
<td>1536 \times 192 \times 768</td>
<td>2048 \times 256 \times 1024</td>
</tr>
<tr>
<td>(\Delta x^+, \Delta y^+, \Delta z^+)</td>
<td>3.00, 0.20–5.93, 3.00</td>
<td>3.29, 0.15–6.52, 3.29</td>
<td>4.00, 0.15–8.02, 4.00</td>
</tr>
</tbody>
</table>

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**FIG. 1.** Distributions of \(\bar{u}_1 \bar{\theta}/u_1' \bar{\theta}'\), \(-\omega_2 \theta_3/\omega_2 \theta_3'\), \(-\omega_2 \theta_3/\omega_2 \theta_3'\), \(u_{1,2} \theta_2\), \(u_{1,3} \theta_3\), and \(u_1 \theta\) for \(h^+=1020\) (Abe et al., 2004), \(h^+=395\), \(h^+=180\), and \(h^+=150\) (Kasagi et al., 1992).
In $u_1$ rather than in $\theta$ [Figs. 3(a) and 3(c)] since contours of $\theta$ do not vary appreciably near the wall [Figs. 3(c) and 3(d)]. Instantaneous fields in the $y$-$z$ plane (not shown here) indicate that the coincidence between $u_1$ and $\theta$ is most likely to occur near the location corresponding to the centroid of the momentum streaks (at $y^*=7$). Below this location, departures from similarity between $u_1$ and $\theta$ appear where $\partial p/\partial x_1$ ($p$ is the pressure fluctuation) is large [see Figs. 3(a), 3(c), and 3(e)]. The same trend was reported by Kong et al. [13] in a perturbed turbulent thermal boundary layer downstream of a spanwise slot at $y^*=5$. Hence, $\partial p/\partial x_1$ should be associated with the dissimilarity in the vicinity of the wall.

The reason for this is the difference between the governing equations for $u_1$ and $\theta$, viz.,

$$\frac{\partial u_1}{\partial t} + u_j \frac{\partial u_1}{\partial x_j} + \bar{u}_j \frac{\partial u_1}{\partial x_j} + u_j \frac{\partial u_1}{\partial x_j} = -\frac{\partial p}{\partial x_1} + \nu \frac{\partial^2 u_1}{\partial x_j^2} + \frac{\partial}{\partial x_j} u_k u_j,$$

(3)

$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} + \bar{u}_j \frac{\partial \theta}{\partial x_j} + u_j \frac{\partial \theta}{\partial x_j} = \kappa \frac{\partial^2 \theta}{\partial x_j^2} + \frac{\partial}{\partial x_j} \theta_{ij},$$

(4)

[note that a term associated with the last term of Eq. (2) is omitted in Eq. (4) since the magnitude is small]. With the assumption that Pr is 1, the major difference between Eqs. (3) and (4) is the appearance of the pressure gradient term in Eq. (3). This difference should be important in the context of the similarity between $u_1$ and $\theta$ close to the wall where the magnitude of $\partial p/\partial x_1$ is much larger than that of $u_1$ (see Antonia and Kim [12]) and is hence large enough to affect the distribution of $u_1$.

To clarify this, an enlarged view of Fig. 3 (at $y^*=0.15$) is shown in Fig. 4, where contours of $u_1$ and $\partial p/\partial x_1$ are included. As inferred from Eq. (3), the positive and negative magnitudes of $u_1$ are amplified when $\partial p/\partial x_1<0$ and $\partial p/\partial x_1>0$, respectively (viz., $u_1$ is negatively correlated with $\partial p/\partial x_1$). They are however decreased when $\partial p/\partial x_1>0$ and $\partial p/\partial x_1<0$, respectively. Consequently, in the vicinity of the wall, unlike thermal streaks, momentum streaks are often intensified or weakened (sometimes broken up) by $\partial p/\partial x_1$ [see Fig. 4(a)], resulting in a weakened similarity between $u_1$ and $\theta$ [see Figs. 4(a) and 4(b)]. The negative correlation between $u_1$ and $\partial p/\partial x_1$ can also be discerned in the correlation coefficient for $y^*<2$ (see Fig. 5). Note that while the

![FIG. 2. Joint pdfs for $h^*=640$ at $y^*=0.15$: (a) $u_1$ and $\theta$, (b) $\omega_x$ and $\theta$. The increment for the line contours is 0.1.](image)

TABLE II. Peak locations and values of the correlation coefficients in Fig. 1(b). The comparison is shown for the three Reynolds numbers. Note that $\omega_x^2/\omega_x^2 \theta_j = \gamma$ is nearly constant from $y^*=0$ to about 7 so that the peak location is not given here. In view of the spatial resolution along $y^*$, the uncertainty in the peak $y^*$ location is within $\pm 1$ in wall unit.

<table>
<thead>
<tr>
<th>$h^*$</th>
<th>Peak $y^*$ location</th>
<th>Peak value</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>180</td>
<td>395</td>
</tr>
<tr>
<td>$u_1 \theta^2 / u_1 \theta^2$</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$u_x \theta_j / u_1 \theta_j$</td>
<td>3</td>
<td>3</td>
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<tr>
<td>$\omega_x \theta_j / \omega_x \theta_j$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$u_1 \theta_j / u_1 \theta_j$</td>
<td>5</td>
<td>5</td>
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<td>$\omega_x \theta_j / \omega_x \theta_j$</td>
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<tr>
<td></td>
<td>180</td>
<td>395</td>
</tr>
<tr>
<td>$u_1 \theta^2 / u_1 \theta^2$</td>
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<td>$u_1 \theta_j / u_1 \theta_j$</td>
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</tr>
<tr>
<td>$\omega_x \theta_j / \omega_x \theta_j$</td>
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</table>
positive correlation appears at $y^+ > 2$ (see Fig. 5) due to the reduction in regions of $u_1$ intensified or weakened by $\partial p/\partial x_3$, the latter regions are detectable below the centroid of the momentum streaks (at $y^+ < 7$). Contours of $\partial p/\partial x_3$ are nearly in perfect alignment with those of $u_3$ [Fig. 4(c)], in contrast to the nonalignment between the contours of $\partial p/\partial x_1$ and those of $u_1$ [Fig. 4(a)] (see also the correlation coefficients in Fig. 5). The same trend was reported by Kim\textsuperscript{14} for the alignments between $\partial p/\partial x_1$ and $\partial \omega_2/\partial x_2$ and between $\partial p/\partial x_2$ and $\partial \omega_1/\partial x_1$ at the wall. Concentrations of $\partial p/\partial x_1$ and $\partial p/\partial x_3$ sometimes overlap [Figs. 4(a) and 4(c)]. Given that contours of $\partial p/\partial x_1$ and $\partial p/\partial x_3$ do not change appreciably near the wall (see Fig. 3) and occur when $Q(=u_i u_j/2)$ is large and positive (the distributions of $Q$, or second invariant of the velocity gradient tensor, are not shown here), the effect of $\partial p/\partial x_3$ on $u_1$ is likely to reflect the presence of the near-wall vortical motions. This observation is related to the finding of Kravchenko et al.\textsuperscript{15} that high skin-friction regions are associated with the strong quasistreamwise vortices above the wall.

With increasing Reynolds number, such as near-wall vortical structures,\textsuperscript{16-18} the regions where the similarity between $u_1$ and $\theta$ close to the wall is relatively poor, tend to become dense and clustered (see, for example, Fig. 3 at $x^+=300–1100$ and $z^+=800–1000$). The latter regions are also associated with clusters of the spanwise wall shear-stress fluctuation (Abe et al.\textsuperscript{19}) since $u_3 = \partial p/\partial x_3$ close to the wall (see also $u_3 p_{3,3}'/u_1' p_3'$ in Fig. 5).

One may wonder if the largest magnitude of $\omega_2 \theta_3/\omega_3 \theta_2'$ in Fig. 1(b) can be discerned in the instantaneous field. Indeed, $\omega_2$, which contains both $u_1$ and $u_3$ components, is unlikely to be affected by either $\partial p/\partial x_1$ or $\partial p/\partial x_3$ close to the wall (see Fig. 6). Consequently, the correspondence between $\omega_2$ and $\theta_3$ (Fig. 6) is nearly perfect and superior to that between $u_1$ and $\theta$ (Fig. 4); the correlation coefficient (0.95) for Fig. 6 is larger than that (0.88) for Fig. 4 [the correlation coefficients are calculated from the instantaneous ($x$-$z$) plane data].

We should also comment on the peak values in the correlation coefficients listed in Table II. The peak locations correspond to the $y$ locations where the correlation coefficient between the pressure fluctuation and either the corresponding velocity, velocity derivative, or vorticity component has a local minimum (see, for example, Fig. 7 where $u_i p_i/u_1' p_1'$ and $u_{1,2} p_i/u_{1,2}' p_1'$ are illustrated). Like $u_i p_i/u_1' p_1'$, their magnitudes become larger as the wall is...
IV. TAYLOR SERIES EXPANSIONS

Some insight into the near-wall similarity between velocity and scalar fields can be gained by writing Taylor series expansions for $u_1$ and $\theta$ in the vicinity of the wall,

$$u_1^* = b_1 x_2^* + c_1 x_2^{*2} + O(x_2^{*3}),$$  \(5\)

$$\theta^* = b_\theta x_2^* + O(x_2^{*3}),$$  \(6\)

where

$$c_1 x_2^{*2} = (1/2)(\partial^2 u_1^*/\partial x_2^* \partial x_2^*) x_2^{*2} = (1/2)(\partial \theta^*/\partial x_1^*) x_2^{*2}. $$  \(7\)

The term $c_1 x_2^{*2}$ (viz., $\partial \rho^*/\partial x_1$) is indeed responsible for the dissimilarity between $u_1$ and $\theta$ close to the wall. One would therefore expect a better correlation if the effect of $\partial \rho^*/\partial x_1$ on $u_1$ is removed.

On the other hand, the Taylor series expansion of $u_3$ near the wall is

$$u_3^* = b_3 x_2^{*2} + c_3 x_2^{*2} + O(x_2^{*3}),$$  \(8\)

where

$$c_3 x_2^{*2} = (1/2)(\partial^2 u_3^*/\partial x_2^* \partial x_2^*) x_2^{*2} = (1/2)(\partial \rho^*/\partial x_3) x_2^{*2}. $$  \(9\)

Given that the relationship between $\partial \rho^*/\partial x_1$ and $\partial \rho^*/\partial x_3$ is such that instantaneous contours for these two quantities sometimes overlap [see, for example, Figs. 4(a) and 4(c) at $x^*=850$ and $z^*=750$, the term $c_3 x_2^{*2}$ (viz., $\partial \rho^*/\partial x_3$) may be effective in the context of attenuating, if not canceling, the effect of $\partial \rho^*/\partial x_1$ on $u_1$.

In this context, $\omega_2$ contains derivatives of both $u_1$ and $u_3$ and its Taylor series expansion near the wall is written as

\begin{align*}
\theta^* &= b_\theta x_2^* + O(x_2^{*3}), \\
\omega_2^* &= b_{\omega_2} x_2^* + c_{\omega_2} x_2^{*2} + O(x_2^{*3}),
\end{align*}

where $c_{\omega_2} x_2^{*2} = (1/2)(\partial^2 \omega_2^*/\partial x_2^* \partial x_2^*) x_2^{*2}$.
where the $O(x^2)$ term, which is related to the pressure gradient, is absent. The instantaneous fields also indicate that close to the wall, $\omega_2$ is unlikely to be affected by the pressure gradient (see Fig. 6).

The final expressions for the near-wall correlation coefficients obtained with the use of the Taylor series expansions close to the wall are

$$\frac{\omega_2 \theta_3}{u_1 \theta_1} = (b_{1,3} - b_{3,1})x^+_2 + O(x^{2+}),$$

$$\frac{\omega_2 \theta_2}{u_1 \theta_1} = - \frac{1}{\sqrt{b_1 b_3}} + \frac{2}{\sqrt{b_1 b_3}} \left( c_{1,0} - \frac{b_{1,0} b_{3,0}}{b_1 b_3} \right) x^+_2 + O(x^{2+}),$$

where in the limit of $y^+ \to 0$, $\omega_2 \theta_3 / u_1 \theta_1$ and $u_1 \theta_1 / u_1 \theta_1$ are identical. Unlike $\omega_2 \theta_2 / u_1 \theta_1$ and $u_1 \theta_1 / u_1 \theta_1$, $\omega_2 \theta_3 / u_1 \theta_1$ does not have the term of $O(x^2)$ [see Eq. (11)], resulting in the plateau seen in the distribution. Its first term is also affected by $\partial u_1 / \partial x_1$, which causes the slight increase in the magnitude. This is why, in the vicinity of the wall, the correlation coefficient between $\omega_2$ and $\theta_3$ has the largest magnitude of all the correlations shown in Fig. 1(b). The present findings imply that the effect of the fluctuating pressure gradient cannot be discounted when assessing the similarity between the velocity and scalar fields close to the wall.

V. CONCLUSIONS

The degree of similarity between velocity and scalar fields and their corresponding derivatives near the wall has been quantified via correlation coefficients, joint pdfs, Taylor series expansions, and instantaneous fields using DNS data for a fully developed turbulent channel flow with passive scalar transport at three values of $h^*$ (180, 395, and 640).

Near the wall, the correlation between $\omega_2$ and $\theta_3$ is larger than that between $u_1$ and $\theta_1$ mainly due to the reduced effect of the fluctuating pressure gradient on $\omega_2$. Joint pdfs and Taylor series expansions are fully consistent with this finding. Instantaneous fields indicate that the impaired similarity between $u_1$ and $\theta_1$ occurs in regions where the magnitude of $\partial p / \partial x_1$ is large due to the presence of near-wall vertical motions. In the latter regions, unlike thermal streaks, momentum streaks are often intensified or weakened by $\partial p / \partial x_1$. While the Reynolds number is unlikely to affect the magnitudes of the correlation coefficients, instantaneous fields associated with the relatively poor similarity between $u_1$ and $\theta_1$ become more clustered as the Reynolds number increases. A firm conclusion regarding the $h^*$ dependence cannot however be made due to the small range of $h^*$ that is currently available.

The largest magnitudes of the correlations between $u_1$ and $\theta_1$, $u_{1,2}$ and $\theta_2$, $u_{1,3}$ and $\theta_3$, and $\omega_3$ and $\theta_2$ are at locations where either $u_1$, $u_{1,2}$, $u_{1,3}$, or $\omega_3$ is least correlated with $p$. Clearly, the effect of the pressure fluctuation cannot be discounted when considering any correlation involving velocity or velocity derivative fluctuations in the near-wall region.

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