A STUDY OF TWO-WAY BENDING IN UNREINFORCED MASONRY

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.
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# Table of Contents

Acknowledgements ........................................................................................................ III

Abstract ..................................................................................................................... VII

Publications ............................................................................................................... IX

List of Symbols ........................................................................................................ XI

Chapter 1: Introduction ............................................................................................. 1
  1.1 Background ........................................................................................................... 2
  1.2 Masonry Walls Subjected to One- and Two-Way Bending ................................. 4
  1.3 Aim of This Work ............................................................................................... 8
  1.4 Structure of the Thesis....................................................................................... 9

Chapter 2: Literature Review ................................................................................... 12
  2.1 Introduction .......................................................................................................... 13
  2.2 Major Achievements before the Early 1980s ....................................................... 13
  2.3 Major Experimental and Theoretical Achievements since the Early 1980s ... 15
  2.4 Major Achievements in Numerical Modelling ..................................................... 28

Chapter 3: Experimental Testing Program ............................................................... 33
  3.1 Introduction .......................................................................................................... 34
  3.2 Test Specimens .................................................................................................. 35
3.3 The Test Rig and Calibration .................................................................36
3.4 Four Brick Unit Specimens Subjected to Vertical Bending .................41
3.5 Four Brick Unit Specimens Subjected to Horizontal Bending ..........49
3.6 Four Brick Unit Specimens Subjected to Biaxial Bending .................53
3.7 Bond Tensile Tests ...............................................................................61
  3.7.1 Bond Wrench Test ..........................................................................61
  3.7.2 Direct Tension Test ........................................................................63
3.8 Compression Test ..................................................................................68
3.9 Brick Modulus of Rupture Test .............................................................71
3.10 Direct Shear and Torsion Shear Tests ................................................73
3.11 Summary .............................................................................................85

CHAPTER 4: DETAILED FINITE ELEMENT MICRO-MODEL ..................... 87

4.1 Introduction ..........................................................................................88
4.2 Micro Modelling Versus Macro Modelling .........................................89
4.3 Modelling Strategy ...............................................................................92
4.4 Parameter Selection .............................................................................103
4.5 Mesh Sensitivity Study ........................................................................110
4.6 Modelling of Four Brick Unit Specimens Subjected to Vertical Bending112
4.7 Modelling of Four Brick Unit Specimens Subjected to Horizontal Bending116
4.8 Modelling of Four Brick Unit Specimens Subjected to Biaxial Bending122
4.9 Numerical Evaluation of the Torsion Shear Test ...............................138
4.10 Summary ...............................................................................................154
CHAPTER 5: MODELLING OF WALL PANELS: SIMPLIFIED FINITE ELEMENT MICRO-MODEL ................................................................. 155

5.1 Introduction ........................................................................................................................................................................... 156

5.2 The Simplified Micro-Model ................................................................................................................................................. 156

5.3 Modelling of Small Walls Subjected to Horizontal Bending ................................................................................................. 158

5.4 Modelling of Wall Panels Subjected to In-Plane Force ........................................................................................................... 170

5.5 Application to Two-Way Bending in Walls .............................................................................................................................. 181

5.6 Summary .................................................................................................................................................................................... 198

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS ....................... 199

6.1 Conclusions ................................................................................................................................................................................. 200

6.2 Recommendations for Future Research ................................................................................................................................ 204

REFERENCES ................................................................................................................................................................................. 206

APPENDIX A: BIAXIAL BENDING TEST RESULTS FOR FOUR BRICK UNIT SPECIMENS ................................................................. 223
ABSTRACT

Masonry walls will almost invariably be required to resist lateral out-of-plane loads due to the action of wind or earthquakes; less commonly walls may be subjected to water or earth pressure or blast loading. Of particular interest is the common case which arises when the walls are supported on two or more adjacent edges. Under these conditions the masonry is subjected to a complex state of biaxial (two-way) out-of-plane bending combined with vertical in-plane compression due to the self weight of the wall and any superimposed loads. Different approaches currently exist for the design of masonry wall panels subjected to out-of-plane loads. However, these approaches are all empirical and often yield widely varying design recommendations and there has been significant criticism by proponents of the different methods regarding the use of alternative approaches.

In this study an extensive program of laboratory testing in parallel with numerical analysis was conducted to examine the bending, biaxial bending in particular, behaviour of masonry walls. The aim was to provide a better understanding of the behaviour at the fundamental level towards ultimately developing a fully rational biaxial-bending failure model that can predict behaviour under any simultaneous combination of bending moments in the two principal directions, along with a superimposed compression force on the bed joints.

Experimentally, “single joint” four brick unit specimens were studied comprehensively, using a newly commissioned test rig, by subjecting them to various vertical and horizontal bending moments both separately and in combinations, along with a superimposed compression force on the bed joints. These tests provided important information about the flexural behaviour of mortar joints and the torsional behaviour of bed joints. In addition, a complete set of characterization tests was also performed to study the fundamental material properties of masonry, which were important input parameters in the numerical modelling.

Numerically, a 3D non-linear finite element micro-model with cohesive contact was proposed and implemented in the ABAQUS/Standard software package. Numerical
analyses were performed to provide rational explanations to the bending behaviours observed in the four brick unit specimen tests and evaluate a newly proposed torsion shear test method. A simplified 3D non-linear finite element micro-model was also proposed to simulate the bending behaviour of small walls. Its effectiveness was clearly demonstrated in its application to masonry walls, with or without openings, subjected to both in-plane and out-of-plane loads.
PUBLICATIONS


**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>The area of brick surface (or bed joint area)</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesive shear strength</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Initial cohesive shear strength</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic modulus of masonry</td>
</tr>
<tr>
<td>$E_b$</td>
<td>Elastic modulus of brick</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Elastic modulus of mortar</td>
</tr>
<tr>
<td>$f_{mt}$</td>
<td>Flexural tensile strength of mortar joint</td>
</tr>
<tr>
<td>$f_t$</td>
<td>Tensile bond strength</td>
</tr>
<tr>
<td>$f_{ut}$</td>
<td>Brick’s modulus of rupture</td>
</tr>
<tr>
<td>$F_h$</td>
<td>Ultimate extreme fibre stresses in the masonry in the horizontal direction</td>
</tr>
<tr>
<td>$F_v$</td>
<td>Ultimate extreme fibre stresses in the masonry in the vertical direction</td>
</tr>
<tr>
<td>$F_f$</td>
<td>Slip tolerance in the contact model</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus of joint</td>
</tr>
<tr>
<td>$G'_I$</td>
<td>The mode I fracture energy</td>
</tr>
<tr>
<td>$G''_I$</td>
<td>The mode II fracture energy</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>Maximum overclosure distance in the contact</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Polar moment of inertia of the circular cross section</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Characteristic contact surface length</td>
</tr>
<tr>
<td>$M_h$</td>
<td>Bending moment around vertical axis</td>
</tr>
<tr>
<td>$M_{hp}$</td>
<td>Peak bending moment around vertical axis</td>
</tr>
<tr>
<td>$M_{hr}$</td>
<td>Residual moment around vertical axis</td>
</tr>
<tr>
<td>$M_v$</td>
<td>Bending moment around horizontal axis</td>
</tr>
<tr>
<td>$M_{vp}$</td>
<td>Peak bending moment around horizontal axis</td>
</tr>
<tr>
<td>$M_{vr}$</td>
<td>Residual bending moment around horizontal axis</td>
</tr>
<tr>
<td>$p_{max}$</td>
<td>Maximum allowable tensile stress across the contacted surfaces</td>
</tr>
<tr>
<td>$P_v$</td>
<td>Vertical pre-compressive force on the four brick unit specimens</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Inner radius of the cylindrical specimen</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Outer radius of the cylindrical specimen</td>
</tr>
<tr>
<td>$t$</td>
<td>The brick thickness</td>
</tr>
<tr>
<td>$T$</td>
<td>The measured torque</td>
</tr>
</tbody>
</table>
νₜ  
Brick Poisson’s ratio
νₘ  
Mortar Poisson’s ratio

w  
The crack width

W  
Weight of brick

Z  
Section modulus of the bed joint

σ, σₙ  
Compressive stress normal to the bed joint

σₚ  
Mean flexural strength of the bed joint of the four brick unit specimen

τ  
Joint shear stress

τ₁, τ₂  
Shear stress component in one of the two perpendicular directions

τₚ  
Critical shear stress in the contact model

τresidual  
Residual shear stress

τₚ  
Peak torsional shear stress

γ₁, γ₂  
Slip component in one of the two perpendicular directions

γ₁ᵖ, γ₂ᵖ  
Elastic predictor strain in one of two perpendicular directions

γₚ  
The allowable elastic slip in the contact model

γeqsl  
Resultant slip

γ₁ᵉˡ  
Elastic tangential slip at the beginning of the increment

γ₁ᵉˡ  
Elastic tangential slip at the end of the increment

γ₁ᵖ  
Elastic predictor strain

µ  
Shear friction coefficient

µᵢ  
Internal shear friction coefficient

µᵣ  
Residual shear friction coefficient

φ  
Angle of linear softening of shear stress

Δγᵢ  
Slip increment
CHAPTER 1
INTRODUCTION
1. **INTRODUCTION**

1.1 **Background**

Masonry has traditionally been the main form of construction in Australia and many parts of the world. It still remains popular due to the excellent fire resistance, insulation and acoustic properties of masonry materials; other building materials cannot quite match up without adding extraneous compounds. Compared to other forms of building materials the durability and ease of service of masonry structures results in lower maintenance costs over the life of the structure as well as a longer life span for the structure.

Nevertheless, being a composite material of brick and mortar, masonry has its own weakness. Although masonry walls have high compressive strength, their brittle and sudden failure under tensile loading makes them vulnerable when subjected to bending load such as wind or earthquake load. For example, on 28 December 1989 a Richter scale 5.6 earthquake struck the city of Newcastle in Australia and caused loss of 13 lives and injuries to more than 100 people. 50,000 buildings were damaged to various degrees, leaving behind a repair bill totalling more than A$1 billion. Most of the damage occurred in unreinforced masonry (URM) buildings built between 1900s and 1950s (Figure 1.1). Elsewhere, a Richter scale 6.2 earthquake on 23 December 1972 killed 5,000 people in Managua, Nicaragua due to collapse of masonry walls.

Despite the catastrophic failure nature exhibited by masonry constructions when they are subjected to severe out-of-plane loads, in countries like Australia where seismic activity historically has been considered low, masonry remains as a major construction material due to its economic benefits. Furthermore, it has been shown that if designed and constructed to appropriate loading and design codes, URM structures should be able to withstand moderate earthquake and wind loads (Page, 1992 and 1995; Scrivener, 1993; Potter, 1994).
Researchers have over the years shown great interest in studying the behaviour of masonry walls subjected to lateral loading such as earthquake and wind loads. Experimental tests have been attempted to cover every likely scenario for support condition, material variety, bond strength and various wall dimensions. As a result of independent research in various parts of the world, several different approaches currently exist for the design of masonry wall panels subjected to lateral out-of-plane loads. These various approaches often yield widely varying design recommendations and there has been significant criticism by proponents of the different methods regarding the use of alternative approaches.

In countries such as the UK, Canada and Australia, design of masonry walls subjected to out-of-plane loads is detailed in their respective design codes. These codes aim to give designers guidance in designing masonry walls subjected to bending. However, the design approaches behind these standards vary considerably and all of them are empirical, either totally or partly. For instance, the current Australian design guidelines, The Australian Masonry Code, AS 3700-2001, are based on the virtual work method (Lawrence and Marshall, 1998) for the design and analysis of URM walls subjected to out-of-plane loading such as wind or earthquake. Although it is one of the most sophisticated design approaches available thus far, it is still partly empirical in that the
diagonal moment capacity is calculated empirically. Furthermore, this empirical method cannot be confidently applied to other scenarios that are not covered by the laboratory tests against which it has been verified. This is certainly not the appropriate design approach in view of safety and reliability issues.

Lawrence and Marshall (2000) have noted that “no completely rational method has yet been developed” and that “further research is necessary to develop a fully rational biaxial bending failure model that can predict behaviour under any simultaneous combination of bending moments in the two principal directions, along with a superimposed compression force along the bed joints”. Therefore, there is a strong and urgent need for studying the behaviour of masonry walls subjected to out-of-plane loads in order to derive a complete rational design method.

1.2 Masonry Walls Subjected to One- and Two-Way Bending

During the past several decades research has been conducted into the behaviour of masonry wall panels subjected to out-of-plane lateral loads. Such loads arise due to the effects of wind and earthquakes as well as earth and water pressure or blast loading. Of particular interest is the complex state of biaxial bending action which arises when the walls are supported on two or more adjacent edges.

Situations where lateral loads play a critical role in masonry walls can be found in:

- A single occupancy house. Masonry walls are usually supported by a timber or steel frame or supported by another leaf of masonry wall.
- Multiple-occupancy domestic units. The design of the walls in upper storeys is often governed by the resistance to out-of-plane loading.
- Low-rise commercial and industrial buildings. The walls must have sufficient flexural resistance to span between concrete or steel frame members.
- Multi-storey framed structures. The cladding walls provide the envelope to protect the interior against weather and are only required to resist lateral out-of-plane wind and earthquake forces.

The difficulty in designing masonry structures lies in the fact that masonry is a composite material consisting of masonry units set in a mortar matrix. It is anisotropic, inhomogeneous and subjected to high variability in terms of its material properties and
workmanship. Therefore, its complex structural behaviour is often poorly understood. Nevertheless, past studies have found that the behaviour of a masonry wall when subjected to lateral loads is closely related to factors such as the bond between mortar and bricks (Baker, 1972a; Sinha and Hendry, 1975; West et al., 1977), support conditions (Lawrence, 1983) and geometry of the wall (Lawrence, 1983; Gairns and Scrivener, 1985; Lovegrove, 1986).

Depending on how the wall is supported, the bending of masonry walls subjected to out-of-plane loads can be classified into three categories:

- One-way vertical bending, where a wall is supported at its upper and lower edges (Figure 1.2a).
- One-way horizontal bending, where a wall is supported at its vertical edges (Figure 1.2b).
- Two-way bending, where a wall is supported on two or more adjacent edges (Figures 1.2c and 1.2d).

Figure 1.2. Various bending conditions of a masonry wall.
When a wall is under one-way vertical bending, the wall panel acts as a simple beam between the upper and lower edges, with the main flexural stresses acting across the bed joints. Resistance to this bending is only provided by the flexural tensile strength of the masonry. The superimposed vertical load on the wall can enhance the flexural strength by relieving the tensile stress due to bending.

When a strip of wall over a window or door opening is subjected to an out-of-plane load such as wind load, horizontal bending occurs if the top of the wall is not supported. How the wall behaves in this kind of bending is influenced by factors such as the flexural strength of brick and the bond strength of brick and mortar in the perpend joints and torsional strength on bed joints.

In most cases walls are supported on at least two adjacent edges. In these cases two-way (or biaxial) bending arises. The failure mechanism of walls under biaxial bending is quite complex since under this bending condition the distribution of bending moment across the wall panel varies. Typical experimentally observed failure patterns for a variety of geometries and support conditions are shown in Figure 1.3 (Lawrence, 1983). Unlike homogeneous materials such as concrete the non-homogeneity and anisotropic characteristics of masonry make prediction of load capacity even harder.

Figure 1.3. Examples of wall crack patterns (Lawrence, 1983).
Over the past few decades researchers have been trying to study the biaxial bending failure mechanism by simulating the full wall behaviour in the laboratory, i.e., the macro approach. Such full-scale tests are expensive and time consuming and test results can be very likely influenced by secondary effects such as flexibility of the support frame and induced in-plane forces. And what is more, limited control tests to determine basic material properties have been reported in conjunction with these full wall tests. As a result, it is often difficult to make comparisons between test data and analytical predictions.

In contrast to the macro approach that has been used by most researchers so far to study the biaxial bending behaviour of masonry walls, in this project the micro approach has been adopted. Our aim was to achieve an understanding of wall behaviour at a fundamental level. To be more specific, instead of examining the response of full walls to lateral loading, this study has focussed on “single joint” four brick unit specimens, as illustrated in Figure 1.4. The author believes that comprehension of this single joint response will play a critical role in understanding the full wall behaviour. As Baker (1979) noted, this “single joint” specimen approximately, if it is assumed that the moment transmitted through the perpend joint is the same as in the perpends of a full wall, represents the actions on the joints in a wall panel when subjected to two-way bending. Therefore, it is believed that systematic studies based on “single joint” four brick unit specimens are warranted.

Figure 1.4. The four brick unit as used in the current study.
1.3 Aim of This Work

Baker (1979) studied four brick unit specimens by subjecting them to both vertical and horizontal bending as well as vertically imposed compression. By testing 250 specimens using different combinations of the horizontal to vertical bending moment ratio as well as different compression levels, Baker established an elliptical failure criterion to describe the interaction between the principal failure stresses in the vertical and horizontal directions. This failure criterion itself is debatable in that as an anisotropic material, the out-of-plane behaviour of masonry cannot be completely described only in terms of principal moments (Lourenço, 1996). Nevertheless, Baker’s approach to isolate the single joints through the study of four unit specimens under different bending conditions certainly provides a starting point towards understanding the full wall behaviour.

Previous research in Australia led to the development, initially by Baker and later by the CSIRO, of an experimental testing rig for simulating biaxial bending in masonry. The rig is capable of subjecting a section of a masonry wall panel to simultaneous horizontal and vertical bending moments as well as vertical pre-compression. It allows attention to be focused on the behaviour of individual bed and perpend joints when subjected to the simultaneous combination of bending and torsion. Furthermore, the apparatus can be used to measure and record both the pre-peak and post-peak joint behaviour, the latter being considered to be also important to the overall masonry response (Lourenço, 1996). To the best of the author’s knowledge the post-peak behaviour has never been explored before with four brick unit specimens and is therefore worthy of further investigation.

Note that prior to this work the test rig mentioned above had never been commissioned. The change in strategic research directions within the CSIRO saw the transfer of this rig to the University of Newcastle. This provided both convenience and the opportunity to study the single joint specimens through experiment, in parallel with numerical analyses.

The ultimate aim of this work is to study the biaxial bending behaviour of masonry at the fundamental level in order to establish the failure criteria under such loading
conditions. The work forms an integral part of an overall aim, that is, a comprehensive understanding of the biaxial bending behaviour of masonry leading to the development of more rational methods for the design of masonry walls subjected to lateral out-of-plane loading and consistency in the way the design of masonry walls is approached. The work has involved both experimental and analytical investigations of biaxial bending behaviour in masonry walls, as occurs under the action of wind and earthquake loads. A comprehensive experimental program has been conducted along with associated numerical modelling in order to duplicate and understand numerically the observed experimental response for small samples of masonry.

More specifically, this work aimed to achieve the following:

- Commissioning of the new test rig.
- Developing a suitable cohesive contact model (micro-model) for use in the ABAQUS/Standard finite element software.
- Subjecting, on the new test rig, specimens of four brick units to combined bending in the vertical and horizontal directions as well as vertical compression in order to simulate a complex combination of bending and torsion acting on the mortar joints between the brick units. Such a loading scenario occurs on the mortar joints of full size wall panels when subjected to out-of-plane loads.
- Obtaining, through laboratory tests, critical parameters required for numerical modelling.
- Conducting numerical modelling using ABAQUS/Standard to duplicate and understand the behaviour observed in the experiments.
- Studying in detail of the post peak residual load behaviour through both experiment and modelling.
- Applying a simplified micro-model to simulate small walls and wall panels that have been studied experimentally in the literature.

1.4 Structure of the Thesis

Chapter 2 provides a literature overview on the progress and achievements made in the area of masonry walls when subjected to bending (vertical, horizontal and biaxial). The focus will be on work performed since early 1980s, since extensive reviews of earlier work are already available in the literature. The current review will be broadly grouped
into experimental and theoretical studies (large wall panels as well as small specimens) and numerical modelling (macro and micro modelling).

Chapter 3 focuses on the experimental program conducted during this research. The special laboratory test rig uniquely designed for biaxial bending tests will be described. Test results from the four brick unit specimens subjected to different bending loads as well as vertical compression load will be presented and discussed. A list of control tests including bond wrench test, direct tension test, prism compression test, brick modulus of rupture test, triplet shear test and couplet torsion test will be described and associated results from each of these test methods will be discussed. Special attention will be given to innovative test methods such as the direct tension test and torsional shear test. Critical parameters required for numerical modelling were obtained using these tests.

Chapter 4 reports on the numerical modelling results from this research. The various numerical modelling approaches used in the literature will be summarized and compared. Justification will be provided on the selection of the micro modelling approach used in this work to simulate the behaviour of four brick unit specimens subjected to biaxial bending. Details of the constitutive principal behind the finite element surface contact model will be presented and its implementation in the ABAQUS/Standard finite element software package explained in detail. Also in this chapter, the performance of the numerical model will be examined by way of two applications. Firstly, the model was used to simulate bending tests of four brick unit specimens subjected to separate as well as simultaneous vertical and horizontal bending, each under pre-compression loading. The numerical simulation was compared with the test results described in Chapter 3. The important parameters used in the model and their impact on the numerical results were investigated. It should be noted that the material parameters used as input to the finite element models were not obtained by fitting the model response to the experimentally observed results of the four unit specimen tests. Rather, the separate control tests in compression, tension and shear were used to characterize the masonry material. These material parameters were then adopted to simulate the four unit specimen tests and comparisons with the experimental results were made. Secondly, the model was used to validate a newly developed torsion shear test method. The simulation results revealed that this new test method would provide
more even stress distributions across the mortar joint and thus more accurate measurement of localized shear stress than existing mortar joint shear test methods.

Chapter 5 describes a version of the simplified micro-model, that is, a model based on the micro-model as described in Chapter 4 but neglecting the mortar thickness. Although the detailed micro-model is applicable to small samples such as four brick unit specimens, it is impractical for applications to large-scale wall panels due to its excessive requirements for computational power. The modified model was examined by applying it to three types of wall panels:

- Small wall panels subjected only to horizontal bending.
- Wall panels with an opening subjected to in-plane shear force.
- Large wall panels subjected to lateral loads with different support conditions.

The numerical simulations are compared to laboratory test results from the literature and the model’s capability in predicting the ultimate load of walls subjected to various bending conditions is demonstrated.

Finally, Chapter 6 summarizes the experimental and numerical methods adopted in this work. Conclusions are drawn based on the results obtained from using these methods. Recommendations are made on potential future work that may be worthy of further investigation.
CHAPTER 2
LITERATURE REVIEW
2. **LITERATURE REVIEW**

2.1 **Introduction**

Research into the behaviour of masonry walls subjected to bending can be dated back to the 19th century. However, significant interest in this area did not occur until the early 1970s. This is reflected by the very limited number of research papers prior to the early 1970s. Since then there has been an increasing number of publications, most of which focussed on tests associated with the flexural properties of small specimens or wall panels. Section 2.2 provides a summary of the major achievements made up to the early 1980s; a more detailed account of these earlier activities can be found in the works of Baker (1981) and Lawrence (1983).

The 1980s saw a renaissance of research activities into masonry walls subjected to bending. Many theories other than the yield line theory (a theory derived indirectly from the design of concrete slabs) were established. In Australia some of these theories were progressively incorporated into its masonry design standards. The theoretical activities were at the same time accompanied by the extensive experimental tests conducted around the world. An overview of the major achievements in experiment and theory since the early 1980s is given in Section 2.3.

While most research work in the 1980s concentrated on experimental aspects, dramatic progress was made in the 1990s in the field of numerical analysis, mainly thanks to the rapid growth of the modern computing technology and the maturity of the finite element method. Micro modelling, in which mortar joints and units are represented separately, and macro modelling, in which no distinction is made between masonry joints and units through averaging the effects of mortar and bricks, were introduced into numerical modelling to simulate masonry wall failure mechanisms. An overview of progress in numerical approaches can be found in Section 2.4.

2.2 **Major Achievements before the Early 1980s**

The interest in the flexural properties of masonry walls can be dated back to the early 19th century and became more actively pursued starting in early 1900s. The first flexural
tests on masonry were carried out by Sir Brunel in England in 1836. A reinforced beam was used to demonstrate the effectiveness of the reinforcement. Since then more tests were carried out to study the masonry material properties, their impact on the bond strength between mortar and bricks, and the effect of the bond pattern and support condition on the strength of wall panels. These early achievements have been summarized by Baker (1981) and Lawrence (1983), which for completeness are highlighted below:

- Identified the bond strength normal to the bed joints as a critical factor in flexural strength (Stang et al., 1925-1926; Kelch, 1931; Richart et al., 1932; Whittemore et al., 1938-1941; Parsons, 1939) and the most important parameter in determining the lateral load resistance of masonry walls (Hallquist, 1969; Dikkers and Yokel, 1970; Baker, 1972a; Cajdert and Losberg, 1973; James, 1975; Sinha and Hendry, 1975; West et al., 1977).

- Recognized the enhanced horizontal flexural strength compared to the vertical flexural strength. The ratio of the two strengths increased as the vertical flexural strength decreased. Two types of failure were identified in horizontal flexure, namely straight failure through bricks and perpend joints and stepped bond failure. A relationship between horizontal and vertical bending was demonstrated for the first time. Brick failures were closely related to its transverse flexural strength (Cox and Ennenga, 1958; Sinha and Hendry, 1975; James, 1975; Lawrence, 1975; Lawrence and Morgan, 1975; West et al., 1977; Drysdale and Hamid, 1980).

- Discovered diagonal flexural strength which is different from the vertical and horizontal flexural strengths (Satti and Hendry, 1973; Cajdert and Losberg, 1973). A failure criterion was proposed based on elliptical interaction diagram (Baker, 1979)

- Discovered the influence of vertical pre-compression on the horizontal bending strength of masonry (Hummel, 1952-1953; West et al., 1971; Page, 1973).
Developed theories to predict the strength of masonry. Four side supported panels were observed to have reserve of strength after initial cracking and the initial cracking load could be predicted from elastic theory (Hallquist, 1969; Hallquist and Granheim, 1969; Hellers, 1969; Losberg and Johansson, 1969; Hallquist, 1970; Nilson and Losberg, 1970). Yield line analysis was applied to estimate the ultimate strength of masonry walls (Nilson and Losberg, 1970; Magdalinski, 1972). Some researchers claimed the method overestimated the ultimate strength (Baker, 1972b; 1973), while others claimed underestimation (Hendry, 1973; West et al., 1973; Haseltine and Hodgkinson, 1973; Cajdert and Losberg, 1973). As a consequence, modified yield line theory (Haseltine et al., 1977; Sinha, 1978) and strip method (Baker, 1973) were proposed.

Identified the concept of statistical variation in strength of vertically spanning panels through experiments (Fisburn, 1961; Structural Clay Products Research Foundation, 1964; 1965; 1966a and 1966b; 1967; 1969).

Identified the arching effect on both ultimate wall strength (Davey and Thomas, 1950; Thomas, 1953; Anderson, 1976) and cracking load (Baker, 1977).

### 2.3 Major Experimental and Theoretical Achievements since the Early 1980s

Most of the research work conducted after the early 1980s were generally extensions of previous studies. Hot debates were (and still are) held over the issue of applicability of the yield line theory to the design of masonry walls when subjected to out-of-plane loading. Extensive laboratory tests were thus carried out around the world to try to experimentally validate this theory’s applicability and comparisons were made to other various approaches such as the newly formed strip method. For example, Essawy and Drysdale (1983) tested a wall of 2.8m high by 6.0m long. The test results were compared to predictions using various design methods. They concluded that although the yield line theory showed reasonable agreement with the experimental results, there was no rational justification for its use. Based on these findings, they emphasized that there was no design method available which was both rational and accurate.
Baker (1982) proposed a new method of analysis based on his earlier work. He calculated the elastic principal moments and used Monte Carlo simulation to assign strengths repeatedly at random for comparison with the calculated moments. Although this new method conservatively predicted the initial cracking load and adequately predicted the ultimate failure loads of panels supported on three and four sides, some of the assumptions used (e.g., load averaging in adjacent joints) lacked support from experimental data.

Sinha (1980) continued his earlier work using the fracture line method. This method is basically the same as the yield line theory; however, it takes into account the orthotropic properties of masonry walls. The fracture line method was proved experimentally to be applicable to wall panels with octagonal and triangular shapes and rectangular wall panels with openings.

Lawrence (1983) carried out extensive tests on 32 single leaf panels of clay brickwork and established a valuable database of walls subjected to biaxial bending. His tests revealed three distinct stages of wall behaviour at which significant changes occurred, although not every stage was present for each wall. The first stage of behaviour was occurrence of first cracking in the panel. The second stage of behaviour was formation of the full crack pattern and the third stage, panel failure by reaching its maximum load carrying capacity and continuing deflect at a slightly reduced load. The occurrence of these three stages was found to be dependant upon the geometry and support conditions of a wall panel. Lawrence combined these three stages in different ways to create four distinct modes of failure.

Lawrence also compared his experimental results with predictions from four major theoretical methods, namely the elastic plate theory, the yield line theory, the empirical strip method and the elastic principal stress theory (Baker et al., 1985). The elastic plate theory was used only to check the cracking load and they found that the experimental results were significantly lower than the theoretical predictions. The discrepancy was explained by the effects of random variation in the material properties due to the scale of the walls. On the other hand, the yield line theory was only used to predict the ultimate loads and was found to overestimate the ultimate strength of the walls consistently, especially for those panels with support on four sides. In contrast to the
inherent ductility of reinforced concrete slabs from which the yield line theory originated, the theory was again found to be invalid for masonry, in which the material exhibits brittle characteristics. Also the yield line theory failed to incorporate the random variation of material and the effect of self weight in its predictions. However, the strip method was shown to be in better agreement when used to predict the ultimate load for those panels simply supported on four sides. Nevertheless, the method was considered to lack a rational basis. The elastic principal stress theory was considered by Lawrence to be thus far the best theory. It not only provided predictions in line with experimental results but was also a rationally based approach, which is preferable to the strip method. Unfortunately, good agreement was only found for clay brick masonry.

Lawrence’s findings mentioned above were later supported by the experimental work of Gairns and Scrivener (1985). These researchers showed that the elastic principal stress theory method was conservative for concrete block masonry, although they pointed out that the difference between the prediction and experiment might be due to the yielding of supports. As a result, a new series of tests was carried out by these authors to study the difference between blockwork and brickwork masonry. The tests were carried out with modified frame supports to prevent yielding. However, the same trend was still found in the new test results. Their focus was then shifted to differences in material properties. This included unit size effect, unit self-weight effect and the failure criterion used in biaxial bending. Comparing the smaller unit size bricks to blocks with the same panel size indicated more joints in the brickwork than in the blockwork. Thus according to the weakest link theory the brick panel will fail sooner. Higher compressive stress of the blockwork due to its larger mass also improved the flexural strength of a wall panel. This effect was attributed to an increase in the effective vertical moment of resistance as well as secondary effects due to shifting of the neutral axis upon failure. The higher compressive stress effect was also supported by Baker’s failure criterion, which suggests that applied compressive stress can greatly enhance the horizontal flexural capacity if failure in horizontal flexure is through the mortar joints.

Essawy and Drysdale (1987) adopted a different approach. Instead of using an experimental approach to examine the accuracy of predictions based on various approaches, they predicted masonry wall behaviour using the non-linear macroscopic finite element method. Their simulation covered a range of panels with length to height
ratios ranging from 0.36 to 4.24. These panels were supported either on all four sides or on three sides with the topside free. The predicted results were compared with the elastic plate theory, the yield line analysis, the empirical strip method, the fracture line method and the British code moment coefficients. They indicated that the accuracy of the calculated panel capacities using the yield line method, the empirical strip method and the British code moment coefficients might be acceptable in some cases but not for the whole range of aspect ratios and boundary conditions. However, the elastic plate theory assuming isotropic properties yielded reasonably accurate predictions for first cracking loads for the whole range of aspect ratios when the panels were supported on all four sides. Thus this theory was regarded as the most promising approach since it is the only method that is rationally acceptable for masonry.

Most researchers believe that the yield line theory applied to masonry is theoretically unsound because of its assumption of the existence of plastic hinges that cannot exist in a brittle material. Interestingly, Lovegrove (1988) pointed out that if this assumption was replaced with consideration of the energy needed to produce a crack whilst retaining all the other assumptions used in the yield line theory, the same final design strength was still obtained. Lovegrove suggested that bending moment and energy were in essence the same. A failure criterion for masonry could thus be obtained by considering the energy required to produce a unit length of crack, rather than the bending moment. However, this author also pointed out that the yield line theory in terms of energy had its own limitation. For example, there will be cases where the energy analysis itself is not applicable to yield line.

In Australia the energy concept was also adopted by Candy (1988) in his newly proposed energy line method. This method is basically similar to the yield line method. However, it differs from the yield line method in that the “energy” lines are not necessarily “yield” lines. In this new energy line method horizontal bending moments are assumed to be fully redistributed along energy lines within the panel but not at vertical clamped edges. The moment distribution on diagonal and vertical lines within the panel is uniform. The moment distribution on vertical clamped edges is assumed to be either linearly varying from zero to the maximum or to be one half of the maximum, uniformly distributed. Vertical bending moments are assumed to be zero along all energy lines. Note that this new method does not consider an orthogonal strength ratio.
Candy applied the energy line method to the analysis of 107 wall panels that had been experimentally tested by other researchers. He showed that the method yielded better predictions for the ultimate pressures than both the yield line method and the strip method.

Later the energy line method was further developed and formulated by Lawrence and Marshall (1998), and renamed as the virtual work method. The virtual work method was verified using data from researchers around the world and was shown overall to have lower variability in predictions of wall lateral load capacities than the strip method. The virtual work method also deals with support configurations that are not covered by the strip method and with walls having door and window openings.

The virtual work method has since been adopted in Australia in the revision of the Masonry Code AS 3700-1998. Nevertheless, it should be pointed out that this method is still partly rational and partly empirical, although the empirical part is much less than in the strip method. In particular, the virtual work method relies on the use of an empirical formula for the diagonal bending moment capacity and does not consider the effect of axial load in the evaluation of the energy absorbed in the bed joints. As a result, it leads to overly conservative strength estimation for walls with high pre-compression (Griffith et al., 2001).

Lawrence and Marshall (2000) note that “no completely rational method has yet been developed” and that “further research is necessary to develop a fully rational biaxial bending failure model that can predict behaviour under any simultaneous combination of bending moments in the two principal directions, along with a superimposed compression force along the bed joints”.

In Canada, the failure line method (Drysdale and Baker, 2003) was developed at McMaster University over the past few years as a variation of the yield line method. A masonry wall is considered as a group of rigid plates connected along crack lines sufficient to form a failure mechanism. So the crack pattern defines the “failure line” mechanism. Although the failure line method is a plastic approach, it differs from yield line method in its rational treatment of first cracking. With this method, the moment resistance across the first crack is ignored in the analysis. The failure line method was
recently introduced into the new CSA Standard, Masonry Design for Building CSA S304.1-04.

Most of the research work published in the 1980s involved validating the theoretical analysis approaches based on the yield line theory, the elastic plate theory and the strip method. However, numerical modelling research had increased dramatically since late 1980s mainly due to the availability of powerful modern computing technology. Consequently, small-scale tests using wallettes became popular since these tests can provide the failure criteria and material parameters that are the required inputs for numerical modelling.

Efforts continued to be made in order to establish a proper failure criterion. Gazzola et al. (1985) tested 25 wallettes made of concrete blocks. The wallettes were subjected to bending at different angles to the bed joints. It was found that all wallettes with bending between 0 and 45 degrees from the bed joints failed in a toothed or stepped pattern. For 75 and 90 degrees failure was simply by debonding along the bed joints. These failure patterns are illustrated in Figure 2.1.

From these test results a failure criterion for the tension-tension range of biaxial stresses for block masonry was proposed, as shown in Figure 2.2. The criterion suggested that the mode of failure was the major factor in determining the degree of interaction for biaxial tension. Where failure was by debonding along the bed joints or by failure through alternate courses of block and head joints, it was suggested that tension parallel to the failure plane would not affect the respective tensile strengths normal to the bed joints. Alternatively, a combined failure along head and bed joints would result in some decrease in these strengths due to tensile stress interaction.
Figure 2.1. Masonry blockwork wallette failure patterns (Gazzola and Drysdale, 1985).
Gazzola and Drysdale (1986) later proposed a refined component failure criterion for blockwork in flexure. This new failure criterion was based on resolving the components of the applied stress in the principal material directions and was governed by six block/mortar strengths, the orientation of the bed joint with respect to applied stress and other specimen properties. The failure criterion was composed of three sub-criteria representing three possible flexural failure modes, each having two cases. Among the six equations provided, the one that gives the lowest strength is considered as the strength of the blockwork in flexure. The component failure criterion was again checked against experimental results from wallette tests and was found to provide reasonably accurate predictions for the flexural strength and mode of failure. These authors suggested that if the proposed criterion for off-axis bending was combined with a pure biaxial bending criterion (principal stress applied in the principal material directions), it would result in a general biaxial bending failure criterion. However, they also recognized that the criterion needed future revision since it only provided an upper bound solution due to the assumption of no interaction between flexural tension and torsional shear.
May and Ma (1986) established a complete failure criterion, which was based on Baker’s elliptical failure criterion (Baker, 1979) for the stress state in the “tension – tension” zone, in combination with assumed linear interaction of stress in the “tension – compression” zone and non-interaction of stress in the “compression – compression” zone. This criterion was used in their finite element model and validated against the experimental results of a few wall panels. Good agreement was found for the prediction of ultimate load. However, no attempt was made by these authors to predict the cracking load.

Sinha et al. (1997) established a failure criterion for finite element analysis based on their biaxial bending test for cross-beam specimens. This is in contrast to Baker’s elliptical failure criterion (Baker, 1979), which was based on the test results from four brick unit specimens. Sinha et al. selected cross-beam specimens that were four courses high and two bricks wide in the centre with four arms built from the same bricks but epoxy sand mortar stretched in two orthogonal directions (Figure 2.3). The specimens were tested as simply supported beams subjected to a central point load. They claimed that the advantage of their test setting was not only to pinpoint the cracking load but also the redistribution of load in two directions after failure in one direction occurred. It was found that the flexural strength in the weaker direction increased under biaxial bending compared to its uniaxial strength for the orthotropic material. The stronger direction did not show any significant improvement in strength under biaxial bending over the uniaxial strength. These results are shown in Figure 2.4. This is contradictory to Baker’s elliptical failure criterion, where the strength in either of the directions is reduced in biaxial bending compared to uniaxial bending. Lawrence (1994) emphasized in his review paper the necessity to develop and verify a failure criterion both experimentally and theoretically.
Figure 2.3. Cross-beam specimens (Sinha et al., 1997).

Figure 2.4. Failure criterion (Sinha et al., 1997).
In addition to the various failure criteria discussed above, composite yield surfaces based on the Rankine-Type Criterion and Hill-Type Criterion have been commonly adopted by modellers (Lourenço et al., 1998). To use these criteria, complex tests involving the collection of strength parameters (both elastic and inelastic) in two material directions are required and strength parameters in directions away from the two material directions need to be calibrated against test data of small wall panels.

Pluijm et al. (1995) also performed some wallette tests to study the flexural behaviour of masonry in different directions. Although the purpose of their tests was to collect data to verify their numerical model, the tests showed clear bi-linear behaviour when bending was not around the horizontal axis. This was explained as being caused by the gradual cracking of the perpend joints. This bi-linear behaviour was also observed earlier by Lawrence (1983), and later by Willis et al. (2002) when they performed horizontal bending tests on small wallettes. The close relationship between this bi-linear behaviour and the bond strength of masonry was once examined by Lawrence (1995) in his tests using wallettes with unfilled perpend joints. He observed an absence of the change of slope in the load-deflection graph. His work highlighted the importance of further study in the torsional behaviour of bed joints.

Study on torsional behaviour of bed joints has been carried out by a number of researchers. Although the importance of torsional shear strength has been identified since the early stages of masonry research on lateral loading, a fundamental understanding of this behaviour is still lacking.

Tests have been carried out by various researchers trying to obtain the torsional strength for differing combinations of brick and mortar using a range of test setups. The torsional strength was found to be proportional to the normal load applied and the coefficient of friction (Baker et al., 1980; Hamid and Tate, 1981). Baker’s work is worthy of particular mention. He tested overlapped bed faces of masonry units and found that the elastic theory was not applicable to the torsional shear behaviour. Instead, a plastic theory was derived to predict the torsional moment of resistance of a rectangular friction area. The assumption used was that the full frictional shear capacity of each elemental area on the surface was mobilized at failure, irrespective of its distance from the centre.
of rotation. This plastic theory was further confirmed by tests using various applied normal loads, eccentricities of load and centres of rotation.

Samarasinghe and Lawrence (1994) tested brick and mortar couplets under combined torsion and various levels of compressive load. They found that the existence of compressive force impacted on the failure pattern as well as the peak and residual torque resistance. While the peak torque resistance was found to increase linearly as the pre-compression stress was raised, correlation between the residual torsional moment and pre-compressive stress was not obvious. This was thought to be due to the geometry of the crack that caused the different interlocking action along the cracked surface.

Willis (2004) performed torsion tests on couplets using a similar setup to the one used by Lawrence (1983). The couplets were built from different batches of bricks and mortar and subjected to various levels of pre-compression. An empirical relationship was established between the torsional strength, flexural strength and axial pre-compression. Willis used this relationship to develop a number of mathematical models which can predict the strength of brick masonry walls in biaxial bending (Willis et al., 2004; Griffith et al., 2005).

The post-peak behaviour of masonry walls has been another important research topic. The post-peak behaviour of shear strength was first identified and tested by Pluijm (1993). In his couplet test of shear strength between brick and mortar a deformation controlled test setting was adopted to measure the load versus displacement behaviour after reaching the peak load. The post-peak behaviour of the shear strength was described by Pluijm using mode II fracture energy. A relationship was established between this energy and the normal compressive stress. Pluijm (1997) also made an effort to measure the tensile strength using a deformation controlled method to catch the post-peak tensile behaviour. The observed behaviour was described by using mode I fracture energy, in a similar way to the post-peak behaviour of shear strength. However, compared to the mode II fracture energy, the results for mode I fracture energy showed much greater variability.

It was Molyneaux et al. (2002) who managed to measure the shear softening process through triplet testing. In their test, triplet samples were loaded using displacement
control at a rate of 60mm/min and a data acquisition system at a scan frequency of 19kHz was used to measure the joint slip. The softening of shear strength was also observed to be exponential. The observation agrees with Pluijm’s test result.

The post-peak behaviour of masonry subjected to horizontal bending was also explored by other researchers such as Hansen (1998). It was found that the failure would be brittle, when bent about the axes perpendicular to the bed joints, if the failure only involved bending failure in bricks and perpend joints. However, when the bed joints participated in the failure, the deformation capacity appeared to increase and a residual load carrying capacity would be present even at large deformations. A slightly modified couplet test setting was later adopted by Hansen to measure the ductility of shear torsion failure in bed joints (Hansen et al., 1998). It was found that mode II fracture energy of bed joints was a function of pre-compression stress.

Other issues that have been researched in the literature and are worthy of brief mention here include prediction of cracking load, the effects of vertical pre-compression and arching and walls with opening.

The issue of predicting cracking load was emphasized by Lawrence and Cao (1988) in their effort trying to compare the cracking load of 32 tested wall panels with their predictions using Monte Carlo simulation and isotropic elastic plate analysis. Their simulation approach took into account the random variation in flexural strength and the plate analysis was used to evaluate the bending moments in the plate. Four failure criteria were applied: no interaction between horizontal and vertical moments; Baker’s elliptical interaction (Baker, 1979) based on his tests of brickwork subjected to moments around two material axes (normal and parallel to the bed joint); assumed linear interaction where the sum of the ratios of horizontal and vertical moments to respective strengths is unity at failure; and Baker’s derived failure criteria applied to the principal moments. It was found that the straight line interaction and principal moment/elliptical criterion yielded lower predictions for the cracking loads than those from the elliptical and no interaction criteria.

The effect of vertical pre-compression on masonry flexural strength was further investigated by some researchers. Garrity and Phipps (1988) tested wallets with pre-
compression subjected to horizontal bending and found that application of a vertical pre-stress increased the horizontal flexural strength of clay brickwork without arching, up to a maximum value equal to the modulus of rupture of the bricks when tested in horizontal flexure. However, no conclusion could be drawn when the arching effect was present, which occurred in their experiments due to the nature of the test setup.

Dawe and Seah (1988) also tested nine concrete block panels in a steel frame subjected to increasing normal lateral pressure up to the ultimate load. The purpose of these tests was to study the effect of arching action on the wall panel behaviour. Meanwhile, an elastic finite element analysis combined with the failure theory proposed by Essawy and Drysdale (1987) was also performed to predict the first crack load. The prediction was found to be in agreement with the experimental values. Furthermore, Dawe and Seah used the yield line analysis by including the arching action to predict the ultimate load and found it improved the prediction compared to using the conventional yield line techniques. As a result, they recommended that for panels in flexible frames the frame flexibility must be accounted for in the analysis in order to obtain good correlation with test results.

Studies have also been carried out to examine walls with openings under out-of-plane load conditions (Chong et al., 1994; Middleton and Drysdale, 1995; de Vekey et al., 1996; Duarte, 1998; Vaculik and Griffith, 2006). These studies mainly followed earlier work conducted in the late 1980s (Southcombe and Tapp, 1988; May et al., 1988), which was trying to address important issues such as the lack of definition in major design codes for the lateral strength of this type of wall.

2.4 Major Achievements in Numerical Modelling

Use of the finite element method to deal with the structural behaviour of masonry can be dated back to four decades ago. Most early analyses considered masonry to be an assemblage of bricks and mortar with average properties; hence the so-called macro-model. Isotropic elastic behaviour was also assumed to simplify the problem (Rosenhaupt, 1965; Saw, 1974). This ignored the influence of mortar joints acting as planes of weakness. These analyses are therefore only useful in predicting deformations at low stress states, since at higher stress levels extensive stress redistribution caused by non-linear material behaviour and local failure may occur. Since then some new
material models, which were still based on average properties and ignored the influence of mortar joints but included the possibility of local failure, were developed (Ganju, 1977; Samarashinghe et al., 1982).

Dhanasekar et al. (1984) proposed a non-linear finite element model for solid masonry. The model was still based on average properties derived from biaxial tests on brick masonry panels, but capable of reproducing the effects of material non-linearity and progressive failure. The masonry was modelled as a continuum with average properties and each element contained several bricks and joints. As such, the model could not catch local effects.

There have since been a very limited number of macro-models implemented due to the complexity of introducing orthotropic behaviour. Note that a complete model with the macro approach must reproduce an orthotropic material with different tensile and compressive strengths and stiffness along the material axes as well as different inelastic behaviour for each material axis. More recently, Lourenço and co-workers (Lourenço et al., 1997 and 1998; Lourenço, 2000) proposed a composite yield criterion by combining the advantages of classic plasticity concepts with a powerful representation of anisotropic material behaviour including the different hardening/softening behaviour along each material axis. The composite yield criterion consists of a Hill-type criterion for compression and a Rankine-type criterion for tension and has been used to describe different failure mechanisms. The Hill-type criterion is associated with a localized fracture process, namely crushing of the material, whereas the Rankine-type criterion is associated with a more distributed fracture process which is usually named cracking of the material. The internal damage due to cracking and crushing failure mechanisms is represented with two internal parameters for damage in tension and compression, respectively. The model was formulated in such a way that each internal parameter is related to two independent fracture energies along each material axis and thus to reproduce different inelastic behaviour along two orthogonal directions. However, the model was formulated on an anisotropic material under plane stress conditions. It was applied to masonry walls subjected to in-plane loading, and later to walls subjected to out-of-plane loading with the assumption of “straight normals” and “zero normal stress”. The first assumption means that the normals to the mid-plane of the element remain straight after deformation, but not necessarily perpendicular to the mid-plane.
The second assumption states that the normal stress component perpendicular to the mid-plane equals zero and the element formulation has been obtained by ignoring the strain energy resulting from this stress.

Modelling of masonry walls with the macro approach has been hindered by the limited laboratory test data available for small masonry wall specimens to obtain the required parameters. Such tests are costly and time consuming and subject to changes of material properties of the masonry components. Because of this and the periodical character of masonry components, a new technique called homogenization has become increasingly popular among the masonry community. The method permits one to establish constitutive relations in terms of averaged stresses and strains from the constitutive relations of the individual components. It works by identifying an element cell, which generates an entire panel by regular repetition. Then, based on the knowledge of the mechanical properties of the constituent materials (bricks and mortar joints) and the geometry of the element cell, an average value can be achieved for the homogenized masonry material. Major studies that contributed to the development of this technique were carried out by Besdo (1985), Bakhvalov and Panasenko (1989), Pande et al. (1989), Muhlhaus (1993), Anthoine (1995 and 1997), Urbanski et al. (1995) and Zucchini and Lourenço (2002).

A method, known as the micro-model, that accounts for the non-linear behaviour of masonry and considers masonry as a two phase material was first developed and applied to solid masonry by Page (1978). It was later used by Ali and Page to study the non-linear behaviour of masonry subjected to concentrated load (Ali and Page, 1985; 1987; 1988). Since then the method was further developed by other researchers such as Anand and Rahman (1990) and Rots (1991). Unfortunately, any analysis with this level of refinement is computationally intensive and only suitable for simulating the fracture behaviour of small laboratory specimens.

To overcome the weakness of this micro-model, Arya and Hegemier (1978) proposed a less refined approach by modelling the masonry units with continuum elements and the mortar joints by means of interface elements. This method was also studied by other researchers such as Rots (1991). Although this method is still computationally intensive
for analysis of large masonry structures, it offers a practical tool to replace the costly and time consuming laboratory tests.

The researchers who have made major contributions to the micro modelling with interface elements include Lotfi and Shing (1994) and Lourenço and Rots (1997). Their models were formulated in a rigorous manner by incorporating the concepts developed in the theory of plasticity for non-standard materials and in fracture mechanics. All these models have adopted a Mohr-Coulomb type failure criterion and used internal variables to describe a post-peak softening behaviour. The constitutive model proposed by Lotfi and Shing (1994), which uses interface elements to model masonry joints and smeared crack elements to model masonry units, is capable of simulating the initiation and propagation of fracture under combined normal and shear stresses in both tension-shear and compression-shear regions. Furthermore, the constitutive model is able to model the experimentally observed dilatancy, which is known to have a significant effect on the response of an interface under confined conditions. On the other hand, Lourenço and Rots’ model adopts a cap model to take into account the joint compression failure due to high compressive stresses. Therefore, it forms a complete failure surface for the mortar joints.

A simpler interface model was later proposed by Giambanco et al. (2001). It has general features similar to those in the earlier models, except that it gave particular attention to the cohesive-frictional joint transition. The model also takes into account the geometrical dilatancy which appears in the pure frictional stage and is caused by the roughness of fracture-slip surface. Note that to the author’s knowledge all the interface models described above have been only validated by simulating two dimensional (2D) problems such as in-plane behaviours of masonry walls.

Efforts have been made recently by other researchers (Martini, 1998; Tzamtzis and Asteris, 2002 and 2003) to implement a three-dimensional (3D) non-linear micro model. The application of the 3D interface model to a wall panel subjected to out-of-plane bending was carried out by Martini (1998). The model was so simplified that the shear strength was assumed to be fully plastic. No quantitative comparison was made since the purpose of Martini’s modelling was to develop a deeper understanding of the
mechanics of unreinforced walls and more practical and approximate methods such as the modified yield line approach.

Tzamtzis and Asteris (2002) also implemented a 3D interface model. However, similar to Martini’s model it assumed elastic-perfect plastic behaviour for the stress-strain relationship. It was applied only to 2D masonry walls under static and dynamic loading conditions due to lack of information about the stress-strain relationship for the masonry components as well as lack of reliable test data for the 3D problems of masonry walls (Tzamtzis and Asteris, 2003).

The numerical modelling approach used in the current research is detailed in Section 4.2.
CHAPTER 3

EXPERIMENTAL TESTING PROGRAM
3. **Experimental Testing Program**

3.1 **Introduction**

This chapter describes the experimental program undertaken during this research. It is common knowledge that mortar joints in masonry are planes of weakness. Failure under biaxial bending can occur by fracture of bed joints (Figure 3.1a) or alternate fracturing of units and perpend joints (Figure 3.1b) or stepped perpend and bed joints (Figure 3.1c) or by fracture along diagonal stepped crack paths through the bed and perpend joints (Figure 3.1d). While most researchers have focussed on full-wall behaviour, in this study the focus was on the behaviour of “single joint” specimens consisting of four brick units. These specimens were bonded together with mortar and subjected to separate as well as simultaneous action of one- and two-way bending, all under vertical compression.

![Possible failure modes under biaxial bending.](image)

Baker (1979) carried out similar tests and noted that this “single joint” specimen approximately, if it is assumed that the moment transmitted through the perpend joint is the same as in the perpends of a full wall, represents the actions on the joints in a wall panel when subjected to two-way bending. This assumption led to the development by Baker and the CSIRO of an experimental testing apparatus for simulating biaxial bending. However, the test apparatus was never commissioned due to changes in strategic research directions at the CSIRO. The apparatus was subsequently transferred to the University of Newcastle and calibrated and commissioned during this thesis work.

Section 3.2 describes in detail the specimens used in the experimental program. In Section 3.3 a description of the testing apparatus is given along with details of its
calibration. A series of tests on four brick unit specimens subjected to vertical \((M_v)\), horizontal \((M_h)\) as well as simultaneous vertical and horizontal bending with different ratios of \(M_v : M_h\) are described in Sections 3.4 – 3.6. Accompanying each series of bending tests, bond wrench tests were conducted as a control measure to monitor the bond strength between mortar and brick for different mortar batches. The bond wrench test, as well as its comparison with direct tension test results, is presented in Section 3.7. In order to implement the numerical model which will be described in Chapters 4 and 5, a series of control tests including compression test, brick modulus of rupture test, triplet shear test and torsion shear test were also conducted. These control tests were used to obtain the parameters necessary to characterize the strength and stiffness properties of the masonry. These test approaches and test apparatus are described in Sections 3.8 to 3.10.

3.2 Test Specimens

150 four unit specimens (Figure 3.2) were constructed. The bricks used were solid extruded clay bricks of dimension 232mm long \(\times\) 75mm high \(\times\) 111mm thick. These dimensions are the standard clay brick size used in Australia, which is nominally 230mm long \(\times\) 76mm high \(\times\) 110mm thick. Solid instead of cored or hollowed bricks were chosen so as to avoid complicating effects such as the behaviour of mortar in the holes or frogs. The mean IRA (initial rate of absorption) of this brick type is 1.14kg/m\(^2\)/min. Although there is no unique relationship between IRA and mortar bond, tests have indicated that IRA values between 0.25 to 1.5 kg/m\(^2\)/min generally produce good bond strength with compatible mortar (Drysdale et al., 1999).

Under horizontal bending \(M_h\), failure can occur alternatively through the units and perpend joints or through the bed and perpend joints depending on the relative strengths of unit and mortar. For the current project it was considered important to focus attention on the more complex stepped joint failure mode. It was therefore necessary to select a mortar composition which, when combined with the chosen brick unit, was of sufficiently low strength to allow the joint failure mode to occur on a significant number of occasions. For this purpose, several series of preliminary tests were conducted using different mortar compositions until joint failures were consistently achieved. The types of mortar used in the trial tests were cement, lime and sand with mixing ratios by
volume of 1:2:9 and 1:0:7, with 5% replacement of sand with ground limestone (to attempt to retain mortar workability). These mortars all proved to be strong in bond since most of the failures occurred by rupture of bricks (Masia *et al*., 2004). Finally a suitable mortar was identified which consisted of cement:lime:sand in proportions of 1:1:6 by volume with eight times the recommended dose of air entraining agent to deliberately create low bond strength. This mortar was adopted for the main series (150 specimens) of the four brick unit bending tests as well as all other control tests. Mortar joints were all 10mm thick and both the bed joints and perpend joints were completely filled.

All specimens used in this thesis were prepared by the same professional bricklayer. The specimens were prepared by first laying the first brick on the floor. Then on each side of the brick, sand was laid as high as the brick so that the second layer of bricks can have half of the bricks sit on the sand. The horizontal level of the second layer of bricks was checked before the top brick was laid on it. The specimens were cured for seven days before testing.

![Sample four brick unit specimen used in this study.](image)

**Figure 3.2. Sample four brick unit specimen used in this study.**

### 3.3 The Test Rig and Calibration

Figure 3.3 shows the apparatus which was used in this study to test the four brick unit specimens. The apparatus was initially designed by Baker and further developed by
Lawrence at the CSIRO for simulating biaxial bending in masonry. It is capable of subjecting four brick unit specimens simultaneously to vertical compression combined with vertical and horizontal bending moments (Figure 3.4). The apparatus consists of a support frame housing a base plate on which the specimen is placed and a support clamp is used to fix the bottom of the specimen.

Figure 3.3. The four unit specimen test apparatus used in this study.
The horizontal moments (applied about the vertical axes), $M_h$, are applied via clamps at each side of the specimen by an actuator at the top of the frame. This actuator either pushes or pulls on the levers attached to the two vertical shafts, therefore applying torque in the shafts (Figure 3.5). The torque in the shafts is transferred to the specimen as a moment about the vertical axis via the side clamps. Each shaft is connected to the side clamp through two universal joints. The shaft is able to slide in the horizontal plane parallel to the specimen to get rid of the shear force. This ensures that only moment is applied to the specimen. The heavy steel side clamps and torque shafts are all counterweighted so that they do not apply any vertical load to the specimen.

**Figure 3.4. Possible actions on the four unit specimen.**

**Figure 3.5. Plan view of the test rig showing hydraulic ram and torque shaft lever arms used to apply $M_h$.**
The vertical moments (applied about the horizontal axes), \( M_v \), at the top and bottom of the specimen are applied via a moving top clamp and the reaction at the base plate support clamp, respectively. The moment applied to the top clamp is applied via a horizontally aligned torque shaft. This shaft has torque applied to it by a second actuator which can either push or pull on a lever attached to the shaft (Figure 3.6). This horizontal torque shaft is also counter-weighted.

![Figure 3.6. Side elevation of the test rig, showing the hydraulic ram and torque shaft lever arms used to apply \( M_v \).](image)

A third actuator is used to apply vertical pre-compression \( P_v \) to the specimen. This actuator is positioned beneath the specimen. It reacts against the underside of the base plate and is attached to rods which pull down on a plate positioned on top of the specimen. The load is applied at the top in such a way that rotation of the specimen due to \( M_v \) is not restrained (Figure 3.7).
All three actuators are able to operate in displacement or load control and in tension or compression. This latter feature means that specimens can be subjected to moments $M_h$ and $M_v$, which apply tension to the same face or opposite faces of the specimen. The level of vertical pre-compression, as well as the ratio of horizontal to vertical moments can be varied and independently controlled to represent the different combinations of actions occurring at various locations in a wall panel subjected to out-of-plane loading. The load and actuator stroke for all actuators can be continuously recorded by a data logger connected to a computer during testing and the ramp rates for actuator stroke (or load) can be pre-set and computer controlled. The moments $M_h$ and $M_v$ applied to the specimen are calculated simply as the appropriate actuator load multiplied by the lever length (150mm) for the levers applying torque to the torque shafts. Another data logger connected to the same computer is also used to record the displacement of the potentiometers, which are placed at the front and back faces of the specimen over the joints to measure the openings of mortar joints.
The apparatus was calibrated using a strain gauged steel specimen to ensure that the applied pre-compression and moments were in fact being applied correctly by the apparatus.

In all tests, compressive force $P_v$ was applied first and held constant (load control). $M_v$ and $M_h$ were then applied gradually in a displacement control mode with a fixed ratio “$M_v$ actuator stroke”: “$M_h$ actuator stroke”. This ratio was varied to achieve different ratios of $M_v:M_h$.

### 3.4 Four Brick Unit Specimens Subjected to Vertical Bending

30 specimens were tested under pure vertical bending $M_v$ (bending moment applied around the horizontal axis) combined with three levels (ten specimens each) of pre-compressive load $P_v$ of 0.424, 2.575 and 5.150kN, respectively. These loads resulted in average compressive stresses across the bed joints of 0.016, 0.1 and 0.2MPa, respectively. The apparatus was initially designed such that the specimen could be subjected to zero pre-compressive stress by counter weighting the top clamp. However, during the preliminary tests and subsequent calibration it was found that when specimens were subjected to vertical bending, a consistent disparity was noticed between the measured residual bending moment after failure and theoretical prediction using a simple static calculation. Subsequent investigations proved that the counter weight of the top clamp did not work as intended and left some unknown portion of compression on the specimen when bending the specimen vertically. Therefore, a decision was made to remove the counter weight of the top clamp and regard the self weight of the top clamp and loading plate as pre-compressive force in subsequent tests. The top clamp and loading plate weigh 0.424kN, which is equivalent to a 0.016MPa vertical compressive stress.

Ten specimens were constructed at a time using one batch of mortar and tested seven days after their preparation. Accompanying each batch of these four brick unit specimens, two stack-bonded prisms with six bricks and five joints each were also constructed for the bond wrench test to be conducted on the same day. The bond wrench test result served as control bond strength for comparison between different batches of specimens. Details of the bond wrench apparatus and test procedure are discussed in Section 3.7.
Figures 3.8a and 3.8b show the typical failure patterns observed in the vertical bending test. The failure was seen to occur by cracking of either the upper or lower bed joint in the brick and mortar interface. After failure, the upper cracked part of the specimen rotated rigidly about a horizontal axis at the compression face of the specimen because of the existence of pre-compressive load at the top of the specimen. Four potentiometers with 30mm range were mounted at the front and back faces of the specimen across the two bed joints to capture the rotation before and after the failure.

![Figure 3.8. Vertical bending failure pattern. (a) Failure through upper joint and (b) failure through lower joint.](image)

Figure 3.9 shows the typical behaviour of the vertical bending moment versus rotation of the joint, where failure occurred under different levels of pre-compression. The specimens behaved almost linearly until the peak moment was reached, after that the resistance moment dropped because of the sudden release of energy after cracking. Then, depending on the magnitude of the compressive load, the moment either dropped or increased to a residual or plateau value. The residual value of moment $M_v$ was that required to maintain equilibrium of the cracked specimen under the action of the pre-compression force $P_v$ which induced a moment acting about an axis along the compression face of the specimen. Figure 3.9 also demonstrates the brittle failure behaviour of the specimen under the lowest pre-compressive load (0.016MPa).
Figure 3.9. Vertical bending moment under different pre-compressive stresses.

Figure 3.10 shows the moment behaviour of the ten specimens tested under pure vertical bending when a fixed small pre-compressive stress of 0.016MPa was applied. It can be seen that the peak moment of these ten specimens varied significantly, ranging from 0.03 to 0.16kNm. There was also significant variation in the rotation (ranging from 0.0025 to 0.018 rad.) before reaching the residual (plateau) moment. This large variation in peak load was thought to be due to the large variability in the bond strength between the brick and mortar amongst these specimens, since the corresponding bond wrench test also showed similar behaviour with a coefficient of variation (COV) of 29%. This is discussed in Section 3.7. The sudden jump in the response from peak moment to residual moment was a direct result of the flexibility of the apparatus. The considerable elastic energy stored in the torque shafts during loading was released suddenly upon specimen cracking. This caused a sudden increase in joint rotation making it impossible to trace the softening response accurately from peak moment to residual moment. The first value of joint rotation recorded after cracking in each test was directly related to the peak moment and therefore these values showed similar scatter to the peak moment values.
The mean peak moment \( (M_{vp}) \) and mean residual moment \( (M_{vr}) \) for the specimens under the different levels of pre-compressive load are summarized in Table 3.1. Only eight out of the ten specimens were tested under the pre-compressive stress of 0.1MPa. Due to weak bonding between mortar and bricks two specimens failed prematurely before the test was even started. Similarly, the mean value of peak moment and residual moment for those specimens under a pre-compressive stress of 0.2MPa are the average values of nine out of ten specimens. One specimen failed during preparation.

**Table 3.1. Vertical bending peak and residual moments, along with the bond wrench test results for each of the three batches of masonry.**

<table>
<thead>
<tr>
<th>Pre-compression (MPa)</th>
<th>No. of Spec.</th>
<th>Peak Moment ( M_{vp} ) Mean (kNm)</th>
<th>COV</th>
<th>Residual Moment ( M_{vr} ) Mean (kNm)</th>
<th>COV</th>
<th>Bond Wrench Test ( f_{mt} ) Mean (MPa)</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.016</td>
<td>10</td>
<td>0.084</td>
<td>50%</td>
<td>0.019</td>
<td>7%</td>
<td>0.33</td>
<td>29%</td>
</tr>
<tr>
<td>0.100</td>
<td>8</td>
<td>0.164</td>
<td>6%</td>
<td>0.145</td>
<td>3%</td>
<td>0.25</td>
<td>17%</td>
</tr>
<tr>
<td>0.200</td>
<td>9</td>
<td>0.254</td>
<td>6%</td>
<td>0.270</td>
<td>4%</td>
<td>0.27</td>
<td>14%</td>
</tr>
</tbody>
</table>
It can be seen from Table 3.1 that the measured residual moments were quite consistent, with very small variations (COV less than 7%) for each level of pre-compressive stress. However, variation in the peak moment was quite substantial (COV = 50%) for the lowest pre-compressive load applied. As the pre-compressive load was increased, the variation became much smaller. The explanation for these observations is the weak and varied bond strength between the chosen type of mortar and bricks. Under a very low compressive stress the peak moment is controlled mostly by this weak and varied bond strength. As the compressive stress increases, especially when this compressive stress becomes comparable to or greater in magnitude than the bond strength, the inherent variation in the specimen’s bond strength becomes less significant and therefore variation in the measured peak moment becomes smaller. More discussions on this topic are given later.

The mean flexural strength ($\sigma_f$) of the bed joint when the joint cracks was estimated from the four unit specimen tests using the simple elastic beam theory:

$$\sigma_f = \frac{M_{vp}}{Z} - \frac{P_v + W}{A}$$

Equation 3.1

In the above equation $P_v$ is the vertical compressive force, $W$ is the weight of one brick, $A$ is the area of the brick surface (or bed joint area), and $Z$ is the section modulus of the bed joint.

For each batch of masonry the mean flexural tensile strength was also calculated from the results of bond wrench tests on ten mortar bed joints. These strength values were calculated in accordance with Standards Australia 2001, AS 3700-2001.

Comparing the mean bond wrench result with the mean flexural strength of the bed joint, calculated using Equation 3.1, reveals a large difference between the two strength values, when the four unit specimens were subjected to the lowest compressive stress (Table 3.2). Under this stress the mean strength from the bond wrench test was almost twice that from the four brick unit test. If this difference can be considered to be statistically meaningful, then it is important to find out what has contributed to this.
Table 3.2. Comparison of flexural strengths of bed joints.

<table>
<thead>
<tr>
<th>Pre-compression (MPa)</th>
<th>Four Unit $M_v$ Test</th>
<th>Bond Wrench Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Flexural Tensile Strength (MPa)</td>
<td>Mean Flexural Tensile Strength (MPa)</td>
</tr>
<tr>
<td>0.016</td>
<td>0.160</td>
<td>0.33</td>
</tr>
<tr>
<td>0.100</td>
<td>0.244</td>
<td>0.25</td>
</tr>
<tr>
<td>0.200</td>
<td>0.333</td>
<td>0.27</td>
</tr>
</tbody>
</table>

There are a large number of factors that could have contributed to the difference between the two mean flexural strengths at the lowest pre-compressive stress applied. First, consideration should be given to the test rig itself to see whether the rig is able to exert the pure moment. The easiest way to find this out is examine the residual moment data to see whether it agrees with the analytical predictions based on the simple static theory. The residual moment can be calculated using the following equation based simply on the assumption that once the joint has cracked the applied moment $M_{vr}$ must be in equilibrium with the moment due to the pre-compression force taken about the compression edge of the joint:

$$M_{vr} = (P_v + W)\times t/2$$  

Equation 3.2

where $P_v$ and $W$ have the same meaning as in Equation 3.1, and $t$ is the brick thickness.

Table 3.3 shows the experimental and analytically calculated residual moments. Except for the 0.016MPa compressive force, results from the two methods show remarkable agreement with differences of less than 6%. Therefore, this result gives confidence about the test rig’s performance and other factors such as the specimen’s geometries should be considered. The marked difference in the residual moments seen at the lowest pre-compressive load may very well be caused by the limited precision of the test rig itself for measuring such small moments. It may also result from the upper part of the specimen moving a little after cracking so that the rotation is not about the brick edge resulting in a lever arm less than $t/2$. Therefore, too much significance should not be attached to this variation.
Another possible explanation for the difference between the two mean flexural strengths at the lowest pre-compressive stress applied is the presence of perpend joints in the four unit specimens and not in the bond wrench piers, which may have an effect on the flexural strength of bed joint for the four unit specimen. To test this scenario ten four-unit specimen as well as two stack bonded prisms, each consisting of six bricks and five joints, were constructed from one batch of mortar on the same day under strict supervision. The four unit specimens and prisms were all tested after seven days using the standard bond wrench test apparatus. For each four-unit specimen both the upper and lower bed joints were tested by clamping the two bricks over the perpend joints as shown in Figure 3.11. The upper joint of one specimen failed before the test was started, due to weak bond. For the remaining nine specimens all joints failed uniformly through de-bonding at the brick and mortar interfaces. Table 3.4 displays the flexural strength of each joint tested and their mean values. Large variations in flexural strength were again found within each of the three types of joints. However, the mean flexural strengths of the three types matched very well. This can be further seen by applying an analysis of variance (ANOVA) procedure to the test data (Table 3.5). The aim of the ANOVA analysis was to try to identify whether the variability of the flexural strength was due to the variability within each group of samples or between groups. The $F$ value is the indicator of this effect. The larger the $F$ value, the stronger the evidence of variability between the batches is. The analysis was performed using Microsoft Excel’s default ANOVA built-in tool. Here an $F$ value of 0.62 and $P$ value of 0.54 strongly suggest that there are no significant differences in terms of the mean flexural bond strength between the three types of joints.

Table 3.3. Comparison of experimental and analytical residual moments.

<table>
<thead>
<tr>
<th>Pre-compression (MPa)</th>
<th>Residual Moment $M_{cr}$ (kNm)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Analytical</td>
</tr>
<tr>
<td>0.016</td>
<td>0.019</td>
<td>0.023</td>
</tr>
<tr>
<td>0.100</td>
<td>0.145</td>
<td>0.143</td>
</tr>
<tr>
<td>0.200</td>
<td>0.270</td>
<td>0.286</td>
</tr>
</tbody>
</table>
Table 3.4. Flexural strength from bond wrench test.

<table>
<thead>
<tr>
<th>Joint No</th>
<th>Four Unit Specimen</th>
<th>Prisms (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper Joint (MPa)</td>
<td>Lower Joint (MPa)</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>5</td>
<td>0.34</td>
<td>0.20</td>
</tr>
<tr>
<td>6</td>
<td>0.20</td>
<td>0.26</td>
</tr>
<tr>
<td>7</td>
<td>0.17</td>
<td>0.27</td>
</tr>
<tr>
<td>8</td>
<td>0.20</td>
<td>0.31</td>
</tr>
<tr>
<td>9</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>10</td>
<td>N.A.</td>
<td>0.16</td>
</tr>
<tr>
<td>Mean</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>St. Dev</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>COV</td>
<td>30%</td>
<td>19%</td>
</tr>
</tbody>
</table>

Table 3.5. ANOVA analysis of bond wrench test result.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>0.004586</td>
<td>2</td>
<td>0.002293</td>
<td>0.621715</td>
<td>0.544817</td>
<td>3.369016</td>
</tr>
<tr>
<td>Within Groups</td>
<td>0.09589</td>
<td>26</td>
<td>0.003688</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now that uncertainties about the test apparatus and specimen geometry have been excluded from the factors that may have resulted in the difference between the two types of test results, we turn our attention to the specimen itself. When a four unit specimen is subjected to vertical bending on the test rig, there are two joints that will be
subjected to the same bending moment and the joint with the weaker bond should fail first. Thus the mean flexural tensile strength obtained from these weaker joints will always be lower than (or at best equal to) that obtained from the bond wrench test where only one single joint is subjected to bending. This phenomenon was observed and explained by Lawrence (1991) using order statistics (Mosteller and Rourke, 1973) in his beam strength tests. Lawrence carried out a series of laboratory tests with one-brick-wide vertical beam and two-brick-wide beams using standard three-point method. The test results showed a significant difference between the two types of beam, with the two-brick-wide beams lower in strength than the one-brick-wide beams. He then used a weakest link hypothesis and an averaging hypothesis to predict the two-brick-wide beam strength. Between these two hypotheses, the weakest link hypothesis which was applied using order statistics resulted in much better prediction. The same approach is used here to examine the data of four-brick unit tests. If we assume the flexural tensile strength follows approximately a normal distribution with the mean and standard deviation, both from the bond wrench test at a pre-compressive load of 0.016MPa, of 0.33 and 0.1MPa, the flexural strength of the weaker joints among the two tested joints in this work should be expected to be around 0.276MPa (i.e., $= 0.33 - 0.1 \times 0.56$, with 0.56 being the correction factor). This is still higher than the experimental value of 0.16MPa derived from the four unit specimen failure moment $M_{vp}$.

While order statistics consideration for the weaker joints may account for, to a small degree, the difference between the two test results under the very small pre-compressive load, other contributing factors may be more important. As already cited earlier in this section, variations in bond strength may very well be the real cause, possibly due to less strict control in the preparation of the four brick unit specimens used for these 0.016MPa pre-compressive load tests since these were the first batch prepared. Better consistency in specimen construction was achieved in subsequent tests, as reflected by the much smaller COVs in corresponding bond wrench tests.

3.5 Four Brick Unit Specimens Subjected to Horizontal Bending

Three batches of specimens, each consisting of ten four-unit specimens, were subject to horizontal bending moment $M_h$ around the vertical axis as well as pre-compressive loads of 0.424, 2.575 and 5.150kN, respectively. These compressive loads are equivalent of 0.016, 0.1 and 0.2MPa compressive stresses. Figure 3.12 displays the two
failure modes observed from this test, i.e., failure through the perpend joints (Figure 3.12a) and failure through the brick itself (Figure 3.12b). Under the pre-compressive load of 0.016MPa eight out of ten specimens were tested while the other two specimens failed during handling. Out of the eight specimens tested, seven failed by cracking of the perpend joints. Under the pre-compressive load of 0.1MPa seven out of ten specimens failed at perpend joints while the other three failed by rupture of the brick. In contrast, under the pre-compressive load of 0.2MPa only three out of nine specimens tested failed at perpend joints while the other six all failed by the rupture of the brick. One specimen failed during handling.

Figure 3.12. Horizontal bending failures. (a) Failure through bed and perpend joints and (b) failure through brick at the right hand side.

Figure 3.13 illustrates the typical moment versus rotation behaviour of the specimens under different pre-compressive stresses when failure occurred in the perpend joints. The rotation was calculated using potentiometers across the perpend joint at the tension and compression faces of the specimens. The moment increased almost linearly until it reached peak moment. After that the moment dropped sharply to zero when there was a small compressive force or reached a plateau value when there was a significant compressive force. This plateau denotes that there existed a substantial residual moment after joint failure caused by the torsional frictional resistance moment of the bed joints when subjected to vertical compression. Both the peak moment and residual moment increased as the vertical stress was increased.
The mean peak moment $M_{hp}$ and mean residual moment $M_{hr}$ obtained from the tests are summarized in Table 3.6. The failure mode has been identified so that the effects of compressive stress on the flexural strength under both joint (J) and brick (B) failure modes can be studied. It can be seen from Table 3.6 that firstly, specimens under horizontal bending showed much less variation in the peak moment as compared to those under vertical bending when specimens were subjected to only a small vertical stress. The COV of horizontal peak moment is only 12% as compared to 50% under vertical bending. This may be due to the lower bond strength variability resulting from specimen preparation for this batch of specimens, since the bond wrench test resulted in only 15% variation in contrast to 29% observed for the specimens under vertical bending. Secondly, it can be seen that for cases where failure occurred in the mortar joints, the mean peak moment increased from 0.472 to 0.736 kNm as the pre-compressive stress was increased from 0.016 to 0.2 MPa. However, there was a minor increase from 0.472 kNm at 0.016 MPa to 0.505 kNm at 0.1 MPa. This may be due to the weaker bond strength of the batch of specimens used in the 0.1 MPa compressive stress test, since their mean flexural bond strength from bond wrench test was 0.28 MPa, which is markedly lower than that for the other two batches used. This may indicate that
if failure is through mortar joints, the horizontal flexural strength of masonry is also closely related to the bond strength between brick and mortar as one would expect.

Table 3.6. The peak and residual moments from horizontal bending test.

<table>
<thead>
<tr>
<th>Pre-compression (MPa)</th>
<th>No. of Spec.</th>
<th>Fail By</th>
<th>Peak Moment $M_{hp}$ Mean (kNm)</th>
<th>COV</th>
<th>Residual Moment $M_{hr}$ Mean (kNm)</th>
<th>COV</th>
<th>Bond Wrench Test $f_{mut}$ Mean (MPa)</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.016</td>
<td>7</td>
<td>J</td>
<td>0.472</td>
<td>12%</td>
<td>NA</td>
<td>NA</td>
<td>0.33</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>B</td>
<td>0.381</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>0.1</td>
<td>7</td>
<td>J</td>
<td>0.505</td>
<td>12%</td>
<td>0.127</td>
<td>7%</td>
<td>0.28</td>
<td>19%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>B</td>
<td>0.563</td>
<td>16%</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>0.2</td>
<td>3</td>
<td>J</td>
<td>0.736</td>
<td>2%</td>
<td>0.242</td>
<td>28%</td>
<td>0.34</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>B</td>
<td>0.542</td>
<td>23%</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

It is worth noting that when failure occurs via brick rupture, the mean peak moments under 0.1 and 0.2MPa compressive loads are very close, being 0.563 and 0.542kNm, respectively. Note that the peak moment under a load of 0.016MPa is somewhat lower (0.381kNm). However, in the latter case there was only one specimen that exhibited this failure mode and hence this datum may not have statistical significance. Overall it appears that the existence of a pre-compressive load has increased the torsional moment capacity of mortar bed joints, but it does not affect the flexural strength of the bricks. This explains the experimental observation that as the pre-compressive load was increased and hence the flexural strength due to torsional shearing of the mortar joints, more failures occurred in the brick (Willis et al., 2004).

The effect of a pre-compressive load on the mortar joints can also be found in the behaviour of the residual torsional moment, which increased as the compressive load was raised. The horizontal residual moment versus compressive stress is shown in Figure 3.14. A linear fit with an $R^2$ of 0.9938 indicates strongly that the frictional torsional stress at the mortar and brick interface is a linear function of the vertical compressive stress.
3.6 Four Brick Unit Specimens Subjected to Biaxial Bending

Nine batches, each consisting of ten four-unit specimens, were constructed for testing under simultaneous vertical and horizontal bending moments, as well as different levels of vertical pre-compressive force. In all cases the vertical and horizontal moments were applied such that tension due to both moments occurred on the same faces of the specimens. The vertical compressive load was applied first using load control and held constant. The moments were then applied using displacement control so that displacement in the $M_v$ clamp control actuator and displacement in the $M_h$ control actuator were kept at ratios of 1:2, 1:4 and 1:8. It should be noted that due to the $M_h$ actuator acting simultaneously on two rotating shafts compared to one for the $M_v$ actuator, the corresponding ratios of moments $M_v:M_h$ were approximately 1:1, 1:2, 1:4. Due to some flexibility in the test apparatus as well as the orthotropic character of the specimens the ratios of moments cannot be related directly to the ratios of actuator displacements. Similar to the vertical bending test and horizontal bending test as discussed before, the three levels of pre-compressive forces were 0.424, 2.575 and
5.150kN, being equivalent to pre-compressive stresses of 0.016, 0.1 and 0.2MPa, respectively. The test results are shown in Tables 3.7 to 3.9 for specimens under the targeted vertical and horizontal bending moment ratios of 1:1, 1:2 and 1:4, each subjected to three different levels of pre-compression. Here the peak moment is defined as the moment at which the specimens failed by cracking. The bed joint and perpend joint can fail at the same or different times depending on the moment ratio. Whichever failed first, the moments \( M_v \) and \( M_h \) at that time were regarded as the failure moments.

Table 3.7. Biaxial bending with a target moment ratio of \( M_v:M_h = 1:1 \).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>No. of Spec.</th>
<th>Fail by</th>
<th>Peak Moment</th>
<th>Residual Moment</th>
<th>Bond Wrench Test ( f_{mt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean ( M_{vp} ) (kNm)</td>
<td>Mean ( M_{hp} ) (kNm)</td>
<td>COV ( M_{vp} )</td>
</tr>
<tr>
<td>0.016</td>
<td>9 J</td>
<td></td>
<td>0.084</td>
<td>0.088</td>
<td>38%</td>
</tr>
<tr>
<td>0.1</td>
<td>10 J</td>
<td></td>
<td>0.143</td>
<td>0.148</td>
<td>14%</td>
</tr>
<tr>
<td>0.2</td>
<td>10 J</td>
<td></td>
<td>0.251</td>
<td>0.246</td>
<td>8%</td>
</tr>
</tbody>
</table>

Table 3.8. Biaxial bending with a target moment ratio of \( M_v:M_h = 1:2 \).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>No. of Spec.</th>
<th>Fail by</th>
<th>Peak Moment</th>
<th>Residual Moment</th>
<th>Bond Wrench Test ( f_{mt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean ( M_{vp} ) (kNm)</td>
<td>Mean ( M_{hp} ) (kNm)</td>
<td>COV ( M_{vp} )</td>
</tr>
<tr>
<td>0.016</td>
<td>8 J</td>
<td></td>
<td>0.123</td>
<td>0.216</td>
<td>23%</td>
</tr>
<tr>
<td>0.1</td>
<td>8 J</td>
<td></td>
<td>0.160</td>
<td>0.321</td>
<td>14%</td>
</tr>
<tr>
<td></td>
<td>2 B</td>
<td></td>
<td>0.202</td>
<td>0.266</td>
<td>NA</td>
</tr>
<tr>
<td>0.2</td>
<td>6 J</td>
<td></td>
<td>0.186</td>
<td>0.361</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td>4 B</td>
<td></td>
<td>0.155</td>
<td>0.302</td>
<td>13%</td>
</tr>
</tbody>
</table>
Table 3.9. Biaxial bending with a target moment ratio of \( M_v:M_h = 1:4 \).

<table>
<thead>
<tr>
<th>Pre-comp.</th>
<th>No. of Spec.</th>
<th>Fail by</th>
<th>Peak Moment</th>
<th>Bond Wrench Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MPa)</td>
<td></td>
<td></td>
<td>Mean ( M_{vp} ) (kNm)</td>
<td>Mean ( M_{hp} ) (kNm)</td>
</tr>
<tr>
<td>0.016</td>
<td>10</td>
<td>J</td>
<td>0.069</td>
<td>0.311</td>
</tr>
<tr>
<td>0.1</td>
<td>10</td>
<td>J</td>
<td>0.070</td>
<td>0.326</td>
</tr>
<tr>
<td>0.2</td>
<td>9</td>
<td>J</td>
<td>0.118</td>
<td>0.594</td>
</tr>
</tbody>
</table>

* No residual moments are listed in this table because in these tests the potentiometers were out of range before the residual moment reached the plateau value. So no meaningful data for the residual moment were recorded.

It can be seen from these tables that although the bending moments were exerted using displacement control, the measured ratio of vertical to horizontal bending moments generally corresponded well with the target ratios of 1:1 and 1:2, although for the target ratio of 1:4, relatively higher ratios of 1:4.5, 1:4.7 and 1:5.0 were obtained. This indicates that the shafts used to transmit the bending moments from lever arms to clamps were not very stiff, but adequately stiff to allow the moment ratios to be controlled. When a vertical to horizontal moment ratio of 1:2 was applied, there were a few cases (two in ten under the compressive stress of 0.1MPa and four in ten under 0.2MPa) in which specimens failed in the brick and at relatively low horizontal moments. Brick failure was, however, not observed for moment ratios of 1:1 and 1:4. This may indicate that the brick failure at the moment ratio of 1:2 was unexpectedly caused by the variability in brick quality.

Figures 3.15a to 3.15i show the typical behaviour of \( M_v \) and \( M_h \) observed under the combination of different moment ratios and pre-compressive stresses. Note that the curves for perpend joint opening (except that for 0.016MPa compression) show the gradual development of crack in the descending branch though these curves may be influenced by the flexibility of the rig itself. Graphs for each specimen can be found in Appendix A. Note that for specimens that were subjected to 0.016MPa compressive stress and with a moment ratio of 1:1, \( M_v \) and \( M_h \) are plotted against rotation, whereas for all other cases \( M_h \) are plotted against crack opening of perpend joint on the tensile side of specimens, with \( M_v \) still against rotation. This is because during the majority of the biaxial bending tests the potentiometer on the compression side across the perpend joint was damaged after the first series of specimens were tested. Unfortunately, this
fault was only identified after completion of the whole series of tests. Consequently, no reliable rotation data around the vertical axis were obtained.

(a) $M_v:M_h = 1:1, \sigma = 0.016\text{MPa}$

(b) $M_v:M_h = 1:1, \sigma = 0.1\text{MPa}$

(c) $M_v:M_h = 1:1, \sigma = 0.2\text{MPa}$
(d) $M_v : M_h = 1:2$, $\sigma = 0.016 \text{MPa}$

(e) $M_v : M_h = 1:2$, $\sigma = 0.1 \text{MPa}$

(f) $M_v : M_h = 1:2$, $\sigma = 0.2 \text{MPa}$
Figure 3.15. Biaxial bending moments $M_v$ and $M_h$. 

(g) $M_v:M_h = 1:4, \sigma = 0.016\text{MPa}$

(h) $M_v:M_h = 1:4, \sigma = 0.1\text{MPa}$

(i) $M_v:M_h = 1:4, \sigma = 0.2\text{MPa}$
Table 3.10 lists the number of cases where failures were initiated by cracking of bed joints ($M_v$ dominant) versus that with failure by cracking of both bed and perpend joints ($M_h$ dominant). The results clearly indicate that as the ratio of $M_v:M_h$ was decreased, more failure occurred in both joints. Also for a given $M_v:M_h$ ratio, more failures were initiated by cracking of both joints as the vertical compression was increased. This seems to suggest that application of a vertical compressive stress enhances masonry vertical bending strength more significantly than horizontal bending strength.

<table>
<thead>
<tr>
<th>Compressive Stress (MPa)</th>
<th>Moment Ratio ($M_v:M_h$)</th>
<th>1:1</th>
<th>1:2</th>
<th>1:4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bed Joint</td>
<td>Both Joints</td>
<td>Bed Joint</td>
<td>Both Joints</td>
</tr>
<tr>
<td>0.016</td>
<td>9</td>
<td>1</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>0.2</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

Baker (1979) reported results from tests conducted using an apparatus designed similar in concept to that used in this study. The tests reported in his work used two different brick sizes and 1:1:6 mortar. A total of 310 specimens were tested. Based on the test results, Baker proposed an empirical failure criterion in biaxial bending in the form of an elliptical interaction of $F_h$ and $F_v$ (Figure 3.16), where $F_h$ and $F_v$ are the ultimate extreme fibre stresses in the masonry under biaxial bending in the horizontal and vertical directions, respectively. This failure criterion was later confirmed by Thürlimann and Guggisberg (1988) using a series of tests on wallettes subjected simultaneously to out-of-plane moments parallel and perpendicular to the bed joints and in-plane compressive force perpendicular to the bed joints. The failure criterion implies a reduction in flexural strength in the presence of moments in the other direction as well as an increase in both $F_h$ and $F_v$ in the presence of vertical pre-compression. Baker noted, however, that nearly all specimens failed through the mortar joints rather than by brick rupture and that if failure is through the bricks, one would expect little increase in the horizontal bending strength as a result of increasing the pre-compression.

Figure 3.17 shows the elliptical interaction line fitted to the experimentally obtained vertical flexural strength and horizontal flexural strength for the current study. The elliptical curves fit reasonably well to the current biaxial test data, especially when the specimens are subjected to relatively high compressive loads where variability of failure
moment is relatively small. This lends further support to Baker’s elliptical failure criterion being applicable to the walls under biaxial bending when brick failure is not dominant.

Figure 3.16. Baker’s elliptical failure criterion.

Figure 3.17. Fitted elliptical failure surfaces from this study.
3.7 Bond Tensile Tests

3.7.1 Bond Wrench Test

The bond wrench test was performed to determine the flexural tensile strength of masonry perpendicular to the bed joints. For each batch of four brick unit specimens tested the bond wrench test was also conducted as the only material control test to examine the difference of bond strength of mortar and bricks between each batch. The bond wrench test was conducted in accordance with Standards Australia 2001, AS 3700-2001. The test apparatus was calibrated to obtain parameters $m_1$, $d_1$ and $d_2$, as illustrated in Figure 3.18.

![Figure 3.18. Schematic illustration of the bond wrench test apparatus used.](image)

- $d_1$ = distance from inside face of tension gripping block to the centre of gravity of the bond wrench
- $d_2$ = distance from inside face of tension gripping block to the loading handle
- $m_1$ = mass of bond wrench
- $m_2$ = mass applied at loading point
- $m_3$ = mass of top unit of the specimen and any mortar adhering thereto.

Altogether 30 prisms were constructed for the bond wrench test. The prisms each consisted of six bricks stack bonded. Two prisms (ten joints) were tested for each batch of four unit specimens. For the joints tested almost all of the failure occurred by debonding at the brick/mortar interface (Figure 3.19). Table 3.11 summarizes the flexural tensile strength obtained from each bond wrench test as well as the mean value for each batch and the overall mean value for all batches. For all batches mean $f_{mt}$ ranged from 0.21 to 0.43MPa. Batch No. 9, 10 and 11 showed the largest departure from the mean.
This may be because before the day batch No. 9 was constructed, the bricks were moved outside the laboratory and heavy rainfall wetted the bricks and this would have altered the absorption characteristics of the bricks. The coefficient of variation (COV) for each batch is included to provide a non-dimensional measure of variability between batches. Except batch No. 1 with a COV of 30%, all other batches have COV of less than 20%, which is within the commonly observed range of 20-30% for laboratory prepared specimens (de Vekey et al., 1994; Drysdale et al., 1999).

![Figure 3.19. Example of failure surfaces in bond wrench test.](image)

**Table 3.11. Bond wrench test results.**

<table>
<thead>
<tr>
<th>Joint No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.26</td>
<td>0.29</td>
<td>0.26</td>
<td>0.24</td>
<td>0.36</td>
<td>0.26</td>
<td>0.31</td>
<td>0.36</td>
<td>0.15</td>
<td>0.34</td>
<td>0.43</td>
<td>0.39</td>
<td>0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.21</td>
<td>0.17</td>
<td>0.26</td>
<td>0.28</td>
<td>0.31</td>
<td>0.32</td>
<td>0.41</td>
<td>0.39</td>
<td>0.33</td>
<td>0.32</td>
<td>0.43</td>
<td>0.29</td>
<td>0.29</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>0.3</td>
<td>0.28</td>
<td>0.36</td>
<td>0.28</td>
<td>0.35</td>
<td>0.35</td>
<td>0.27</td>
<td>0.47</td>
<td>0.14</td>
<td>0.28</td>
<td>0.31</td>
<td>0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.52</td>
<td>0.22</td>
<td>0.26</td>
<td>0.38</td>
<td>0.33</td>
<td>0.35</td>
<td>0.27</td>
<td>0.47</td>
<td>0.26</td>
<td>0.27</td>
<td>0.34</td>
<td>0.31</td>
<td>0.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>0.24</td>
<td>0.29</td>
<td>0.32</td>
<td>0.22</td>
<td>0.34</td>
<td>0.29</td>
<td>0.36</td>
<td>0.29</td>
<td>0.21</td>
<td>0.35</td>
<td>0.37</td>
<td>0.41</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>0.29</td>
<td>0.22</td>
<td>0.37</td>
<td>0.37</td>
<td>0.36</td>
<td>0.24</td>
<td>0.29</td>
<td>0.33</td>
<td>0.36</td>
<td>0.23</td>
<td>0.22</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.28</td>
<td>0.24</td>
<td>0.22</td>
<td>0.29</td>
<td>0.27</td>
<td>0.39</td>
<td>0.28</td>
<td>0.3</td>
<td>0.45</td>
<td>0.4</td>
<td>0.19</td>
<td>0.29</td>
<td>0.41</td>
<td>0.39</td>
<td>0.3</td>
</tr>
<tr>
<td>8</td>
<td>0.35</td>
<td>0.26</td>
<td>0.25</td>
<td>0.25</td>
<td>0.31</td>
<td>0.27</td>
<td>0.34</td>
<td>0.28</td>
<td>0.51</td>
<td>0.54</td>
<td>0.24</td>
<td>0.23</td>
<td>0.43</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>9</td>
<td>0.31</td>
<td>NA</td>
<td>0.35</td>
<td>0.39</td>
<td>0.32</td>
<td>0.25</td>
<td>0.26</td>
<td>0.3</td>
<td>0.51</td>
<td>0.42</td>
<td>0.21</td>
<td>0.4</td>
<td>0.34</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.4</td>
<td>NA</td>
<td>0.29</td>
<td>0.29</td>
<td>0.2</td>
<td>0.33</td>
<td>0.36</td>
<td>0.27</td>
<td>0.39</td>
<td>0.58</td>
<td>0.21</td>
<td>0.25</td>
<td>0.34</td>
<td>0.31</td>
<td>0.25</td>
</tr>
<tr>
<td>Mean</td>
<td>0.33</td>
<td>0.25</td>
<td>0.27</td>
<td>0.33</td>
<td>0.29</td>
<td>0.34</td>
<td>0.31</td>
<td>0.31</td>
<td>0.41</td>
<td>0.43</td>
<td>0.21</td>
<td>0.28</td>
<td>0.38</td>
<td>0.35</td>
<td>0.3</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.10</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.08</td>
<td>0.09</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>COV</td>
<td>0.30</td>
<td>0.17</td>
<td>0.14</td>
<td>0.15</td>
<td>0.19</td>
<td>0.15</td>
<td>0.17</td>
<td>0.10</td>
<td>0.19</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

| Overall Mean | 0.32 |
| Overall St. Dev. | 0.08 |
| Overall COV   | 0.23 |
The variability between batches was also of interest since this affects the ability to compare test results of specimens constructed from different batches directly. Since all the specimens were constructed using the same bricks and mortar with the same cement, lime and sand ratio and constructed by the same mason, an investigation was conducted using an analysis of variance (ANOVA) to enable comparison of flexural strengths between the batches. The results are shown in Table 3.12. The aim of the ANOVA analysis was to try to identify whether the variability in the flexural strength was due to the variability within each batch or between batches. The $F$ value is the indicator of this effect. The larger the $F$ value, the stronger the evidence of variability between the batches is. The ANOVA in Table 3.12 was performed on the test results of all batches but excluding batches No. 9, 10 and 11. These three batches were excluded due to being exposed to rain and hence likely to be unrepresentative of typical experimental conditions used. In Table 3.12 the $F$ value of 4.75 and $P$ value of 6.78E-6 indicate that variance between each batch may have played an important role in causing the fluctuation in the test results. This could be due to factors such as workmanship or variation in the quality of mortar ingredients used for each mortar batch, despite the fact that the specimens were prepared by the same mason using the same mix proportions.

Table 3.12. ANOVA analysis of bond wrench test results.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>$F$</th>
<th>$P$-value</th>
<th>$F$ crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>0.135696</td>
<td>11</td>
<td>0.012336</td>
<td>4.745706</td>
<td>6.78E-06</td>
<td>1.880108</td>
</tr>
<tr>
<td>Within Groups</td>
<td>0.275538</td>
<td>106</td>
<td>0.002599</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.7.2 Direct Tension Test

Following ANOVA analysis of flexural tensile strengths obtained from bond wrench testing, especially the observation of variability between mortar batches, direct tension tests were also conducted to study the true local tensile bond strength. Jukes et al. (1998) investigated numerically the effects of tensile strain softening on bond tensile test results. They concluded that the assumption of a linear elastic stress distribution at peak load in flexural tests could result in significant overestimation of the true local tensile bond strength. However, for direct tension tests the effect is expected to be less significant due to the more uniform stress distribution across the joint. Importantly, Jukes et al. (1998) also highlighted that test methods which resulted in non-uniform stress distributions across the mortar joint could produce results which suggested strain
softening even if little softening existed and in any case might lead to overestimation of
the amount of strain softening. Therefore, direct tension tests were used to define more
accurately the true tensile bond strength at a material point for use in the subsequent
finite element modelling. Although the purpose of the direct tension tests in this project
was to obtain the local tensile bond strength of the masonry joints, effort was also made
to measure the softening of tensile strength by measuring reduction of the load and the
opening of the cracks after peak load.

The test arrangement is shown in Figure 3.20. The cylindrical specimen with 70mm
diameter was fabricated by first coring solid circular cylinders, using a water cooled
hollow diamond drill, from two solid extruded clay masonry bricks with dimensions of
232mm long × 76mm high × 110mm thick. Then the two cored brick cylinders were
bonded on the bedding faces with a 10mm thick mortar joint. To prevent the specimens
from being bent during the curing period, the specimens were laid on a steel pipe which
has a half circle cross section (Figure 3.21a). The pipe was raised at one end to
strengthen the bond between mortar and brick by the self weight of the specimen
(Figure 3.21b). It is preferable to cast a couplet specimen first and then core the
complete specimen through brick/joint/brick. However, the approach was tried
unsuccessfully mainly due to the very weak bond strength of the mortar. During the
coring of the couplet the mortar dissolved in the cooling water. The approach was
abandoned.

The cylindrical specimens were then epoxy glued between two circular steel plates at
the top and bottom. The steel pads were sized to be thick enough to avoid bending, thus
providing a uniform axial displacement. It is also recognized the importance of trying to
obtain a uniform crack opening of the joint in order to derive consistent crack energy.
This means the cross sections of cored bricks composing the specimen must ideally be
perfectly parallel and mortar joints must have uniform thickness everywhere. This is
difficult to achieve. Care was taken to minimize the effect of rotation on the joint
opening by using a very stiff rod to connect the upper pad to the load cell. Each
specimen was left sitting in the test apparatus for a few hours before testing to allow the
epoxy to achieve adequate strength.
Three potentiometers were used to measure the deflection across the joint. To catch the crack opening as precisely as possible, instead of gluing the potentiometers directly on the specimens, a specially designed frame made of aluminium was used (Figure 3.20b). Two aluminium rings were held by three aluminium plates at a gauge length of 70mm. The rings were mounted to the specimen with screws and placed across the mortar.
joints in such a way that the joint was approximately in the mid-span between the two rings. The potentiometers were attached to the rings with screws. Before the test the screws attaching the aluminium plates to rings were loosened to allow vertical movement. To minimize the effect of loading speed on the tensile strength the loading speed was deliberately set to low (0.01mm/sec). Two data loggers with logging speed 1 Hz and 5kHz respectively were used to measure the deflection across the joint. The purpose of using one relatively slow logger to measure the pre-peak deflection was to reduce the quantity of data recorded to avoid data overflow for the computer system. A trigger in the data logging system was used to detect a value which was set to very close to the expected residual strength. The trigger then started the second fast data logger when the load dropped to this value. This fast logger was able to trace back and fetch the portion of data including the peak load from the temporary memory of the data logging system and store it permanently. By composing the pre-peak data from the slow logger and post-peak data from the fast logger, the full loading versus deflection graph was derived for each specimen in an efficient way.

12 specimens cured for more than 28 days were tested. The mean flexural tensile strength obtained through bond wrench testing for this batch was 0.14MPa. All of the direct tension test specimens failed at the brick and mortar interface. Table 3.13 shows the peak tensile stresses obtained from the test. The tests show a large variability of tensile strength with coefficient of variation of 41%. The mean tensile strength is 0.32MPa. This is different from the result obtained by Pluijm (1997). It was found in his tests that flexural strength is 1.5 and 1.2 times of tensile bond strength from direct tension tests for two types of the brick mortar combination. He explained that the difference of tensile bond strength resulting from the two test methods was due to the non-linear stress distribution in the bending test at failure. In the current study, the ratio of flexural tensile strength to direct tensile strength is only 0.44. This lower ratio is most likely caused by the substantial differences in the specimen preparation.
Table 3.13. Tensile strength results from the direct tension test.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Tensile Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>0.42</td>
</tr>
<tr>
<td>3</td>
<td>0.48</td>
</tr>
<tr>
<td>4</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
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</tr>
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<td>0.13</td>
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<td>11</td>
<td>0.49</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>St. Dev.</td>
<td>0.13</td>
</tr>
<tr>
<td>COV.</td>
<td>41%</td>
</tr>
</tbody>
</table>

Among the 12 specimens tested, the crack opening of the first five specimens was measured with LVDT. However, because of the noise of the data logger, no sensible data of displacement could be obtained for the softening branch. So the LVDTs were replaced later by potentiometers for the last seven specimens. Figure 3.22 plots the tensile stress versus the crack opening displacement of the seven specimens. Note that the crack opening in the graph is the average of the measurements of three potentiometers. Along with the test results the prediction based on Lourenço and Rots’ exponential softening rule (Lourenço and Rots, 1997) using Equation 3.3 is also plotted.

\[
\frac{\sigma}{f_t} = \frac{f_t w}{G_f^I} e^{\frac{w}{G_f^I} \frac{f_t}{G_f^I}}
\]

Equation 3.3

where \(\sigma\) is the tensile stress, \(f_t\) is the tensile bond strength, \(w\) is crack width and \(G_f^I\) is the mode I fracture energy.

The prediction in Figure 3.22 is based on mean tensile bond strength \(f_t\) of 0.32MPa and mode I fracture energy \(G_f^I\) of 0.002N/mm. For the different combinations of bricks and mortar tested by Pluijm (1997) with the tensile bond strength ranging from 0.22 to 0.36MPa, \(G_f^I\) varied between 0.0017 to 0.0068N/mm. The \(G_f^I\) obtained from the current test is well within that range. Note that although effort was made to keep the test setup as rigid as possible, due to the very brittle nature of tensile failure, the post peak
responses captured in the experiment may have resulted partly from the elastic response of the load cell. However, the magnitude of this effect of the load cell is unknown. Due to this uncertainty the post peak response of tensile strength is not included in the numerical model.

![Figure 3.22. Tensile stress vs cracking opening displacement obtained from the direct tension test.](image)

### 3.8 Compression Test

To determine the modulus of elasticity of the brick and mortar as well as the masonry compressive strength, the same type of brick and mortar as used in the four brick unit tests was used to construct five prisms for the compression test. The prisms, each consisting of seven bricks, were chosen to achieve height to thickness ratios larger than five to reduce effects of platen restraint. They were prepared in accordance with the AS 3700-2001 requirement (Clause C3.2), that is, height-to-width ratio must be between two and five and not less than three courses. The test setup is shown in Figure 3.23. To provide an even loading platform to avoid stress concentrations, plywood capping was used at top and bottom and a spherical seat was provided at the top of the specimens. Three potentiometers, No.1 to 3, were mounted on the front side of each specimen to
measure the masonry, mortar and brick deformation in the vertical direction under compressive load. Another three, No.4 to 6 were mounted on the back side of each specimen. Instead of glue mounting the gauges directly to the specimen, special support brackets were built to be attached to the specimens with pointed bolts to make the measurement of gauge length more precise. Gauges were mounted on the brackets over the gauge lengths, as shown in Figure 3.23. Although the Tinius Olsen machine is not strictly a displacement control machine, crosshead displacement was monitored to attempt to achieve a constant displacement rate.

The specimens were loaded and unloaded three times before being loaded to failure. Displacement on the first loading was ignored. Displacement on 2\(^{nd}\), 3\(^{rd}\) and 4\(^{th}\) loading measured from both sides of the specimens were averaged to obtain the elastic modulus of masonry \((E)\) values. For the first specimen, the load was increased to 200kN before unloading. The failure load of the first specimen was 524kN. For the subsequent specimens loads were increased to 40% of the failure load obtained from the first specimen, around 200kN before unloading. All five specimens failed by crushing of mortar and splitting of bricks. Figure 3.24 shows a specimen failed by splitting of the top three bricks and crushing of the mortar on the right hand side.

Figure 3.23. Compression test setup. Figure 3.24. Prisms failed by splitting.
Table 3.14. Elastic modulus of masonry components.

<table>
<thead>
<tr>
<th>Spec No.</th>
<th>$P_u$ (kN)</th>
<th>$f_m$ (MPa)</th>
<th>$E$ (GPa)</th>
<th>$E_m$ (GPa)</th>
<th>$E_b$ (GPa)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>20.3</td>
<td>15.7</td>
<td>2.3</td>
<td>53.3</td>
</tr>
<tr>
<td>2</td>
<td>497</td>
<td>19.3</td>
<td>12.9</td>
<td>1.6</td>
<td>17.3</td>
</tr>
<tr>
<td>3</td>
<td>524</td>
<td>20.3</td>
<td>22.4</td>
<td>5.5</td>
<td>28.4</td>
</tr>
<tr>
<td>4</td>
<td>568</td>
<td>22.1</td>
<td>13.4</td>
<td>1.8</td>
<td>29.2</td>
</tr>
<tr>
<td>5</td>
<td>466</td>
<td>18.1</td>
<td>26.3</td>
<td>2.7</td>
<td>48.5</td>
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<tr>
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<td>1.5</td>
<td>6.0</td>
<td>1.6</td>
<td>15.1</td>
</tr>
<tr>
<td>COV</td>
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<td>0.07</td>
<td>0.33</td>
<td>0.57</td>
<td>0.43</td>
</tr>
</tbody>
</table>

In Table 3.14 the modulus of elasticity of masonry, $E$, is calculated for each test as secant modulus using the stress and strain obtained between 5% and 33% of the ultimate strength (Drysdale et al., 1999). The stress behaves approximately linearly with strain in this region. Equation 3.4 is used to calculate $E$ and the modulus of elasticity of mortar $E_m$ values for all the tests. For calculating $E$ the average value of potentiometers No.1 and No.4 readings is used for $\delta$ in Equation 3.4. $L$ is the distance between top bracket and bottom bracket. For calculating $E_m$, $\delta$ is the average value of potentiometers No.2 and No.5 readings. Instead of using the measured distance between the two gauges, the 10mm of mortar thickness is used for $L$ in the calculation since the brick is much stiffer than mortar and the deflection of brick within this small range can be ignored. The elastic modulus of brick $E_b$ is not calculated directly using readings from the potentiometers because the potentiometers used in these tests were not sensitive enough to measure the very small deflection of bricks and they did not record sensible data. So $E_b$ is calculated indirectly from $E$ and $E_m$.

$$E = \frac{PL}{A\delta}$$  \hspace{1cm} \text{Equation 3.4}

Traditionally, $E$ is estimated from Equation 3.5, with $k$ obtained from experimental results and ranging from 210 and 1,670. The significant scatter in $E$ values can be attributed to variations in test methods including prism configuration, loading setup, instrumentation and method of calculation. The value $k$ obtained from Table 3.14 is approximately 900, well within this range. In AS 3700-2001 design values for elastic modulus for masonry under short-term loading in the absence of test results are given as $700f_m$ and $1000f_m$ depending on the clay units’ compressive strength and mortar type. The $k$ calculated using the mean values from Table 3.14 is within this range.
3.9  Brick Modulus of Rupture Test

When a wall is subjected to horizontal bending, failure may occur in the bricks when the torsional shear strength of the mortar bed joints is high compared to the flexural strength of the brick unit. Also, as the vertical compressive load increases, the torsional shear capacity of the bed joints increases and there are greater chances of failures occurring in the bricks. This phenomenon was also observed in the four brick unit tests. To model this typical failure mechanism numerically the lateral modulus of rupture of the brick units is one of the important parameters that needs to be measured. To test for this material property for the bricks used to construct the four brick unit specimens, a four-point loading test setup was adopted in accordance with Standards Australia 1997 AS/NZS 4456.15.1997 as shown in Figure 3.25. Three units were glued together end-to-end with epoxy resin and were supported with two rollers at a span of 570mm with another two rollers placed within the central brick unit at a span of 190mm, leaving a distance between centre of the loading rollers and the joint of around 20mm. This satisfies the requirement that the specimen shall have a span to depth ratio not less than 5 and a total length of at least 50mm greater than the span. Also the distance between the two loading lines is a third of the span. The bedding faces are kept vertical to reflect the behaviour of a brick in a real wall when the wall is subjected to out-of-plane loading.

Figure 3.25. Brick modulus of rupture test.
Ten specimens were built and left for four days before testing so that the epoxy resin was able to develop its strength. Vertical load was applied to the steel bar placed above the two loading rollers using an Instron Universal Testing Machine. The load was increased at a uniform rate of 6kN/min until failure. Unfortunately, among the ten test specimens, only four failed by rupture in between the two loading lines. Another four failed at locations between the loading line and support with the remaining two failing in the epoxy joints. Although strictly speaking, the specimens which failed outside the ranges between two loading lines should not be included in the calculation of the average flexural strength of bricks, their failure moments were calculated to provide additional data. These values represent a lower bound for the strength of the central brick. The large number of specimens failing outside the maximum moment region reflects the high variability of flexural strength of the bricks that were used in four brick unit tests (Table 3.15). The specimens’ modulus of rupture $f_{ut}$ is calculated using Equation 3.6. Here failure moment $M$ was calculated as half of the failure load times the distance between the support and loading lines when failure occurred between the two loading lines. When failure occurred outside the range between two loading lines, $M$ was taken as half of the failure load times the distance between the support line and closest point to the support line where cracking initiated.

$$f_{ut} = \frac{M}{Z}$$  \hspace{1cm} \text{Equation 3.6}

**Table 3.15 Brick modulus of rupture.**

<table>
<thead>
<tr>
<th>Specimen No</th>
<th>$f_{ut}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>9</td>
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<td>2.24</td>
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<tr>
<td>Std. Dev (MPa)</td>
<td>1.1</td>
</tr>
<tr>
<td>COV</td>
<td>0.30</td>
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</table>
3.10 Direct Shear and Torsion Shear Tests

When a wall is subjected to pure horizontal bending or diagonal bending, some part of the total moment resistance is contributed to by torsional shearing action on the bed joints. Also there will be direct shear acting near the supports for vertical bending. So both shear bond strength and frictional shear strength of bed joints are important parameters in out-of-plane bending that need to be investigated. Tests to determine the shear strength of mortar joints are amongst the most difficult to perform. It is extremely difficult to achieve a state of pure shear stress in a specimen without inducing bending effects as a result of the nature of the applied load. Numerous testing arrangements have been proposed for the determination of joint shear strength in the masonry composite. The following is a list of possible test methods adopted generally in the laboratory to obtain the shear strength of masonry joint:

- **Couplet test**
  In a couplet test two units connected by a single mortar joint are subjected to loading so as to cause shear sliding failure along the joint plane. Although this testing arrangement is easy and economical relative to other methods discussed later, it is extremely difficult to provide loading and restraint conditions that result in uniform stress distributions on the plane of failure.

- **Triplet test**
  The basic principle of this test is to subject two joints to load simultaneously, producing a symmetrical arrangement with consequent simplification of the apparatus and testing procedure. The major disadvantage associated with using a triplet arrangement is, however, that the measured shear resistance will be based on the weaker of the two joints. The average strength from a series of tests will therefore underestimate the average shear strength of the tested population.

- **Four-unit test**
  Similar to the triplet test but using four units, this test arrangement has the same disadvantage in that the measured shear strength is the minimum strength of the four half joints.
• Uniaxial compression/shear test
With this method specimens are cut from four-unit high, stack-bonded masonry prisms. Through variation of the joint angle with respect to the axis of loading, it is possible to generate different ratios of normal to shear stress on the joint plane and thus derive the shear strength versus normal stress relationship. The two side joints are constructed using a mortar of higher strength to make sure that only the central joint fails. The major disadvantage of this testing arrangement is that only the peak load response of the joint is measurable. No residual shear strength can be measured.

• Torsion test
With this test a section of bed joint between two units is subjected to torsion about an axis normal to the joint to measure the torsional stiffness and the torque at failure. Generally a couplet specimen similar to that in the shear couplet test is used. The advantage of this test is that it relates more closely to the joint shear behaviour which occurs when the masonry is subjected to horizontal bending or diagonal bending. However, if the area of joint subjected to torsion is square or rectangular, then complex, non-uniform stress distribution results.

Experimental investigations have shown that the shear corresponding to slip along one or more bed joints is strongly related to the combined shear bond strength and normal compressive stress. The relationship can be described most commonly with Mohr-Coulomb rule with which it assumes that the joint shear strength is composed of initial shear bond strength (cohesive shear strength) between the mortar and the masonry unit plus a shear friction capacity, which is considered to be proportional to the compressive stress applied normal to the bed joints. The relationship is expressed as:

$$\tau = c + \mu \sigma_n$$  \hspace{1cm} \text{Equation 3.7}

where
\begin{align*}
\tau &= \text{joint shear strength} \\
c &= \text{cohesive shear strength} \\
\mu &= \text{coefficient of friction} \\
\sigma_n &= \text{compressive stress normal to the bed joints.}
\end{align*}
To measure the shear strength between the brick and mortar used in the four brick unit specimens, 12 triplet specimens were built and tested in accordance with European Committee for Standardization EN1052-3 (2002). These specimens were constructed from a new batch of mortar using the same combination of sand, lime and cement as that used for four brick unit specimens. The flexural strength from the bond wrench test was 0.25MPa. Due to the weakness of bond between the brick and mortar, one specimen failed even prior to the test. The remaining 11 specimens were divided into four groups with each group being subjected to one level of normal compressive stress as well as shear force. The four levels of compressive stress used in the tests were 0, 0.1, 0.2 and 0.5MPa. Figure 3.26 illustrates the apparatus used to test the triplet specimens. Each specimen with two stiff steel plates glued to the outer bricks was placed on the two steel rollers. Another stiff steel plate was glued to the top of the middle brick. The shear load was applied to the steel plate through the two loading bars that are attached to the pad of the loading machine. A hydraulic ram was used to apply normal compressive force across the bed joints (not shown in Figure 3.26). Effort was made to keep the pre-compression force constant even after the failure took place. Figure 3.27 shows the gauge arrangement on the specimen. Six potentiometers were mounted on each side of the specimens, with four mounted on top and bottom of the specimens to measure the dilatancy in horizontal direction and two mounted in the middle of specimen to record the relative shearing displacements between the middle and outer bricks.

![Figure 3.26. Triplet shear test setup.](image)
Although considerable care was taken to place the specimen so that the support and loading bars were symmetrically located, for most specimens the two joints failed at different times. This is thought to result primarily from strength variability between the two joints. Figure 3.28 shows the failure surfaces of one of the triplet specimens. Most of the specimens had similar failure surfaces with partially unfilled mortar. This indicates that this batch of specimens may have relatively lower bond strength than those used in four unit tests as all failures in the latter showed completely filled joints. It is unfortunate that such poor workmanship was used for these specimens. Such bed furrowing would certainly have reduced the bond strength.

Figure 3.27. Gauge arrangement.  Figure 3.28. Failed specimens.

Figure 3.29 shows a plot of shear force versus joint slip for four of the tested specimens under 0, 0.1, 0.2, and 0.5MPa pre-compressive stresses. Except for the specimen under zero pre-compressive stress, the two joints were observed to fail at different times, showing two turning points in the curve. The load under which the second joint failed can be larger or smaller than the load under which the first joint failed. Nevertheless, the load at which the first joint fails is conventionally regarded as the failure load. This is because before the point where the first joint failed, both joints were considered to be contributing evenly to the resistance and thus the load at that point was more representative than the load when the second joint failed. The load at failure of the second joint would comprise the inherent strength of the second joint plus the residual frictional resistance of the first joint.
The average shear bond strength and residual frictional shear strength of all tested specimens were calculated and are listed in Table 3.16, showing that both strengths increased as the compression was increased. The shear strength is calculated using half of the failure load divided by the area of the brick bed surface. Figure 3.30 depicts the average shear strength of the 11 specimens tested versus the normal compressive stresses. Also shown is the strong linear relationship of the shear bond strength versus normal stress ($R^2=0.988$) and residual frictional shear strength versus normal stress ($R^2=1$). From the equations derived from the linear relationship a slope before failure of 0.682 (internal friction coefficient) agrees very well with the one after failure, namely, 0.696 (residual friction coefficient). The shear bond strength was found to be around 0.12MPa.

While the triplet test described provided important data for the cohesive shear strength and friction coefficient, the test failed to obtain another critical shear property, namely shear fracture energy $G_f^{II}$. In the triplet test only one load cell was used to measure the total shear force applied to the two joints. Therefore, the shear stress versus single joint slip cannot be plotted. This makes measurement of $G_f^{II}$ impossible.
Table 3.16. Shear strength from the triplet tests.

<table>
<thead>
<tr>
<th>Compressive Stress (MPa)</th>
<th>Spec No</th>
<th>Shear Bond Strength (MPa)</th>
<th>Mean Shear Strength (MPa)</th>
<th>Residual Shear Strength (MPa)</th>
<th>Mean Residual Shear Strength (MPa)</th>
</tr>
</thead>
<tbody>
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<td></td>
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<td>0.18</td>
<td>0.11</td>
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<td>NA</td>
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<td>0.53</td>
<td>0.46</td>
<td>0.4</td>
<td>0.4</td>
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</table>

\[
\tau = 0.682\sigma + 0.122 \\
R^2 = 0.988 \\
\tau_{res} = 0.696\sigma + 0.016 \\
R^2 = 1
\]

Figure 3.30. Average shear strength versus normal stress from triplet tests.

An innovative new torsion shear test method was proposed by Masia et al. (2006a and 2006b). This method, though not applicable to perforated bricks, enables the assessment of the mortar joints shear behaviour under combined shear and compression loading. This method is adopted here to obtain characterization of the shear behaviour.
The test subjects an annular cylindrical specimen containing a single mortar joint (Figure 3.31) to a combination of compressive force normal to the mortar joint and torsion about the longitudinal axis of the specimen. Masia et al. (2006a and 2006b) were able to show that this test setup resulted in very close to uniform distributions of shear and normal stresses across the mortar joint.

The test apparatus, illustrated in Figure 3.32, was mounted within an Instron Universal Testing Machine. The annular cylindrical specimen was first epoxy glued between two end plates. One end plate is fully fixed. The other end plate is attached to a shaft which is free to rotate about and move along the longitudinal axis of the specimen. Bearings are used to minimize any resistance to both of these degrees of freedom. The specimen was glued in situ into the apparatus to avoid any disturbance and/or residual stress which may arise from gluing to the end plates first and then bolting into the apparatus.
The specimen was first subjected to a compressive force normal to the mortar joint. This force is applied by a hydraulic jack. A load cell is positioned between the jack and the rotating shaft and the hydraulic pressure is automatically controlled during testing so that the compressive force remains constant at all stages. Also the dilation of the joint is not prevented.

Figure 3.32. Torsion shear test apparatus.

The rotation of the specimen is introduced by moving at a constant rate, the cross head of the testing machine which contacts at a point on a lever that is fixed to the rotating shaft of the apparatus. The apparatus is very stiff and so controlling the machine cross head displacement is assumed to result in very close to a constant rate of increase of joint rotation. The rotation is increased until the mortar joint is cracked and is increased further until the load reaches a plateau value. The rotation is applied very slowly to result in quasi-static conditions.

Through all stages of loading the following information is continuously logged: normal compressive force, machine cross head displacement and applied force, shearing displacement across the mortar joint from which the relative rotation is calculated and normal displacement across the mortar joint.
Ten annular specimens with outer radius of 47.5mm and inner radius of 36.0mm were prepared for testing. For each specimen, a pair of annular bricks were made first by coring circular cylinders, using a water cooled hollow diamond drill, from two solid extruded clay masonry bricks with dimension of 232mm long × 76mm high × 110mm thick. Efforts were made to try to make the thickness of the annular bricks as even as possible. Then the pair of annular bricks was bonded together with a 10mm mortar joint and placed in an inclined half pipe for curing (Figure 3.21). Thus the overall length of each specimen was 162mm (76 + 10 + 76). Bond wrench tests were also carried out to obtain the mean flexural bond strength of the batch, which is 0.14MPa with a COV of 26%.

Although the specimens were cured for more than 28 days before testing, due to the weak bond strength of the mortar two of the ten specimens failed even before the tests. Among the eight specimens tested, one failed during test setup. Only residual torque due to the friction was measured for this specimen. For another specimen, the normal compressive force suddenly increased during the post-peak stage of the test due to the hydraulic pump malfunctioning resulting in crushing of the joint. Therefore, only the peak load for that specimen was obtained. Among the other specimens, some failed by diagonal cracking across the mortar joints followed by failure along the mortar and brick interfaces (Figure 3.31a). For others failure occurred only at the mortar and brick interfaces (Figure 3.31b).

Figures 3.33a and 3.33b show the torque versus rotation response recorded from a typical test. Figure 3.33a shows the full range behaviour of the torque versus rotation, including pre-peak response and post-peak response. Figure 3.33b gives a clear view of the response just before the peak. It can be clearly shown from Figure 3.33b that the torque initially increases almost linearly as the rotation is increased, followed by the gradual reduction of the stiffness as the rotation is further increased. Figure 3.33a shows that after reaching the peak load the torque reduces sharply at first and then more gradually until it reaches the residual torque capacity.
Figure 3.33. Typical recorded torque vs rotation response.

(a) Full range of test

(b) Peak torque
Table 3.17 lists the normal compressive stress $\sigma_n$, the peak torsional shear stress $\tau_u$, residual shear stress $\tau_{\text{residual}}$ for each of the specimens tested. $\tau_u$ and $\tau_{\text{residual}}$ are both calculated using Equation 3.8.

$$\tau = \frac{3T}{2\pi(r_o^3 - r_i^3)}$$

Equation 3.8

In this equation $T$ is the measured torque (peak or residual as appropriate), $r_o$ is the outer radius of the cylindrical specimen and $r_i$ is the inner radius of the cylindrical specimen. The derivation of this equation is based on the plastic theory for shear stress distribution. Detailed description of this theory can be found in Chapter 4 in the section discussing the evaluation of the torsion shear test (Section 4.9).

Table 3.17. Torsional shear tests.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\sigma_n$ (MPa)</th>
<th>$\tau_u$ (MPa)</th>
<th>$\tau_{\text{residual}}$ (MPa)</th>
<th>$G_f^{II}$ (N/mm)</th>
<th>$G$ (MPa)</th>
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</thead>
<tbody>
<tr>
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<td>0.49</td>
<td>NA</td>
<td>NA</td>
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</tr>
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<td>0.6</td>
<td>0.31</td>
<td>0.05</td>
<td>350</td>
</tr>
</tbody>
</table>

Figure 3.34 shows the peak $\tau_u$ and residual $\tau_{\text{residual}}$ shear stresses plotted against the normal stress $\sigma_n$. Note that the unusually large value of $\tau_u$ for specimen 5 has been excluded from this figure. Linear regression lines were fitted to both data series of $\tau_u$ and $\tau_{\text{residual}}$. It can be seen from this figure that even with the limited tests conducted, a sensible Mohr-Coulomb relationship can be established between the peak shear stress and normal compressive stress. From this relationship the cohesion $c_0$ of 0.22 MPa and internal friction coefficient $\mu$ of 0.9 can be obtained. Note that the ratio of shear bond strength to the flexural bond strength in this case is 1.6, which is quite comparable to the ratios obtained by other researchers (Pluijm, 1993). The regression line fitted to the residual shear stress data gives a residual friction coefficient $\mu_r$ of 0.56. Ideally the line of residual shear stress against normal compressive stress should pass through the origin since once the joint is fully cracked there should be no shear resistance under zero compressive stress. However, the small non-zero value of 0.0779 was thought to have resulted from the small number of specimens tested.
The shear fracture energy $G_{f}^{II}$ for each test was also calculated and listed in Table 3.17. The fracture energy against the normal compressive stress is shown in Figure 3.35. The expected increase in fracture energy with normal compressive stress can be observed. A linear regression line is fitted to the data series. However, a negative $y$-axis intercept is obtained, which has no physical meaning. Again this may be due to the limited amount of test data available at the higher compressive stresses which results in considerable variability in the data.

Efforts were also made to determine the elastic shear modulus $G$ for each test by computing the slope of the loading branch of the shear stress $\tau$ versus displacement response using a secant drawn between 5% and 33% of peak shear stress. The shear strain is calculated by dividing the shear displacement by the joint thickness (10mm). However, as illustrated in Table 3.17, the $G$ obtained shows considerable scatter. The large scattering may be caused by the variations in the mortar joint thicknesses between specimens as well as insufficient accuracy in the measurement of the very small shear displacements occurring in the early stages of loading.
3.11 Summary

An extensive test program was carried out to study the behaviour of four brick unit specimens subjected to separate vertical, horizontal as well as biaxial bending in combination with various levels of vertical compressive stress. These tests enable one to gain a better fundamental understanding of the flexural and shear behaviour of mortar joints when subjected to various bending conditions. Furthermore, the tests provided valuable data for validating the accuracy of the numerical model proposed in Chapter 4.

An extensive experimental study of the material properties of masonry components was also carried out. These included tests to obtain flexural tensile strength, direct tensile strength, shear strength and torsional shear strength between brick and mortar, compressive strength and stiffness of the masonry components and brick modulus of rupture. In particular, an innovative shear test method was developed as a result of this work. This test will eventually enable accurate measurement of the shear strength of a mortar joint, which plays a crucial role in the biaxial bending of masonry walls. A series of preliminary test data have been reported with this new method and it has been shown
that the method, though it may not be suitable for routine testing due to its complexity, provides pre and post cracking data to allow characterization of shear behaviour. This method, though still in its developmental stage, will eventually allow accurate determination of the parameters required for the finite element micro modelling of mortar joint shear behaviour.
CHAPTER 4

DETAILED FINITE ELEMENT MICRO-MODEL
4. DETAILED FINITE ELEMENT MICRO-MODEL

4.1 Introduction

Historically, masonry structures have been designed and constructed based on rule-of-thumb methods, which relied heavily on the masons’ experience, building skills and some basic understanding of the action of forces within the structure. Although there are countless historical buildings which have withstood centuries of natural disasters and human wars, masonry structures are generally prejudicially portrayed as an expensive and fragile building type as compared with modern structures such as concrete or steel. However, over the past few decades researchers have been gradually drawing their attention to masonry structures because of their heritage value and popularity. In particular, considerable efforts have been made in recent years on masonry bending behaviours, and as a result, various design standards have been developed and adopted in many countries. Nevertheless, these standards are either empirical or semi-empirical due to lack of fundamental understanding of the complex behaviour of this composite material.

Masonry, which is a periodical composition of two physically contrasting materials, namely very “stiff” bricks and relatively “soft” mortar, poses significant difficulties on conventional analytical methods. However, the advent of modern computing technology over the past two decades has provided researchers with a very powerful tool to gain an insight into masonry structural behaviour and aided the derivation of more rational design rules. In particular, the finite element method is one of the most widely used approaches that have been adopted by researchers to simulate the behaviour of masonry structures. The method itself has been refined so much over the past two decades that it has now become a standard modelling tool in a significant number of software packages. Nevertheless, how the properties of masonry and its constituents should be implemented within these software packages is up to the individual researchers to decide. Furthermore, modelling progress has been hindered by lack of communication between modellers and experimentalists (Lourenço, 1996), notwithstanding the obvious importance of reliable and useful experimental data in successful modelling of masonry structures.
Over the past several years various numerical implementation methods have been proposed. These methods range from treating each brick and mortar individually and studying the interaction of the brick and mortar so that the overall behaviour of the wall can be derived (i.e., micro modelling), to considering the masonry composites as a single estimated anisotropic material so that more application-oriented analysis can be applied (i.e., macro modelling). Some of the major numerical approaches adopted in the literature are briefly described and compared in Section 4.2.

In this study numerical modelling of the “single joint” four brick unit specimens of masonry as examined experimentally in Chapter 3 has been carried out. The specimens were subjected to biaxial bending as well as vertical pre-compression. A discrete micro modelling method was proposed and adopted. This model was considered to be more suitable than a macro approach since the focus of the current research was on the detailed behaviour of the “single joint” specimens. The results are presented and discussed in this chapter. The results from the finite element modelling are closely compared with the experimental data reported in Chapter 3.

A detailed account of the modelling strategy used in this research is provided in Section 4.3. Parameter selection and mesh sensitivity are discussed in Sections 4.4 and 4.5. The numerical results and discussions for four brick unit specimens subjected to vertical, horizontal and biaxial bending are presented in Sections 4.6 – 4.8, respectively. Finally, as detailed in Section 4.9, the numerical model is applied to evaluate a newly proposed torsion shear test method. The usefulness and power of this model in the field of masonry research is demonstrated.

4.2 Micro Modelling Versus Macro Modelling

Micro modelling is a detailed analysis of masonry wherein each unit, mortar and unit/mortar interaction is represented. The main advantage of the micro-model lies in its capability to capture point-to-point prediction of the stress and strain state on a masonry panel. If proper knowledge is known about the properties of each component and their interaction, this model is capable of representing the masonry more closely than any other model. The main drawback of the micro-model is that it is difficult to apply the model to structural analysis of complex walls of existing buildings, because it goes too much into the details of each brick and mortar. Thus it is computationally expensive and
takes long simulation time. Having said that, rapid progress in modern computing technology may make what is impossible now become a reality in the not-too-distant future.

Improvements to the micro-model are still being sought. One widely used method is the so-called simplified micro method, where units are expanded and represented by continuum elements, and the behaviour of the mortar joints and uni-mortar interface is represented together by discontinuous elements. However, this simplified model has so far only been applied to laboratory specimens and small masonry panels.

In contrast, macro modelling is intended for large-scale structural analysis because it requires reduced computation time compared to micro modelling. This macro method focuses only on the global structural behaviour. The knowledge about units, mortar and their interaction is neglected. The macro-model does not make any distinction between units and mortar joints. Consequently, the material is simply smeared as an anisotropic composite that possesses an average stress versus strain relationship. The main disadvantage of the macro-model lies in the fact that the material parameters must be obtained through masonry tests of a sufficiently large specimen under homogeneous stress state, which is very difficult to achieve in the laboratory. Furthermore, a complete macro-model must reproduce an orthotropic material with different tensile and compressive strengths and stiffness along the material axes as well as different inelastic behaviour along each material axis. The complexity of orthotropic behaviour makes it difficult to obtain robust numerical tools, despite their attractiveness due to the simplicity of the mechanical assumptions used in the approach.

There is a third technique which comes somewhere in between the micro and macro methods. This method is called homogenization which works by identifying an elementary cell or unit cell, which generates an entire panel by regular repetition. Starting from knowledge about the mechanical properties of the constituent materials (bricks and mortar joints) and the geometry of the unit cell, a series of empirical analytical expressions of constitutive relations in terms of average stresses and strains can be obtained by fitting the experimental data. Therefore, a field problem can be written on the unit cell so that average values for the homogenized masonry material can be obtained.
Three approaches have been used to model the unit cell. The first one is to use a simplified geometry to represent the actual complex geometry of the unit cell. This approach has the advantage of only requiring limited parameters. The latter include Young’s modulus and Poisson’s ratio of brick and mortar. However, this approach can only be used to derive the linear characteristics of masonry. In the presence of non-linear behaviour the technique is likely to yield large errors (Anthoine, 1995; Lourenço, 1996).

The second approach is to model the unit cell as a periodic composite continuum. The finite element method is used to obtain a numerical solution. This approach is applicable to the analysis of non-linear behaviour of a complex masonry unit cell by solving for all possible macroscopic loading histories (Anthoine, 1997; Wu & Hao, 2006). Here the modelling parameters required include Young’s modulus and Poisson’s ratio of brick and mortar and their fracture parameters including fracture energy, compressive and tensile strengths and strength softening or hardening.

The third approach is to simulate the composite of the unit cell in detail (Zucchini & Lourenço, 2002). Apart from simulating the properties of brick and mortar, it adopts the micromechanical and microstructural model to simulate the interactions between brick and mortar. In addition to the parameters that are also required in the second approach, the model needs extra parameters such as the bond strength (tensile and shear) and the fracture energy and strength softening at the brick and mortar interfaces. These parameters are quite difficult to determine in the micromechanical model for macroscopic analysis.

The advantage of the homogenization method is that it provides an alternative to the costly and time consuming laboratory experiments of masonry specimens. Its disadvantage lies in the fact that similar to the micro-model this method requires intensive computation since the field problem on the unit cell has to be solved numerically for every time step, although the time required is much less than the micro-model. More importantly, the homogenization method cannot provide fracture mechanisms.
The choice between the micro- or macro-model depends very much on the field of application. The micro-model finds its application in the simulation of laboratory studies on small wall panels or specimens where a better understanding of local masonry behaviour is important. On the other hand, the macro-model is generally applicable when emphasis is on the structural response of large-scale walls and complex masonry structures. A compromise must be made between accuracy and efficiency in macro modelling.

4.3 Modelling Strategy

In this study the complex behaviour of masonry walls has been investigated in detail through experimentally and numerically studying the “single joint” four brick unit specimens of masonry subjected to horizontal, vertical and biaxial bending as well as vertical pre-compression. Since the scope of this research was mainly on the out-of-plane behaviour of these “single joint” four brick unit specimens, especially the behaviour of crack formation, peak load capacity and residual load capacity, a discrete micro-model was considered to be suitable and therefore adopted for simulation of the small specimens used in this work.

Page (1978) was the first to adopt a micro-model in the simulation of masonry behaviour. In his work units were modelled with elastic continuum elements and mortar with interface elements. The yield surfaces separating elastic and plastic ranges were obtained experimentally and represented three different failure modes: tension mode, pure shear failure mode in the joint and a change of failure mode from pure shear failure in the joint to combined joint/unit failure. The tension failure was brittle, whereas shear strength was assumed to be hardened following an experimental curve. The model was used to reproduce the stress distribution in bending tests on a deep masonry beam under vertical load. Based on this model, attempts were made by other researchers to use interface elements to model masonry structures. These later models included tensile (brittle) and shear failure (brittle or elastic/ideal plastic) for the mortar joint. Lourenço (1996) provided a summary of all previous models and proposed a multi-surface plasticity envelope. In Lourenço’s strategy the elastic domain is bounded by a composite yield surface which includes tension, shear and compression failure.
It should be pointed out that when a wall is subjected to out-of-plane load, failure patterns are generally dominated by cracking and slipping at the unit/mortar interface and cracking of the units. Nevertheless, in real situations joint failure may also be due to cracking within the mortar itself. To simplify the problem in the current study the joint was assumed to be elastic, and any joint failure was assumed to occur in the bond at the mortar/brick interface.

A more detailed description of the modelling approach is provided in the remainder of this section. To assist the discussion some of the important points that have direct relevance to the modelling used have been recapitulated from ABAQUS/Standard (2002). Where appropriate, any modifications and assumptions used in this work are also highlighted.

An important feature of the micro modelling used in the current study was the adoption of contact mechanics. Contact mechanics, which initially found its application in civil engineering for rock mechanics, has been used to solve many engineering problems. It is so widely used that most finite element software packages include it as a standard analysis tool. In this study a commercially available software, ABAQUS/Standard was used as the numerical analysis tool. However, the default contact model within ABAQUS/Standard is not fully applicable to masonry analysis because it does not allow the inclusion of the adhesive strength between brick and mortar. Therefore, in this work a special user-defined model had to be introduced and incorporated into the software.

One type of contact model offered by ABAQUS/Standard is the so-called surface contact. It works by defining a pair of contacted surfaces, namely master surface (usually for stiffer surface) and slave surface (usually for softer surface). The slave surface can come into contact with the master surface (close) or separate from the master surface (open). When the slave surface comes into contact with the master surface, ABAQUS/Standard defines the contact conditions between the two bodies using a strict “master-slave” algorithm in which the slave nodes are constrained not to penetrate into the master surface; however, the nodes of the master surface can, in principle, penetrate into the slave surface. The contact direction is always normal to the master surface. Only the master surface is used as a surface, where its geometry and orientation are taken into consideration. The only data that ABAQUS/Standard needs
from the slave surface is the location of its nodes and the surface area associated with each node. The change of states from closed to open is governed by an appropriately defined contact law, which defines the normal and tangential force interactions.

In ABAQUS/Standard the surface contact problem can be formulated in both 2-dimensions (2D) and 3-dimensions (3D). To properly model the out-of-plane behaviour of masonry, especially the torsion behaviour in the brick bedding surface, a 3D finite element approach was regarded as more appropriate and was adopted in this work. With this 3D model linear elastic continuum elements are chosen to represent both bricks and mortar. Each element has eight nodes and four integration points. Non-linear contact surfaces were chosen to represent the bond between brick and mortar (at the bed and perpend joints) and at the potential vertical failure interface at the mid-length of each brick, as illustrated in Figure 4.1. Such a discrete model allows for numerical representation of crack formation and propagation at these interfaces.

The tangential and normal interaction at the brick and mortar interface was defined by initial and residual failure surfaces (Figure 4.2). Since the cases considered in this work are failures initiated by debonding or slipping of brick and mortar components, with very few cases of failure by crushing of mortar, the contacted surfaces in the normal direction were assumed to have infinite capacity under compressive forces. The initial failure surface was defined by a Mohr-Coulomb failure surface under both compression

Figure 4.1. Representation of brick, mortar and bond in the micro modelling of a four brick unit specimen.
and tension with a tension cut off. As the failure progressed, all the tensile strength capacity was consumed. The residual failure surface was then defined by a Mohr-Coulomb failure surface with zero tensile strength.

![Graph showing Initial Yield Surface and Residual Yield Surface](image)

**Figure 4.2. Failure surfaces of brick and mortar interface.**

Another assumption made in the modelling was that bed and perpend joints had the same flexural capacity and the same shear torsional capacity. This assumption was based on the test results reported by Willis *et al.* (2004). These authors built two batches of specimens of two brick units with mortar on the surfaces where perpend joint forms. They tested the flexural tensile strength of these specimens using a two point bending test setup with the bedding faces in the vertical direction, to emulate the orientation of the units in a wall subjected to out-of-plane bending. They found that the results were comparable to those from bond wrench tests of bed joints. Similar specimens were also built for shear strength testing and were found to have approximately the same ultimate shear strength as that of bed joints.

ABAQUS/Standard provides a few default surface constitutive models for defining the normal force versus normal displacement behaviour. The normal contact pressure between two surfaces at a point, \( p \), is usually defined as a function of the
interpenetration of the surfaces, $h$. One of the models, called “Hard” Contact, is defined as

$$p = 0 \text{ for } h < 0 \text{ (open)}, \quad h = 0 \text{ for } p > 0 \text{ (closed)}. \quad \text{Equation 4.1}$$

The “hard” contact minimizes the penetration of the slave nodes into master surface and does not allow the tensile stress across the interface. When surfaces are in contact, any contact pressure can be transmitted between them. The surfaces are separated if the contact pressure reduces to zero. Separated surfaces come into contact when the clearance between them reduces to zero. The hard contact constraint is enforced through a Lagrange multiplier (classical Lagrange multiplier method and the augmented Lagrangian method) and representing the contact pressure in a mixed formulation. For the case when a slave surface consists of a node-based surface, the contact pressure is equal to the normal contact force divided by the cross-sectional area at the contact node. In this work the default cross-sectional area of 1.0 was adopted.

ABAQUS/Standard also provides a “modified hard contact” model, which is useful for cases where negative pressure values, i.e., tension (surface cohesion) may be allowed. The model permits some limited penetration of the slave nodes into the master surface and some transfer of tensile stress across the interface. The pressure and overclosure relationship is illustrated in Figure 4.3. Note that overclosure is defined in this figure as the depth of penetration of one node from the master surface into the slave surface. Clearance is defined as the distance of a node from the slave surface away from the master surface.

The modified hard contact pressure-overclosure relationship differs from the strict hard contact in that the former is implemented to allow up to $n$ points on a surface to “overclose” by a certain distance, $h_{\text{max}}$, before contact pressure is transmitted. If the overclosure exceeds $h_{\text{max}}$, the contact state is changed from open to close. The slave node is moved back to the master surface, and “hard” contact is enforced. It is also implemented to allow $n$ points on the surfaces to violate the strict contact condition to transmit “tensile” contact pressures up to a particular value, $p_{\text{max}}$, before they separate, as shown in Figure 4.3. If either $h_{\text{max}}$ or $p_{\text{max}}$ is exceeded at a node, ABAQUS/Standard will change the contact status.
The modified hard contact model was used in this work to simulate the bond between mortar and brick unit in the normal direction. In modelling the interaction between mortar and brick under the compressive load, \( h_{\text{max}} \) was defined as zero so that no overclosure was allowed. The nodes on the mortar and brick surfaces were initially bonded together by defining \( h = 0 \). At this moment any compression can be transmitted between brick and mortar. \( p_{\text{max}} \) was defined using the maximum tensile stress between the node pairs before they can be separated from each other. The nodes on brick and mortar surfaces stay bonded before the normal stress reaches \( p_{\text{max}} \). After reaching \( p_{\text{max}} \) the bonded nodes separate from each other to simulate the crack formation. When this happens, the tensile stress between the node pairs drops to zero instantly.

In the tangential direction the maximum allowable shear stress across an interface is calculated based on the contact pressure between the contacting bodies through a Coulomb friction model. The default Coulomb friction model implemented by ABAQUS/Standard allows two contacting surfaces to carry shear stresses up to a certain magnitude across their interface before they start sliding relative to one another. The state before sliding is called sticking. The shear stress at which sliding initiates is called the critical shear stress \( \tau_{\text{crit}} \). This stress is defined by ABAQUS/Standard as a fraction of the contact pressure, \( \sigma \), between the surfaces \( \tau_{\text{crit}} = \mu \sigma \). The stick/slip
calculations determine when a point transitions from sticking to slipping or from slipping to sticking. The fraction, $\mu$, is commonly known as the coefficient of friction.

$\mu$ is assumed to be same in all directions (i.e., isotropic friction) on the contacted surfaces. For 3D simulation there are two orthogonal components of shear stress, $\tau_1$ and $\tau_2$, along the interface between the two bodies. These components act in the slip directions for the contact surfaces. The two shear stresses are combined into an equivalent shear stress, $\tau$, for the stick/slip calculations, where $\tau = \sqrt{\tau_1^2 + \tau_2^2}$. Also the two slip components, $\gamma_1$ and $\gamma_2$, are combined into an equivalent slip, $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$.

The slope of the shear stress versus total slip relationship may be defined as finite or infinite while in the “sticking” state. With a finite relationship some incremental slip may occur even though the model determines that the current friction state is “sticking”. With an infinite relationship this incremental slip is not allowed. When the mortar joint thickness is explicitly modelled, the infinite relationship should be more applicable for modelling the brittle tangential behaviour of the brick and mortar interface. However, in order to obtain a stable solution/convergence the finite relationship was adopted in this work. This may cause an overall soft response if the simulation result is compared to the experimental result.

Figure 4.4a illustrates the ABAQUS default relationship for shear stress versus slip, which is analogous to elastic-plastic material behaviour. However, to model the post-peak response of masonry in bending accurately, the shear softening process has to be incorporated into this relationship. ABAQUS/Standard has a subroutine which facilitates the customization of the shear behaviour by the user to suit their specific purpose. This subroutine was utilized here to define a modified Coulomb friction model. The modified shear stress versus slip relationship, which includes the shear softening process, is illustrated in Figure 4.4b.

In the modified model the tangential shear stress is defined as a function of slip. The shear stress first increases linearly as the relative displacement of adjacent nodes on the mating contact surfaces increases. In this elastic range the allowable elastic slip $\gamma_{\text{crit}}$ must be selected. $\gamma_{\text{crit}}$ is calculated by ABAQUS/Standard as a small fraction of the
“characteristic contact surface length”, \( l_i \). ABAQUS/Standard scans all facets of the slave surfaces when calculating \( l_i \). The allowable elastic slip is given as

\[
\gamma_{\text{crit}} = F_f l_i
\]

Equation 4.2

where \( F_f \) is the slip tolerance.

![Diagram of tangential shear stress versus slip](image)

(a) ABAQUS/Standard default model  (b) Modified model by including shear softening

Figure 4.4. Tangential shear stress versus slip.

Use of a large value for \( F_f \) in the simulation makes the program converge more quickly, albeit at the expense of solution accuracy. On the other hand, if \( F_f \) is chosen to be very small, convergence problems may occur. The default value in ABAQUS/Standard is 0.005. However, in this study this value was changed to 0.0001 in order to get as close as possible an infinite relationship between \( \tau \) and \( \gamma \) while retaining numerical stability. The elastic stiffness, \( k \), required in ABAQUS/Standard, was calculated as \( \tau_{\text{crit}} / \gamma_{\text{crit}} \). The critical shear strength, \( \tau_{\text{crit}} \), was assumed to be isotropic across each contact surface. Between each contacted pair of nodes \( \tau_{\text{crit}} \) was assumed to follow the classic Mohr-Coulomb rule, as defined in Equation 4.3.

\[
\tau_{\text{crit}} = \mu \sigma + c
\]

Equation 4.3
where \( \mu_i \) is internal friction coefficient, \( c \) is cohesion and \( \sigma \) is the normal stress (positive for compression and negative for tension). In the elastic range \( c \) is equal to \( c_0 \), with \( c_0 \) being the initial cohesive shear strength.

The softening process in shear stress after reaching its peak was implemented by decreasing \( c \) either exponentially (Equation 4.4 and Figure 4.5a) or linearly (Equation 4.5 and Figure 4.5b), as slip progressed.

\[
c = c_0 \exp\left(-\frac{c_0}{G_{f}^{II}}(\gamma - \gamma_{\text{crit}})\right) \quad \text{Equation 4.4}
\]

\[
c = c_0 - \tan \phi(\gamma - \gamma_{\text{crit}}) \quad \text{Equation 4.5}
\]

In the above equations \( G_{f}^{II} \) is called mode II fracture energy, which is the area defined between the shear stress-displacement diagram and the residual dry friction shear level (Figure 4.5a). \( \phi \) is the angle of linear softening of shear stress (Figure 4.5b).

![Shear Stress vs Displacement Diagram](image)

(a) Exponential softening  
(b) Linear softening

Figure 4.5. Assumed shear softening modes.

During softening the shear strength drops and approaches a residual plateau value. From this point on only the frictional shear stress \( \tau_{\text{res}} \), calculated using \( \mu_r \sigma \) where \( \mu_r \) is the residual shear frictional coefficient, dominates the relative tangential movement of the nodes of the pair. The initial and residual friction coefficients, \( \mu_i \) and \( \mu_r \), have been reported to be different by most literature. However, for simplicity the two coefficients
were assumed, in this work, to have the same value. The consequence of this assumption is discussed in Section 4.7.

In ABAQUS/Standard the elastic tangential slip, $\gamma_i^{el}$, for the elastic stick formulation is defined as the reversible relative tangential motion from the point of zero shear stress. Here $i$ stands for one of the two orthotropic directions on the contact surface. The elastic shear slip is related to the interface shear stress through Equation 4.6.

$$\tau_i = k \gamma_i^{el}$$  \hspace{1cm} \text{Equation 4.6}

where $k$ is the (current) “stiffness in stick”, which follows from the following relation:

$$k = \frac{\tau_{\text{crit}}}{\gamma_{\text{crit}}}$$  \hspace{1cm} \text{Equation 4.7}

Since $\tau_{\text{crit}}$ is dependent on contact pressure at the contact point, $k$ may change during the analysis. The behaviour remains elastic as long as the equivalent stress does not exceed the critical stress. Hence,

$$\gamma_i^{el}(t + \Delta t) = \gamma_i^{el}(t) + \Delta \gamma_i$$  \hspace{1cm} \text{Equation 4.8}

where $\Delta t$ is the time increment in a static analysis.

The above expressions hold if the equivalent shear stress remains less than the critical stress. If the equivalent stress exceeds the critical stress, slip must be taken into consideration so that the condition $\tau = \tau_{\text{crit}}$ is satisfied. Let the starting situation be characterized by the elastic slip $\gamma_i^{el}$ and resultant slip $\gamma_{eq}^{sl}$. The critical stress at the end of the increment follows from the contact pressure, $p$. Let the (as yet unknown) elastic slip at the end of the increment be $\gamma_i^{el}$ and the slip increment be $\Delta \gamma_i^{sl}$. Compatibility requires that

$$\Delta \gamma_i = \gamma_i^{el} - \gamma_i^{el} + \Delta \gamma_i^{sl}$$  \hspace{1cm} \text{Equation 4.9}

and the shear stress at the end of the increment follows from the elasticity relation,
\[ \tau_i = k \gamma_i^{el} = \frac{\tau_{crit}}{\gamma_{crit}} \gamma_i^{el} \quad \text{Equation 4.10} \]

The slip increment is related to the stress at the end of the increment with the backward difference approach:

\[ \Delta \gamma_i^{sl} = \frac{\tau_i}{\tau_{crit}} \Delta \gamma_{eq}^{sl} \quad \text{Equation 4.11} \]

With these equations and the critical stress equality \( \tau = \tau_{crit} \) it is possible to solve for \( \gamma_i^{el} \), \( \Delta \gamma_i^{sl} \) and \( \tau_i \). Elimination of \( \gamma_i^{el} \) and \( \Delta \gamma_i^{sl} \) from the above equations yields

\[ \Delta \gamma_i = \frac{\tau_i}{\tau_{crit}} \gamma_{crit} - \gamma_i^{el} + \frac{\tau_i}{\tau_{crit}} \Delta \gamma_{eq}^{sl} \quad \text{Equation 4.12} \]

or

\[ \tau_i = \frac{\gamma_i^{el} + \Delta \gamma_i}{\gamma_{crit} + \Delta \gamma_{eq}^{sl}} \quad \tau_{crit} \quad \text{Equation 4.13} \]

It is convenient to define the “elastic predictor strain”

\[ \gamma_i^{pr} = \gamma_i^{el} + \Delta \gamma_i \quad \text{Equation 4.14} \]

which simplifies the expression for the stress to

\[ \tau_i = \frac{\gamma_i^{pr}}{\gamma_{crit} + \Delta \gamma_{eq}^{sl}} \quad \text{Equation 4.15} \]

Substitution in the critical stress equality yields

\[ \Delta \gamma_{eq}^{sl} = \gamma_{eq}^{pr} - \gamma_{crit} \quad \text{Equation 4.16} \]
\[ \gamma_{eq}^{pr} = \sqrt{\left(\gamma_i^{pr}\right)^2 + \left(\gamma_2^{pr}\right)^2} \]  

Equation 4.17

Substitution in the expression for \( \tau_i \) and introduction of the normalized slip direction \( n_i = \gamma_i^{pr}/\gamma_{eq}^{pr} \) furnishes the final result

\[ \tau_i = n_i \tau_{crit} \]  

Equation 4.18

### 4.4 Parameter Selection

Table 4.1 provides a summary of all the parameters required for simulating four brick unit specimens subjected to various bending conditions using the detailed micro-model described in Section 4.3. Values for these parameters were generally obtained from the tests described in Sections 3.7 to 3.10, conducted in conjunction with the four unit specimen tests. However, due to the complex nature of some of the laboratory test methods used and the presence of significant variability in masonry material properties, some of the parameters measured did not appear to be realistic. Under those situations assumptions had to be made based on literature reports.

This section discusses in some detail the selection of the following parameters: flexural tensile bond strength \( (f_{mt}) \) and cohesive shear strength \( (c_0) \) between brick and mortar, friction coefficient \( (\mu) \) and mode II shear fracture energy \( (G_f^{II}) \). The impact of other parameters on the simulation outcome will be discussed briefly.
Table 4.1. Summary of parameters used in the micro modelling of four brick unit specimens subjected to various bending conditions in this study.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{mt}$</td>
<td>Flexural tensile bond strength between brick and mortar (MPa)</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Cohesive shear strength between brick and mortar (MPa)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>$\gamma_{crit}$</td>
<td>Allowable elastic slip between brick and mortar interface (mm)</td>
</tr>
<tr>
<td>$E_b$</td>
<td>Brick elastic modulus (MPa)</td>
</tr>
<tr>
<td>$v_b$</td>
<td>Brick Poisson’s ratio</td>
</tr>
<tr>
<td>$f_u$</td>
<td>Brick modulus of rupture (MPa)</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Mortar elastic modulus (MPa)</td>
</tr>
<tr>
<td>$v_m$</td>
<td>Mortar Poisson’s ratio</td>
</tr>
</tbody>
</table>

- **Flexural tensile bond strength between brick and mortar ($f_{mt}$)**

As described in Chapter 3, two five-joint stack bonded prisms were built in this study for every batch of four brick unit specimens in order to obtain the flexural tensile strength ($f_{mt}$) between brick and mortar using the bond wrench test. The flexural strength obtained varied between batches from 0.21 to 0.49MPa. Statistical analysis conducted in Chapter 3 showed that variance between batches might have played an important role in causing the variability in this result.

As described in Section 3.7 direct tension tests using a separate sample batch were also conducted and a mean direct tensile strength of 0.32MPa was obtained. However, a mean flexural tensile strength of 0.14MPa was obtained for the same batch through the bond wrench test. It is worth noting that the mean flexural strength of this batch was significantly lower than the mean values obtained for the other batches discussed above. This being the case, we strongly believe that the low value obtained for this separate batch must be simply reflective of the variability between the different batches. Bond wrench testing has been widely recognized as being a standard, simple and reliable method for assessing the variability in tensile bond strength.

The above observation that the mean flexural tensile strength is less than half the mean direct tensile strength appears contradictory to earlier findings. Baker (1981) and Pluijm (1997) noticed that the flexural tensile strength in their tests was generally larger than
the direct tensile strength, with the ratio of the two being approximately 1.4:1. The author believes that this apparent discrepancy may be caused by the way the direct tension test specimens were prepared for the current study and also by the limited number of specimens tested. The cylindrical specimens used in the direct tension tests here were constructed by first cutting the bricks and then bonding with mortar. Ideally these specimens should be cored directly from couplet specimens that are constructed using the standard procedure. The limited number of cylindrical specimens tested also resulted in a very large coefficient of variation (41%) in the current tests. In short, it is believed that there may have been significant uncertainties associated with the direct tension tests used in the current study.

In theory direct tensile strength is more appropriate for use in the numerical model to simulate the tensile bond strength between the nodes (that is, at a material point) on contacted surfaces. Nevertheless, in light of the difficulties in obtaining reliable direct tensile strength data, for the current study the flexural bond strength obtained for each individual four brick unit specimen batch was used as the tensile bond strength in the numerical modelling. This enabled the strength variability between batches to be accounted for much more accurately.

- **Cohesive shear strength \( (c_0) \) and friction coefficient \( (\mu) \)**

In the numerical model the shear strength between brick and mortar \( (\tau_{\text{crit}}) \) was assumed to follow a Mohr-Coulomb rule and calculated as \( \mu \sigma + c_0 \). After reaching the shear strength the shear stress softens to the residual frictional shear strength, \( \mu_r \sigma \). For simplicity, in the current numerical modelling \( \mu_i \) and \( \mu_r \) were assumed to have the same value known as \( \mu \).

Two tests were carried out to obtain values of \( c_0 \) and \( \mu \), namely, triplet shear tests in accordance with European Committee for Standardization EN1052-3 (2002) and innovative torsion shear tests using annular cylindrical specimens. Ideally these tests should be conducted alongside each batch of four brick unit specimens so that strength variability between batches can be represented properly. However, due to the complex nature of these tests, especially with the torsion shear test being only at its developmental stage, tests were only carried out for specimens prepared using batches of masonry different from those for the four brick unit specimens. However, for these
batches the mortar used was identical to the one used before. That is, the same ratio of cement : lime : sand was used and the same amount of air entraining agent was added. Note that the specimens tested in the torsion shear tests and in the direct tension test discussed previously used the same batch of masonry.

From the triplet shear test mean values of 0.12MPa, 0.70 and 0.68 were obtained for $c_0$, $\mu_i$ and $\mu_r$, respectively. Note that almost identical values for $\mu_i$ and $\mu_r$ were obtained here. Bond wrench test for this batch of specimens yielded a mean flexural tensile strength $f_{mt}$ of 0.25MPa. From the torsion shear test of annular cylindrical specimens mean values of 0.22MPa, 0.90 and 0.56 were obtained for $c_0$, $\mu_i$ and $\mu_r$, respectively. The flexural tensile strength $f_{mt}$ for this batch of specimens was only 0.14MPa. Note that the ratio of $c_0$ to $f_{mt}$ from the triplet test is 0.48, which is very different to the ratio of 1.57 as obtained from the torsion shear test. Although the latter agrees reasonably well with the test result of other researchers (Pluijm, 1993; Molyneaux et al., 2002), it needs to be pointed out that the torsion shear test data were based only on very limited number of tests. Also the specimens used for the torsion shear test were prepared by first cutting the annular cylindrical brick and then bonding with mortar. This may result in the joint shear strength measured from the torsion shear test being unrepresentative of that in the four brick unit specimens. For convenience, a single value of 1.0 for the ratio of $c_0$ to $f_{mt}$ was assumed here, which is the average of 0.48 and 1.57.

As a first approximation, the same value of 1.0 for the ratio of $c_0$ to $f_{mt}$ was assumed for each batch of four brick unit specimens. Once the flexural bond strength $f_{mt}$ was measured for each batch, the cohesive shear strength $c_0$ for that batch could be calculated. As a result, the variability in shear bond strength ($c_0$) between each batch was represented by the variability in its flexural bond strength.

As a second approximation, a value of 0.73, which is the average value of 0.90 ($\mu_i$) and 0.56 ($\mu_r$) as obtained in the torsion shear test, was used in modelling the friction coefficient $\mu$. This assumption is for the simplification of the model since no data are available to define the transition from $\mu_i$ to $\mu_r$. The impact of this assumption will be discussed in detail later in this chapter. Note that this average is very close to $\mu_i$ and $\mu_r$ as obtained in the triplet test.
• **Mode II shear fracture energy \( (G_{II}) \)**

From the torsion shear test (cf. Figure 3.35) a linear relationship of \( 0.75\sigma - 0.09 \) was established between the mode II shear fracture energy \( (G_{II}) \) and normal compressive stress \( (\sigma) \). According to this relationship negative fracture energy would be obtained under zero compressive stress. This is physically meaningless and would pose problems in the numerical analysis procedure for small compressive stresses. This strange behaviour can be attributed to the large variance and limited number of data points measured. Therefore, this relationship was not used in the simulation. Instead, a function of \( G_{II} = 0.13\sigma + 0.03 \) (Pluijm, 1993; Molyneaux et al., 2002) was adopted and used throughout the simulations. This function gives fracture energy of 0.03N/mm under zero compressive stress, 0.048N/mm under 0.14MPa compressive stress (as compared to our torsion shear test average of 0.045N/mm, Table 3.17) and 0.074N/mm under 0.34MPa compressive stress (as compared to our torsion test average of 0.13N/mm, Table 3.17). Noticeably, as the compressive stress was increased, the difference between the calculated and experimental values became larger, notwithstanding the large variance in the experimental measurements.

To assess the degree of error likely to be introduced to the simulated horizontal bending moment due to improper selection of the fracture energy, a sensitivity study was carried out to simulate four brick unit specimens subjected to horizontal bending using shear fracture energy values of 0.06 and 0.12N/mm. The simulated behaviour in horizontal moment versus rotation is plotted in Figure 4.6a for pure horizontal bending and Figure 4.6b for biaxial bending where the ratio of vertical to horizontal moments used was 1:4. In the latter case, the failure mode was observed to be mostly governed by shear failure after cracking of perpend joint.
Figure 4.6. Impact of shear fracture energy on the predicted horizontal moment under zero compressive stress.
It can be seen from Figure 4.6 that doubling the shear fracture energy from 0.06 to 0.12N/mm only resulted in moderate increase in the peak horizontal moment, with only about 10% increase in pure horizontal bending and 2% in biaxial bending. Increasing the shear fracture energy had a much more pronounced effect on the softening process. The higher the fracture energy, the milder the softening process is. Therefore, it can be concluded that the impact of using the above-mentioned literature equation can be expected to be marginal for peak horizontal moment predictions.

- Other parameters (\(\gamma_{\text{crit}}\), \(E_b\), \(E_m\), \(\nu_b\), \(f_u\) and \(\nu_m\))

Another important parameter that describes the shear behaviour is the allowable elastic shear slip (\(\gamma_{\text{crit}}\)), which is a fraction of the average element length. Ideally the \(\gamma_{\text{crit}}\) value chosen should represent closely the experimental shear stiffness. However, if \(\gamma_{\text{crit}}\) is chosen to be too small (i.e., high stiffness), it can cause numerical convergence problems. For the current simulation of four brick unit specimens, \(\gamma_{\text{crit}}\) was taken as 0.0001 \(\times\) average element length. This was reached based on trial and error to ensure that numerical stability could be achieved.

As described in Section 4.3, brick and mortar were both assumed to be linear elastic in the numerical model. This assumption of mortar elasticity was based on the experimental results from Jukes and Riddington (2001). These authors found that for normal pre-compression less than approximately 2MPa mortar nonlinearity was not significant. Since our entire four brick unit specimen tests were carried out under compressive stresses of \(\leq 0.2\)MPa, this assumption was considered to be reasonable.

In the simulations the elastic moduli of brick and mortar, \(E_b\) and \(E_m\), were obtained through standard compressive strength tests. Five prisms were used to get an average value of 35.4GPa and 2.8GPa for \(E_b\) and \(E_m\). These were used in the subsequent numerical modelling of the four brick unit specimen tests. However, the poisson’s ratios of \(\nu_b\) and \(\nu_m\) for brick and mortar were simply assumed to be 0.15 and 0.125, based on a survey of literature findings (Lourenço, 1996).

In Section 4.7 the brick modulus of rupture \(f_u\) is used to model the observed brick failure mode in the horizontal bending of four brick unit specimens. The value of \(f_u\) resulted from the brick modulus of rupture tests has been described in Section 3.9.
4.5 Mesh Sensitivity Study

To study the mesh sensitivity of the finite element model, four brick unit specimens under separate vertical and horizontal bending without pre-compressive load were simulated with different mesh densities (Figure 4.7). As both the geometry of the specimen and imposed actions are symmetric, only half of the specimen was modelled. The boundary conditions and application of moment (through prescribed rotation) will be described in Sections 4.6 to 4.8. The elements were all eight node solid brick elements with each having four integration points. The masonry units with dimensions of 230×111×75 (mm) were each discretized using two different mesh densities, namely 8×4×4 (i.e., number of elements long × number of elements thick × number of elements high) and 16×8×8. Exceptions were those top and bottom units where an extra layer of elements was at the left side. These elements were modelled such that the length of the element was 5mm, so that it matched half of the thickness of perpent joint. 10mm thickness of bed joint mortar was modelled with two layers, each layer being 5mm thick. At the brick and mortar interfaces the nodes on all the master and slave surfaces were perfectly matched to avoid oscillations in contact stress which may occur in the contact modelling. Figure 4.8 shows the numerically generated curves of vertical moment $M_v$ versus rotation around the horizontal axis (Figure 4.8a) and horizontal moment $M_h$ versus rotation around the vertical axis (Figure 4.8b) for a specimen subjected only to separate vertical and horizontal bending. The curves were generated using two levels of discretization.

Figure 4.7. Four brick unit specimen, showing two densities of mesh used in the mesh sensitivity study.
Figure 4.8. Effect of mesh density on the bending moment.
As can be seen in Figure 4.8a, for vertical bending the coarse mesh may result in slight overestimation for the cracking load compared to the fine mesh. Also there are differences between the two meshes in the transition region from the peak moment to the residual moment. The fine mesh provides a smoother and steadier transition than the coarse mesh. Large fluctuations in the moment from the coarse mesh are due to greater release of energy with a large element and thus represent more brittle behaviour. However, the residual vertical moment is not affected by mesh density since when the vertical moment reaches the residual state, the brick movement is governed by the rigid body movement only and is not mesh sensitive.

Figure 4.8b indicates that for horizontal bending the fine mesh provides a stiffer shear modulus than the coarse mesh. This is caused by the way the allowable elastic slip $\gamma_{\text{crit}}$ is calculated, as discussed in the previous section. $\gamma_{\text{crit}}$ is calculated as a fraction of the model’s average characteristic element length and is different for the two meshes considered. The fact that the peak moments from the two meshes match very closely indicates that peak moment is insensitive to mesh density for horizontal bending.

As the overall response of the two meshes was very similar, it was decided to adopt the coarse mesh. All bending simulations reported in the remainder of this chapter for four brick unit specimens were performed using the coarse mesh, namely 8×4×4.

4.6 Modelling of Four Brick Unit Specimens Subjected to Vertical Bending

Before describing the numerical modelling procedure, it may be helpful to recapitulate the experimental method used for measuring the response of four brick unit specimens subjected to vertical bending, as reported in Chapter 3. The four brick unit specimens were fixed on the support frame with a clamp at the bottom brick. Vertical pre-compressive stresses equalling 0.016, 0.1 and 0.2MPa were added and held constant. The top clamp was then rotated around the horizontal axis which is movable to exert rotation and hence moment on the specimen. The horizontal shafts and side clamps were connected through the universal joints to get rid of shear forces so that a pure moment was applied. Both the top and bottom clamps were very rigid.
The half specimen used in the finite element modelling is shown in Figure 4.9. Here all the nodes on the left side (the vertical plane of symmetry) were all restrained in direction 1. The action of the top clamp was modelled by completely fixing the upper masonry unit to a rigid body as denoted by the white elements. The bottom clamp was modelled by restraining all nodes across the base of the specimen in direction 2. The simulation took two steps. The first step was to add a constant compressive load to reflect the load control process when pre-compression was added in the experiment. In the second step the rigid body was rotated around its reference node as illustrated until failure occurred and a distinct rigid body movement of the upper part of the specimen occurred. The reference node is only restrained in direction 1, but movable in direction 2 and 3. Figure 4.9 also shows the simulated failure mode of the four brick unit specimen, showing cracking of the upper joint.

![Figure 4.9. Predicted failure mode from the numerical modelling.](image)

The vertical bending simulation was run for specimens under compressive stresses of 0.016, 0.1 and 0.2MPa. The corresponding flexural tensile strengths of 0.33, 0.25 and 0.27MPa, obtained from the bond wrench tests, were used in the simulations. As discussed in Section 4.4, the same value of 1.0 for the ratio of $c_0$ to $f_{mt}$ was assumed for each batch. As a result, for each batch the cohesive shear strength was taken to be the same as its flexural tensile strength ($f_{mt}$). A prescribed rotation of 0.003 radians was then
given to allow the rigid body to turn around the horizontal axis after a compressive load was added.

The predicted vertical bending moment versus rotation is shown in Figure 4.10a, with the corresponding experimental data shown in Figure 4.10b. Clearly the prediction successfully captured the main bending features as observed in the experiment. The curves show that the specimen behaved almost linearly until it reached the peak load. After that, depending on the magnitude of the compressive load applied, the resistance moment either dropped or increased to a plateau value. This plateau value is the residual moment required to equilibrate the moment due to the vertical pre-compression as the specimen rotates rigidly around the horizontal axis at the compression face of the specimen after failure occurs in the upper or lower bed joint. Note that it is widely accepted that masonry walls may preserve a certain amount of residual load resistance after reaching their peak load capacity and this property is important for a thorough understanding of the complex bending behaviour of masonry walls.

In Figure 4.10a the peak resistance moments are those points just before the moment drops for the first time. The fluctuation between the peak and residual moments was caused by the coarse mesh of the elements. Finer meshes would have provided a smoother transition.

The peak and residual moments (averages) from the simulation and experiment are shown in Table 4.2 for a closer comparison. Table 4.2 shows that for 0.1 and 0.2MPa pre-compression the simulation and experiment are in excellent agreement, with differences being only 1 - 6%. However, under a very low compression load of 0.016MPa the difference became very large, particularly in the peak moment. The large difference in the peak moment may be explained by the weak bond strength (and hence large variations) between the brick and mortar for the specimens tested. A more detailed explanation can be found in Section 3.4.

Note that under the compression load of 0.016MPa, a peak moment of 0.176Nmm is obtained whereas under the compression load of 0.1MPa, a smaller peak moment of 0.16Nmm is obtained. That is because under 0.016MPa compression load, the flexural
strength of the joint is 0.33MPa whereas under 0.1MPa compression load, the flexural strength of the joint is only 0.25MPa.

As also discussed in Section 3.4 over prediction of the residual moment may be due to the limited precision of the load cell used to measure $M_v$ when extremely low compression loads are applied. Overall, the numerical model provided good simulation of the behaviour of the four brick unit specimens subjected to vertical bending.
(b) Typical data from experiment

Figure 4.10. Vertical bending moment vs rotation of a four brick unit specimen.

Table 4.2. Comparison of peak and residual moments between prediction and experiment for four brick unit specimens subjected to vertical bending and under varying compressive loads.

<table>
<thead>
<tr>
<th>Pre-compression (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>Residual</td>
<td>Peak</td>
</tr>
<tr>
<td>0.016</td>
<td>0.176</td>
<td>0.025</td>
<td>0.084</td>
</tr>
<tr>
<td>0.1</td>
<td>0.160</td>
<td>0.144</td>
<td>0.164</td>
</tr>
<tr>
<td>0.2</td>
<td>0.246</td>
<td>0.285</td>
<td>0.254</td>
</tr>
</tbody>
</table>

4.7 Modelling of Four Brick Unit Specimens Subjected to Horizontal Bending

The half specimen used in the finite element modelling is shown in Figure 4.11. Here all the nodes on the left side (the vertical symmetrical plane) were all restrained in direction 1. The action of the side clamp was modelled by completely fixing the side masonry unit to a rigid body (not shown in Figure 4.11). The bottom clamp was modelled by
restraining all nodes across the base of the specimen in direction 2. The simulation took two steps. The first step was to add a constant compressive load to reflect the load control process when pre-compression was added in the experiment. In the second step the rigid body was rotated around the vertical axis until failure occurred (brick failure) or a residual moment plateau was reached (joint failure).

As discussed in Chapter 3, when the four brick unit specimens were subjected to horizontal bending, two failure modes were observed: failure through mortar and failure through brick units. These failure modes were precisely seen in the simulations as shown in Figures 4.11a and 4.11b. It has been argued in the literature (Baker et al., 1985) that the mode of failure depends on the relative flexural strength of brick to the bond strength between brick and mortar. Without pre-compression, if the bricks have a relatively high flexural strength compared to the joint bond strength, then failure through the joints occurs. On the other hand, if the bond between brick and mortar is relatively high compared to the flexural strength of the brick, then brick rupture occurs. However, the presence of pre-compression can complicate the situation since pre-compression can change the bond strength between brick and mortar at the bed joints but has insignificant effect on the flexural strength of the brick. In the horizontal bending tests for the four brick unit specimens, brick failure occurred in two out of ten specimens under 0.1MPa pre-compression. However, failures were predominantly by brick rupture under 0.2MPa pre-compression, with only three out of ten specimens showing failure through the mortar joints. The small number of brick failures under 0.1MPa pre-compression is most likely due to the variability of the brick flexural strength and weak bricks were used in those specimens, as pointed out in Section 3.5. So special consideration of brick failure was only given to specimens under 0.2MPa pre-compression.
Flexural tensile strengths of 0.33, 0.28 and 0.34MPa from the bond wrench test were used in the simulation between the nodes of brick and mortar contact surfaces. As discussed before, the cohesive shear strength was taken to be the same as the flexural tensile strength. Also, an average value of 0.73 was assumed for the shear friction coefficient for both $\mu_i$ and $\mu_r$. The limited test data obtained using the torsion shear test indicated that $\mu_i$ could be significantly greater than $\mu_r$; see discussions in Section 4.4. The consequence of the assumption of a single value of $\mu$ is that it would lead to underestimation for the peak bending moment and overestimation for the residual frictional resistance moment. The last parameter used in the simulation was the mode II fracture energy. The relationship $G_{II} = 0.13\sigma + 0.03$ was used.

Figure 4.12a shows the predicted behaviour of horizontal bending moment versus rotation around the vertical axis for the four brick unit specimens under pre-compression loads of 0.016, 0.1 and 0.2MPa, where failure all occurred in the mortar joints. For specimens under 0.016 and 0.1MPa pre-compression, a mean flexural strength of 3.7MPa, as obtained from the brick rupture test, was used to model the tensile strength between the potential failure surfaces of the brick. For specimens under 0.2MPa pre-compression, this parameter was deliberately increased to 4.0MPa (still within the range of variability in the measured brick flexural strength) in order to enable failure to occur in the mortar joints.

Figure 4.11. Predicted failure modes in horizontal bending.

(a) Perpend joint failure  
(b) Brick failure
Comparing the modelling results in Figure 4.12a with the experimental curves in Figure 4.12b shows that the numerical model is capable of capturing the main features as seen in the horizontal bending tests. The horizontal bending moment first increases almost linearly until the peak moment is reached. After that, due to the gradual softening of the shear strength, the resistance moment decreases until it reaches a plateau value. The simulation also shows that the existence of pre-compression increases both the peak moment and residual resistance moment substantially.

Table 4.3 presents a comparison between the predicted and experimentally obtained peak moments and residual moments. Overall the differences between the prediction and experimental results are all within ±20%.

Table 4.3 also shows that there is consistent underestimation for the peak moment and overestimation for the residual moment. This is to be expected since the model does not differentiate between the internal friction coefficient and residual friction coefficient. Instead of using an internal friction coefficient (0.90) to predict the peak moment and a residual friction coefficient (0.56) to predict the residual moment, an average value of the two was used in the simulation. To illustrate this point, Figure 4.13 shows the predicted horizontal moment behaviour under 0.1MPa compressive stress using friction coefficients of 0.90, 0.56, as well as 0.73 as a reference. It can be seen that use of $\mu = 0.90$ results in the peak moment being only 6.7% above the experimental value, while the predicted residual moment is only 6.3% below the experimental value if $\mu = 0.56$ is used.
Figure 4.12. Horizontal bending moment versus rotation of a four brick unit specimen where failure occurs in the joints.
Table 4.3. Comparison of peak and residual moments between prediction and experiment for four brick unit specimens subjected to horizontal bending and under varying compressive loads.

<table>
<thead>
<tr>
<th>Pre-compression (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>Residual</td>
<td>Peak</td>
</tr>
<tr>
<td>0.016</td>
<td>0.401</td>
<td>0.0275</td>
<td>0.472</td>
</tr>
<tr>
<td>0.1</td>
<td>0.448</td>
<td>0.145</td>
<td>0.505</td>
</tr>
<tr>
<td>0.2</td>
<td>Mortar failure</td>
<td>0.617</td>
<td>0.285</td>
</tr>
<tr>
<td>0.2</td>
<td>Brick failure</td>
<td>0.436</td>
<td>NA</td>
</tr>
</tbody>
</table>

Figure 4.13. Effect of friction coefficient on the horizontal bending moment.
The case of brick failure was also listed in Table 4.3 when the specimens were subjected to 0.2MPa pre-compression. To induce brick failure the brick flexural tensile strength used in the simulation had to be reduced to 3.0MPa to ensure numerical convergence, when the properties of mortar were kept unchanged. This value is within the range of $3.7\pm1.1$MPa as observed from the brick rupture test described in Section 3.9. Underestimation in the peak moment was, in this case, simply due to the lower flexural tensile strength used. The convergence problem is attributed to the model’s limitation that the shear fracture energy can only apply to one type of interface. In the model, instead of using brick’s shear fracture energy to model the shear softening process of a brick and brick interface, the fracture energy of the mortar joint was used.

### 4.8 Modelling of Four Brick Unit Specimens Subjected to Biaxial Bending

Simulation of biaxial bending of four brick unit specimens was achieved by first applying a vertical compression to the rigid body attached to the top brick. Then the rigid body attached to the top brick and the rigid body attached to the side brick were each given simultaneous rotations about the horizontal and vertical axes respectively. In the experiment, as reported in Section 3.6, three ratios (1:1, 1:2 and 1:4) of vertical bending moment $M_v$ to horizontal bending moment $M_h$ were used, together with three levels of compression (0.016, 0.1 and 0.2MPa), thus giving rise to nine loading conditions. Each of these conditions was simulated here in order to verify the performance of the numerical model. Figure 4.14 illustrates the typical failure mode as predicted from the numerical model.

![Figure 4.14. Typical failure mode from simulation of a four brick unit specimen subjected to biaxial bending.](image)
• $M_v : M_h = 1:1$

The vertical bending moment and horizontal bending moment versus vertical rotation under the moment ratio of 1:1 are shown in Figures 4.15a - 4.15c for vertical compressive stresses of 0.016, 0.1 and 0.2MPa. Note that only half of the specimen was modelled, so the simulation was run by giving a prescribed rotation around the horizontal axis and rotation around the vertical axis at a ratio of 1 to 2 to achieve the moment ratio $M_v : M_h = 1:1$ in the experiment. To identify this moment ratio clearly as captured by the simulation, Figures 4.15a - 4.15c are purposefully plotted only against vertical rotation to reflect the loading time history. The flexural tensile strengths used in the simulation between the nodes of brick and mortar contact surfaces were, respectively, 0.31, 0.30 and 0.41MPa, which were obtained from the bond wrench test. In all three cases the modelling shows consistently that the vertical bending moment reached its peak value before the horizontal bending moment. This suggests that under all three compression loads the specimens should fail by cracking of the bed joint first followed by failure of the perpend joint. This matches the experimental observation on the failure behaviour of these specimens under 0.016MPa pre-compression. However, in the experimental work only half of the specimens failed first at a bed joint under 0.1MPa compression and only two in ten specimens under 0.2MPa compression. Nevertheless, the ratio of the predicted rotation at peak horizontal moment to the predicted rotation at peak vertical moment did show the trend of decreasing as the pre-compression was increased. This suggests that the specimens were closer to having both bed and perpend joints fail simultaneously as the pre-compression was increased. This matches the trend captured in the experiment.
(a) $\sigma = 0.016\text{MPa}$

(b) $\sigma = 0.1\text{MPa}$
The predicted and experimentally obtained failure moments are shown in Table 4.4. It should be pointed out that in the numerical modelling, if failure was initiated by the vertical bending moment, the moment where the vertical bending moment first showed a decrease was regarded as the failure moment. This is to reflect the brittle failure nature of bed joint. As can be seen in Table 4.4, the numerical simulation overestimates the experimentally observed failure moment. Under pre-compression loads of 0.016 and 0.1MPa the simulation over predicted the experimental averages by around 50%. Under 0.2MPa pre-compression the over prediction was 30% or less, which is more reasonable in view of the variability in the material properties of masonry. As also shown in Table 4.4, the difference between the prediction and experimental results decreased as the pre-compression was increased.
Table 4.4. Comparison of peak moment between prediction and experiment for four brick unit specimens subjected to biaxial bending and under varying compressive loads ($M_v : M_h = 1:1$).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
<th>Bond Wrench</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{vp}$</td>
<td>$M_{hp}$</td>
<td>Mean $M_{vp}$</td>
<td>Mean $M_{hp}$</td>
</tr>
<tr>
<td>0.016</td>
<td>0.116</td>
<td>0.137</td>
<td>0.084</td>
<td>0.088</td>
</tr>
<tr>
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<td>0.217</td>
<td>0.143</td>
<td>0.148</td>
</tr>
<tr>
<td>0.2</td>
<td>0.307</td>
<td>0.321</td>
<td>0.251</td>
<td>0.246</td>
</tr>
</tbody>
</table>

The numerical model gives predictions of the residual moment until the rotation reaches about 0.16 radians or 9°. These are shown in Figures 4.16a and 4.16b, which are essentially the same as Figures 4.15b and 4.15c except showing the complete moment versus rotation records under larger rotations. Table 4.5 provides a comparison between the simulation and experiment for pre-compression loads of 0.1 and 0.2MPa. Note that no comparison could be made for 0.016MPa pre-compression since no residual could be measured due to the brittle failure of the specimens. As can be seen from this table, the model over predicted the vertical residual moment by about 20% and horizontal residual moment by about 100%. The overprediction for the vertical residual moment is within the reasonable limit. The significant overprediction for the horizontal residual moment may be due to the use of an average value for $\mu$ (0.73), instead of using a real measured value.
Figure 4.16. Complete moment vs rotation record for biaxial bending

(a) $\sigma = 0.1$MPa

(b) $\sigma = 0.2$MPa

$(M_v:M_h = 1:1)$. 
Table 4.5. Comparison of residual moment between prediction and experiment for four brick unit specimens subjected to biaxial bending and under varying compressive loads ($M_v:M_h = 1:1$).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{vr}$ ($\mu=0.73$)</td>
<td>$M_{hr}$ ($\mu=0.56$)</td>
<td>Mean $M_{vr}$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.143</td>
<td>0.083</td>
<td>0.058</td>
</tr>
<tr>
<td>0.2</td>
<td>0.285</td>
<td>0.158</td>
<td>0.118</td>
</tr>
</tbody>
</table>

To examine more closely the effect of $\mu$ on the horizontal residual moment under biaxial bending conditions, a simulation was run using values of 0.56, 0.73 and 0.90 for $\mu$. Initially, to magnify the effect a ratio of 1:2 was used for $M_v:M_h$. Figure 4.17 shows that varying $\mu$ by $\pm 24\%$ about its average (0.73) resulted in variations in the horizontal residual moment between -22% and +34%. Obviously, smaller values of $\mu$ would result in lower residual moments. However, varying $\mu$ appeared to have much less impact on the horizontal peak moment than on the residual moment.

![Figure 4.17. Effect of $\mu$ on horizontal bending moment under biaxial bending.](image)
The simulation was re-run using \( \mu = 0.56 \) for \( M_v:M_h = 1:1 \) under pre-compression loads of 0.1 and 0.2MPa. It was found that the residual horizontal moment dropped from 0.083kNm to 0.058kNm for 0.1MPa pre-compression and from 0.158kNm to 0.118kNm for 0.2MPa pre-compression. These new results showed significant improvement when compared with the experimental values. Nevertheless, there were still about 29% and 69% overestimation under the two pre-compressive loads (see Table 4.5). The changing nature of \( \mu \) due to variation in the nature of contact under biaxial bending failure mode may account for the remaining difference between simulation and experiment. By the time the vertical bending moment in biaxial bending reaches its residual value, the contact status between brick and mortar changes from surface-surface contact to edge-surface contact. Consequently, being a parameter that reflects contact property, \( \mu \) can be reasonably expected to change as well. In the current case the true value for \( \mu \) will be difficult to obtain, but it could be much smaller than 0.56, the latter being obtained from the torsion test which only reflects surface to surface contact between brick and mortar.

Another possible reason for the discrepancy of residual moments between the model prediction and experimental result may be attributed to the lack of modelling for dilatancy in the current study.

- \( M_v:M_h = 1:2 \)

Figures 4.18a - 4.18c show the simulated behaviour of the vertical and horizontal bending moments versus vertical rotation before reaching the residual moment. Note that the simulation was run by giving a prescribed rotation around the horizontal axis and rotation around the vertical axis at a ratio of 1 to 4 for \( M_v:M_h = 1:2 \) and under pre-compression loads of 0.016, 0.1 and 0.2MPa. The data are only plotted against vertical rotation so that moment ratio can be clearly identified.

In the experiment it was observed that for specimens subjected to 0.016MPa pre-compression failure was initiated by bed joints. In contrast, under the other two pre-compression loads failure occurred by both bed joint and perpend joint. These observations are also reflected in Figures 4.18a - 4.18c, which show that both vertical and horizontal moments reached the peak capacity almost at the same time. The bed
joint failures under 0.016MPa in the experiment were most likely attributed to the weak bond strength under a low compressive force.

(a) $\sigma = 0.016\text{MPa}$

(b) $\sigma = 0.1\text{MPa}$
Table 4.6 provides a quantitative comparison between the measured and predicted peak moments for each of the pre-compression loads applied. Overall, the model overestimates the experimental peak moments. The worst case is for 0.016MPa pre-compression where overprediction was 34% for $M_v$ and 48% for $M_h$. However, as the pre-compression was increased, the prediction improved significantly. The large overestimation under 0.016MPa can again be explained by the variability in the bond strength between brick and mortar. Under a lower pre-compression, especially under the state of biaxial stress, the bending behaviour can be significantly influenced by this variability in bond strength. However, as pre-compression is increased, this variability is reduced as the applied compressive stress becomes the more dominant portion of the total joint capacity.
Table 4.6. Comparison of peak moment between prediction and experiment for four brick unit specimens subjected to biaxial bending and under varying compressive loads ($M_v:M_h = 1:2$).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
<th>Bond Wrench</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{vp}$</td>
<td>$M_{hp}$</td>
<td>Mean $M_{vp}$</td>
<td>Mean $M_{hp}$</td>
</tr>
<tr>
<td>0.016</td>
<td>0.165</td>
<td>0.32</td>
<td>0.123</td>
<td>0.216</td>
</tr>
<tr>
<td>0.1</td>
<td>0.187</td>
<td>0.374</td>
<td>0.160</td>
<td>0.321</td>
</tr>
<tr>
<td>0.2</td>
<td>0.230</td>
<td>0.4</td>
<td>0.186</td>
<td>0.361</td>
</tr>
</tbody>
</table>

Figures 4.19a and 4.19b show the vertical and horizontal residual moments versus vertical rotation under the pre-compression loads of 0.1 and 0.2MPa. A quantitative comparison between the model prediction and experiment is shown in Table 4.7. The model is seen to provide close predictions for the residual vertical bending moment with the maximum difference being only 15%. The residual horizontal bending moments predicted using $\mu = 0.73$ and $\mu = 0.56$ are both shown in the table. Although the simulation using $\mu = 0.56$ gives a better prediction for the residual horizontal moment, it is still around 46% higher than the experimentally obtained value. Possible explanations for this overestimation can be found in the discussion for the case of $M_v:M_h = 1:1$ and will not be repeated here.
Figure 4.19. Residual moment for biaxial bending ($M_v:M_h = 1:2$).

(a) $\sigma = 0.1\text{MPa}$

(b) $\sigma = 0.2\text{MPa}$
Table 4.7. Comparison of residual moment between prediction and experiment for four brick unit specimens subjected to biaxial bending and under varying compressive loads ($M_v:M_h = 1:2$).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{vr}$</td>
<td>$M_{hr}$ ($\mu=0.73$)</td>
<td>$M_{hr}$ ($\mu=0.56$)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.144</td>
<td>0.087</td>
<td>0.0714</td>
</tr>
<tr>
<td>0.2</td>
<td>0.285</td>
<td>0.173</td>
<td>0.122</td>
</tr>
</tbody>
</table>

$M_v:M_h = 1:4$

Figures 4.20a - 4.20c illustrate the predicted vertical bending moment and horizontal bending moment versus vertical rotation response before the residual moment is reached. The moments are plotted against vertical rotations so that moment ratio can be clearly identified. From the prediction it can be clearly seen that in all the cases the horizontal bending moment reached its peak capacity first. This indicates that for all the cases perpend joint would initiate the failure then followed by failure of the bed joint. In experiment this normally manifests itself by both joints failing at the same time. The simulation results appear to be supported by the experimental observation that only three out of the 30 specimens tested failed by bed joint first.

Table 4.8 presents a quantitative comparison between the predicted and experimentally obtained peak moments. Except for 0.016MPa pre-compression, the predicted peak moment only showed a maximum of 18% difference as compared to the test result. Similar to the cases of $M_v:M_h = 1:1$ and $M_v:M_h = 1:2$, the higher prediction for the peak moment under the lower compression of 0.016MPa can be attributed to the variability in the bond strength between brick and mortar.

The predicted residual moment behaviour under 0.1 and 0.2MPa pre-compression is shown in Figures 4.21a and 4.21b. No comparison can be made here between the simulation results and experimental data since residual moments could not be successfully measured in the experiment.
(a) $\sigma = 0.016\text{MPa}$

(b) $\sigma = 0.1\text{MPa}$
(c) $\sigma = 0.2\text{MPa}$

Figure 4.20. Moment versus rotation for biaxial bending ($M_v: M_h = 1:4$).

Table 4.8. Comparison of peak moment between prediction and experiment for four brick unit specimens subjected to biaxial bending and under varying compressive loads ($M_v: M_h = 1:4$).

<table>
<thead>
<tr>
<th>Pre-comp. (MPa)</th>
<th>Prediction (kNm)</th>
<th>Experiment (kNm)</th>
<th>Difference</th>
<th>Bond Wrench (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{vp}$</td>
<td>$M_{hp}$</td>
<td>Mean $M_{vp}$</td>
<td>Mean $M_{hp}$</td>
</tr>
<tr>
<td>0.016</td>
<td>0.107</td>
<td>0.414</td>
<td>0.069</td>
<td>0.311</td>
</tr>
<tr>
<td>0.1</td>
<td>0.079</td>
<td>0.333</td>
<td>0.070</td>
<td>0.326</td>
</tr>
<tr>
<td>0.2</td>
<td>0.117</td>
<td>0.49</td>
<td>0.118</td>
<td>0.594</td>
</tr>
</tbody>
</table>
Figure 4.21. Residual moment for biaxial bending ($M_v:M_h = 1:4$).

(a) $\sigma = 0.1\text{MPa}$

(b) $\sigma = 0.2\text{MPa}$
4.9 Numerical Evaluation of the Torsion Shear Test

For years researchers have put enormous effort into trying to understand the shear behaviour in masonry walls. One piece of information that is of particular interest is the shear stress versus shear displacement response at a point on the masonry joint both before and after joint cracking. This information is critical for the numerical simulation of masonry wall behaviour. Various test methods have been developed and proposed by researchers to pursue this information. However, these test methods all fail, by varying degrees, to achieve uniform distributions of shear and normal stress across the mortar joint. Joint failure will usually initiate at a point at a shear stress higher than the average value calculated from the failure load. So the strength values of mortar joints based on these test methods will represent underestimation of the true local joint shear strength.

Recently a new test method was proposed by a group of researchers at the University of Newcastle (Masia et al., 2006a and 2006b). The proposed test method is to subject a single joint specimen of annular circular cross section to normal compressive force combined with torsion. The detailed test setup is discussed in Section 3.10. The choice of annular circular cross section of small thickness results in predictable, and importantly, uniform distributions of normal and shear stresses across the mortar joint, allowing shear behaviour at a material point to be characterized. However, during the development of the new test method, it was necessary to determine how thin the annular specimens needed to be in order for the assumption of uniform shear stress to be reasonable. This aspect of the test development was assessed numerically and formed part of the current PhD project. The following sections discuss the background theory on which the test is based and detail the finite element micro modelling performed to assess the test method. The same finite element micro-model as used for the simulation of the four brick unit specimen tests was used here to simulate the torsion shear test.

When a shaft with a circular cross section (solid or annular) is subjected to torsion about its longitudinal axis, transverse sections which were plane before twisting remain plane and diameters remain straight. This means that shear strain over the cross section varies linearly with the radius from the axis of the shaft. For linear elastic material behaviour this results in a distribution of shear stress over the cross section which also varies linearly with the radius (Equation 4.19) (Craig, 2000).
where $\tau$ is the shear stress (which at all points acts normal to the radial line passing through the point), $T$ is the torque applied to the shaft, $r$ is the radial distance from the axis of the shaft and $I_p$ is the polar second moment of area of the cross section given by

$$I_p = \frac{\pi(r_o^4 - r_i^4)}{2}.$$  

The terms $r_i$ and $r_o$ are the inner and outer radii respectively of an annular section, with $r_i = 0$ for solid circular sections. Note that Equation 4.19 is valid only at some distance from a sudden change in cross section, the distance typically being considered to be approximately equal to the diameter of the shaft (Craig, 2000). The shear stress distributions for solid and annular circular cross sections under linear elastic conditions are depicted in Figure 4.22.

![Figure 4.22. Theoretical distributions of shear stress in (a) solid and (b) annular circular sections subjected to torsion (linear elastic).](image)

Under the elastic theory, it is assumed that at peak torque the shear stress distribution in the mortar joint remains linear elastic so the shear strength can be calculated using Equation 4.19, replacing $T$ with maximum torque obtained from the experiment and using $r = r_o$. However, this elastic theory is unlikely to be valid at peak torque and has been questioned by researchers in relation to bending tests on masonry. In contrast to elastic theory, under plastic theory, it is possible and likely that the peak torque resisted by the specimen will occur when the distribution of shear stress over the cross section is partially linear elastic (at small radii) and partially non-linear softening (at large radii) (Figure 4.23.). This assumption makes the calculation of shear strength impossible using Equation 4.19 because it is impossible to locate where maximum shear stress is on the circular cross section.
However, with fully plastic theory, shear strength can be calculated using Equation 4.20 below, assuming that at the peak torque, the maximum shear stress is reached at all radii simultaneously (Figure 4.24). In Equation 4.20, $\tau$ is shear strength, $T$ is torque, $r_o$ is the outer radius of annular specimen, and $r_i$ is the inner radius of annular specimen.

$$\tau = \frac{3T}{2\pi(r_o^3 - r_i^3)}$$  
Equation 4.20

The only way to achieve a condition of uniform shear stress with radius is to make the thickness of the annular cross section as small as possible so that uniform distributions of normal stress and shear stress can be assumed and achieved experimentally.

To support this theoretical assumption behind the experiment method, the numerical model proposed in Section 4.3 was adopted as an evaluation tool to study the cross
sectional stress distribution. A circular specimen with solid cross section was also numerically analysed for comparison with the annular cross section to study the thickness effect on the stress distribution on the annular cross section. Figure 4.25 illustrates schematically a cylindrical specimen with solid cross section and the actions applied to the specimen, namely normal compressive force $P$ and Torque $T$.

![Diagram](image)

**Figure 4.25. The normal force and torque applied to the cylindrical specimen.**

Jukes and Riddington (1997) used finite element analyses and experimental results to show that mortar non-linearity begins to affect the joint stress distribution in triplet tests only for normal pre-compression stresses greater than about 2MPa. As the current study focuses on behaviour at pre-compression stress levels less than 2MPa, the bricks and mortar were simulated using 3D 8-noded linear elastic, isotropic and homogeneous continuum elements. The finite element meshes used in the current study are illustrated in Figures 4.26a and 4.26b. It was assumed that all non-linear behaviour was concentrated in the brick/mortar interfaces. However, the maximum principal tensile stress within the mortar was also monitored to assess any likelihood that mortar failure would precede bond slip. The non-linear surface contact relationship was specified only at one of the brick/mortar interfaces since bond strength variability is expected to result in only one interface failing. The other brick/mortar interface was assumed to remain perfectly bonded.
Figure 4.26. Finite element mesh for the numerical evaluation of the torsion shear test.

(a) Solid cross section

(b) Annular cross section
The finite element model was used to simulate the following test cases:

(i) Solid circular cross section, radius \( r_o = 50 \text{mm} \) subjected to increasing rotation with zero normal pre-compression.

(ii) As for (i) except normal pre-compression was first applied resulting in an average normal stress of 1.0MPa.

(iii) Annular cross section, inner radius \( r_i = 35 \text{mm} \), outer radius \( r_o = 50 \text{mm} \) subjected to increasing rotation with zero normal pre-compression.

(iv) As for (iii) except normal pre-compression was first applied resulting in an average normal stress of 1.0MPa.

The annular dimensions chosen for cases (iii) and (iv) represent a cross section that can realistically be cored from solid bricks in the laboratory. In fact, specimens with inner radius \( r_i = 36 \text{mm} \) and outer radius \( r_o = 47.5 \text{mm} \) have been successfully fabricated.

Table 4.9 displays the input parameters used for the numerical model. Most of the parameters are based on actual test data obtained at the University of Newcastle. The shear softening is assumed exponential. The shear fracture energy \( G_f^{II} \) and Poisson’s ratio of brick and mortar were chosen from the values derived by other researchers through their experimental programs (Lourenço, 1996).

**Table 4.9. Material properties assumed for finite element modelling.**

<table>
<thead>
<tr>
<th>Surface contact parameters at brick/mortar interface</th>
<th>Values</th>
<th>Source of data (UoN = University of Newcastle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_t )</td>
<td>0.244 MPa</td>
<td>Bond wrench (UoN)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.776</td>
<td>Triplet shear test (UoN)</td>
</tr>
<tr>
<td>( \tau_o )</td>
<td>0.310 MPa</td>
<td></td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>0.776</td>
<td></td>
</tr>
<tr>
<td>( G_f^{II} )</td>
<td>( 0.13\sigma_n + 0.03 ) N/mm</td>
<td>Estimated from literature</td>
</tr>
<tr>
<td>( \gamma_{crit} )</td>
<td>0.000477 mm</td>
<td>0.0001 x average element length</td>
</tr>
</tbody>
</table>

Elastic parameters for brick and mortar

| \( E_b \)                                         | 35360 MPa       | Prism compression tests (UoN)                 |
| \( E_m \)                                         | 2772 MPa        |                                               |
| \( v_b \)                                         | 0.15            | Literature                                    |
| \( v_m \)                                         | 0.125           |                                               |
Figures 4.27a - 4.30a show respectively the predicted torque versus rotation responses for cases (i) to (iv). Figures 4.27b – 4.30b show the regions up to and including the peak torques in more detail. These numerically generated plots represent what is measured directly during the experimental program. Qualitatively, the plots agree with intuition based on the observations reported by other researchers using various shear testing approaches (Pluijm, 1993; Willis, et al., 2002). An initial linear response is followed by some loss of stiffness as the peak torque is approached. This results from cracking and softening at the outer fibres while at points at smaller radii, the shear stress continues to increase. Eventually sufficient points on the cross section exceed their peak shear stress and the total torque response softens until all points have reached their residual shear strength and the total torque reaches a residual (frictional sliding) value.
Figure 4.27. Numerically predicted torque versus rotation response, case (i).
Figure 4.28. Numerically predicted torque versus rotation response, case (ii).
Figure 4.29. Numerically predicted torque versus rotation response, case (iii).
(a) Full range

(b) Up to peak torque

Figure 4.30. Numerically predicted torque versus rotation response, case (iv).
Table 4.10 shows, for each of cases (i) to (iv), values of torque calculated by substituting the material parameters from Table 4.9 into Equations 4.19 and 4.20. The values of torque calculated using $\tau = \tau_u$ in Equation 4.19 assume linear elastic behaviour for all radii and therefore represent the torque resisted by the cross sections when first cracking occurs at the extreme outer fibres. In each case, these torque values are less than the peak torque resisted and lie close to the point at which the torque versus rotation plots first become non-linear. The torques calculated using $\tau = \tau_u$ in Equation 4.20 assume that shear stress is uniform with radius and represent an upper limit on the peak torque resisted. In each case, the values indeed bound the finite element predictions (Figures 4.27 to 4.30) and in cases (iii) and (iv) they agree very closely with the numerically predicted values of $T_{\text{peak}}$. Equation 4.20 was also used to calculate values of $T_{\text{residual}}$ by substitution in the residual shear strength $\tau_{\text{res}}$ for $\tau$ and comparison with the plots shows excellent agreement for all four cases.

Table 4.10. Analytical torque predictions using elastic and plastic theories and numerical predictions.

<table>
<thead>
<tr>
<th>Case</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_u$ (MPa)</td>
<td>0.310</td>
<td>1.086</td>
<td>0.310</td>
<td>1.086</td>
</tr>
<tr>
<td>$\tau_{\text{residual}}$ (MPa)</td>
<td>0</td>
<td>0.776</td>
<td>0</td>
<td>0.776</td>
</tr>
<tr>
<td>$T_{\text{linear}}$ max (Nmm) using Equation 4.19 ($T_{\text{linear}} = \tau_u \frac{I_p}{r_o}$)</td>
<td>60870</td>
<td>213240</td>
<td>46250</td>
<td>162040</td>
</tr>
<tr>
<td>$T_{\text{peak}}$ (Nmm) using Equation 4.20 ($T_{\text{peak}} = \tau_u 2\pi (r_o^3 - r_i^3)/3$)</td>
<td>81160</td>
<td>284310</td>
<td>53320</td>
<td>186790</td>
</tr>
<tr>
<td>$T_{\text{peak}}$ (Nmm) using numerical method</td>
<td>77000</td>
<td>279288</td>
<td>52151</td>
<td>186000</td>
</tr>
<tr>
<td>$T_{\text{residual}}$ (Nmm) using Equation 4.20 ($T_{\text{residual}} = \tau_{\text{residual}} 2\pi (r_o^3 - r_i^3)/3$)</td>
<td>0</td>
<td>203160</td>
<td>0</td>
<td>133470</td>
</tr>
<tr>
<td>$T_{\text{residual}}$ (Nmm) using numerical method</td>
<td>0</td>
<td>201500</td>
<td>0</td>
<td>133000</td>
</tr>
</tbody>
</table>

For each of cases (i) to (iv), Figures 4.31 - 4.34 show distributions of shear stress at the brick/mortar interface versus radius through the various stages of loading. For each of the figures the various lines represent the shear stress distribution at a different value of...
relative rotation. Similarly, Figures 4.35 and 4.36 show the distributions of normal stress across the brick/mortar interface versus radius at different rotations.

Figures 4.35 and 4.36 indicate that the assumption of uniform normal stress over the cross section is satisfied closely for cases (i) and (iii) with zero normal force. For cases (ii) and (iv) with normal pre-compression the assumption is still reasonable with the normal stress deviating from the mean value by a relatively small amount at the extreme fibres.

The distributions of shear stress clearly show the expected linear variation with radius at small rotations, with softening at the outer fibres and extending inwards as rotation is increased. The single line of heavier shading in each of Figures 4.31 to 4.34 represents the relative rotation corresponding to the peak torque resisted. Figures 4.33 and 4.34 indicate that at peak torque, the distributions of shear stress are very close to uniform with radius for the annular cross section. Furthermore, this uniform shear stress is equal to the shear strength $\tau_u$ predicted using the Coulomb relationship (Table 4.10). As identified above from the torque versus rotation plots, this will allow $\tau_u$ to be obtained directly from the experimentally measured $T_{peak}$ using Equation 4.20, even for this relatively “thick” 15mm annular section. For the solid cross section the distributions of shear stress at peak torque are approximately constant across a significant portion of the cross section but vary back to values close to zero at small radii.

In all of the plots the shear stresses approach the residual values reported in Table 4.10 for large values of rotation. For the 15mm thick annular cross section, the distributions of shear stress with radius during the softening phase of the response are also reasonably close to uniform, particularly in case (iv) with pre-compression. The use of Equation 4.20 to obtain values of shear stress in the softening branch is therefore also reasonable.

The results of this study therefore confirm that the torsion shear test is able to produce the desired conditions of uniform shear and normal stress across a mortar joint during both the pre and post peak stages of behaviour. This allows the $\tau$ versus displacement response at a material point to be accurately measured.
Figure 4.31. Shear stress at brick/mortar interface versus radius, case (i).

Figure 4.32. Shear stress at brick/mortar interface versus radius, case (ii).
Figure 4.33. Shear stress at brick/mortar interface versus radius, case (iii).

Figure 4.34. Shear stress at brick/mortar interface versus radius, case (iv).
Figure 4.35. Normal stress across brick/mortar interface versus radius, cases (i) and (ii)

Figure 4.36. Normal stress across brick/mortar interface versus radius, cases (iii) and (iv)
It should be noted that no attempt was made to specifically simulate the preliminary experimental series of torsion shear tests described in Section 3.10. The simulations reported here were conducted using different geometry and strength parameters than those observed for the specimens described in Section 3.10. Therefore, the numerical and experimental results cannot be compared directly. However, the numerically predicted torque versus rotation response agrees well with the experimentally observed response in a qualitative sense.

4.10 Summary

A 3D non-linear finite element micro-model was used to predict the behaviour of four brick unit specimens subjected to vertical, horizontal and biaxial bending loads, along with simultaneous vertical pre-compression. This was made possible by introducing cohesive contact between brick and mortar into the ABAQUS/Standard software package. The model describes brick and mortar with linear elastic elements and the bond between brick and mortar as a non-linear contact surface.

It was shown that the micro-model could be used successfully to simulate the peak and post-peak behaviours of four brick unit specimens subjected to separate vertical and horizontal bending. The resistance moment behaviour of four brick unit specimens subjected to biaxial bending was also satisfactorily captured. In most cases the model also provided good prediction of peak moment capacities. However, to match the residual horizontal moment capacity in biaxial bending, the model would need to be improved to take into account the softening effect of the residual friction coefficient.

The model was also used to evaluate a newly developed torsion shear test method. It was shown that the model satisfactorily captured the main features expected from the test, namely, the shear stress distribution, normal stress distribution and torque versus rotation behaviour. It proved that close to uniform normal and shear stress distributions in the mortar joint could be achieved with the new test method and the latter could be confidently introduced as a useful tool to allow shear behaviour to be characterized at a material point.
CHAPTER 5
MODELLING OF WALL PANELS: SIMPLIFIED
FINITE ELEMENT MICRO-MODEL
5. **MODELLING OF WALL PANELS: SIMPLIFIED FINITE ELEMENT MICRO-MODEL**

5.1 **Introduction**

In Chapter 4 a finite element micro-model was proposed and described in detail. The model has been shown to be capable of simulating the behaviour of four brick unit specimens subjected to various out-of-plane bending conditions as well as in-plane vertical pre-compression. While the micro-model has been proved to be very efficient for modelling such small specimens, it is not practical to apply this model to wallettes or large-scale walls directly due to the intensive computation required. In this chapter this detailed micro-model has been further simplified so that its applicability can be extended to small wall panels, for which experimental data are available in the literature for comparison.

Section 5.2 provides a brief outline of the simplified micro-model. Its applicability is first demonstrated in Section 5.3 by modelling a small wall subjected to horizontal bending. In Section 5.4 the model is further modified to include brick tensile failure as well as brick crushing failure. The efficacy of the simplified model is then examined by simulating the coupled shear wall behaviour observed in tests conducted at The University of Newcastle. The results show that the model is capable of capturing the same characteristic failure mechanism as observed in the laboratory tests. In Section 5.5 the model is further extended to the case of a large-scale wall. It is shown that under well defined boundary conditions the model may enable estimation of the load capacity of a masonry wall subjected to out-of-plane load.

5.2 **The Simplified Micro-Model**

The micro-model described in Chapter 4 is capable of modelling in detail individual bricks, mortar and the bond between brick and mortar. While accurate description of masonry components (small specimens) is vital to gain fundamental understanding of masonry behaviour, the ability for direct prediction on larger scales cannot be overstated. However, intensive computational effort will be required if the micro-model
is used without simplification to model the behaviour of a real wall. It can also be expected that the discrete micro-model, which incorporates modelling of brick mortar interface, would take even longer than the continuum model.

In this section the micro-model as detailed in Chapter 4 is simplified in such a way that the units are expanded and still represented by continuum elements, but the behaviour of mortar and bond between mortar and unit is “smeared” with zero thickness non-linear surface contact. Although this simplified model may lose a certain degree of accuracy since the Poisson’s ratio effect of mortar is ignored, under circumstances where not all of the model parameters are available experimentally, this compromise between efficiency and accuracy is considered reasonable.

Lourenço (1996) made a comparison between the micro-model and simplified micro-model. He modelled a wall made up of 18 courses of clay bricks and 10mm thick mortar joints subjected to in-plane shear force with both the continuum model and simplified micro-model. Both models predicted the displacement in the linear elastic range very closely. However, the computational time required using the simplified micro-model was only 8% of that for the continuum model.

Due to the larger size of the brick elements and the assumption of zero thickness for mortar, the elastic stiffness of the “expanded unit” needs to be adjusted in the simplified micro-model to yield correct results. Lourenço (1996) proposed two methods to derive this property. The first approach was to assume that the elastic stiffness of the unit remains unchanged. However, the normal and shear stiffness of the interface between the bricks were adjusted using a formula (CUR, 1994) derived based on the assumption of stack bond, a serial chain connection of the components, and uniform stress distributions both in unit and mortar. The second approach was to reduce the stiffness of the unit to the stiffness of masonry and use interface elements with dummy stiffness to avoid interpenetration of the continuum.

In this study, Lourenço’s second approach has been adopted, mainly because there is a lack of separate data associated with bricks and mortar that accompany wall tests reported in the literature. In this approach the masonry properties were obtained through a series of control tests accompanying those wall tests. These control tests yielded
elastic stiffness of the wall, which can be directly used for the “expanded bricks” in the simplified micro-model.

5.3 Modelling of Small Walls Subjected to Horizontal Bending

Willis et al. (2002) tested wallettes that were six courses high and three and a half bricks in length. Each of the wallettes was supported vertically and subjected to horizontal bending in a four point bending arrangement over a span of 800mm with 250mm between load lines. At the same time vertical pre-compression was applied via concrete blocks to help distribute the pressure evenly. Teflon / grease interfaces were used to prevent out-of-plane restraint by the pre-compression application. Figure 5.1 illustrates the laboratory setup used by Willis et al.

![Figure 5.1. Horizontal bending test arrangement used by Willis et al. (2002).](image)

Prior to the wallette tests, Willis et al. (2002) also tested couplets that were subjected to torsion in order to estimate the shear strength of single joints. Their test arrangement consisted of specimens of two brick units including a mortar bed joint being subjected to various levels of pre-compression combined with a torsional moment acting about the centre of the bed joint. The specimens were prepared such that the mortar on the bedding face was reduced by a distance of one-quarter of the total unit length from each
end (Figure 5.2a). This rectangular section of joint represented the plan area of mortar between brick units of adjacent courses in half overlap stretcher-bonded masonry.

![Torsion test arrangement (Willis et al., 2002)](image1) ![Numerical mesh](image2)

**Figure 5.2. Couplets subjected to torsion.**

From the torsion test a relationship (Equation 5.1) was determined using multiple linear regressions relating the mean torsional shear strength ($\tau_u$) to a function of the flexural strength ($f_{mt}$) of the masonry and the level of compressive stress ($\sigma_v$):

$$\tau_u = 0.139 + 0.824\sigma_v + 2.088f_{mt}$$

Equation 5.1

The inclusion of flexural strength as a variable enables calculation of the ultimate torsional shear stress accounting for the variability between mortar batches. The value of $\tau_u$ was calculated as the peak shear stress on the rectangular mortar joint cross section, assuming linear elastic conditions at peak torque using an equation based on the theory of elasticity (Equation 5.2)

$$\tau_u = \frac{T_u}{kab^2}$$

Equation 5.2

Use of this equation is questionable for two reasons: (i) The sudden change in cross section from a full brick to half brick length mortar joint violates one of the key assumptions upon which Equation 5.2 is based, and (ii) Equation 5.2 assumes linear
elastic behaviour at peak torque when this may not be valid. The result is that Equation 5.2 is unlikely to represent the true shear strength at a material point. The latter is what is required for use in the contact interface properties for the simplified micro modelling strategy. For these reasons the shear strength derived from Equation 5.1 has not been adopted here to model Willis et al.’s wallette test.

The approach adopted in the current study was to model a set of Willis et al.’s torsion tests directly to obtain the shear properties through a calibration process. The strategy used to obtain the input parameters, namely, shear bond strength and frictional shear strength, was to first model a couplet torsion test under one level of pre-compression. This enabled the determination of essential parameters such as the shear bond strength and internal friction coefficient by adjusting these parameters to match the experimentally observed peak and residual torque values. Then these fitted parameters were further tested by applying the model to the torsion tests of the same couplet batch subjected to other levels of pre-compression. Figure 5.3 shows a typical relationship of torque versus rotation observed in the experiment. In these tests three levels of pre-compression (0, 0.25 and 0.5MPa) were added before the upper brick was rotated around the vertical axis. This set of experimental data was used initially to obtain the basic parameters for the simplified micro-model.

![Figure 5.3. Torque versus rotation behaviour for varying levels of pre-compression](Willis, 2004).
The brick was modelled using 8 nodes 3D continuum elements with eight elements in length, four elements in width and four elements in height (cf. Figure 5.2b). The white elements at the top represented a rigid plate and rotation was prescribed about a vertical axis through the central node of that plate. A preliminary model with linear softening of shear strength was first chosen to simulate the torsion behaviour and the results have been published elsewhere (Han and Masia, 2004). With linear softening the model predicted the peak torque satisfactorily. Nevertheless, prediction for the residual torque did not match the experiment well. Further study has shown that this discrepancy was due to the fact that in the linear softening procedure the effect of pre-compression on fracture energy was not accounted for.

In the current simulation an exponential softening procedure for the shear strength was used. The predicted torque versus rotation relation is shown in Figure 5.4. The fracture energy used in the exponential softening was $0.13\sigma + 0.14$ N/mm, where $\sigma$ represents the normal compressive stress across the mortar joint. The fracture energy was obtained based on a trial and best fit approach.

![Graph showing torque-rotation behaviour](image)

**Figure 5.4.** Predicted torque-rotation behaviour under varying levels of compressive stress using the simplified micro modelling strategy.
In summary, the detailed procedure used here to obtain the basic parameters for the simplified micro-model was as follows:

- Firstly, simulation was performed to establish the torque-rotation relationship for the couplet tested under no pre-compressive stress, so that the shear bond strength could be obtained.
- Secondly, the shear bond strength combined with the trial value of the frictional coefficient was used in the model to get the best fit of the torque-rotation relationship for the couplet tested under the 0.25MPa compressive stress.
- Thirdly, the shear bond strength and frictional coefficient obtained from the previous two steps were entered into the model to see how well the predicted curve fitted the experimental torque-rotation relationship for the torsion test under 0.5MPa.
- Given the possible variability of the both shear bond strength and frictional coefficient, the steps from one to three were iterated until the outcome was considered satisfactory.

From the above iteration process a friction coefficient of 0.45 and shear bond strength of 0.42MPa were obtained. These values were much lower than those (0.824 and 1.25MPa) derived from Equation 5.1 using a flexural strength of 0.53MPa for the same batch of couplets. This disparity further highlights the inappropriateness of using the rectangular specimen for the torsion test to obtain the shear properties through Equation 5.1, because of the complex shear stress distribution across the rectangular section as well as possible non-uniform normal stress distribution.

The friction coefficient of 0.45 and shear bond strength of 0.42MPa obtained from the iteration approach can then in theory be used as the basic parameters to model the wallets subjected to horizontal bending. Note that the obtaining of these parameters was based on the simulation of torsion tests of the couplet specimens with flexural bond strength of 0.53MPa. The batches of masonry used in the wallets under horizontal bending tests, however, all have the flexural bond strength of 0.56MPa. Before the values of 0.45 and 0.42MPa can be used for the simulation of the wallette horizontal tests, they need to be adjusted to reflect the batch differences. In this study only the shear bond strength was adjusted based on the flexural tensile strength, assuming a
linear relationship exists between the shear bond strength and flexural tensile strength. Under this assumption shear bond strength of 0.445MPa is obtained and used in the subsequent modelling of the wallettes subjected to horizontal bending.

Figure 5.5a shows the finite element mesh for a wall that was simulated using the simplified micro-model. The brick was modelled with eight elements in length, four elements in width and four elements in height (cf. Figure 5.2b). The support bars, as shown in Figure 5.1, were not explicitly modelled. Instead, the nodes on the walls where the support bars are located were fixed so that no movements were allowed along the direction perpendicular to the wall panel. The loading process was modelled by giving the nodes where the loading bars are located a prescribed uniform displacement perpendicular to the wall which was increased incrementally up to a value of 8mm. Figure 5.5b illustrates the numerically modelled wallette with cracks formed in the perpend joints in the wall when subjected to horizontal bending. Note that in the numerical modelling, brick cracking failure was not included though in the real wall test, this mode of failure occurred. The occurrence of brick failure is mostly dependent on the ratio of flexural strength of bricks to the bond strength between bricks and mortar. These two strength values normally have high variability. Furthermore, the existence of vertical compressive stress can even complicate the situation. This was also shown in Willis et al.’s tests. For the five walls tested under no pre-compression, stepped failure through the joints occurred. However, as the pre-compression was increased from 0.075 to 0.25MPa, there were more cases of failure occurring in both bricks and bed joints. Despite the fact that various combinations of failure modes can be present in one wall, the deterministic model can only capture two failure modes, either step failure (failure all through bed and perpend joints) or line failure (failure all through bricks and perpend joints). Since most cases of failure in Willis et al.’s tests were through bed joints, attempt was only made here to model the stepped failure mode.
Figure 5.5. Simulation of a wallette, showing the wall mesh used and crack formation under a flexural load.
Figure 5.6 illustrates the typical load deflection behaviour recorded in the laboratory wallette tests. The eight parameters that had been used to describe the flexural response in the horizontal bending tests are indicated in the figure. These are:

- Primary slope $S_1$, indicating elastic behaviour
- Load $P_1$, where cracking of the perpend joints first occurred
- $\Delta_1$, deflection at change of slope
- Secondary slope $S_2$
- Ultimate load $P_u$
- $\Delta_u$, deflection at ultimate load
- $P_f$, frictional load resistance
- $\Delta_f$, deflection at which $P_f$ is reached.

Figure 5.6. Experimental load-deflection behaviour in horizontal bending test (Willis et al., 2002).

Although deflection is a very important characteristic for the description of wall response, in this work comparison could not be made between the model and experiment due to lack of data on fracture energy, the latter being a crucial factor required in the modelling of deflection. Instead, focus was drawn only on the load behaviour, including $P_1$, $P_u$ and $P_f$. 
The load-deflection behaviour of the wall predicted by the numerical model is shown in Figures 5.7a and 5.7b. The numerical modelling shows three stages of loading, which are consistent with the wall behaviour observed experimentally (cf. Figure 5.6). In the first stage the wall is loaded almost linearly until the load reaches $P_1$ where cracking of the perpend joints first occurs. This can be seen as a marked change in slope (Lawrence and Morgan, 1975; Lawrence, 1983). The wall has only minor deflection at this stage so the torsional strain in the bed joint is low and thus torsional stress is quite low. The wall strength is dominated by the flexural strength of perpend joints. In the second stage as the wall further deflects, the perpend joint cracks further causing the increased torsional strain and thus the wall behaviour is switched to be governed by bed joint torsional shear behaviour. The load increases until it reaches the ultimate load capacity $P_u$, where most of the bed joints reach their maximum shear strength. After reaching the ultimate load, the load resistance of the wall drops gradually as further increase in the deflection of the wall causes progressive damage to the bed joints and the shear strength in the bed joints decreases. Finally, in the third stage upon the existence of vertical pre-compression, when the load capacity reaches $P_f$, the residual strength is dominated by the torsional frictional resistance of bed joints. Note that if there is a pure line failure through bricks and joints, there is no frictional resistance.

![Figure 5.7. Numerically predicted load-deflection behaviour.](image)

(a) Before peak load  (b) Full range

The predicted behaviour of the walls with a flexural strength of 0.56MPa subjected to horizontal bending as well as various levels of pre-compression is shown in Figure 5.8. The impact of the pre-compressive force on the wall’s load capacity is clearly illustrated in the prediction. Just like what was captured in the experiment (Figure 5.9), Figure 5.8
demonstrates clearly that the horizontal bending capacities (both ultimate load and residual load) increase as the compression is increased. However, based on the model prediction, the existence of pre-compression seems to have no impact on the first cracking load and stiffness. This trend was not reflected in Figure 5.9. This difference can be due to the fact that although the first cracking load is closely related to the flexural strength of masonry, the masonry stiffness is closely related to the vertical pre-compression. The first half of this argument seems to be supported by the test data shown in Table 5.1. The test data show quite consistently with only one exception ($P_1 = 5.8\text{kN when } \sigma = 0$) that the first cracking load for the walls with a flexural strength of 0.56MPa varied only between 3.72 and 3.94MPa. The second half of the argument is supported by the work of Vermeltfoort (2005). He has shown that the edge of a mortar joint has a zone where there is no bond between the mortar and the unit caused by bleeding of water as the moisture moves between mortar and unit when the unit is laid. After that, there is an area of partial or broken bond, and finally an inner portion with complete bond. Thus when the joint is compressed, the fissures and areas of no bond close up. The contact area increases and consequently so does the stiffness. This feature of changing stiffness cannot be modelled with the current deterministic model.

Table 5.1 compares the $P_1$, $P_u$ and $P_f$ obtained from the experiment with those from the numerical simulation. Note that experimental $P_f$ values for the cases of pre-compression of 0.15MPa and 0.25MPa were calculated using the actual torsional frictional resistance reported by Willis et al. divided by the actual number of bed joints failed times five, assuming five bed joints all failed so that comparison could be made between experiment and numerical prediction. In the experiment, three out of the five bed joints failed.
Figure 5.8. Predicted horizontal bending load-deflection behaviour of a wallette under various levels of pre-compression.
Table 5.1. Measured and predicted load capacity of walls.

<table>
<thead>
<tr>
<th>$\sigma$ (MPa)</th>
<th>$P_1$ (kN)</th>
<th>Diff.</th>
<th>$P_u$ (kN)</th>
<th>Diff.</th>
<th>$P_f$ (kN)</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp.</td>
<td>Mod.</td>
<td></td>
<td>Exp.</td>
<td>Mod.</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>5.8</td>
<td>4.28</td>
<td>-26%</td>
<td>11.21</td>
<td>12.6</td>
<td>12%</td>
</tr>
<tr>
<td>0.075</td>
<td>3.88</td>
<td>4.28</td>
<td>10%</td>
<td>13.95</td>
<td>13.7</td>
<td>-2%</td>
</tr>
<tr>
<td>0.15</td>
<td>3.72</td>
<td>4.28</td>
<td>15%</td>
<td>14.93</td>
<td>14.7</td>
<td>-2%</td>
</tr>
<tr>
<td>0.25</td>
<td>3.94</td>
<td>4.28</td>
<td>9%</td>
<td>11.16</td>
<td>16.1</td>
<td>44%</td>
</tr>
</tbody>
</table>

Overall, except for one case ($P_1 = 5.8$kN when $\sigma = 0$) the predicted value of 4.28$kN$ for the cracking load agrees with the experimental results reasonably well, with the maximum difference being only 15%. The predicted ultimate load for walls also fits the experimental data well. With one exception (44%) all the predictions are within a range of 12% difference from experiment. The agreement for the walls under the 0.075 and 0.15MPa pre-compression was even more remarkable; only 2% difference was noticed. However, prediction for the wall under 0.25MPa pre-compression resulted in a 44% difference from the experiment. This may be due to the variability of the laboratory test.
itself, since close examination of the whole test data for this wall clearly shows that this particular value is against the trend from all the other data in this series.

The predicted residual loads are, however, on average much lower than those obtained from the experiment except for the case when the wall is under no pre-compression. However, the difference is still within a 30% range. The overall lower prediction may have resulted from the assumption in the modelling that the friction coefficient remains constant for all batches of walls. As a result, no correction for the friction coefficient was made to reflect the masonry batch variability. Note that the exceptionally high prediction for $P_f$ under zero compression should not be given too much attention since the precision of the test data under zero pre-compression is questionable.

5.4 Modelling of Wall Panels Subjected to In-Plane Force

In 2001, experiments were carried out at The University of Newcastle to study the in-plane shear behaviour of unreinforced masonry walls with openings. The experiments consisted of a series of tests including a test of a single panel without openings and two panels with openings (Corrêa and Page, 2003). The panels with openings were built in such a way that the piers on each side of the opening had the same dimensions as the single panel. All the panels were built with clay bricks with dimensions 230mm x 110mm x 76mm and a 1:1:6 mortar (cement: lime: sand) by volume.

These experiments were not part of the current PhD program. However, in the current modelling study the above-mentioned walls were simulated using the simplified finite element micro-model as discussed in Section 5.2. The purpose of this modelling exercise was to provide an additional means to validate a proposed plastic hinge method, that is, a simple method for estimating the collapse capacity of flexurally dominant shear walls with openings (Corrêa et al., 2006). In doing this it was found that good agreement was achieved between the experimental data from Corrêa and Page (2003) and the current model prediction. This lends further support to the applicability of the modified micro-model for simulating the behaviour of wall panels.

In the experiment the single panel (without openings) was built 14 courses high and two bricks wide (Figure 5.10a). The wall was first loaded with a vertical load of 87kN. Then this load was kept constant while the horizontal load was slowly increased.
Experimentally, the first visible crack appeared at the joint of the second course at the back of the wall. The horizontal load at that time was 14kN. At 19.5kN the same crack increased and extended to the front toe of the wall. When the horizontal load reached 22kN, the crack opened and propagated down the first vertical joint near the front region just before toe crushing occurred. The maximum horizontal force was 22.8kN.

In modelling the single panel wall, the masonry units were expanded so that the joint was modelled with zero thickness. Units were simulated by 8-noded 3D solid elements with each unit containing only two brick elements (Figure 5.10b).

The ABAQUS default “concrete smeared cracking” criterion was adopted to model the non-linear behaviour of the units. The purpose of using a concrete material model was to capture the brick crushing failure which was observed to be occurring at the toe of the single panel. “The concrete smeared cracking” model is designed for applications in which the concrete is subjected to essentially monotonic straining at low confining pressures. It consists of an isotropically hardening yield surface that is active when the stress is dominantly compressive and an independent “crack detection surface” that determines if a point fails by cracking. This failure surface is illustrated in Figure 5.11. When concrete is loaded in compression, it initially exhibits elastic response. As the
stress is increased, some non-recoverable (inelastic) straining occurs and the response of the material softens. An ultimate stress is reached, after which the material loses strength until it can no longer carry any stress. When a uniaxial concrete specimen is loaded in tension, it responds elastically until, at a stress that is typically 7 - 10% of the ultimate compressive stress, cracks form. The model assumes that cracking causes damage, in the sense that open cracks can be represented by a loss of elastic stiffness. It is also assumed that there is no permanent strain associated with cracking. This will allow cracks to close completely if the stress across them becomes compressive. 14.1MPa is assumed to be the compressive strength of bricks for using this model. The default tensile strength of 1.4MPa, which is 10% of the compressive strength, is assumed for the bricks.

![Figure 5.11. ABAQUS concrete smeared cracking model.](image)

Both bed joints and perpend joints were modelled using non-linear interface elements with the tangential interaction being defined by a Mohr-Coulomb rule with a friction coefficient of 0.64 and cohesion of 1.5MPa. Normal interaction was represented as elastic brittle under tensile forces with a limit of 0.45MPa and infinite capacity under compression. Shear strength was assumed to soften linearly with $\tan \phi$ taken as 0.5, where $\phi$ is the angle between the softening branch of the shear stress $\tau$ versus displacement $\Delta$ plot and the horizontal axis. Although exponential softening would have been more appropriate, linear softening was adopted here instead, since no experimental
softening data were available. Therefore, a more computationally efficient softening strategy has been adopted here.

The predicted and the actual failure mechanisms are shown in Figures 5.12a and 5.12b. It is obvious that the finite element model closely reproduced the experimentally observed failure mechanism. However, when the vertical pre-compression was held at 87kN, the maximum horizontal load obtained from the numerical prediction was only 17.9kN, which is almost 21% less than the experimental value (Figure 5.13). It is hypothesized that this disparity may have been caused experimentally by an observed increased vertical load and an increased compression of the vertical jack against the reaction frame due to rocking of the wall at large displacements during the laboratory test.

(a) Detail of the modelled wall at collapse      (b) Detail of the wall at collapse
Figure 5.12. Finite element simulation of a single wall subjected to in-plane force.

To verify the above hypothesis the finite element model was further modified by including a vertical bar pinned at the ends to simulate the jack and a solid element to represent the top steel beam (see Figure 5.14). Figure 5.15a is a diagram showing the axial force in the vertical bar versus horizontal load from the numerical analysis. Comparing this to the experimental curve shown in Figure 5.15b, both curves show a similar pattern, although the characteristics of the bar simulating the jack only approximately represented the jack since the stiffness of the jack itself was unknown. This verifies the hypothesis and confirms that the most important aspect of the experimental results corresponds to the point before the increase of vertical load. To verify this further, a case where the wall was subjected to a vertical force of 150kN
instead of 87kN was simulated. This time a maximum horizontal force of 24.3kN was obtained, only 6% higher than the laboratory test value.

Figure 5.13. The horizontal load versus top displacement.

Figure 5.14. The single wall, showing the simulated top beam and vertical jack.
Table 5.2, based on Figure 5.15, gives a better verification of the accuracy of the finite element model by comparing the value of the horizontal force at different stages of the test for the single wall. Note that in the table comparison of the horizontal load at the end of the vertical load threshold is made just before the vertical load starts to increase from 87kN. It can be seen that important indicators are closely predicted by the numerical model, with the largest difference being less than 10% as compared with the experiment.
Table 5.2. Comparison of horizontal loads (kN) at different events of the single wall.

<table>
<thead>
<tr>
<th>Result</th>
<th>1st crack</th>
<th>2nd change of slope</th>
<th>3rd change of slope</th>
<th>End of vertical load threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>9</td>
<td>14.2</td>
<td>16.9</td>
<td>18</td>
</tr>
<tr>
<td>FE model</td>
<td>9.8</td>
<td>13</td>
<td>16.7</td>
<td>17.9</td>
</tr>
<tr>
<td>Difference</td>
<td>9%</td>
<td>-8%</td>
<td>-1%</td>
<td>-1%</td>
</tr>
</tbody>
</table>

The second panel modelled (see Figure 5.16) was a coupled wall with an opening between two single panels. The coupled panel had a flat steel bar lintel supporting the masonry above the opening, a common construction practice in Australia. In the laboratory test the loading procedure was similar to the single panel wall. First, a vertical load of 40kN on each pier was applied. In the second phase the vertical load was increased slowly from zero to the final value of 87kN each. During the loading process a small vertical crack appeared above the centre of the lintel. Therefore, the wall was unloaded to nearly 64kN to avoid the loss of the specimen and then horizontal load was applied on the rear pier as illustrated in Figure 5.16. When the horizontal load was increased from 10.5 to 12.6kN, the vertical loads were increased to 88.7kN and the application of horizontal load continued as usual. When the horizontal load reached 32kN, a small crack appeared over the opening, near the front pier. That crack propagated progressively until the horizontal load reached its maximum value of 64.4KN, when cracks near the heels of the piers occurred. At this stage the vertical load on each pier had increased to 129.2kN. After that the cracking increased with horizontal load until the crushing of the toes. The load was then reduced.
The coupled wall was simulated here with the modified finite element model similar to the single panel, except for the inclusion of an interface element between the two 3D elements representing each brick unit in order to simulate potential vertical failure surfaces at the unit mid-length. These interface elements were located at the critical regions (at the toe of the piers and at the interface planes of piers and lintel). The Coulomb criterion was also used for the tangential contact between brick and brick interfaces. The cohesion and friction coefficient adopted for these interfaces are consistent with the compressive and tensile strengths shown in Table 5.3. The parameter values for the mortar joints and for the material “concrete” of the unit were reduced by 7%, consistent with the values obtained for the prisms as compared to the single panel.

Figure 5.17 shows the modelled wall as well as the theoretical failure pattern of the wall. The predicted failure pattern can be seen to be very similar to the experimentally observed failure pattern as shown in Figure 5.18. From the modelling work, a horizontal force ($H$) of 50kN was obtained for the intended vertical load of 87kN on the piers and $H = 67.6$kN was obtained for the actual vertical load of 129.2kN. Note that the value of 67.6kN is only 5% larger than the experimental result of 64.4kN.
Table 5.3. Summary of the material tests.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Number of tests</th>
<th>Test</th>
<th>Average strength (MPa)</th>
<th>C.O.V (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick</td>
<td>10</td>
<td>Compression</td>
<td>18.6</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Indirect tension</td>
<td>1.8</td>
<td>14</td>
</tr>
<tr>
<td>Joint</td>
<td>6</td>
<td>Direct tension</td>
<td>0.45</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>Shear</td>
<td>$c_o=1.5$, $\mu=0.64$</td>
<td>-</td>
</tr>
<tr>
<td>Wallettes</td>
<td>3</td>
<td>Compression</td>
<td>14.1</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>perpendicular to bed joint</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>Concrete</td>
<td>6</td>
<td>Compression</td>
<td>42.5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>Indirect tension</td>
<td>4.3</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 5.17. Finite element simulation of the coupled wall without pre-cast beams at collapse.
The third wall panel modelled in this study was a coupled panel with pre-cast concrete beams above the opening as shown in Figure 5.19. Experimentally the vertical load was increased slowly from zero to 87kN on each of the piers and maintained constant until the end of the test. Then the horizontal load acting upon the rear pier was increased slowly from zero until the failure of the panel occurred. The first crack appeared between the pre-cast beam and the rear pier at a load of 29kN. When the horizontal load reached 43kN, some cracks were observed above the opening, crossing both the brickwork and the beams. Eventually horizontal cracks became visible at the base of the two piers, near the heel when the horizontal load was 58.8kN. That was the maximum horizontal load obtained. Figure 5.19 shows the final experimental failure pattern of the panel.
To simulate the coupled panel with pre-cast concrete beams using the finite element model, additional parameters needed to be obtained. The first one was the property of the Coulomb law for the interface elements within the reinforced concrete beams and this was obtained from the compression and the indirect tension tests of the concrete specimens. The second parameter was the shear property of the interfaces between the concrete beams and bed joints. Since no test was carried out to measure these values, they were obtained by taking the values from Mullins and O’Connor (1989) for the interface between trowelled concrete and 1:1:6 mortar, i.e., shear bond strength of 0.75MPa and friction coefficient of 0.54.

Figure 5.20 shows the numerically predicted failure pattern. This is very similar to the experimentally observed failure pattern as shown in Figure 5.19. The maximum horizontal load obtained with the finite element model was 64.2kN, which is 9% larger than the experimental result of 58.8kN. By changing the vertical load from an intended value of 87kN to the actual recorded value of 84.4kN, the predicted peak horizontal load reduced to 64.0kN, slightly closer to the experimental value.
Table 5.4 provides a comparison between the experimental results and the numerical prediction for the three types of wall panels studied above. The fact that the numerical predictions compare well with all of the experimental results clearly demonstrates that despite its simplicity, the modified finite element micro-model exhibits good accuracy even when a coarse mesh is used.

Table 5.4. Comparison of horizontal peak loads.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Mechanism Type</th>
<th>Experimental $H$ (kN)</th>
<th>Prediction (kN) (Under intended vertical load)</th>
<th>Prediction (kN) (Under recorded vertical load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>Hinge at base</td>
<td>22.8</td>
<td>17.9 (-21%)</td>
<td>24.3 (+6%)</td>
</tr>
<tr>
<td>Coupled without pre-cast beams</td>
<td>Weak beam</td>
<td>64.4</td>
<td>50 (-22%)</td>
<td>67.6 (+5%)</td>
</tr>
<tr>
<td>Coupled with pre-cast beams</td>
<td>Mixed</td>
<td>58.8</td>
<td>64.2 (+9%)</td>
<td>64 (+9%)</td>
</tr>
</tbody>
</table>

5.5 Application to Two-Way Bending in Walls

To test the model’s applicability to simulating how walls behave in two-way bending further, a wall with dimensions of $2.5m \times 2.5m$ was modelled here using the simplified micro-model as discussed in Section 5.2.
During his PhD research, Lawrence (1983) performed extensive laboratory tests of wall panels subjected to biaxial bending. These tests included walls of various dimensions and the tests were carried out under different combinations of boundary conditions. The walls were supported in a stiff frame. Uniform lateral loads were applied through air bags, which were supported by a reaction frame. In conjunction with each wall test, small specimen tests were also carried out to obtain basic parameters for comparison between different theories. Among the walls tested, those with a $2.5m \times 2.5m$ dimension were tested using five categories of support conditions (Figure 5.21). The latter include: (1) four sides simply supported (SS), (2) four sides with built-in support (BI), (3) two sides with built-in support but top and bottom edges simply supported, (4) two sides with built-in support and bottom edge simply supported and top edge unsupported, and (5) the two sides and bottom edge all simply supported but the top edge free. The built-in condition was achieved by restraining the wall panel in its own plane at two opposite edges with a quite stiff steel frame. This stiff support gave restraint by the development of in-plane arching forces.

The above tests revealed three significant stages of cracking that had occurred in the wall. These were the occurrence of the first crack, formation of the full crack pattern, and failure of the wall panels. Based on these three stages of cracking behaviour, the test walls were classified into four distinct failure modes resulting from different combinations of the three stages. The first and simplest failure mode, called type A, is defined as having all three stages of behaviour coincident. There is no progression of behaviour through the three stages. Failure follows immediately as soon as the first crack forms. The second failure mode, called type B, is defined as having distinct first and third stages of behaviour, but no second stage. The third failure mode, called type C, is defined as having a distinct first stage, followed by coincident second and third stages. In other words, formation of the first crack is distinct, followed by an increase in applied load until formation of a more extensive crack pattern at which time failure occurs. The last mode of failure involves all three separate stages and is called type D. For this case the initial crack is similar to that in type C and is similarly followed by an increase in load capacity. However, formation of the full crack pattern constituting mechanisms is not immediately followed by failure in this case. The wall is still capable of resisting an increased load and failure follows later at a higher pressure.
Figure 5.21. Support conditions of the walls tested in Lawrence’s (1983) two-way bending tests.
To assess the ability of the simplified model to duplicate the cracking behaviour observed in Lawrence’s tests, a 2.5m × 2.5m wall was modelled with three types of supporting conditions, namely, Category 1, 2 and 5 (Figure 5.21).

The simplified micro-model as discussed in Section 5.2 was adopted here to simulate the wall panel behaviour. Use of the simplified micro-model, in which the mortar is not explicitly modelled, significantly reduced a large number of degrees of freedom so that the computation time could be kept to within an acceptable limit. In the model the bricks were expanded from their real dimension of 234 × 112 × 78mm to 250 × 112 × 88mm. The 2.5m × 2.5m wall comprised 28 courses of bricks in height and ten bricks in width. The mortar was assumed to have zero thickness and its material properties were replaced by horizontal and vertical interactions between the interfaces of the brick layers using contact interface relationships. Bricks were assumed to behave linear-elastically. Since the model was initially developed to model the four brick unit specimen tests and brick crushing was not observed in the four-unit specimen experiments, the compression failure of bricks was not included in the model. Figure 5.22 shows the finite element mesh of the wall panel modelled. The bricks were each modelled with four elements in length, two elements in height and two elements in thickness.

Figure 5.22. Meshes of the wall used in the finite element model.
Accompanying each wall test, Lawrence also tested small specimens to assess the brick and mortar batch variability. These included vertical beam rupture tests, vertical beam compression tests and torsion tests, similar to those conducted by Willis et al. (2002) as described in Section 5.3. While the test results from vertical beam rupture tests and vertical beam compression tests can be directly used as model parameter inputs for normal tensile strength between the “expanded units” and elastic modulus for the “expanded units”, particular care was taken for the shear strength from the torsion test. Earlier in Section 5.3, questions were raised over the way the shear bond strength was obtained through the torsion test by Willis et al. (2002). Finite element modelling was performed to assess the torsion tests and it was found that the shear bond strength derived from the finite element modelling was much lower than that derived using Equation 5.1. Based on this assessment, the shear bond strength derived from the simulation of the torsion test was used in the numerical modelling of small wall panels.

Ideally the above-mentioned approach should be adopted here to assess the torsion tests performed by Lawrence since very similar torsion test methods were adopted by both Lawrence and Willis et al. However, since there is no detail given by Lawrence about the torque versus rotation relation for each torsion test, simulations were not performed on each torsion test accompanying the wall panel test. Instead, in the simulation of wall panels the torsional shear strength given by Lawrence was used directly in the modelling. Table 5.5 summarizes the parameters used in the modelling below.

**Table 5.5. Parameters used in the simulation of walls.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Wall Category 1</th>
<th>Wall Category 2</th>
<th>Wall Category 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick elastic modulus (GPa)</td>
<td>19.34</td>
<td>17.35</td>
<td>20.49</td>
</tr>
<tr>
<td>Tensile bond strength (MPa)</td>
<td>0.81</td>
<td>0.86</td>
<td>1.03</td>
</tr>
<tr>
<td>Shear bond strength (MPa)</td>
<td>2.18</td>
<td>2.18</td>
<td>2.62</td>
</tr>
<tr>
<td>Shear friction coefficient</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Shear softening tanφ (linear)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Case 1 – Modelling of a wall simply supported on the four edges – support Category 1

The first case modelled here was for a 2.5m × 2.5m wall which was simply supported on the four edges and was observed to exhibit failure pattern C in the laboratory tests. Experimentally, it was observed that for this type of support the first crack was usually horizontal, through the bed joint near the mid-height of the panel. However, when the length-to-height ratio was low, vertical cracking near the mid-length occurred first. For cases where the first crack was horizontal, the pressure-deflection plot was linear up to this point, but there was noticeable non-linearity in cases where a vertical crack formed first. A cracking pressure of 7.6kPa and a full crack pattern pressure and ultimate pressure of 8.6kPa were recorded for the wall undergoing the test. Along with this test, several small specimen tests were also conducted to obtain the vertical beam strength (0.81MPa), torsional shear strength (2.18MPa) and vertical beam elastic modulus (19.34GPa). These values were used in the model as the normal tensile strength, tangential shear strength between interfaces and elastic modulus of bricks, respectively. Since no data for the shear softening are available, the shear strength was assumed to soften linearly with \( \tan \phi = 0.5 \), where \( \phi \) is the softening angle.

Figures 5.23a and 5.23b illustrate the simulated crack formation at the early stage and final stage of cracking. It can be seen that with the simply supported square panel the vertical cracks appeared first in the perpend joints near the mid length of the wall. These cracks were accompanied by those that appeared at the base of the wall just above the first course of bricks. As the load was further increased, several horizontal and diagonal cracks formed in the upper part of the wall. After that the wall could still withstand a further load increase until it failed. No other cracks were formed during this stage.

Comparing the final crack pattern illustrated in Figure 5.23b with the experimentally observed pattern (Figure 5.24) shows that the two are generally in good agreement. Note that the diagonal crack formation at the lower part of the wall, although present, is not as distinct in the simulation as in the experiment. This may be due to the self-weight effect of the bricks which strengthens the lower part of the wall. However, in a real wall this effect may be neglected by the variability of individual joints which can cause the weak joints at the lower part of the wall to crack first.
Figure 5.23. Cracking progress for wall with support Category 1 (numerical).

Figure 5.24. Final crack pattern for wall with support Category 1 (Lawrence, 1983).
The model gives a value of 7.58kPa for the pressure at the first obvious vertical cracks. This is in excellent agreement with the experimental cracking pressure of 7.6kPa. In addition, the model shows that there is a small difference between the full crack pattern pressure (8.0kPa) and the ultimate pressure (8.15kPa), whereas the experiment has given the same value of 8.6kN for the two, as mentioned before. The simulated ultimate pressure is only 5% lower than the experimental value.

Figure 5.25a shows the modelled behaviour of lateral pressure versus deflection in the middle point of the wall. For the purpose of easy comparison with experimental result the same scales of X and Y axes that are from Lawrence’s graph (Figure 5.25b) are used in Figure 5.25a. Note that in Figure 5.25b, the unit of the deflection is 10mm. Also shown in the insert of Figure 5.25a is the magnified graph which gives a clearer view of the behaviour of the wall panel. It shows clearly a linear behaviour prior to the initial cracking at about 7.1kPa. Beyond this point a non-linear behaviour is observed and this becomes more pronounced when more vertical cracks, and particularly horizontal cracks and diagonal cracks, are formed. The onset of this non-linear behaviour at about 7.1kPa can also be seen in the experimental pressure-deflection curve. However, comparison of the two results shows that the predicted deflection of the wall at the ultimate pressure is approximately half the experimental value. This may be due to the fact that the boundary conditions applied in the model assume no movement is allowed in the wall plane for all the support nodes. However, in the experiment this may not be the case. Possible wall support movement in the experiment may lead to a lower stiffness.
Figure 5.25. Pressure-deflection for wall with support Category 1.

(a) Prediction

(b) Experiment (Lawrence, 1983)
Case 2 – Modelling of a wall supported on four sides with built-in supports – support Category 2

The second case considered in the modelling study was a wall supported on four sides with all edges built-in hard against the support frame. Two 2.5m × 2.5m wall panels had been experimentally tested under such boundary conditions. The built-in conditions were achieved by restraining the wall panel in its own plane at two opposite edges with quite stiff steel frame. This stiff support gave restraint by the development of in-plane arching forces. However, the plotted profiles of the deflections of the wall panels at all stages clearly showed that the built-in supports did not provide significant restraint against rotation. Therefore, the strengthening effect of the built-in supports was mainly due to the induced arching forces rather than the clamping moments at the edges. Due to a lack of information about the stiffness of the supporting frames used in the experiment, in the numerical model the panel could be only assumed to be rigidly supported in-plane at the edges. No rotational restraint was added to the edges of the wall. So the wall was actually idealized to be pin supported at all edges as shown in Figure 5.26. These idealized support conditions make it difficult to attempt any quantitative comparison between the numerical analysis and experimental result because the finite stiffness of the experimental support must have some effect on the experimental behaviour. The main purpose for the numerical modelling here was to demonstrate the ability of the model to capture the characteristic cracking behaviour observed in the experiment, especially the arching effect.

![Figure 5.26. Idealized built-in support in the numerical modelling.](image)

Experimentally, one of the walls could not continue to be loaded due to failure of the air bag at 12.1kPa after the full crack pattern was formed. For this wall the initial cracking pressure and the full crack pattern pressure were observed to be the same, at 9.1kPa. The wall was classified as failure type B. The other wall tested started to crack at a
similar pressure of 9kPa but a slightly higher full crack pattern pressure of 10.7kPa was observed. This wall failed at an ultimate pressure of 24kPa and this was classified as failure type D. Figure 5.27 illustrates the failure pattern recorded for the two walls. Clearly both failure patterns are characterized by a vertical crack line in the mid-length of the wall, followed by diagonal cracking.

![Image of full crack pattern](image)

**Figure 5.27. Full crack pattern for walls with support Category 2 (Lawrence, 1983).**

For the numerical modelling the control test results from the first wall were used to obtain input parameters. A vertical beam strength of 1.25MPa and torsional shear strength of 2.18MPa were used, respectively, as the tensile bond strength and tangential shear strength between “expanded bricks”. The elastic modulus (17.35GPa) of the vertical beam was used as the elastic modulus of “expanded bricks”.

Figure 5.28a illustrates the predicted pressure versus mid-wall deflection behaviour of a wall subjected to out-of-plane pressure load under the idealized rigid in-plane support and free rotational restraint. For comparison, the experimental result is shown in Figure 5.28b. In Figure 5.28a the first change in slope corresponds to the loading point where the first vertical crack is formed (Figure 5.29a). The second change in slope corresponds to the point where almost the full crack pattern is formed (Figure 5.29b). After that the pressure increases indefinitely until the deflection is too large for the whole wall to maintain equilibrium (Figure 5.29c). During this stage cracks may form around the four corners at the loading side of the wall (Figure 5.29d).
Figure 5.28. Pressure-deflection for wall with support Category 2.

(a) Prediction

(b) Experiment (Lawrence, 1983)
The numerical simulation predicts a cracking pressure of 25.3kPa and full cracking pattern pressure of 25.5kPa. These are almost three times the values observed during the experiment (9.1kPa). This discrepancy may be attributed to the fact that the compressive failure of masonry was not included in the model analysis but this type of failure (which will reduce the resistance of the wall) was observed in the experiment. The model shows that at this stage the compressive stress at the edge of the wall is about 5MPa. This suggests that before getting to the full cracking pattern pressure, the wall may have already failed by crushing of the mortar. However, this mode of failure is not included in the current model to catch this failure behaviour. Another contributing factor may be
the rigid in-plane stiffness assumption, the arching effect, for the supporting frame, whereas the experimental frame would exhibit a less stiff response therefore providing less arching restraint. This comparison clearly demonstrates the effect of properties of the supporting frame on the overall wall behaviour.

It is worth noting that based on the numerical simulation the arching effect appears to be important for correctly estimating the cracking load. Nevertheless, Lawrence (1983) concluded that the existence of arching effect had no impact on the initial cracking load.

Case 3 – Modelling of a wall simply supported on both sides and the bottom edge – support Category 5

In the third case modelled here the wall was simply supported on both sides and the bottom edge with the top edge unsupported. For this type of support, it has been observed experimentally that failure follows immediately as soon as the first cracks are formed. The final failure pattern observed in the laboratory test for this type of wall is shown in Figure 5.30d. The cracking pressure, full crack pattern pressure and ultimate pressure were all measured to be 7.8kPa.

A vertical beam strength of 1.03MPa, torsional shear strength of 2.62MPa and vertical beam elastic modulus of 20.49MPa, all obtained through small specimen tests in the laboratory, were used in the numerical analysis for the values of tensile bond strength, tangential shear strength at the joint interfaces and elastic modulus of bricks, respectively. The crack development, predicted from the numerical analysis, as the lateral pressure was increased is shown in Figures 5.30a to 5.30c. The first cracks were seen to appear in the perpend joints between the bricks on the top of the wall panel. Right after that, vertical cracks formed in the upper part of the wall, followed by formation of the diagonal cracks.
The final crack pattern compares very well with that from the experimental wall test (cf. Figures 5.30c and 5.30d). However, in contrast with the experimentally observed phenomenon, there was a significant increase in pressure from 3.37kPa to 9.18kPa between formation of the first cracks and occurrence of the failure. The numerically predicted failure pressure is 18% higher than the experimental value (7.8kPa). Examination of the maximum principal stress in the bricks modelled shows this stress had reached up to 4.32MPa in some of the bricks. Therefore, overestimation in the load capacity of the wall is quite likely due to the fact that the numerical model did not
include the possibility of flexural failure of bricks. In the experiment, failure of the bricks was observed. Although the mean modulus of rupture of the bricks was greater than 4.32MPa, variability in this property in the tested wall may have resulted in the observed brick failures at a lower stress. Such variability was not accounted for in the deterministic numerical model. Some attempt was made to model this failure mode by adding to the bricks in the mid-length of the wall potential failure surfaces using interface elements. Unfortunately, algorithm convergence problems caused termination of the calculation in its early stage. Better algorithms will be needed to obtain more stable numerical solutions. In any case, unless material strength variability is accounted for, the numerical model would still predict joint failure, as the mean brick strength was greater than the stress observed when the joints failed.

The predicted pressure-deflection behaviour of the wall is plotted in Figure 5.31a. The wall was initially behaving linearly until the pressure reached 3.37kPa. This compares well with the value of 3.5kPa where the experimental curve appears to exhibit a change in slope. This perhaps indicates that some minor cracks formed at this pressure which were not recorded by Lawrence (Figure 5.31b). After that, more vertical cracks were formed and the wall behaved non-linearly until diagonal cracks appeared and the failure load capacity was reached. The smooth appearance of the non-linear numerically predicted curve clearly shows the progressive cracking nature of the perpend joints and torsion on the bed joints.
Figure 5.31. Pressure-deflection for wall with support Category 5.

(a) Prediction

(b) Experiment (Lawrence, 1983)
5.6 Summary

The micro-model described in Chapter 4 was simplified to assume zero thickness for the mortar joints and expanded bricks. The simplified model was applied to the cases of wallettes subjected to pure horizontal bending, wall panels subjected to in-plane shear force and wall panels subjected to two-way out-of-plane bending under various support conditions. Despite its simplicity, the model has been shown to be capable of providing satisfactory prediction for wall behaviour.

For the case of wallettes subjected to one-way horizontal bending, the model proved to be capable of capturing the characteristic behaviour of the wall from crack formation to peak load and eventually the residual load. Quantitatively, although the model prediction was based on parameters that were obtained through modelling the control tests due to a lack of direct data, it proves that the predicted load capacity, especially the cracking load and ultimate load, was in reasonable agreement with the experimental value. However, the model underestimated the residual load capacity.

For the case of walls, including both single wall panel and wall panel with openings, which were subjected to in-plane shear force, the model not only captured the failure pattern very well but also accurately predicted the ultimate load (only 5% difference between the prediction and experimental result).

For the case of walls subjected to two-way out-of-plane bending, the model predicted both cracking load and ultimate load quite well for the walls with four sides simply supported or three sides simply supported with one side free. It is believed that inclusion of interface elements in the potential brick failure surfaces may further improve the simulation of real failure patterns. For those walls supported with built-in edges, either two sides or all four sides, due to the difficulties in simulating the built-in effect in the real test scenarios, the model could only provide a qualitative comparison with the experiment. Nonetheless, the model could still capture the failure pattern quite well, although it overestimated the wall load capacity due to the idealized assumption of rigid supports in the model. The model will need to incorporate the masonry crushing failure mode to further improve its applicability to walls with built-in support.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS
6. **CONCLUSIONS AND RECOMMENDATIONS**

6.1 **Conclusions**

Extensive tests were carried out in this work using a newly commissioned test rig to study the behaviour of four brick unit specimens subjected to various bending conditions as well as vertical pre-compression. Accompanying these tests were a wide range of control tests conducted to obtain the material properties of masonry. These properties, which included the flexural bond strength, direct tensile strength, direct shear and torsional shear strengths between brick and mortar, the flexural strength of brick, the masonry compressive strength and the elastic stiffness of units and mortar, were important input parameters in the numerical modelling.

In parallel with this experimental work extensive numerical analysis was also conducted to provide rational explanations to the experimental observations. A 3D non-linear finite element micro-model, which is based on the discrete contact modelling concept, was proposed and incorporated into the ABAQUS/Standard software package. Implementation of this model empowers the ABAQUS/Standard so that the software can be applied to simulate not only in-plane but also out-of-plane behaviours of masonry walls. The model was validated by application to two case studies. In the first case it was used to simulate the four brick unit specimen tests. Results obtained from the numerical analyses were compared to the experimental data from the four brick unit specimen tests and it was shown that in most cases the numerical model reproduced the experimental observations satisfactorily. In the second case the model was used, partly as an evaluation tool, to assess a newly developed innovative torsion shear test method. Numerical simulation for the torsion shear test provided sound theoretical support for this new method to be introduced into the masonry community.

Based on the micro-model described above, a simplified micro-model was also proposed in this study to enable simulation of the behaviour of large-scale masonry wall panels. The simplified micro-model was also validated by application to three cases. The first one involved application to a small wall panel subjected to horizontal bending and the second one to shear wall panels with openings. These applications demonstrated
the capability of the simplified model to simulate both in-plane and out-of-plane behaviors of masonry walls. Finally, the model’s capability for simulating complex two-way out-of-plane bending behaviour of masonry walls was demonstrated through application to large wall panels with various boundary conditions.

Specific conclusions that can be drawn from this study are summarized below:

**Experimental**

- A new test rig was successfully commissioned and proved to be suitable for studying four brick unit specimens subjected simultaneously to vertical compression combined with vertical and horizontal bending moments. This apparatus possesses three separate loading systems and as such is rather complex. Yet the rig is well arranged so that transfer of interactive moment arising from one of the loading systems to other planes, causing moment or restraint in the latter, can be effectively prevented.

- Tests on four brick unit specimens subjected to vertical bending revealed that the flexural strength of bed joints had significant variability. This was attributed to the variability in masonry as a material as well as inconsistency of workmanship. However, the existence of vertical compression not only increased the flexural strength of the bed joint but also significantly reduced the variability in the flexural strength.

- The presence of perpend joints did not have significant effects on the flexural strength of the bed joints.

- Tests on four brick unit specimens subjected to horizontal bending indicated that the flexural strength parallel to the bed joint had much less variability than that normal to the bed joint. The existence of vertical compression could increase the horizontal bending moment capacity when no brick failure occurred. This was likely caused by the enhancement of the torsional moment capacity due to an increase in the frictional shear component of the torsional resistance. However,
this compression did not affect the flexural strength of the brick and therefore did not enhance the strength for cases where failure occurred through the bricks.

- The residual horizontal bending moment was able to be captured. It showed a linear relationship with vertical compression.

- Tests on four brick unit specimens subjected to biaxial bending showed that application of a vertical compressive stress enhanced the vertical bending moment capacity more significantly than horizontal bending moment capacity.

- Under the biaxial bending condition the interaction of vertical flexural strength and horizontal flexural strength could be described with elliptical curves, as proposed by Baker (1979). This elliptical interaction was preserved as the vertical compression was increased, indicating that increasing vertical compression enhances both vertical and horizontal moment capacities.

- Direct tension tests showed that the tensile bond strength between brick and mortar had high variability (COV of 41%). Softening of tensile strength might exist and follow an exponential trend.

- Triplet shear tests and torsion shear tests showed that the Mohr-Coulomb relationship could be established between the peak shear stress and normal compressive stress. Both the shear strength and residual shear strength between brick and mortar increased linearly as the normal compressive stress was increased.

- In the torsion shear test, although the shear fracture energy increased as the normal compressive stress was increased, no sensible relationship could be established between the two due to the limited number of data points available. Nevertheless, the data suggested that softening of shear strength could be described exponentially.
Numerical

- By incorporating cohesive contact, the 3D micro-model, which describes brick and mortar with linear elastic elements and the bond between brick and mortar with non-linear contact surface, was successfully used to simulate the peak and post-peak behaviours of four brick unit specimens subjected to separate vertical and horizontal bending. It captured qualitatively the moment resistance behaviour of four brick unit specimens subjected to biaxial bending. In most cases the model also provided good prediction of peak moment capacities. However, to match the residual horizontal moment capacity in biaxial bending, the model will need to be improved to take into account the softening effect of the residual friction coefficient.

- The 3D micro-model was used to evaluate a new torsion shear test method. The model satisfactorily captured the main features expected from the test, namely, the shear stress distribution, normal stress distribution and torque versus rotation behaviour. Simulation of this test proved that close to uniform normal and shear stress distributions in the mortar joint can be achieved with the new test method.

- The simplified 3D micro-model, which describes the brick as linear elastic material, the bond between brick and mortar as non-linear contact surface and the mortar with zero thickness, was shown to be able to simulate both in-plane and out-of-plane behaviours of masonry wall panels. The model successfully captured the failure pattern and load capacities. In particular, in the case of small wall panels subjected to horizontal bending, the model was capable of capturing the characteristic behaviour of the wall from crack formation to peak load and eventually the residual load.

To sum up, an extensive program of laboratory testing in parallel with numerical analysis was conducted in this study to examine the vertical, horizontal as well as biaxial bending behaviours of masonry structures. This study helped to gain a better understanding at the fundamental level. It is hoped that the outcomes will assist future research aimed at ultimately developing a fully rational biaxial-bending failure model that can predict behaviour under any simultaneous combination of bending moments in
the two principal directions, along with a superimposed compression force on the bed joints.

6.2 Recommendations for Future Research

Based on the knowledge gathered from this work, the following closely related topics are considered to be worth further investigation:

Laboratory Testing

- The existing test rig, which allows the bending of four brick unit specimens around its material axes (parallel and perpendicular to bed joints), may be modified so that bending moment around axes with any angle to the bed joints can be applied in the test. This may be very important for a good understanding of the principal moments in biaxial bending.

- Tests should be conducted of four brick unit specimens in which the bending moments $M_h$ and $M_v$ are applied so that tension due to each moment occurs on the opposite faces of the specimen. This will help to establish a complete failure criterion so that the stress interaction of tension and compression can be described. It is noted that no modification to the test rig should be required for such tests to be conducted.

- The direct tension test setup will need to be improved so that specimens can be loaded and unloaded more stiffly. This will enable accurate measurement of the tensile fracture energy.

- The relationship between the direct tensile strength, flexural tensile strength, shear strength and torsional shear strength will need to be further investigated. More direct tension and torsion shear tests will need to be conducted. Such tests should cover a wide range of combinations of various types of bricks and mortar and include a larger number of specimens for each combination. Also, cylindrical specimens used in both the direct tension test and torsion shear test should be cored directly from the standard couplet specimens so that the strength characteristics in the mortar joints are truly representative of the joints in a masonry wall.
Numerical Modelling

- With both the proposed micro-model and simplified micro-model, contact modelling may be refined to incorporate tensile softening in the normal direction so that cracking between brick and mortar can be more precisely represented. In the tangential direction, both the initial and residual friction coefficients will need to be implemented separately so that the change of this coefficient can be represented.

- The simplified micro-model may be modified to include compression failure in the failure surface so that the model can catch the ultimate load caused by crushing of bricks and mortar. Such failure modes can occur when a wall has rigid support and arching effects play a critical part.
REFERENCES
REFERENCES


APPENDIX A

BIAXIAL BENDING TEST RESULTS FOR FOUR BRICK UNIT SPECIMENS
- $M_v : M_h = 1:1$, $P = 0.016$ MPa
- \( M_v : M_h = 1:1, \ P = 0.1 \text{MPa} \)
- $M_v : M_h = 1:1$, $P = 0.2\text{MPa}$

### Spec 1

![Graph Spec 1](image1)

### Spec 2

![Graph Spec 2](image2)

### Spec 3

![Graph Spec 3](image3)

### Spec 4

![Graph Spec 4](image4)
$M_v : M_h = 1:2, \ P = 0.016 \text{MPa}$
- $M_v : M_h = 1:2$, $P = 0.1\text{MPa}$
$M_v : M_h = 1:2$, $P = 0.2\text{MPa}$
- $M_v:M_h = 1:4$, $P = 0.016MPa$
$M_v : M_h = 1 : 4$, $P = 0.1 \text{MPa}$
- $M_v : M_h = 1:4$, $P = 0.2\text{MPa}$