Approaching Euclidean limits: a fractal analysis of the architecture of Kazuyo Sejima

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ABSTRACT: Fractal Geometry evolved in mathematics during the late 1970’s and early 1980’s; building on Benoit Mandelbrot’s proposal that natural systems frequently possess characteristic geometric complexity over multiple scales of observation. In 1996 Carl Bovill demonstrated a method for determining an approximate fractal dimension, or characteristic visual complexity, of architectural elevations and plans. This “box-counting” method, as used by Bovill, is the only known way to calculate and compare the fractal geometries of buildings. Significantly, past architectural analysis using the box-counting approach has only been applied to the works of Frank Lloyd Wright, Le Corbusier and a limited selection of ancient buildings. This paper will expand the set of cases tested by the box-counting method to incorporate five house designs by Kazuyo Sejima, a famous, late 20th century minimalist architect. The fractal dimensions are calculated using a combination of Archimage and Benoit software, the former of which uses an extrapolation of Bovill’s box-counting method for the fractal analysis of house designs. Being the first examples of minimalist architecture tested by this method, this paper will also explore the extremities of this method, and Bovill’s suggestion that modern architecture with minimal visual complexity should result in a more Euclidean, rather than highly fractal, results.

Keywords: fractal architecture, computational tools, design assessment, Kazuyo Sejima

INTRODUCTION

Fractal geometry began to inform a number of approaches to measuring and understanding non-linear and complex forms several years after it emerged during the late 1970’s and early 1980’s. It was Benoit Mandelbrot’s proposal, that natural systems frequently possess characteristic geometric complexity over multiple scales of observation, which initiated mathematical studies of fractal geometry (Mandelbrot 1982). As the uses of fractal geometry spread across different disciplines, architectural designers adopted fractal geometry as an experimental alternative to Euclidean geometry. However, it was not until the late 1980s and the early 1990s that fractal dimensions were applied to the analysis of the built environment (Ostwald 2001). For example, Batty and Longley (1994) and Hillier (1996) have each developed methods for using fractal geometry to understand the visual qualities of urban space. Oku (1990) and Cooper (2003; 2005) have used fractal geometry to provide a comparative basis for the analysis of urban skylines. Yamagishi, Uchida and Kuga (1988) have sought to determine geometric complexity in street vistas and others groups have applied fractal geometry to the analysis of historic street plans (Hidekazu and Mizuno 1990). Eglash (1999) has also compared the geometric patterns and formal relationships found in indigenous architecture with fractal geometry. While all of these works rely on a range of approaches, the majority of the examples of the fractal analysis of architecture possess a more common lineage.

In 1996 Carl Bovill developed a version of the box-counting method, originally proposed by Mandelbrot, for the analysis of architecture. Bovill’s work produced numerical results for line drawings of elevations of different buildings as comparable fractal dimensions. This method is potentially useful because, in architecture, there are only a small range of quantifiable approaches to the analysis of the visual qualities of buildings and landscapes. Although others have used Bovill’s approach to analyse vernacular architecture (Bechhoefer and Appleby 1997), urban layouts and landscape (Makhzoumi and Pungetti 1999), and ancient architecture (Burkle-Elizondo, Sala and Valdez-Cepeda 2004), any evidence of the testing of this method and its parameters is rare, and the comparisons Bovill makes between buildings have been tested only once (Lorenz 2003). In 2007 the authors began to address this deficiency when they undertook a detailed examination of the method to determine its validity. Bovill’s original results for the Robie House and the Villa Savoye were re-tested, and the set of cases was expanded to include five houses by Frank Lloyd Wright and five by Le Corbusier (Ostwald, Vaughan and Tucker 2008).

Carl Bovill’s 1996 publication builds in part on Mandelbrot’s reflection that “[a] Mies van der Rohe building is a scale bound throwback to Euclid, while a high period Beaux Arts Building is rich in fractal aspects” (Mandelbrot 1992). According to Bovill, “Mandelbrot is referring to the presence of a progression of interesting detail from the large to the small scale in the Beaux Arts buildings and a lack of this progression in modern buildings” (Bovill 1996:115-116) Bovill demonstrated this idea by using the box-counting method to calculate and compare the fractal dimensions of the architecture of Frank Lloyd Wright with that of Le Corbusier. From his own results, Bovill proposes that the
differing styles of their architecture are reflected in their fractal dimensions. The authors of the paper found the fractal dimensions of the different architects' works to be significantly closer than Bovill concluded. The results of testing on Le Corbusier and Frank Lloyd Wright were not entirely convincing, however the clustering pattern of the results indicated that Bovill's proposal has potential merit and that further analysis would be useful. As part of a larger project to expand the testing of Bovill's method, the present research undertakes a detailed analysis of an extreme example of architectural style. Kazuyo Sejima's Japanese minimalist architecture, with its typical low visual complexity is a useful test subject for Bovill's method, as, according to conventional thinking, the fractal dimension should be relatively close to $D=1.0$.

In this paper the fractal dimensions of the elevations and plans of five of Kazuyo Sejima's houses are calculated using TruSoft's Benoit (vers. 1.3.1) program and Archimage (vers. 2.1); a program developed by the authors. The following sections explain what is meant by fractal dimension and provide an overview of the box-counting method. Thereafter, the paper describes how the present study was undertaken and a background to Sejima's work in general, and the five houses in particular. The paper concludes with a review of the results of the study and any questions raised by these results.

1. FRACTAL DIMENSION

Mandelbrot argues that Euclidean geometry, the traditional tool used in science to describe natural objects, is fundamentally unable to fulfill this purpose. Mandelbrot's contention is that mountains are not conical in form, clouds are not spherical and rivers are not orthogonal. Every attempt to abstract a complex natural feature into a pure geometric form loses some of the essential qualities of that feature and renders the abstraction meaningless. Instead, Mandelbrot proposes that a new model is required which allows scientists to approach “in rigorous and vigorous quantitative fashion,” a range of shapes that have previously been defined as too “grainy, hydrailike, in between, pimply, pocky, ramified, seaweedy, strange, tangled, tortuous, wiggly, wispy [and] wrinkled” to be seriously considered (Mandelbrot 1982:5). Mandelbrot's breakthrough was to consider that objects in nature often possess characteristic complexity: this implies that natural objects frequently look alike when they are viewed at different scales. Thus a tree, when viewed from a distance, will often look similar to a branch of that same tree or a leaf from that tree, when viewed at a closer scale. Thus a tree may possess a form of consistent complexity, or characteristic irregularity and if this is the case, then that irregularity can be measured. Mandelbrot then suggests that the characteristic irregularity of a natural form may be measured by imagining that the increasingly complicated path of the form's outline is actually somewhere between a one-dimensional line and a two-dimensional surface. The more complicated the line, the closer it comes to being a two-dimensional surface. Therefore irregular lines and forms can be viewed as being fractions of integers, or what Mandelbrot describes as “fractal geometric forms”. Thus, fractal geometry describes irregular or complex lines, planes and volumes that exist between whole number integer dimensions. This implies that instead of having a dimension, or $D$, of 1, 2, or 3, fractals might have a $D$ of 1.51, 1.93 or 2.74.

1.1 The box-counting method

One method for determining the approximate fractal dimension of an object is to apply the box-counting approach. Consider a drawing of an elevation of a house. A large grid is placed over the drawing and each square in the grid is analysed to determine whether any lines from the façade are present in each square. Those grid boxes that have some detail in them are recorded. Next, a grid of smaller scale is placed over the same façade and the same determination is made of whether detail is present in the boxes of the grid (see figures 1 – 3). A comparison is then constructed between the number of boxes with detail in the first grid and the number of boxes with detail in the second grid; this comparison is made by plotting a log-log diagram for each grid size (Bovill 1996; Lorenz 2003; Ostwald, Vaughan, Tucker 2008). By repeating this process over multiple grids of different scales, an estimate of the fractal dimension of the façade is produced (see figure 4). While this process can be done by hand, the software programs Benoit and Archimage automate this operation.

There are many variations of the box-counting approach that respond to known deficiencies in the method. The following points describe the primary issues and how the present research responds to each.

- **White space.** The volume and distribution of white or empty space around the source image can alter the result. To solve this, Foroutan-Pour, Dutillieu and Smith (1999) offer an algorithm to optimise the way in which an image is positioned against its background and suggestions on how to derive an ideal analytical grid. Careful sizing of the initial images in combination with solutions to the starting grid problem (described hereafter) limits the impact of this problem.

- **Image proportion.** If the original image is not pre-sized to produce a clear starting grid, then an additional step must be added to ensure that a divisible grid is determined. Benoit solves this problem by cropping the image size to achieve a whole-number starting grid. Archimage enlarges the image by adding small amounts of empty space to the boundaries. While neither of these variations changes the elevation in the source image, they produce subtle variations in the resultant $D$ (partially explaining the former program’s tendency to produce lower results that the latter’s).

- **Line width.** The wider the lines in the source image, the more chance they have of being counted twice when grid sizes become very small, leading to artificially increased $D$ values. To counter this situation, Archimage software pre-processes images using a line-detection algorithm that reduces all lines to one pixel width. Benoit overcomes this problem by allowing the analytical grid to be rotated or resized to minimize the impact of line weight.

- **Scaling coefficient.** The factor by which successively smaller grids is produced is called the scaling coefficient.

Bovill, in his original examples, halved the grid dimension for each comparison (a ratio of 2:1) whereas Benoit and
Archimage use a lower scaling coefficient (a ratio of 1.3:1) to more gradually reduce the grid size and generate a more accurate result.

- **Statistical divergence.** The average slope of the log-log graph may be the approximate $D$ value, but the points generating the line are not always consistent with it (for example, see the result for the largest grid dimension in figure 4). The $D$ value is only a reasonable approximation when most of the points in the chart correspond with the resultant average line. The question then becomes, how are divergent points handled? While there is no definitive answer to this question, divergent results tend to occur at the extremes of the graph; with the largest and smallest grid sizes. For the present research, initial trials allowed similar settings for starting grid proportion, size and scaling coefficient to be chosen that minimise the number of divergent results. This has the advantage of ensuring consistently produced results, but the disadvantage of occasionally producing inaccurate readings at the lower scaling limits of the analytical grids.

![Figures 1 and 2. First grid (left) and second grid (right) placed over Elevation 1 of the Y-House showing box-counting.](image)

![Figures 3 and 4. Third grid (left) placed over Elevation 1 of the Y-House showing box-counting. Log-log diagram (right) of the comparison between the number of boxes counted in a grid and the size of the grid (Archimage result for Elevation 1 of the Y-House).](image)

2. RESEARCH METHOD

The early house designs of Kazuyo Sejima are the focus of the present study. Houses were chosen for the previous research on the box-counting method in architecture because they typically possess similar scale, program and materiality. In order to build on the results of this past research, five of Sejima’s house designs were selected for analysis. In accordance with the method employed in past research, preference was given for houses that had been completed within a relatively tight period of time. In the case of the Sejima houses, less than ten years separates the design of the earliest from the most recent. Preference was also given for single houses (rather than pavilions or estates), completed works (ensuring a similar level of development) and houses with a relatively tight geographic distribution (to limit the impact of climate on the form of the house). Three of the chosen houses are in Tokyo, and the others are located in the urban prefectures of Japan. The five houses are: Y-House (1994), S-House (1996), M-House (1997), Small House (2000) and House In A Plum Grove (2003).

As there are no consistent sets of plans and elevations available for Sejima’s work, a number of sources were used as the basis for the present analysis. The majority of the figures for the Y-House were published in *El Croquis* and the missing two elevations were recreated with reference to the drawings and photographs in the same issue (Sejima 1996). Jun Aoki’s adapted drawings of the S-House and M-House and Yuko Hasagawa’s plans for the Small House and the House in a Plum Grove were used for the analysis (Aoki 2003; Hasegawa 2006). The elevations of the House in a Plum Grove were also sourced from an issue of *El Croquis* (Sejima 2003). Using these materials, new drawings of Sejima’s houses were prepared for the analysis, each with consistent graphic conventions and scale. The lines in each drawing typically record changes in form, not changes in surface or texture. Thus, for example, major window reveals, thickened concrete edge beams, and steel railings are all drawn, while brick coursing and control joints are not. In most cases four elevations and a single plan were developed for testing. The plan was of the primary living level regardless of whether it was at ground floor, first floor or basement level. In the case of the M-
3. The complexity of houses is as follows.

2. Each view of the house is analysed using Archimage and Benoit programs producing, respectively, a  \( D(\text{Arch}) \) and a  \( D(\text{Benoit}) \) outcome. The settings for Archimage and Benoit, including scaling coefficient (determining the ratio by which grids reduce in scale) and scaling limit (the smallest grid where data is collected), are preset to be consistent between the programs. The starting image size (IS (Pixels)), largest grid size (LB (Pixels)), and number of reductions of the analytical grid (\( G_{(e)} \)), are recorded so that the results can be tested or verified. Archimage results are typically slightly higher than those produced by Benoit although the variation is consistent.

3. The  \( D(\text{Arch}) \) and  \( D(\text{Benoit}) \) results for the elevation views are averaged together to produce a separate  \( D(\text{Elev}) \) result for each program for the house. These results are a measure of the average fractal dimension of the exterior facades of the house. Past research suggests that  \( D(\text{Elev}) \) results tend to be relatively tightly clustered leading to a high degree of consistency.

4. The  \( D(\text{Arch}) \) and  \( D(\text{Benoit}) \) results for the elevation and plan views are averaged together to produce a separate  \( D(\text{Plan + Elev}) \) result for each program for the house. This result is a measure of the average fractal dimension of both the exterior form of the house and its interior spatial arrangement. Past research questions the validity of this variation, because of the assumption that the  \( D \) results for plans will differ markedly from those of elevations. With this assumption still unproven, the reporting of  \( D(\text{Plan + Elev}) \) values allows for a larger body of data to be available for future testing.

5. The  \( D(\text{Elev}) \) results produced by Archimage and Benoit are averaged together to produce a composite result,  \( D(\text{Comp}) \), for the house. The composite result is a single  \( D \) value that best approximates the characteristic visual complexity of the house.

6. This process (steps 2 to 5) is repeated for each house producing a set of five  \( D(\text{Comp}) \) values. These values are averaged together to create an aggregate result  \( D(\text{Agg}) \) which is a reflection of the typical, characteristic visual complexity of the set of the architect’s works.

Importantly, this method does not produce a  \( D \) result for the three-dimensional form of the house rather, it generates a series of average  \( D \) results for the two-dimensional visual qualities of a structure.

### Table 1. Abbreviations and definitions.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>( D )</td>
<td>Approximate Fractal Dimension.</td>
</tr>
<tr>
<td>( D(\text{Arch}) )</td>
<td>( D ) calculated using Archimage software</td>
</tr>
<tr>
<td>( D(\text{Benoit}) )</td>
<td>( D ) calculated using Benoit software.</td>
</tr>
<tr>
<td>( D(\text{Plan + Elev}) )</td>
<td>Average  ( D ) for a set of elevation views of a house using a specified program.</td>
</tr>
<tr>
<td>( D(\text{Comp}) )</td>
<td>Composite  ( D ) result averaged from both Archimage and Benoit outcomes for the elevations of a house.</td>
</tr>
<tr>
<td>( D(\text{Agg}) )</td>
<td>Aggregated result of five composite values used for producing an overall  ( D ) for a set of architects’ works.</td>
</tr>
<tr>
<td>IS(( e ))</td>
<td>The size of the starting image measured in pixels.</td>
</tr>
<tr>
<td>LB(( e ))</td>
<td>The size of the largest box or grid that the analysis commences with, measured in pixels.</td>
</tr>
<tr>
<td>( G_{(e)} )</td>
<td>The number of scaled grids that the software overlays on the image to produce its comparative analysis.</td>
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</tbody>
</table>

### 3. KAZUYO SEJIMA AND THE FIVE HOUSES

Born in 1956 in the Ibaraki Prefecture in Japan, Kazuyo Sejima worked for the Japanese architect Toyo Ito for six years after completing her Masters degree in architecture at the Japan Women’s University in 1981. In 1987 Sejima left Ito’s office and formed her own practice, Kazuyo Sejima & Associates. While maintaining her own practice, in 1995, Sejima began collaboration with Japanese architect Ryue Nishizawa, forming SANAA (Sejima + Nishizawa and Associates). Sejima, both on her own and in collaboration with Nishizawa, has since designed many award-winning projects including houses, museums, commercial centres and apartment buildings. Sejima’s and Nishizawa’s work has been exhibited and published in journals and books internationally. Sejima lives and works in Japan although, since 2000, she has designed public and commercial projects throughout Europe and most recently in America.

According to Ito, Sejima’s “expression is pure, simple and geometrical” (1996:18); she arranges the functional conditions which the building is expected to hold, in a final diagram of the space, then she immediately converts that scheme into reality (1996: 20).

Stan Allen (1996) describes “the apparent simplicity of the extruded sections” in Sejima’s work as being “complicated by involuted circulation paths and unfamiliar objects lodged within these rectilinear volumes.” Her work is an architecture of intentionally limited formal means that seeks to engender in the experience of the viewer [...] an explosive play of light, reflection, sign, and movement. (1996:103).

For Allen, Sejima’s work is “the most radical exemplar of present-day minimalism” in architecture (1996:103). Sejima’s earliest independent projects display the genesis of her architectural design method that is based on people’s movements within the functional zones of the building. Sejima’s early concern was for architectural forms not impeding the continuity of human motion. This initially resulted in her buildings becoming “more and more complex” until, in 1991, with the design of Castelbajac Sport and Saishnkan Seiyaku Women’s Dormitory, Sejima...
began to approach design from the point of view of “how ideas of simple volume and mixture of use could interact” (Sejima 1999:118); it suggests that this was a turning point in her design method (1996:23). By the time she designed the Y-House in 1994, Sejima argued that the setting strategy, which indicates the setting up of relations between the building occupants and the world, could determine volume, locations of functions (and) circulation (Sejima 1999:118).

Kristine Guzman’s interpretation of this approach is that it creates numerous spaces and openings that provide their inhabitants the freedom to generate new relationships or sociological behaviour. In most cases, these spaces look out onto the exterior […] or onto interior gardens (2007:170-171).

Sejima’s houses are typical of her simple, geometrical designs, which are derived from a subtle and complex understanding of the inhabitant’s life-patterns. According to Hasegawa, Sejima focuses on the ‘temporal sequence’, actions and events caused by living in that building. When the movements of the people inside the building are visible from without, the sequence of events becomes a part of its external appearance (2006:9).

The five houses by Kazuyo Sejima, that are the focus of the current research, were built between 1994 and 2003 in dense, residential areas in Japan. All five houses are designed for families and, as is often the tradition for Japanese families, for up to three generations of the one family. With integral courtyard spaces, which are accessed from most areas of the home, these houses demonstrate Sejima’s use of circulation as a design strategy. Guzman (2007:167) describes Sejima’s work as “an architecture defined by visual lightness”. These five houses are typical of her small, seemingly transparent houses with thin walls, monochromatic finishes and flat roofs. Luis Fernandez-Galiano (2007:175) sees these houses as an “architecture in the negative, achieved through a stripping-down” process, her “buildings strive to divest themselves of thickness, dispense with inertia, rid themselves of density.”

The Y-House in Katsuura (1994) is located in the Chiba prefecture, in the Greater Tokyo Area of Japan. The typical house in Katsuura is two stories high, with white walls and a dark sloping tiled roof. In contrast, the Y-House is a three-storey, flat-roofed structure with two, almost fully glazed, walls and a tall, green marble tiled wall to the street. A private residence for a couple and their two children, the Y House is built on a site which has an area of 172m²; the house has a footprint of 70m² on the site and a total floor area of 152 m². The rooms in the house are all connected by way of various circulation routes to one of the external courtyards on the long sides of the house. According to Sejima (1999), the Y-House was a chance for exploration of her ideas about the equality of circulatory and non-circulatory spaces and the relationship between the interior and the exterior.

Designed in conjunction with Ryue Nishizawa, the S-House (1996) in Okayama, is a home for an extended family including two children and grandparents. The extended family shaped both the brief and the form of the S-House when they requested that this cohabitation of two families be reflected in the design […] to encourage communication between generations (Chermayeff, Perez Rubio and Sakamoto 2007:90).

The S-House, a small two-storey cubic volume, has an external skin of clear corrugated polycarbonate sheeting on a timber frame, a footprint of 67 m² and a total floor area of 142m². Programatically, the S-House is reminiscent of a duplex with a shared living room that reflects the extended family’s close relationship. The external skin of the S-House creates a double storey void that acts as circulation space on the ground floor and a spatial connection to the top floor. As a result of this planning strategy, all rooms of the S-House open into this single, indoor/outdoor space. The S-House requires few openings in the facade as it draws light and air through the external skin. The elevations have no ornamentation, each appearing as a corrugated plane punctuated by small windows. The few windows that do exist are unassumingly framed with minimal steel flashings and no roof is visible from the exterior.

The M-House (1997), completed in conjunction with Nishizawa, was designed for a couple to provide space for guests, entertainment and for future children. This house is located in Shibuya, an area of Tokyo which Sejima describes as once having “large residences” but where now the density is increasing as lots in the vicinity are being subdivided into many sites. In most of the residences the south side faces the street, creating a situation of permanently drawn curtains and high fences which often hide large windows (Sejima 1998:20).

In response to the setting, the M-House utilises a mixture of corrugated metal external cladding and transparent sheeting to suggest an internal space behind the bare walls. Appearing from the street as a single storey, the house has a basement level which contains the living room, dining room, study and studio, and a series of double storey courtyards flood this lower level with light. With a similar external appearance and materiality to the S-House, with uninterrupted corrugated surfaces and sunken doorways framed with a minimum of expression, the M-House appears to have no windows to the outside at all. This house has a footprint of 112m², and an indoor area of 215m². The building is viewed from the roadside and only the street elevation is available for analysis.

Set on a tiny 60m² site, the Small House (2000) is located in leafy Aoyama, a place Sejima describes as “one of the most attractive areas of Tokyo” (2004:242). With a compact total floor area of 77m², this house rises from its 34m² footprint in an undulating four-storey volume of steel and glass. The house is for a small family who had a clear idea of the functions they required of their home. The resulting form of the house takes the shape of these functions. According to Ito, the phrase that best describes the
spirit of [Sejima’s] structures, it would be [...] ‘diagram architecture’. In other words [Sejima’s] building is ultimately the equivalent of the diagram of the space used to abstractly describe the mundane activities presupposed by the structure (1996:18).

The Small House, where each of its concrete floors serve a different purpose in accordance with the client’s brief, is the embodiment of this idea. Sloping lightweight walls then connect the shifted slabs of each floor, forming the overall gentle, swaying shape of the building.

Built in Tokyo on a small site bordered with established plum trees, Sejima’s House In A Plum Grove (2003) meets the requirements of an extended family who wanted a home which “felt like a connected space” and which would “save the landscape” so that they would eventually be able to “enjoy its characteristic plum trees” (Chermayeff, Perez Rubio and Sakamoto 2007:278). This three-storey house has an external skin of steel panels with insulation and gypsum board, and (16mm) structural steel walls internally. Of a similar shape to the S-House, the House In A Plum Grove differs dramatically in its appearance. With white painted, steel walls, the house is covered on all elevations by thinly framed openings of seemingly random locations and sizes which are described by Hasegawa as where elements of the exterior scenery are converted to something like visual signs of a painting, and people inside the building, experiencing the total effect of these windows, become joined in a relationship with exterior scenery of refined metaphysical character (2007:186).

These shapes continue to puncture the interior of the building where, according to Hasegawa, every room has windows that can be viewed from every other space, making the inside of the house one large space (2007: 186).

This openness, combined with the thin white walls, illustrates Fernandez-Galiano’s description of Sejima’s work as possessing “an immaterial appearance,” and of being “metaphysical in that they transcend the realm of the senses’ [and] standard conventions.” They are “dreamlike in so far as they reside along the vague border between sleep and waking” (2007:175).

Table 2. Y-House data and results.

<table>
<thead>
<tr>
<th>Views</th>
<th>IS (Pix)</th>
<th>LB (Pix)</th>
<th>GI (Pix)</th>
<th>$D_{(Comp)}$</th>
<th>$D_{(Elev, Arch)}$</th>
<th>$D_{(Plan + Elev, Arch)}$</th>
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<tr>
<td>Plan</td>
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<td>300 13</td>
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<td>1.289</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 1</td>
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<td>400 14</td>
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<td>1.264</td>
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<td></td>
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<tr>
<td>Elev 2</td>
<td>1200x815</td>
<td>400 14</td>
<td>1.615</td>
<td>1.505</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 3</td>
<td>1200x686</td>
<td>400 14</td>
<td>1.275</td>
<td>1.169</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 4</td>
<td>1200x815</td>
<td>400 14</td>
<td>1.584</td>
<td>1.490</td>
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<tr>
<td>$D_{(Plan + Elev, Arch)}$</td>
<td>1.4484</td>
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<td>$D_{(Comp)}$</td>
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Table 3. S-House data and results.

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<th>IS (Pix)</th>
<th>LB (Pix)</th>
<th>GI (Pix)</th>
<th>$D_{(Comp)}$</th>
<th>$D_{(Elev, Arch)}$</th>
<th>$D_{(Plan + Elev, Arch)}$</th>
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</thead>
<tbody>
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<td>300 13</td>
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<td>1.357</td>
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</tr>
<tr>
<td>Elev E</td>
<td>1200 x 772</td>
<td>400 14</td>
<td>1.151</td>
<td>1.043</td>
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<td></td>
</tr>
<tr>
<td>Elev N</td>
<td>1200 x 868</td>
<td>400 14</td>
<td>1.258</td>
<td>1.113</td>
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</tr>
<tr>
<td>Elev S</td>
<td>1200 x 800</td>
<td>400 14</td>
<td>1.363</td>
<td>1.233</td>
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</tr>
<tr>
<td>Elev E/W</td>
<td>1200 x 772</td>
<td>400 14</td>
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<td>1.13</td>
<td></td>
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</tr>
<tr>
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<td>1.1752</td>
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</tr>
<tr>
<td>$D_{(Elev, Arch)}$</td>
<td>1.2045</td>
<td>1.12975</td>
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</tr>
<tr>
<td>$D_{(Comp)}$</td>
<td>1.1921</td>
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<td></td>
</tr>
</tbody>
</table>

Table 4. M-House data and results.

<table>
<thead>
<tr>
<th>Views</th>
<th>IS (Pix)</th>
<th>LB (Pix)</th>
<th>GI (Pix)</th>
<th>$D_{(Comp)}$</th>
<th>$D_{(Elev, Arch)}$</th>
<th>$D_{(Plan + Elev, Arch)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan</td>
<td>1200 x 686</td>
<td>400 14</td>
<td>1.404</td>
<td>1.254</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev S</td>
<td>1200 x 272</td>
<td>75 9</td>
<td>1.361</td>
<td>1.257</td>
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<td></td>
</tr>
<tr>
<td>$D_{(Plan + Elev, Arch)}$</td>
<td>1.3825</td>
<td>1.2555</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$D_{(Elev, Arch)}$</td>
<td>1.361</td>
<td>1.257</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{(Comp)}$</td>
<td>1.309</td>
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</tbody>
</table>

Table 5. Small House data and results

<table>
<thead>
<tr>
<th>Views</th>
<th>IS (Pix)</th>
<th>LB (Pix)</th>
<th>GI (Pix)</th>
<th>$D_{(Comp)}$</th>
<th>$D_{(Elev, Arch)}$</th>
<th>$D_{(Plan + Elev, Arch)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan</td>
<td>1200 x 1200</td>
<td>300 13</td>
<td>1.326</td>
<td>1.252</td>
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<td></td>
</tr>
<tr>
<td>Elev 1</td>
<td>688 x 1200</td>
<td>344 13</td>
<td>1.629</td>
<td>1.517</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 2</td>
<td>809 x 1200</td>
<td>405 14</td>
<td>1.376</td>
<td>1.289</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 3</td>
<td>789 x 1200</td>
<td>395 14</td>
<td>1.504</td>
<td>1.401</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 4</td>
<td>789 x 1200</td>
<td>395 14</td>
<td>1.506</td>
<td>1.400</td>
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<tr>
<td>$D_{(Plan + Elev, Arch)}$</td>
<td>1.4664</td>
<td>1.3676</td>
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<tr>
<td>$D_{(Elev, Arch)}$</td>
<td>1.50425</td>
<td>1.3965</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{(Comp)}$</td>
<td>1.4503</td>
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</tr>
</tbody>
</table>

Table 6. House in a Plum Grove data and results

<table>
<thead>
<tr>
<th>Views</th>
<th>IS (Pix)</th>
<th>LB (Pix)</th>
<th>GI (Pix)</th>
<th>$D_{(Comp)}$</th>
<th>$D_{(Elev, Arch)}$</th>
<th>$D_{(Plan + Elev, Arch)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elev 1</td>
<td>1200 x 1288</td>
<td>300 13</td>
<td>1.349</td>
<td>1.405</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 2</td>
<td>1200 x 1484</td>
<td>317 13</td>
<td>1.221</td>
<td>1.112</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 3</td>
<td>1200 x 1369</td>
<td>263 13</td>
<td>1.150</td>
<td>1.074</td>
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<td></td>
</tr>
<tr>
<td>Elev 4</td>
<td>1200 x 1319</td>
<td>278 13</td>
<td>1.297</td>
<td>1.247</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elev 5</td>
<td>1200 x 1269</td>
<td>283 13</td>
<td>1.244</td>
<td>1.200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{(Plan + Elev, Arch)}$</td>
<td>1.2522</td>
<td>1.1756</td>
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<td></td>
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</tr>
<tr>
<td>$D_{(Elev, Arch)}$</td>
<td>1.228</td>
<td>1.15825</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$D_{(Comp)}$</td>
<td>1.1931</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 7. Composite and aggregate results for Sejima’s Houses.

<table>
<thead>
<tr>
<th>House</th>
<th>$D_{(Comp)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y-House</td>
<td>1.4087</td>
</tr>
<tr>
<td>S-House</td>
<td>1.1821</td>
</tr>
<tr>
<td>M-House</td>
<td>1.3090</td>
</tr>
<tr>
<td>Small House</td>
<td>1.4503</td>
</tr>
<tr>
<td>House in a Plum Grove</td>
<td>1.1931</td>
</tr>
</tbody>
</table>

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of the S House was \( D_{\text{Elev, Archi}} = 1.2545 \) and \( D_{\text{Elev, Benoit}} = 1.12975 \). The composite result for the S-House was 
\( D_{\text{Comp}} = 1.1921 \); a very low figure for the visual complexity of a three-dimensional object (see table 3).

Sejima described the S-House as having an “extremely abstract exterior” (Sejima 1999:119); a description which is consistent with the results, the lowest result of all of the five houses analysed. The S-House is so stripped of detail, that the South Elevation, which is a minimal, flat wall with one large and small one window shown, has the highest \( D \) result (\( D_{\text{Archi}} = 1.363 \)) for any of the views of the building; higher than the average result of the elevations by \( D = 0.11 \) or 8.7%.

The M-House results (\( D_{\text{Comp}} = 1.309 \)) are compromised by the single façade of the house. However, the final composite result is consistent with the rest of Sejima’s houses (see table 4). With the highest composite value, the Small House (\( D_{\text{Comp}} = 1.450 \)) generally also has the highest individual results for elevations; (\( 1.378 < D_{\text{Archi}} < 1.629 \) and \( 1.268 < D_{\text{Benoit}} < 1.517 \)) (see table 5).

With only a 0.001 difference to the \( D_{\text{Comp}} \) result for the S-House, another low composite \( D \) value was found in the House in a Plum Grove (\( D_{\text{Comp}} = 1.193 \)). The elevations for this house have consistently low average fractal dimensions (\( 1.150 < D_{\text{Archi}} < 1.297 \) and \( 1.074 < D_{\text{Benoit}} < 1.247 \)) (table 6).

The final composite \( D \) results for the five houses by Kazuyo Sejima range between the lowest result, for the S-House (\( D_{\text{Comp}} = 1.192 \)), and the highest, for the Small House (\( D_{\text{Comp}} = 1.450 \)). Overall, the aggregated result for all of Sejima’s five houses was \( D_{\text{Agg}} = 1.3175 \) (see table 7).

5. DISCUSSION AND CONCLUSION

Following the standard method, the analysis for all five houses was undertaken using drawings that show the significant lines of the house. The resultant line-work depicts a change in form but does not express subtle changes in materials. If Elevation 1 of the Y-House was drawn as presented in El Croquis, the lines depicting the green marble tiles to the street elevation would be shown (Sejima 1996). This change would increase the low reading of Elevation 1 (\( 1.368_{\text{Archi}} \) and \( 1.264_{\text{Benoit}} \)) and the overall visual complexity for this building. If the elevations of the House in a Plum Grove were drawn showing the materials on the exterior (smooth flat metal sheet), no additional lines would be noticed on the facades of this building. Thus, the \( D \) result would remain the same (\( D_{\text{Comp}} = 1.193 \)) as the elevations truthfully depict the minimal house exterior (see figs. 5 – 6).

Ultimately, there is a difference between the approximate fractal dimension of Sejima’s architecture and the work of other architects analysed in previous research. Compared to the Modernist architects Eileen Gray and Le Corbusier, the sample of Kazuyo Sejima’s houses (1996-2003) exhibits a lower \( D \) result for both individual elevations and for the aggregated result for all of her houses. For example, the aggregate result for five houses (1926-1934) by Gray is \( D_{\text{Agg}} = 1.378 \); this is approximately 4.4% higher than the aggregate result for Sejima (\( D_{\text{Agg}} = 1.3175 \)). The aggregated result for five houses (1922-1928) by Le Corbusier was \( D_{\text{Agg}} = 1.481 \); approximately 10.8% higher than the aggregated result for Sejima. Thus, Sejima’s Minimalist architecture has less characteristic visual complexity (less

\[
\begin{align*}
D_{\text{Archi}} & = 1.150\text{ and }D_{\text{Benoit}} = 1.074 \\
D_{\text{Archi}} & = 1.629\text{ and }D_{\text{Benoit}} = 1.517 \\
D_{\text{Archi}} & = 1.363\text{ and }D_{\text{Benoit}} = 1.233
\end{align*}
\]

Figures 5, 6 and 7. Elevation 2 of the House In A Plum Grove (left), Elevation 1 of the Small House (centre) and South Elevation of the S-House (right).
detail and less formal modulation) than the Modernist works of Gray and Le Corbusier; this is the anticipated result and it supports the previously untested assumptions about the correlation between fractal dimension and intuitive readings of architectural styles. For example, Bovill notes that the fractal dimension of Le Corbusier’s Villa Savoye “dropped off to \( D = 1.0 \), indicating a lack of progression of detail. This difference is due to the difference in design approach.[…] Le Corbusier’s purism called for materials to be used in a more industrial way, always looking for efficiency and purity of use” (Bovill 1996:143). Modernism, which is typified by functional architectural forms, has clear parallels to Minimalism. However, Minimalism with its monochromatic finishes and unadorned surfaces should have a \( D \) value that is even closer to 1.0. The significantly lower fractal dimension of Kazuyo Sejima’s architecture supports this assumption.

In the case of two of Sejima’s houses, Small house and Y House, the individual calculations for each iteration of the grid show a significant statistical divergence from the average slope of the graph. As previously discussed in this paper, this is a known problem in the box-counting method, and the results for these two houses may show an artificial increase in their \( D \) value. By analysing a larger number of samples, the errors are flattened out, with Sejima’s final \( D \) result consistent with her minimalist design aesthetic. When the results are further analysed to compare only three of Sejima’s houses, M House, S house and House in a Plum Grove, which cluster tightly (1.192<\( D <1.309 \)), their aggregate value (\( D = 1.231 \)) produces the lowest known fractal dimension for houses.

The final question then is; how low can an fractal result (\( D_{\text{comp}} \)) for a completed building go, or what is the lower limit for a set of houses? The tightest clustering of the three lowest house results by Sejima should be a clear indicator of the lower scaling limits of domestic architecture. This would imply that the practical lower limit for housing is in the order of \( D = 1.2 \). This is because, a completely smooth, blank wall, with no evidence of construction or materiality, may have a \( D \) of around 1.0, but any addition, of even the most minimal kind, will marginally raise the \( D \) result to around 1.19. The practical upper limit for the fractal dimension of a set of houses has not yet been determined by any researchers; this will be the topic of future research.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


