MULTI-MODAL MODULATED AND DEMODULATED VIBRATION CONTROL OF FLEXIBLE STRUCTURES USING PIEZOELECTRIC TRANSDUCERS

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Abstract: We propose a novel method for controlling vibrations within a resonant structure equipped with piezoelectric transducers. The scheme uses a parallel connection of modulated and demodulated controllers, each designed to damp the transient oscillation corresponding to a single mode. This technique allows multiple modes to be controlled with a single actuator. Simulation results are provided for a laminate beam.

Keywords: Amplitude modulation, active control, vibration damping, transient oscillations

1. INTRODUCTION

There has been significant research interest in utilising piezoelectric transducers for vibration suppression in mechanical structures, see e.g. (Moheimani and Goodwin, 2001; Moheimani, 2003; Fuller et al., 1996; Clark et al., 1998). By bonding piezoelectric transducers to the surface of a structure, these transducers can be used for vibration control, where they can be deployed as actuators, as sensors or both. For that purpose several control algorithms have been proposed, see e.g. (Lazarus et al., 1991; Hagood and von Flotow, 1991).

Given the fact that piezoelectric transducers can be accurately described with linear models (Moheimani, 2000), both active and passive LTI (linear time invariant) methods have been proposed, see e.g. the survey (Moheimani, 2003). Restricting controllers to be LTI is certainly attractive, since the design problem can be cast in the well studied LTI control systems framework, see e.g. (Goodwin et al., 2001). On the other hand, it is well known that resonant systems are not always easy to control with LTI methods (Serón et al., 1997; Goodwin et al., 2001). As a consequence, depending upon the application, non-LTI methods may be worth studying, see e.g. (Corr and Clark, 2003) for work on switching (nonlinear) methods within this context.

In the present work, we propose a time varying (periodic) vibration control method for resonant systems. It makes use of the fact that signals within a resonant structure are of an oscillatory nature, and hence concentrate their energies around a set of discrete frequencies, which correspond to the modes of the mechanical system. The method utilises concepts from (amplitude-modulated) communication systems in order to shift the spectrum of the high-frequency oscillations down to baseband. It then operates on these low frequency signals. This corresponds to controlling the amplitude of the oscillations directly and has strong conceptual and computational advantages, when compared to operating directly on high frequency signals.

The methodology is based upon so-called modulated and demodulated control, which has proven to be effective in a number of applications, see e.g. (Bode, 1945; Chen et al., 2003; Leland, 2001; Gerber, 1978). It has also previously been used in (Chang, 1993) for vibration suppression of flexible structures. In (Chang, 1993), however, external oscillators are not used to modulate the control signal up to the vibration frequency. Instead, the plant output is passed through a nonlinear gain adjust module to generate the modulation signal. This technique is based on the assumption that the output has sufficient energy at the modulation frequencies. This may limit the performance of the scheme (particularly when disturbances are present), since the aim of the controller is vibration suppression.

This paper extends (Lau et al., 2004) to the control of multiple modes within a resonant structure, which we assume fixed and known. Conceptual relations also exist to filter-banks, which are widespread in signal processing applications, see e.g. (Vaidyanathan, 1993).

The remainder of the paper is organised as follows: In the next section, we describe the system to be controlled. Then, in Section 3, we present some background results on modulated and demodulated control systems. In Section 4, we investigate the application of modulated control to vibration damping and provide simulation results.
2. STRUCTURE TO BE CONTROLLED

Piezoelectric materials can transform electrical energy into mechanical energy and vice versa, and can be deployed as actuators, as sensors or both. More precisely, due to their permanent dipole nature, piezoelectric materials strain when exposed to a voltage and conversely produce a voltage when strained. More details can be found e.g. in (Fuller et al., 1996; Moheimani, 2003).

In the present work, we concentrate upon the piezoelectric laminate beam schematized in Fig. 1. In this configuration, the beam is fixed at one end and free at the other. Mounted on each side of the beam is a piezoelectric patch. These are collocated and one is used as an actuator, i.e. it transforms the voltage $u(t)$ into strain. The other functions as a sensor, providing a voltage $v(t)$, which depends upon the (local) beam deflection. A controller for this system specifies a relationship between the voltages $v(t)$ and $u(t)$. We consider the control of transient oscillations at $v(t)$. The response $z(t)$, the displacement of some other point $Z$ on the beam, is also considered.

![Fig. 1. Piezoelectric Laminate Beam.](image)

Following the methodology outlined in (Pota et al., 1999) and without including external disturbances explicitly, one can develop the following description:

$$v(t) = G_v(s)u(t), \quad z(t) = G_z(s)u(t). \quad (1)$$

In this expression, $G_v(s)$ and $G_z(s)$ are linear transfer functions, which reflect the resonant nature of the beam. The relation between $u(t)$ and $v(t)$ can be described as:

$$G_v(s) = \sum_{i=1}^{M} \frac{\gamma_i}{\xi^2 + 2\xi\omega_i s + \omega_i^2}, \quad (2)$$

where $\omega_i$ are the modes and $\xi_i$ are their (uncontrolled) damping factors. The scalars $\gamma_i$ depend upon the position of the piezoelectric patches and, in the collocated case of Fig. 1, are always nonnegative. $M$ is the number of modes taken into account in the model, see also (Moheimani, 2003).

The transfer functions $G_z(s)$ and $G_v(s)$ usually differ only in the location of their zeros i.e. in the values adopted by the factors $\gamma_i$. In the case of $G_z(s)$, we denote these factors by $\gamma_{zi}$.

3. MODULATED AND DEMODULATED CONTROL

In this section, we present some background results on modulated and demodulated control systems. This material is, to a large extent, a review of results from (Lau et al., 2004). However, our emphasis here is on modulated controllers rather than on modulated processes. Also, the quadrature scheme is not discussed in (Lau et al., 2004).

3.1 Notation

Arg $z$ denotes the principal argument of $z$. Thus $-\pi < \text{Arg} z \leq \pi$. Upper case is used to denote the Laplace transform of a signal. A symbol with a tilde denotes a modulated signal. If $f$ is a modulated signal, then $\tilde{f}$ is its envelope.

3.2 Fundamental Concepts

Consider the modulated and demodulated system shown in Fig. 2. In this figure, $G(s)$ is the transfer function of an LTI system and $d(t)$ is an output disturbance. The input to $G(s)$ is $\cos \omega_0 t$ modulated (i.e., multiplied) by $u(t)$. The output $\hat{y}(t)$ is demodulated by correlating it with $\cos(\omega_0 t + \phi)$ and passing the resulting signal through a low pass filter $F(s)$. The angle $\phi(\omega_0)$ is defined as $\text{Arg}[G(j\omega_0)]$. We note that we omit the argument of $\phi$ when it is clear from the context. If $d(t)$ is set to zero, then the filter output $y_f(t)$ is the envelope of $\hat{y}$ filtered by $F(s)$ under the following assumptions:

**Assumptions**

1. $u(t)$ is a band-limited signal having bandwidth $\omega_b$ rad/s (By this we mean that $|U(j\omega)|$ is small for $\omega > \omega_b$).
2. $\omega_0 > \omega_b$.
3. $\omega_0 j$ is not a pole or zero of $G(s)$ (i.e., $\phi(\omega_0)$ is well defined).
4. $F(s)$ is a low pass filter which rolls off between $\omega_b$ and $2\omega_0 - \omega_b$.

Suppose that $d(t) = 0$, that $u$ stabilises the modulated system, and that the above assumptions hold. It then follows that

$$\hat{y}(t) \approx y(t) \cos(\omega_0 t + \phi),$$

where $y(t)$ is a band-limited signal with a bandwidth of $\omega_b$ rad/s. We refer to $y(t)$ as the envelope of $\hat{y}$, and note that $y(t)$ may be positive or negative. It follows that the input to the filter, i.e., $2\hat{y}(t) \cos(\omega_0 t + \phi) \approx y(t)[1 + \cos(2(\omega_0 t + \phi))]$. Taking the Laplace transform of this signal yields

$$Y(s) + \frac{1}{2} e^{-2j\phi} Y(s + 2j\omega_0) + e^{+2j\phi} Y(s - 2j\omega_0).$$

From the model, we have that this signal also equals

$$G_m(s, \omega_0)U(s) + \frac{1}{2} [e^{-j\phi} G(s + j\omega_0)U(s + 2j\omega_0) + e^{+j\phi} G(s - j\omega_0)U(s - 2j\omega_0)], \quad (3)$$

where

$$G_m(s, \omega_0) = \frac{1}{2} [e^{-j\phi} G(s + j\omega_0) + e^{+j\phi} G(s - j\omega_0)].$$

Since $u$ and $y$ are band-limited and Assumption 2 holds, we can safely approximate $Y(j\omega)$ by $G_m(j\omega, \omega_0)U(j\omega)$, and hence, the approximate transfer function from $U(s)$ to $Y(s)$ is $G_m(s, \omega_0)$. It can also be seen that when (3) is filtered by $F(s)$, the terms containing $U(s \pm 2j\omega_0)$ are attenuated significantly,
Fig. 2. Block diagram of modulated and demodulated system.

and so the approximate transfer function from \( U(s) \) to \( Y_f(s) \) is \( G_m(s, \omega_0) F(s) \).

Now suppose that
\[
\bar{d}(t) = d_I(t) \cos(\omega_0 t + \phi) - d_Q(t) \sin(\omega_0 t + \phi),
\]
where \( d_I(t) \) and \( d_Q(t) \) are band-limited with bandwidths \( \omega_0 \). Note that we refer to \( d_I \) and \( d_Q \) as the in-phase and quadrature components of \( \bar{d} \). In this case, we have \( \bar{y}(t) = y_I(t) \cos(\omega_0 t + \phi) - y_Q(t) \sin(\omega_0 t + \phi) \). It is clear that \( y_I \) is controllable from \( u \) and that \( y_I \) is \( y_I \) filtered by \( F(s) \). However, \( y_Q \) cannot be controlled from \( u \) and is unobservable from \( y_I \). It follows that the scheme in Fig. 3 can be used to control both components of \( \bar{y} \).

We note that \( \bar{y}(t) \) can also be written as \( y_I(t) \cos(\omega_0 t + \phi + \varphi(t)) \), where \( \varphi(t) \) is a continuous function of time. Similarly, \( \bar{u}(t) \) can be written as \( u(t) \cos(\omega_0 t + \varphi(t)) \). If \( \varphi(t) \approx \varphi(t) \), then the quadrature scheme in Fig. 3 is mathematically equivalent to the one shown in Fig. 4. If \( \varphi(t) \approx 0 \), then \( Y(s)/U(s) \approx G_m(s, \omega_0) \) and so the baseband system can be modelled as shown in Fig. 5.

Remark 1. We note that, in practice, the low pass filter \( F(s) \) may introduce a significant delay (relative to \( 1/\omega_0 \)) into the system. This would seem to imply that it is difficult to control the system at \( \omega_0 \) rad/s. However, it is clear from Fig. 5 that the filter delay limits only the speed of response of the baseband system, not the full system. Thus, the delay limits only the speed of response of the envelope of \( \bar{y} \). This fact is analysed more fully in (Lau et al., 2004).

\[
3.3 \text{ Properties of } G_m(s, \omega_0)
\]

Let \( G(s) = N(s)/D(s) \), where \( N(s) \) and \( D(s) \) are coprime polynomials. Then
\[
G_m(s, \omega_0) = \frac{1}{2} \frac{N_m(s, \omega_0)}{D_m(s, \omega_0)},
\]
where
\[
N_m(s, \omega_0) = e^{-j\omega_0} N(s + j\omega_0) D(s - j\omega_0)
+ e^{j\omega_0} N(s - j\omega_0) D(s + j\omega_0),
\]
and
\[
D_m(s, \omega_0) = D(s + j\omega_0) D(s - j\omega_0).
\]

We refer to the zeros of \( N_m(s, \omega_0) \) as the zeros of \( G_m(s, \omega_0) \) and the zeros of \( D_m(s, \omega_0) \) as the poles of \( G_m(s, \omega_0) \). Thus, \( G_m(s, \omega_0) \) may contain pole zero cancellations.

It can easily be shown that \( G_m(s, \omega_0) \) has the following properties (Lau et al., 2004, Lem. 5.1, 5.6, 5.7):

**Property 1** The poles of \( G_m(s, \omega_0) \) are the poles of \( G(s) \) shifted by \( j\omega_0 \) and \( -j\omega_0 \).

**Property 2** If \( G(s) \) has poles at \( p \in \mathbb{C} \) and \( p + 2j\omega_0 \), then \( G_m(s, \omega_0) \) has two poles and one zero at \( p + j\omega_0 \).

**Property 3** If \( G' = e^{-s\tau} G(s) \), \( \tau > 0 \), then \( G'_m(s, \omega_0) = e^{-s\tau} G_m(s, \omega_0) \).

Remark 2. If \( G(s) \) has a pair of poles at \( a \pm j\omega_1 \) and \( \omega_0 = \omega_1 \), then \( G_m(s, \omega_0) \) has a double pole at \( a \) and a pair of poles at \( a \pm 2j\omega_2 \) (Property 1). \( G_m(s, \omega_0) \) also has a zero at \( a \) which cancels one of the poles (Property 2). If \( \omega_0 = \omega_1 \), then there is an approximate cancellation.

Remark 3. We have already noted (Rmk. 1) that the filter delay limits the bandwidth of the baseband system, not that of the full system. Property 3 implies that this observation also holds for plant delays.

4. APPLICATION OF MODULATED CONTROL TO FLEXIBLE BEAM

In this section, we show that modulated control may be used to damp the transient modes of a flexible beam. We consider the control of the first two modes, but note that the extension to a higher number of modes is straightforward.

Let \( \omega_1 \) and \( \omega_2 > \omega_1 \) be the frequencies of the two modes and let \( \Delta \omega = \omega_2 - \omega_1 \). Since we are interested in only these modes, we approximate \( G(s) \) by
\[
G(s) = \frac{\gamma_1}{s^2 + 2\kappa_1 \omega_1 s + \omega_1^2} + \frac{\gamma_2}{s^2 + 2\kappa_2 \omega_2 s + \omega_2^2},
\]
i.e., we let \( M = 2 \) in Equation (2).

We control the modes in a decentralised manner by using one modulated controller per mode. The resulting control scheme is shown in Fig. 6. We note that, in this figure, the \( \cos \) terms in the demodulators are both replaced by \( \sin \) terms because \( \text{Arg}[G(j\omega_1)] \approx \text{Arg}[G(j\omega_2)] \approx -\pi/2 \). The initial conditions corresponding to the first and second modes are modelled by \( d_1 \) and \( d_2 \), respectively.

An underlying assumption of the scheme is that the closed loop output \( v(t) \) satisfies
\[
v(t) = \tilde{v}_1(t) + \tilde{v}_2(t)
= v_1(t) \sin(\omega_1 t + \varphi_1(t)) + v_2(t) \sin(\omega_2 t + \varphi_2(t))
\]
is small, then an equivalent condition is

\[ F_i(s) \approx G_m(s, \omega_i) \]

where \( v_1, v_2 \) are band-limited signals, and \( \varphi_1, \varphi_2 \) are continuous. It is also assumed that \( u_i \) is bandlimited. Let \( \omega_{b_i} \) be an upper bound on the bandwidths of \( u_i \) and \( u_i \). We require \( F_i(s) \) and \( \omega_{b_i} \) to satisfy the following conditions:

1. \( \omega_{b_1} < \min\{\Delta \omega, \omega_1\} \) and \( \omega_{b_2} < \Delta \omega \).
2. \( F_i(s) \) is a low pass filter which rolls off for \( \omega > \omega_{b_i} \).
3. \( |F_i(j\Delta \omega)G(j\omega_k)| \approx |F_i(0)G(j\omega_1)| \) for \( i, k \in \{1, 2\} \) and \( i \neq k \).\(^2\)

These conditions ensure that the approximate transfer function from \( U_i(s) \) to \( V_i(s) \) is given by \( G_m(s, \omega_i) \). They also ensure that the coupling between the two loops in Fig. 6 is not significant. In particular, the third condition implies that the gain from \( u_i \) to \( v_{bf} \) is small.

5. CASE STUDY AND SIMULATION RESULTS

We provide simulation results for the control of a flexible beam of length 0.775 [m] and width 0.05 [m].

\[ G_m(j\omega_2). \] It follows that an equivalent condition is

\[ |F_i(j\Delta \omega)G_m(j\Delta \omega, \omega_i)| \approx |F_i(0)G_m(0, \omega_i)| \]

Similarly, if \( G(j\omega_2 + \Delta \omega) \) is small, then an equivalent condition is

\[ |F_2(j\Delta \omega)G_m(j\Delta \omega, \omega_2)| \approx |F_2(0)G_m(0, \omega_2)| \].
This beam is modelled in (Pota et al., 1999). We have $\gamma_1 = 32.29, \gamma_2 = 584.39, \zeta_1 = \zeta_2 = 0.008, \omega_1 = 50.18 \text{ rad/s}$ and $\omega_2 = 314.46 \text{ rad/s}$. The model $G(s)$ has poles at approximately $-a_1 \pm j\omega_1$ and $-a_2 \pm j\omega_2$, where $a_1 \approx 0.4$ and $a_2 \approx 2.52$. It follows that the time constant of the (envelope of) the first mode is $1/0.4 [s]$ and that of the second mode is $1/2.52 [s]$.

The baseband model $G_m(s, \omega_1)$ has a pair of poles and a zero at approximately $-a_1$ (Rmk. 2). Thus, we have an approximate pole zero cancellation at $-a_1$. Since the rest of the poles are large, it follows that $G_m(s, \omega_1)$ has a dominant pole at $-a_1$. Similarly, $G_m(s, \omega_2)$ has a dominant pole at $-a_2$. We note that the transient (of $G_m(s, \omega_1)$) associated with $a_1$ corresponds to the envelope of the $i$th mode of $G(s)$.

We take $F_1(s)$ and $F_2(s)$ as fourth order Butterworth filters with bandwidths of $\omega_{b1} = 20$ and $\omega_{b2} = 50$, respectively. The Bode magnitude plots of $G_m(s, \omega_1)F_1(s)$ and $G_m(s, \omega_2)F_2(s)$ are shown in Figs. 7 and 8, respectively. From these diagrams, it is clear that $F_1(s)$ and $F_2(s)$ have sufficient attenuation at $s = j\Delta\omega$.

The upper plots in Figs. 9 and 10 show the transient (uncontrolled) responses $\tilde{v}_1(t)$ and $\tilde{v}_2(t)$ and also the outputs of $F_1(s)$ and $F_2(s)$. Notice that $v_{1f}(t)$ and $v_{2f}(t)$ are the filtered envelopes of the first and second modes, respectively. The lower plots in the figures show the controlled responses with $C_1(s) = C_2(s) = 5$ (i.e., proportional control in the baseband) and both of the loops closed. We observe that the closed loop responses are significantly faster than the open loop ones. Note that $C_1(s)$ shifts the dominant pole of $G_m(s, \omega_1)$ from $-a_1 \approx -0.4$ to $-2.65$. This is equivalent to shifting the poles of $G(s)$, corresponding to the first mode, to the left (increasing the damping). $C_2(s)$ shifts the dominant pole of $G_m(s, \omega_2)$ from $-2.52$ to $-10.33$.

As discussed in Section 2, we are often interested in the vibration measured at another point on the beam. Fig. 11 contains plots of the displacement, $z(t)$, measured at a point 0.76 [m] from the fixed end. The open and closed loop responses are both shown. In this case, we have, $\gamma_{z1} = 0.0029$ and $\gamma_{z2} = -0.0013$ (Pota et al., 1999). We note that the response is dominated by the first mode.

Figs. 12 and 13 show the open and closed loop frequency response (magnitude only) from the input disturbance $d_u$ (shown in Fig. 6) to $v$ and to $z$, respectively. The responses up to the third mode are shown. We note that the open loop responses were obtained.

\[^3\] The imaginary parts of the poles are actually $\pm \sqrt{1-\zeta^2\omega_1} \approx \pm \omega_1$. 

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**Fig. 7.** Bode magnitude plot of $G_m(s, \omega_1)F_1(s)$ showing the attenuation of the peak at $\Delta\omega$.

**Fig. 8.** Bode magnitude plot of $G_m(s, \omega_2)F_2(s)$ showing the attenuation of the peak at $\Delta\omega$.

**Fig. 9.** First mode time responses.

**Fig. 10.** Second mode time responses.

**Fig. 11.** Time responses for $z$. 

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The low frequency signal is then manipulated directly, and finally shifted (modulated) up to the original frequency of oscillation. Through simulation, we demonstrated the effectiveness of the proposed control methodology in adding damping to the two lowest frequency oscillatory modes of a piezoelectric laminate cantilever beam. Future research involves experimental implementation of the proposed controller on a laboratory scale apparatus.

4 It is perhaps worth noting that the magnitude of the closed loop gain at $\omega_i$, $i = 1, 2$ is $\approx \frac{G_m(0, \omega_i)}{1 + G_m(0, \omega_i)C(0))}$. 

7. REFERENCES


