Granular Contact Dynamics for the Probabilistic Stability Analysis of Slopes

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Abstract. Slope instability and landslides can be catastrophic events often leading to loss of life and property. To assess the risks of slope failures, it is often desired that the dynamic process of slope failure can be simulated. This paper proposes a Granular Contact Dynamics (GCD) approach based on variational principles and implicit time integration to simulate the failure process of slopes. The method has the ability to simulate fracture initiation and development (i.e., progressive failure process) and allows the consequence of slope failures to be assessed quantitatively. A two dimensional example is given to demonstrate the method.

Keywords: landslide, risk assessment, contact dynamics, consequence

1. Introduction

The evaluation of the stability of natural or constructed slopes has traditionally been based on deterministic approaches and is quantified by a safety factor. In such an approach, geotechnical engineers try to deal with uncertainties by choosing reasonably conservative parameters. However, it is common to use the same safety factor for different type of applications without any regard to the degree of uncertainty involved in its calculation. In regulation or tradition, the same safety factor is often applied to conditions that involve widely varying degree of uncertainty. Since this is not a very logical strategy as noted by Duncan (2000), numerous studies have been undertaken in recent years to develop probabilistic methods that deal with uncertainties in a systematic way (e.g., Low et al. 1998; Griffiths and Fenton 2000; 2002; Griffiths and Fenton 2004; Griffiths et al. 2009; Huang et al. 2010; Li et al. 2011; Zhang et al. 2011).

It is noted however, most of the studies only aimed at obtaining a more rigorous estimation of probability of failure ($P_f$). Huang et al. (2013) proposed that the consequences should be assessed individually. The consequences of slope failures were assumed to be proportional to the volume of sliding mass. A more rigorous approach should simulate the dynamic process of slope failures. However, the probabilistic methods for geomechanics based on continuum mechanics is incapable of modelling the most general rock slope failures, which usually involve complex interactions between pre-existing discontinuities and brittle fracture propagation through intact rock bridges, resulting in a step-path failure mode involving both sliding on existing discontinuities and brittle fracturing of intact rock.

In this paper, the contact dynamics approach developed by Krabbenhof et al (2012a; 2012b) and Huang et al (2013) is further developed to model the dynamic process of slope failure. This approach was original developed to model granular materials. In order to model cohesive soil slopes and rock slopes, a new bond model is developed. The bond model is placed in between particles, which can resist normal forces, shear forces and rolling moments. A slope failure process in two dimensional was adopted to show the method.

2. Granular Contact Dynamics of Frictional Contact

The so-called non-smooth contact dynamics (CD) method (Jean 1999) originally developed by Moreau (1988, 1998) is one type of discontinuum based method, having advantages in modelling dynamic behaviors and in modelling initiation and motion of landslide. In contrast to the traditional DEM, the contact forces of particles in CD method are determined through collision effects of particles, such as...
Signorini’s condition. Following Krabbenhoft et al. (2012a; 2012b) and Huang et al. (2013), the force based problem of frictional contact can be formulated as

$$\min \quad \frac{1}{2} \mathbf{t}^T \mathbf{M}^{-1} \mathbf{t} + \frac{1}{2} \mathbf{r}^T \mathbf{J}^T \mathbf{r} + g_0^T \mathbf{p}$$

s.t. \quad \mathbf{t} + \mathbf{N}_0 \mathbf{p} + \mathbf{\hat{N}}_0 \mathbf{q} = \mathbf{f}_0$$
$$\mathbf{r} + \mathbf{R}_0 \mathbf{q} + \mathbf{\tau} = \mathbf{m}_0$$
$$p \geq 0$$
$$\|\mathbf{q}\| \leq \mu p$$
$$\|\mathbf{r}\| \leq \mu \|\mathbf{R}_0 p\|$$

where \(\mathbf{t}, \mathbf{r}\) are the dynamic forces and moments, \(\mathbf{M}^{-1}\) and \(\mathbf{J}^{-1}\) are the function of the mass and moment of inertia terms, \(g_0\) is the inter-particle gap, \(\mathbf{p}, \mathbf{q}\) are the contact normal and shear forces (refer to Fig.1), and \(\mathbf{m}_0, \mathbf{f}_0\) represent the external forces and moments. \(N_0\) and \(\mathbf{\hat{N}}_0\) describe the contact force topology, and \(\mathbf{\tau}\) is the rolling resistance.

3. Granular Contact Dynamics with bond model

3.1. Bond model

The bond model with finite size envisioned as rectangular cross section between particles can resist tensile forces, shear forces and rolling torques (Fig.3). When the limit of forces or moments that the bond provides is reached, bond model is considered as failure and then the contact turns to purely frictional behavior. Accordingly, three failure modes can be introduced to the bond fracture: tensile fracture, shear fracture and torque fracture.

Before each time step's calculation, the potential contacts are identified by Delaunay triangulation based on present particles' position. Fig.2. shows an example.

Conic programming provides a convenient framework for above discussed problem. The Coulomb friction law and rolling resistance can be treated as the second order cone constrains for the mathematical programming. At present, there are a number of efficient and robust solvers (Andersen, 2003) for such problems.

In tension, the maximum tensile force \(f_i\) is defined based on tensile strength, \(\sigma\), such that
$$f_i = \sigma_i A_b$$
where \(A_b\) is the bond area between two particles which is function of the particle's radius, \(r\), as \(A_b = 2r \times 1\).
Fig. 4 (a) shows the maximum tensile force that bond can provide and local frictional constitutive relation for the contact.

The modified Mohr-Coulomb model is implemented in the framework of Contact Dynamics by defining the parameter cohesion \( c \) and friction angle \( \phi_b \) and then the shear force threshold \( f_s \) of the bond is provided as
\[
f_s = cA_b + \mu_b p
\]
where \( c \) is cohesion and friction coefficient of the bond \( \mu_b = \tan \phi_b \).

In the same way, the shear fracture can be defined when the maximum shear force is reached and then the contact turns purely friction (see Fig. 4 (a)).

The bond model requires also torque transmission by defining rolling resistance as we have already discussed above. The largest rolling moment that the bond can be provided \( m_b \) is presented as
\[
m_b = \mu_{br} r (p + f_s)
\]
where \( \mu_{br} \) is the coefficient of torques transmission.

The value of \( \mu_{br} \) can be larger than 0.3 while the reasonable limits of \( \mu_c \) for the non-bonded granule material is from 0 to 0.21. The loss of bond due to torque facture is shown in Fig. 4 (b).

3.2. Formulation

The Granular Contact Dynamics can be extended to bond model with the description above. Interestingly, the formulation for the bonded particles is easily obtained simply by removing last item, \( g_0 p \) to the objective function and by adding constrain conditions above discussed, provided as
\[
\begin{align*}
\min \ & \frac{1}{2} r^T \bar{M}^{-1} r + \frac{1}{2} r^T \bar{J}^2 r \\
\text{s.t.} \ & t + N_0 p + \bar{N}_0 q = \bar{f}_0 \\
& \bar{r} + R_0 q + \tau = \bar{m}_0 \\
& -f_s \leq p \\
& \|q\| \leq \mu_c p + cA_b \\
& \|r\| \leq \mu_{br} R_0 (p + f_s)
\end{align*}
\]

4. Example

In this study, the slope model was employed to implement above formulation. 25072 particles with radius varying from 0.05m to 0.2m were first randomly generated inside the slope boundary. After the equilibrium state for the granular assemble was reached, the bond was assigned to the contact of particles. Finally, the slope model was created as shown in Fig. 5.

The local parameters for the bond model are listed as: tensile threshold \( \sigma_t = 0.6 \text{kPa} \), friction angle \( \phi_b = 35^\circ \), cohesion \( c = 2 \text{kPa} \) and a coefficient of rolling friction \( \mu_{br} = 0.3 \). For the slope properties, the input parameters, the unit weight, the slope height and the slope depth are 20 kN/m\(^2\), 20 m and 20 m, respectively.

The yellow cross in the particles’ contact point represents the bond between particles so the failure process of the slope, therefore, can be seen directly from the figure of the bonds location (Fig. 6).

Figure 4. Strength criterion of the bond model (a) tensile fracture and shear fracture (b) torque fracture. The subscript \( b \) indicates the local properties for the bond.
The progressive failure process of the above example was studied, as shown in Fig. 6. Micro-cracks firstly occur at the toe region of the slope model. With the increase of time step, failures gradually expand upward and it is worth noting that the chainlike structures of contact forces exist along with the failure region. When the time step reaches 15, with the failure progressively extend toward to the crest, the sliding mass forms. Following the initiation of slope failure, i.e., in the transportation stage, the movement of sliding mass is main feature of the landslide. This progressive failure agrees well with the method of strain-softening slopes presented by Zhang (2013). During failure process, the sliding mass gradually increases accompanied by erasing bonds. In a word, the present method is a realistic solution, as it represents the behavior of progressive failure.

The calculation results at the time step of 88 are shown in Fig. 7. The sliding mass of the slope is 98.06 m², which was obtained simply by identifying the area of particles with loss of bonds. The horizontal displacement of slope failure is summarized in Fig. 7 (b). The consequence of slope failure can be estimated by analyzing sliding mass, run-out distance and damage forces during sliding in the further study.

5. Conclusion

To quantitatively assess the landslide risk, the consequence of landslide can be estimated by simulating the dynamic failure process. The consequence therefore can be estimated by calculating the sliding mass, run-out distance and damage forces. In this paper, the dynamic process of slope failure was numerical studied with Granular Contact Dynamics.
Two-dimensional formulations of Granular Contact Dynamics of frictional contact particles have been extended to the bond model by introducing modified Mohr-Coulomb model in the frame of Contact Dynamics. The bond between particles can resist tensile forces, shear forces and rolling moments, which enables the method to be applied to the study of common geomaterial rather than purely frictional granular materials.

An example has been adopted to show the potential ability of our method. A result of the bond progressive failure in the slope can be summarized as: from the initial intact slope, to fractures initiating from the toe region and to formatting the failure surface.

References