Spectrum of a passive scalar in stretched grid turbulence at low Reynolds numbers

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Abstract. Approximately homogeneous isotropic turbulence is obtained by stretching a wind-tunnel grid flow with a 1.36:1 contraction. The flow is mildly heated so that temperature serves as a passive scalar. For three different grids, the dissipation rates and spectra of velocity and temperature fluctuations are obtained from simultaneous hot-wire and cold-wire measurements. The dissipation rates follow a power-law decay. Comparison with an unstretched grid flow shows that the contraction improves the isotropy and reduces the effect of grid shape on the decay exponents. At low Reynolds numbers, there is a significant scaling range for the temperature spectrum but not for the velocity spectrum. With stretching, the temperature spectrum shows a wider scaling range, and that the scaling range exponent is closer to 5/3. The scaling exponent for the temperature spectrum ($m_\theta$) is represented by a power-law function of Reynolds number, and it approaches 5/3 faster than that for the velocity spectrum ($m_u$). Results show that the ratio between the velocity and temperature scaling range exponents, $(5/3+m_u)/m_\theta$, is about 1.98.

1. Introduction

For approximately homogeneous isotropic turbulence, the one-dimensional spectral density for the streamwise velocity ($u$) and the transported passive scalar ($\theta$) indicates a power-law behaviour in the inertial scaling range of wavenumber ($\kappa$):

$$
\Phi_u(\kappa) = C_u \varepsilon^{2/3} \kappa^{-m_u} \quad \text{and} \quad \Phi_\theta(\kappa) = C_\theta \varepsilon^{-1/3} \chi \kappa^{-m_\theta},
$$

where $C_u$ and $C_\theta$ are empirical coefficients. The variance for the streamwise component of velocity is $u'^2 = \int_0^\infty \Phi_u(\kappa) \, d\kappa$ and the variance for the passive scalar is $\theta'^2 = \int_0^\infty \Phi_\theta(\kappa) \, d\kappa$. The mean dissipation rates for $u'^2$ and $\theta'^2$ are $\varepsilon$ and $\chi$, respectively. For very large Taylor micro-scale Reynolds numbers ($Re_\lambda = \lambda u'/\nu$), the theory of Kolmogorov (1941), Obukhov (1949) and Corrsin (1951) predicts that the power-law exponents $m_u$ and $m_\theta$ are both equal to 5/3, while available experimental data (on geophysical flows) suggest $C_u \sim 0.5$ (Sreenivasan, 1995) and $C_\theta \sim 0.4$ (Sreenivasan, 1996) for $Re_\lambda \approx 10^5$-$10^6$. For moderately high Reynolds numbers $Re_\lambda \approx 10^2$-$10^3$ (of an “active” grid flow), Mydlarski & Warhaft (1998) show that there is a relatively narrow scaling range for the spectra but $m_u < m_\theta < 5/3$. Danaila & Antonia (2009) describe the difference between $m_u$ and $m_\theta$ by proposing that the transfer of energy to higher wavenumbers in the scalar field depends on the Batchelor time scale of the velocity field. The time scale is defined such that eddies at wavenumber $\kappa$ are strained by larger eddies at wavenumbers between zero and $\kappa$ (i.e. $\tau(\kappa) \sim [\int_0^\kappa p^2 \Phi_\theta(p) \, dp]^{-1/2}$). Danaila & Antonia (2009) have used this in their analysis
to establish a relationship between $\Phi_u$ and $\Phi_\theta$, and therefore a relationship between the scaling exponents. Their analysis suggests that, for low/moderate Reynolds numbers with gaseous mixing at Prandtl number near one, the scaling range exponents $(m_u, m_\theta)$ follow the relationship:

$$m_\theta = \left(\frac{5}{3} + m_u\right)/2.$$

Measurements for $m_u$ and $m_\theta$ obtained at low-to-moderate Reynolds numbers $Re_\lambda \approx 10 \rightarrow 10^2$ for a grid flow not stretched by a contraction are available from Danaila & Antonia (2009). In the present work, the test section of these authors is replaced by the 1.36:1 contraction tunnel of Lavoie et al. (2007). The purpose of the contraction is to stretch the flow in the streamwise direction and therefore to reduce the anisotropy of the grid turbulence. Particularly for a grid of round bars with a fine wire helically wrapped around each bar (denoted “Rd44w”), Lavoie et al. (2007) show that stretching this grid flow produces nearly isotropic turbulence, where the measured ratio of streamwise to cross-stream velocity fluctuations is $u'^2/w'^2 \approx 0.99$. For this stretched Rd44w grid flow, Lee et al. (2010) found that the passive scalar is also nearly isotropic, where the measured skewness for temperature fluctuation is almost zero and approximately constant (i.e. $S_\theta \approx 0.05$) over the full length of the test section. For the same grid flow but without the wire wrapped around each bar and without the contraction, Lee et al. (2010) observed that the skewness for temperature fluctuation tends to be less constant and larger (with $S_\theta$ reaching up to about 0.14).

This paper reports the dissipation rate and the spectrum for the velocity and the passive scalar of three different grid flows (including Rd44w) at low Reynolds numbers. The aim is to study the effect of grid geometry and stretching on the dissipation rates and the scaling range of the spectra, and to compare results with Equation 2.

2. Experimental method

In this experiment, the passive scalar is temperature, where a wind-tunnel grid flow is mildly heated ($\Delta T \approx 2^\circ$C) with a “mandoline” of 0.5-mm-diameter Chromel-A wire located at 1.5-mesh-lengths downstream of the turbulence-generating grid. Figure 1 shows the three biplanar grids which match earlier reports (Lavoie et al., 2007; Lee et al., 2010), one of square bars at 35% solidity (Sq35) and two of round bars at 35% and 44% solidity (Rd35, Rd44w). The mesh pitch for each grid is $M = 24.76$ mm. To reduce the anisotropy of grid turbulence, the 1.36:1 contraction is located at $11M$ downstream of the grid. The velocity and the temperature are obtained simultaneously from signals produced by hot- and cold-wires etched from a fine
begins with measurements of the dissipation rates. In grid turbulence, in Equations 1, the spectra are scaled according to the mean dissipation rates and so this analysis to data points in the range \(22 < \tau_u/M < 110\), where \(\tau_u = \frac{1}{U_0} \int_0^x U_d/\overline{U(x)} \, dx\) is distance converted about 10\(\eta\) in \(\theta\), is in the range 25 to 3.0 Kolmogorov lengths (\(\eta\)), with \(l_{hw}/\eta \approx 0.7 \rightarrow 1.7\) and \(l_{cw}/\eta \approx 0.9 \rightarrow 2.1\). The signals are low-pass filtered at half the sampling rate and the duration for each measurement is equivalent to 10\(^6\) samples. Spectra of the signals are estimated by fast Fourier transform (Welch, 1967). To remove visible electronic/high-wavenumber noise, the (compensated) spectra are manually fitted with cubic splines and are then carefully extrapolated, after Pope (2000): 

\[
\tau^{-2/3} \kappa^{-1/3} \Phi(\kappa \eta) \quad \text{or} \quad \tau^{-2/3} \bar{\kappa}^{-1/3} \Phi(\bar{\kappa} \cdot \eta) = C_1 \cdot \exp \left\{ -C_2 \left[ \left( \kappa_0 \eta \right)^4 + C_3^4 \right]^{1/4} - C_3 \right\}
\]

where the coefficients \(C_1, C_2\) and \(C_3\) are chosen to minimise the r.m.s. difference between the curve fit and the spectra. The examples in Figure 2 show the typical effect of smoothing. To ensure adequate extrapolation, the fitted curves are checked by plotting the spectra weighted by the wavenumber. Measurements are obtained for mesh Reynolds number \((Re_M = MU_0/\nu)\) of about 10,400 up to 26,000 \((U_0\text{ is the free-stream velocity; } \nu\text{ is the kinematic viscosity})\). The Taylor micro-scale, \(\lambda = \sqrt{u'^2/(\overline{u^2})}\), is in the range 25 < \(\lambda u'/\nu\) < 55.

### 3. Decay of mean dissipation rates

In Equations 1, the spectra are scaled according to the mean dissipation rates and so this analysis begins with measurements of the dissipation rates. In grid turbulence, \(\tau\) and \(\overline{\tau}\) are determined by the streamwise decay rates of \(q'^2\) (approximated by \(3u'^2\)) and \(\theta'^2\), after Zhou et al. (2000):

\[
\tau_d \approx \frac{1}{2} U_0 \frac{d}{dx} 3u'^2 \quad \text{and} \quad \overline{\tau}_d = \frac{1}{2} U_0 \frac{d}{dx} \theta'^2.
\]

The streamwise functions \(u'^2(x)\) and \(\theta'^2(x)\) are obtained from fourth-order polynomial curve fits to data points in the range 22 ≤ \(\tau_u/M\) ≤ 110, where \(tU_0 = \int_0^x U_d/\overline{U(x)} \, dx\) is distance converted.
Figure 3. Decay of the mean dissipation rates ($\varepsilon$, $\chi$) downstream of each grid; $Re_M \approx 10,400$. The calculation methods are shown in the legend. The lines of best fit are power-law functions of the form $\varepsilon \sim t^{m-1}$ and $\chi \sim t^{n-1}$. For the Sq35 data ($\varepsilon$ ■; $\chi$ □) taken from Figure 4 of Zhou et al. (2000), the grid flow is not stretched by a contraction. Lavoie's (2006) dissipation rates × (without contraction) and + (with the 1.36:1 contraction) are estimated from his measurements of the streamwise decay rates of $q'^2$.

Figure 4. The anisotropy ratio for the mean dissipation rates of each grid: (a) $\varepsilon_d/\varepsilon_s$ and (b) $\chi_d/\chi_s$. (c) Measurements of the dissipation-amplitude ratio ($\chi/\Delta T^2)/(\varepsilon/U_0^2)$ are compared with those (◆) calculated from the Sq35 data of Zhou et al. (2000).
Table 1. Decay exponents of velocity and temperature from the same wind tunnel with and without the 1.36:1 contraction: $Re_M \sim 10^4$. The uncertainty in the decay exponents is for a 95\% confidence level. The range of curve fit data is shown as $tU_0/M$. The anisotropy ratio is $u'^2/w'^2$.

<table>
<thead>
<tr>
<th>Grid flow stretched by a 1.36:1 contraction</th>
<th>Grid flow not stretched by a contraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref.</td>
<td>Measurements from Figure 3</td>
</tr>
<tr>
<td>Probe</td>
<td>Hot- and cold-wires</td>
</tr>
<tr>
<td>$tU_0/M$</td>
<td></td>
</tr>
<tr>
<td>Grid</td>
<td>$m(\tau)$</td>
</tr>
<tr>
<td>Sq35</td>
<td>$-1.21 \pm 0.05$</td>
</tr>
<tr>
<td>Rd35</td>
<td>$-1.31 \pm 0.05$</td>
</tr>
<tr>
<td>Rd44w</td>
<td>$-1.12 \pm 0.06$</td>
</tr>
</tbody>
</table>

| Probe | Hot- and cold-wires | Hot- and cold-wires | Cross hot-wires |
| $tU_0/M$ | 30 to 80 | 20~30 to 80 | 29~32 to 80 |
| Grid | $m(q^2)$ | $n(\theta^2)$ | $m(q^2)$ | $n(\theta^2)$ | $m(q^2)$ | $u'^2/w'^2$ |
| Sq35 | $-1.33$ | $-1.36$ | $-1.3$ | $-1.4 \sim -1.46$ | $-1.04$ | 1.45 |
| Rd35 | No data | No data | No data |
| Rd44w | No data | No data | No data |

to a decay time to account for acceleration of the grid flow through the contraction. The selected range of $tU_0/M$ is sufficiently far downstream of the regions of initially developing turbulence and accelerated decay in the contraction, and avoids possible effects of the duct exit (Lee et al., 2010). Equations 4 are compared with the following method(s) for estimating isotropic dissipation rates:

\[
\bar{\varepsilon}_1 = 15\nu \int \frac{du}{dt}^2 / U_0^2 \quad \text{and} \quad \bar{\chi}_1 = 3\gamma \int \frac{d\theta}{dt}^2 / U_0^2
\]

or

\[
\bar{\varepsilon}_a = 15\nu \int_0^\infty \kappa_\theta^2 \Phi_\theta(d\kappa) \, d\kappa_\theta \quad \text{and} \quad \bar{\chi}_a = 3\gamma \int_0^\infty \kappa_\theta^2 \Phi_\theta(d\kappa) \, d\kappa_\theta
\]

where $\kappa_\theta = 2\pi f_j / U_0$ is the one-dimensional wavenumber and $\gamma$ is the thermal diffusivity (the Prandtl number is $Pr = \nu/\gamma \approx 0.71$). The Taylor hypothesis ($dx = U_0 dt$) is applied to Equations 4 and 5. Figure 3 shows good agreement between the different methods for estimating the dissipation rates, and indicates that the grid turbulence is approximately isotropic. Figure 4(a, b) shows the anisotropy ratios $\bar{\varepsilon}_d/\bar{\varepsilon}_a$ and $\bar{\chi}_d/\bar{\chi}_a$, particularly for Rd44w, are reasonably close to 1.

A comparison of Zhou’s (2000) and Lavoie’s (2006) data with the results in Figure 3 shows that the contraction slightly lowers the amplitude of $\varepsilon$ and $\chi$. The overall effect tends to bring the dissipation-amplitude ratio $(\bar{\chi}/\Delta t^2)/(\bar{\varepsilon}/U_0^2)$ nearer to 1 (Figure 4(c)). Also, an amplitude ratio near 1 may suggest a weak large-scale periodic shedding/mixing behind the grid, since the amplitude ratio is closest to 1 for grid Rd44w and the purpose of wire wrapping the bars of Rd44w is to reduce shedding (Lavoie et al., 2007).

In Figure 3, the dissipation rates are well represented by the power-law decays $\varepsilon \sim t^{m-1}$ and $\chi \sim t^{n-1}$, where $m, n < -1$. With the least-square curves of best fit shown in Figure 3, the r.m.s.
Figure 5. Spectra of the velocity ($\Phi_u$, $\Phi_w$, $\Phi_q$) and temperature ($\Phi_\theta$) at $Re_M \approx 10,400$ and $tU_o/M \approx 70$. The $q$ spectrum of Lavoie (2006) is approximated by $\Phi_u + 2\Phi_w$. $Re_\lambda \approx 38$ (grid Sq35), 27 (Rd35) and 34 (Rd44w).

Figure 6. The effect of inlet conditions on the scaling range; the Sq35 grid flow (a) with and (b) without stretching at $tU_o/M \approx 30 \rightarrow 40$; (c) an unstretched active grid flow at $tU_o/M \approx 70$.

difference between the power law and the present measurements ($\varepsilon$, $\chi$) is no more than 18%. Table 1 shows that the stretched velocity and temperature decay rates follow the same trends ($m$, $n$: Rd35 $<$ Sq35 $<$ Rd44w) and are consistent with earlier reports. The data of Lavoie et al. (2007) suggests that, by stretching the flow to improve isotropy, the decay exponents ($m$, $n$) are less dependent on grid shape. For the stretched grid flows, an improvement in isotropy tends to increase $m$ and $n$, since Rd44w generates the most isotropic turbulence with $u'^2/w'^2 \approx 0.99$ (Table 1). If turbulence is isotropic, the theory of Kármán & Howarth (1938) predicts a decay exponent of $-1$. However, Zhou et al. (2000, 2002), Lavoie et al. (2007) and Lee et al. (2010) have not obtained exponents equal to $-1$, which suggests that, with or without stretching by the contraction, perfect isotropy is rather difficult to achieve in grid turbulence.

4. Scaling the spectra

Figure 5 shows examples of one-dimensional spectral density for the velocity and passive temperature in weak turbulence ($Re_\lambda < 100$). The spectra are scaled by the Kolmogorov length.
Over the full length of the test section (for the three grids), where smaller changes in Re results in Figure 8(a, b) show that the inertial range and the scaling exponents (where $\beta$ and isotropy in grid turbulence improves with increasing Re) trends in Figure 9 show that, with stretching, the velocity and temperature scaling ranges are not significantly affected by the different grids. Comparison of the results in Figure 8(b, c) with those of Danaila & Antonia (2009) shows the effect of stretching is to increase the scaling exponents.

In Figure 7, the optimally compensated spectra $C_u = \xi^{1/3} \kappa_{\eta}^{m_u} \Phi_{\theta}(\kappa_{\eta})$ and $C_{\theta} = \xi^{1/3} \kappa_{\varphi}^{m_{\theta}} \Phi_{\Phi}(\kappa_{\varphi})$ are obtained by re-plotting Figure 5, where the choice for $m_u$ (or $m_{\theta}$) and $m_{\theta}$ presents the velocity and temperature scaling ranges as the widest (horizontal) “plateaus”. For $m_u$, $m_{\theta}$=1.3, the plateaus are at $C_u \sim 0.04$ and $C_{\theta} \sim 0.12$. For $m_u$=1.5, the plateau is at $C_{\theta} \sim 0.5$. Figure 7 shows that, with stretching, the velocity and temperature scaling ranges are not significantly affected by the different grids.

Since Lavoie et al. (2007) have shown that Rd44w produces the more isotropic turbulence, and isotropy in grid turbulence improves with increasing $Re_\lambda$, this grid flow is run at higher flow speeds and measurements are obtained for a fixed streamwise distance ($tU_* M \approx 70$). The Rd44w results in Figure 8(a, b) show that the inertial range and the scaling exponents ($m_u$, $m_{\theta}$) are both increasing functions of $Re_\lambda$. They are consistent with the measurements obtained over the full length of the test section (for the three grids), where smaller changes in $Re_\lambda$ and in the scaling exponents are indicated by the horizontal and vertical error bars in Figure 8(b). Comparison of the results in Figure 8(b, c) with those of Danaila & Antonia (2009) shows the effect of stretching is to increase the scaling exponents.

To compare between $m_u$ and $m_{\theta}$ with the effect of (higher) $Re_\lambda$, the measurements in Figure 8(b, c) are plotted with those of an active grid flow in Figure 9. The active grid data is used here to bridge between the scaling exponents of low- and high-$Re_\lambda$ turbulence. The trends in Figure 9 show $m_u < m_{\theta} < 5/3$, and that both $m_u$ and $m_{\theta}$ depend on Reynolds number. Mydlarski & Warhaft (1996) have shown that the Reynolds-number dependence correlates well with a “$-3/2$” power law (i.e. $m_u \propto Re_\lambda^{-3/2}$), and their results have indicated that a $5/3$ scaling range does not occur until $Re_\lambda \sim 10^4$. The formulas used to describe the scaling exponents are of the form, after Mydlarski & Warhaft (1996) and Danaila & Antonia (2009):

$$m_u = 5/3 - \beta Re_\lambda^{-3/2} \quad \text{and} \quad m_{\theta} = (5/3 + m_u)/\xi,$$

where $\beta$ and $\xi$ are numerical coefficients which depend on the initial/boundary conditions.
Figure 8. (a) Spectra for the velocity and temperature for stretched (Rd44w) grid flow at increasing $Re_{\kappa}$ at $t\bar{U}/M \approx 70$. The scaling exponents $m_u$ and $m_\theta$ as functions of $Re_{\kappa}$ for grid turbulence (b) with and (c) without stretching. The data with error bars are for velocity (Sq35 ■; Rd35 ●; Rd44w ♦) and temperature (Sq35 □; Rd35 ○; Rd44w △) taken over the length of the duct at $Re_M \approx 10,400$. The Rd44w data ($u$ ♦; $\theta$ △) with no error bars are for $t\bar{U}/M \approx 70$ with increasing flow speed. For the Sq35 data of Danaila & Antonia (2009) ($m_u \times; m_\theta +$), the grid flow is not stretched by a contraction. The least-square curves of best fit for $m_u$ and $m_\theta$ are shown as solid lines. The dashed lines for $m_\theta$ are determined by Equation 2.

Figure 9. Comparison of the scaling exponents. The curves [A], [B], [C] and [D] are taken from Figure 8. The data of Mydlarski & Warhaft (1996, 1998) ($m_u \blacktriangle; m_\theta \blacktriangledown$) are obtained by using an active grid, and the least-square curve fits are labelled [E] and [F]. All dashed lines for $m_\theta$ are determined by Equation 2.
Table 2. Summary of curve fits for the scaling exponents in Figure 9; $\sigma_r$ is the r.m.s. difference between the data and the best fit; $\sigma_r|\xi=2$ is the r.m.s. difference between the data and Equation 2.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Measurements from Figure 8(b)</th>
<th>Danaila &amp; Antonia (2009)</th>
<th>Mydlarski &amp; Warhaft (1996,98)</th>
</tr>
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<tr>
<td>Grid</td>
<td>Rd44w</td>
<td>Sq35</td>
<td>An “active” grid</td>
</tr>
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<td>Stretching</td>
<td>1.36:1 contraction</td>
<td>~30 to 35</td>
<td>~30 to $10^2$</td>
</tr>
<tr>
<td>$Re_\lambda$</td>
<td>~30 to 55</td>
<td>~30 to $10^2$</td>
<td>~$10^2$ to $10^3$</td>
</tr>
<tr>
<td>$m_u = 5\beta - \beta Re_\lambda^{-2/3}$</td>
<td>[A]</td>
<td>[C]</td>
<td>[E]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>4.12</td>
<td>5.77</td>
<td>5.25</td>
</tr>
<tr>
<td>$\sigma_r(%)$</td>
<td>1.3</td>
<td>3.0</td>
<td>1.3</td>
</tr>
<tr>
<td>$m_\theta = (5\beta + m_u)/\xi$</td>
<td>[B]</td>
<td>[D]</td>
<td>[F]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.98</td>
<td>2.02</td>
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<tr>
<td>$\sigma_r(%)$</td>
<td>1.3</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_r</td>
<td>\xi=2(%)$</td>
<td>1.6</td>
<td>2.4</td>
</tr>
</tbody>
</table>

the flow. With least-square curves of best fit, the r.m.s. difference between Equations 7 and the data in Figure 9 is no more than 3%. The results, summarised in Table 2, show that the ratio $(5\beta + m_u)/\xi$ is approximately 2 (i.e. $1.98 \leq \xi \leq 2.03$) for Reynolds number in the range $10 < Re_\lambda < 10^3$. If Equation 2 is used, where $\xi = 2$, the r.m.s. difference between the curve fit and the data remains at no more than 3% (Table 2).

5. Conclusions

This paper reports the effect of stretching on the dissipation rates and spectra of velocity and passive-temperature variances in decaying grid turbulence. Stretching of the grid flow is produced by a 1.36:1 contraction. Measurements of velocity and temperature are obtained by using simultaneous hot- and cold-wires.

For three different grid flows, the dissipation rates are represented by a power-law decay, and that the decay exponents ($m - 1$ for the velocity variance; $n - 1$ for the temperature variance) depend on the geometry of the grid. The measurements are in the range $-1.31 \leq m \leq -1.12$ and $-1.45 \leq n \leq -1.34$, where the higher limits of $m$ and $n$ are obtained with the grid of round bars (Rd44w). If turbulence is perfectly isotropic, Kármán & Howarth (1938) predict a decay exponent of “$-1$”. Measurements and review data show that the effect of stretching lowers the anisotropy of grid turbulence, reduces the effect of grid geometry on the decay exponents ($m$ and $n$), and tends to bring the dissipation-amplitude ratio $(\langle \nabla \Delta T^2 \rangle / (\overline{\langle T^2 \rangle}^2))$ closer to 1.

For weak turbulence ($Re_\lambda < 100$), the temperature spectrum has a more significant scaling range than the velocity spectrum. Stretching the grid flow makes the scaling range of the temperature spectrum slightly wider and more universal (or less dependent on grid shape). The combined effect of the contraction and increasing $Re_\lambda$ is to further widen the scaling range and bring the scaling range exponents ($m_u$ for the velocity spectrum; $m_\theta$ for the temperature spectrum) closer to the isotropic value of “$5\beta/3$” predicted by Kolmogorov (1941), Obukhov (1949) and Corrsin (1951). With or without stretching, the scaling exponents follow the trend $m_u < m_\theta < 5\beta/3$ and depend on $Re_\lambda$. The $Re_\lambda$-dependence is well represented by the “$-23$” power law of Mydlarski & Warhaft (1996). With least-square curves of best fit, $m_\theta$ is about a factor of 2 larger than $5\beta/3 + m_u$ and is consistent with the prediction by Danaila & Antonia (2009).
Acknowledgments

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References


