Finite Element Algorithms for Dynamic Analysis of Geotechnical Problems

by

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(signed) ________________________
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Abstract

The objective of this study is to document the development of a computational procedure for the analysis of coupled geotechnical problems involving finite deformation, inertia effects and changing boundary conditions. The procedure involves new finite element (FE) algorithms that were formulated and implemented into SNAC—a FE code developed by the geomechanics group at the University of Newcastle, Australia. The numerical scheme was then utilised to analyse some important offshore geotechnical problems.

The first development concerns the implementation of the governing equations of two-phase saturated porous media in a mixed form, allowing predictions of solid displacement, pore fluid pressure and Darcy velocity. The generalised-$\alpha$ method was chosen to integrate the governing equations in the time domain. The formulation was extended to consider geometrical nonlinearity within the framework of the Arbitrary Lagrangian–Eulerian approach. Suitable absorbing boundary conditions were also incorporated to model the radiation of bulk waves towards infinity at the truncated FE mesh boundaries. Some closed-form solutions were also developed, which are suitable to verify the implementation of dynamic consolidation algorithms.

The second development involves the formulation and implementation of a high-order contact algorithm for solid–fluid mixtures accounting for large deformations and inertia effects. The contact algorithm is based on a mortar segment-to-segment approach formulated for cases of frictionless and frictional interfaces. The node-to-segment approach was also employed to compare and highlight the merits of the mortar method when dealing with dynamic coupled problems.

The computational procedure was evaluated by modelling some numerical exercises and comparing the predicted results with alternative numerical and analytical solutions where possible.

In the last part of the thesis, the computational framework was employed to successfully model the problems of dynamically penetrating anchors and offshore pipeline-seabed
interactions. The analysis of dynamically penetrating anchors comprises the simulation of the penetration process and consolidation of the soil surrounding the penetrometer. The analysis of the pipeline-seabed interaction involves the simulation of the laying process and the large-amplitude lateral motion of the pipe.
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Preface

The research work presented in this thesis was conducted in the Department of Civil, Surveying and Environmental Engineering at the University of Newcastle from July 2010 to August 2014. This work was performed under the supervision of Dr. Majid Nazem, Laureate Prof. Scott Sloan and Prof. John Carter.

The author claims originality for the entire work described in this thesis, except the information or ideas derived from the many references and sources which have been acknowledged in the text. In particular, originality of the following works is claimed:

Chapter 2

i. The field equations for two-phase porous media were derived in light of the mixture theory extended by the concept of a volume fraction. Although these equations may have been applied in earlier studies, the equivalent arrangement introduced in the derivation of the equation system facilitates the description of frictional contact in terms of the effective normal stress component on the contact interface.

ii. A numerical solution of the governing differential equations for the dynamics of saturated soils was obtained by the finite element method. A U-P-V formulation was selected to describe both incompressible and compressible fluids, in which the resulting mixed formulation predicted all field variables, including solid displacement U, pore-fluid pressure P and the Darcy velocity of the pore fluid V. This dynamic consolidation scheme was implemented by the author into the existing in-house finite element program, SNAC. The implemented scheme provided a rigorous solution to the governing differential equations considering the convective terms of the fluid acceleration.

iii. A simplified solution was also outlined in the form of the U-P approximation, which ignores the acceleration of the fluid component. This scheme was also implemented into SNAC by the author.
iv. The ALE operator split technique and the mesh refinement strategy presented by Nazem et al. (2009) was incorporated in this thesis to consider the effects of finite deformations and to avoid possible mesh distortions. Application of the ALE scheme within the dynamic consolidation framework is specifically claimed to be original.

v. A literature review was presented for some of the available boundary conditions for solving wave-propagation problems in an unbounded domain.

vi. The cone boundary of Kellezi (2000) was adopted and implemented in the U-P-V consolidation algorithm.

vii. Closed-form solutions were developed in collaboration with others (Carter et al. 2015) for some one-dimensional problems. These solutions were useful for validating FE codes for the dynamic consolidation of soil.

Chapter 3

A new contact algorithm based on the mortar method was formulated and implemented for solid-fluid mixtures in the spatial frame that can accommodate inertia effects together with finite deformation and contact sliding. Both frictionless and frictional contact formulations were addressed for two different forms of the dynamic consolidation formulations, including U-P-V and U-P schemes.

Chapter 4

A number of validation exercises were presented to evaluate the performance of the developed numerical scheme. These results are claimed as original.

Chapter 5

i. A brief literature review of the available computational methods and available model tests on Dynamically Penetrating Anchors (DPAs) was presented.

ii. The numerical scheme developed in this thesis was then employed to conduct coupled analysis of DPAs. These results are claimed as original.

Chapter 6

The computational scheme was utilised to analyse a few pipeline-seabed interaction problems. These results are claimed as original.
Chapter 7
The conclusions and recommendations for future work.

The candidate used the existing node-to-segment (NTS) contact algorithm in SNAC to analyse some problems and compare the results with the mortar contact algorithm. However, the modification of the NTS scheme and application of the method for dynamic coupled consolidation analyses is claimed to be original.

During the term of the candidature, a number of papers and reports were published and some awards were granted. These are listed below:


*Awards:*

**Jun 2014**  
Australian Geomecanics Society (AGS) NSW research award.

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**Sep 2013**  
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