Investigation of the Mechanics of Funnel Flow in Relation to
Draw-down and Loads on Buried Structures in Stockpiles

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(M.Eng, B.Eng)

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for the award of the degree of

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Discipline of Mechanical Engineering
The University of Newcastle
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Date of Submission: April 2014
Declaration

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institute and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

(Signed) ____________________________

Yanyan He

The University of Newcastle

May 2014
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>b</td>
<td>Width of the column cross-section</td>
<td>[m]</td>
</tr>
<tr>
<td>b_1</td>
<td>Exponential-fitted constants for bulk density</td>
<td>[-]</td>
</tr>
<tr>
<td>B</td>
<td>Diameter of silo opening</td>
<td>[m]</td>
</tr>
<tr>
<td>c_1</td>
<td>Exponential-fitted constants for the effective angle of internal friction</td>
<td>[-]</td>
</tr>
<tr>
<td>C_A</td>
<td>Coefficient defining build-up of bulk solid on column surfaces</td>
<td>[-]</td>
</tr>
<tr>
<td>C_S</td>
<td>Shear friction coefficient defining lateral shear force on the side walls</td>
<td>[-]</td>
</tr>
<tr>
<td>d</td>
<td>Length of the column cross-section</td>
<td>[m]</td>
</tr>
<tr>
<td>d_1</td>
<td>Exponential-fitted constants for the wall friction angle</td>
<td>[-]</td>
</tr>
<tr>
<td>D</td>
<td>Depth of the buried column</td>
<td>[m]</td>
</tr>
<tr>
<td>D_f</td>
<td>Critical rathole diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>D_R</td>
<td>Diagonal of the rectangular stockpile outlet/reclaim hopper transition</td>
<td>[m]</td>
</tr>
<tr>
<td></td>
<td>in Equations (3-12)</td>
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<tr>
<td>F_c</td>
<td>Total vertical compressive shear force on all column faces</td>
<td>[N]</td>
</tr>
<tr>
<td>F_f</td>
<td>Normal force acting on rear face of column</td>
<td>[N]</td>
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<tr>
<td>F_s1, F_s2</td>
<td>Traction force due to shear on column left and right faces</td>
<td>[N]</td>
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<tr>
<td>F_l</td>
<td>Total lateral force on column faces</td>
<td>[N]</td>
</tr>
<tr>
<td>F_u</td>
<td>Normal force acting on front or upper face of column</td>
<td>[N]</td>
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<tr>
<td>FF</td>
<td>Flow function of a bulk material: ( \sigma_c = FF(\sigma_i) )</td>
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<tr>
<td>h_D</td>
<td>Draw-down head of solids</td>
<td>[m]</td>
</tr>
<tr>
<td>H</td>
<td>Filling height</td>
<td>[m]</td>
</tr>
<tr>
<td>j</td>
<td>Factor defined by Equation (4-2)</td>
<td>[-]</td>
</tr>
<tr>
<td>k_j</td>
<td>Ratio of horizontal to vertical pressure in the bin, normally assumed to be 0.4</td>
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</tr>
<tr>
<td>K_c</td>
<td>Pressure ratio at column surface</td>
<td>[-]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
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<tr>
<td>Ks</td>
<td>Pressure ratio at bulk solids failure surface</td>
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<tr>
<td>Kca, Ksa</td>
<td>Pressure ratios for active stress state</td>
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<tr>
<td>Kcp, Ksp</td>
<td>Pressure ratios for passive stress state</td>
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</tr>
<tr>
<td>L</td>
<td>Length of the stockpile outlet/reclaim hopper transition</td>
<td>[m]</td>
</tr>
<tr>
<td>pnc</td>
<td>Normal pressure on column face, in Figures 4.3, 4.4 and 4.5</td>
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</tr>
<tr>
<td>pns</td>
<td>Normal pressure on the inclined face of the wedge shape</td>
<td>[Pa]</td>
</tr>
<tr>
<td>pns1, pns2</td>
<td>Normal pressure on left and right faces of column</td>
<td>[Pa]</td>
</tr>
<tr>
<td>pnl</td>
<td>Normal pressure on column rear face</td>
<td>[Pa]</td>
</tr>
<tr>
<td>pnu</td>
<td>Normal pressure on column front or upper face</td>
<td>[Pa]</td>
</tr>
<tr>
<td>pv</td>
<td>Vertical pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>py</td>
<td>Pressure in y direction</td>
<td>[Pa]</td>
</tr>
<tr>
<td>pyo</td>
<td>Surcharge pressure in y direction</td>
<td>[Pa]</td>
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<tr>
<td>r</td>
<td>Critical rathole radius</td>
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<tr>
<td>R</td>
<td>Characteristic radius</td>
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<td>W</td>
<td>Width of the stockpile outlet/reclaim hopper transition</td>
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<tr>
<td>y</td>
<td>Buried depth in the direction of centre line of the wedge</td>
<td>[m]</td>
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<tr>
<td>yh</td>
<td>Dimension defining the wedge height</td>
<td>[m]</td>
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<tr>
<td>z</td>
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<td>[m]</td>
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<td>Buried depths for front, rear, left and right faces of column</td>
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<tr>
<td>αc, αs</td>
<td>Half angles of wedge</td>
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<tr>
<td>β</td>
<td>Rathole sloughing angle close to the flow channel: β=45°+ϕ/2</td>
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<tr>
<td>γ</td>
<td>Bulk-specific weight of bulk solid</td>
<td>[N/m³]</td>
</tr>
<tr>
<td>γy</td>
<td>Bulk-specific weight in y direction</td>
<td>[N/m³]</td>
</tr>
<tr>
<td>δ</td>
<td>Effective angle of internal friction</td>
<td>[deg]</td>
</tr>
<tr>
<td>δD</td>
<td>Effective angle of internal friction at the buried column base</td>
<td>[deg]</td>
</tr>
<tr>
<td>εc</td>
<td>Rathole expansion angle on conical wall/end wall of a rectangular outlet</td>
<td>[deg]</td>
</tr>
<tr>
<td>εp</td>
<td>Rathole expansion angle on side wall of a rectangular outlet</td>
<td>[deg]</td>
</tr>
<tr>
<td>η</td>
<td>Angle defining the direction of major principal stress</td>
<td>[deg]</td>
</tr>
</tbody>
</table>

xvi
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_R$</td>
<td>Angle of repose</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Friction coefficient for bulk material in contact with column surface</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Bulk density</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Major consolidation stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_{1D}$</td>
<td>Major consolidation stress at the buried column base</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Minor consolidation stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_{av}$</td>
<td>Average of the major and minor consolidation stresses</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Rathole unconfined yield stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_{c\theta}$</td>
<td>Rathole unconfined yield stress in the circumferential direction</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>Rathole hoop stress in the circumferential direction</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>Shear or traction stress at column surface</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>Shear or traction stress at column rear surface</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Shear stress at failure surface of wedge</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_u$</td>
<td>Shear or traction stress on front or upper column surface</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>Static angle of internal friction</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Wall friction angle</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\phi_{ws}$</td>
<td>Friction angle due to internal shear at failure surface of the wedge or at the flow channel</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Angle for the wedge shape as defined in Figures 4.3, 4.4, 4.6 and 5.3</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Rathole sloughing angle from the free surface: $\phi_t &lt; \psi \leq \delta$</td>
<td>[deg]</td>
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Abstract

Silos and gravity reclaim stockpiles are widely used to store and handle bulk materials in industry. The rathole mechanism for funnel flow in silos or gravity reclaim stockpiles, and the loads on support structures buried in stockpiles, play a decisive role in determining the dimensions of silos and their support structures, and are closely related to the stress conditions developed within the bulk solid handled. Therefore, the reliability and efficiency of these storage facilities can be enhanced by employing optimal designs based on a good understanding of the stress conditions. However, thus far, the stress conditions in silos or stockpiles during storage and flow have not been satisfactorily examined in the field of bulk solids handling. Therefore, the aim of this thesis is to investigate these stress conditions.

In order to investigate the performance of funnel flow, laser devices were employed to depict rathole profiles occurring in silos and gravity reclaim stockpiles. Different silo geometrical configurations and different bulk materials were applied in the experimental investigation. In addition, the experimental outcomes obtained by a probe-profile gauge in 1987 were also used in the comparison regarding rathole geometries and draw-down heads between all the experimental results and the theoretical predictions from Roberts and Jenike’s theories. The results reveal that Roberts’ hoop stress theory is capable of predicting the rathole geometry, especially for funnel flow occurring in gravity reclaim stockpiles.

The work presented also investigates the loads exerted on support structures, such as a column buried in a stockpile. A laboratory scale test rig was established with two different column setups employed to determine the loads acting on the columns. Tekscan tactile pressure sensors and load cells were used to measure the normal pressure distributions on the column faces and total vertical and lateral forces exerted on the column, respectively. Discrete Element Method (DEM) simulation work corresponding to the experiments was also undertaken to explore the load conditions. Although deficiencies exist in the current cohesive contact model in the DEM software package's
applied, the normal loads and shear loads were obtained from both the experimental and simulation work, and the comparison between them showed reasonable agreement. This verifies Roberts’s load analysis theory in terms of the loads on buried structures.
Chapter 1: Introduction

1.1 General Review of Bulk Solids Handling

There is currently an increase in the application of technology related to bulk solids handling in a wide variety of industries around the world. The field of bulk solids handling includes the storage, flow, transport and processing of particulate materials. The costs of handling operations are substantial, and often constitute a high percentage of overall production costs. Therefore, to achieve maximum reliability and efficiency, as well as maximum economy of handling operations, all bulk handling and storage facilities should be well designed and properly operated.

Bulk solids are commonly processed through storage, flow and transportation processes in nearly all industries, from powder coating to food; from nanoscale powders and pharmaceutical substances to products such as cement, coal, and ore; from dry materials such as fly ash to moist bulk solids such as filter cake and clay [1]. Silos and stockpiles are widely used to store these bulk materials. A bulk material is filled continuously or discontinuously into a silo or onto the ground to form a pile, and discharged later at a predetermined time in the desired quantities. Apart from the storage of bulk solids, the most important function of silos and stockpiles is to ensure the stored bulk materials can flow out continuously with the maximum reclaim efficiency on the premise of structural reliability and economy. However, problems such as arching and ratholing often occur in many practical applications, which can lead to irregular discharge or complete blockage.

During the past 50 years, a strong foundation has been built for the professional discipline of bulk solids handling in regard to the geometrical designs and theoretical research on these storage facilities based on pioneering research work [2-8] and follow-on work [9-16]. Nevertheless, the heterogeneous nature of bulk solids—due to great variations in particle shape, particle size and the manner of packing—can create
uncertainty in the stress conditions developed in stored bulk solids. This generates difficulties in the design of silos and stockpiles, with poor designs resulting in the occurrence of severe problems such as flow obstructions, segregation, unsteady flow and even construction failures.

It is difficult to achieve trouble-free operation designs in practical applications because the storage and handling of bulk solids are still poorly understood in terms of how to analyse and design to avoid problems, such as caking, segregation, rathole analysis [17] and loads analysis. Therefore, it is essential to explore the principles and mechanisms occurring in silos and stockpiles where bulk solids are being handled, by involving both theoretical and practical knowledge in order to provide an improved guide to reliable industrial designs.

1.2 Bulk Solids’ Properties

A proper design of a silo or stockpile requires the flowability data of the handling bulk material because the material properties can vary significantly among different bulk solids, which makes the silo and stockpile design highly material dependent. Common properties of bulk solids include moisture content, angle of repose, bulk density, particle size distribution (PSD), flow function (FF), time FF, effective angle of internal friction, static angle of internal friction and wall friction angle. Bulk solids’ properties are measured with laboratory equipment, such as shear testers, to measure the effective and static angle of internal friction, wall friction angle, solids density and permeability of the solids. These parameters of the strength and flowability of bulk solids can help predict the flow behaviour of the handling material. As a result, bulk solids’ properties provide a dominant role in the determination of silo geometry, such as outlet dimensions and hopper half angle, and stockpile outlet dimensions. However, it is difficult to obtain bulk solids’ properties that are representative of all the handling material, due to the heterogeneity of bulk solids. This is the major obstacle in the design of trouble-free operation facilities. To solve this issue, the material properties for the worst case which may occur in practice are usually adopted in industrial applications to achieve the reliability of designs.
There are a number of testers available to measure bulk solids’ properties, such as the Jenike shear cell, annular shear cells, triaxial tester, true biaxial shear tester, Johanson indicisers, torsional cell, uniaxial tester, oedometer, lambdameter, Jenike and Johanson quality control tester, Hosokawa tester and others [18]. These testers are commonly used in laboratories to determine bulk solids’ properties for both academic research and consulting work, and produce more reliable results than those that follow from empirical tests or the application of so-called ‘simple testers’. However, most of these testers are suitable only for specific scientific investigations, and the Jenike shear cell has proven to be the most versatile shear tester for design applications and has become an industrial standard because the shear plane is defined and it is amenable to time-consolidation tests, despite the disadvantage of limited travel for shear [19,20].

1.3 Mass Flow in Silos

Since Jenike [3,7] published his theory about the flow behaviour of bulk solids in silos in the early 1960s, it has been acknowledged that it greatly helps to understand the principles of the flow of bulk solids. Consequently, a comparatively reliable method has been developed for the geometric design of silos. In Jenike’s major publications [3,4,7], there are two main flow regimes: mass flow and funnel flow. In mass flow, as shown in Figure 1.1, all the bulk solid stored in the silo is in motion, and the significant characteristic is that flow of bulk solids within a silo takes place along the walls of both the cylindrical part and the hopper, which means that it guarantees ‘total live capacity’ performance. It is a ‘first in, first out’ flow pattern, as indicated in Jenike’s studies [3,7].
A series of design charts of the two flow types was created to give hopper geometrical and bulk material properties, but this is not always satisfactory when used to predict the flow regime. Jenike’s following modified theory [21], radial stress field theory—based on the quasi-static equilibrium conditions—replaced the previous modified Tresca yield pyramids theory with conical yield surfaces. It provides a better prediction of the flow regime; however, Kruyt [22] stated that Jenike’s new theory introduces a much higher flow factor, and thereby greatly overestimates the critical outlet dimension. In terms of mass-flow criteria, Drescher’s [23] criteria defines the transition state from mass flow to funnel flow, and relates the mass-flow boundary to the orientation of the shear bands observed in mass flow. Nevertheless, clear criteria for mass flow have not yet been developed.

Arching is the major problem occurring in the operation of mass-flow silos. Based on the fundamental work provided in the last 50 years about bulk solids handling, a great deal of research work [9,11,24-26] has been conducted to investigate arching behaviour in mass-flow hoppers. This work has been based on different hypotheses and predictions for arching model and equilibrium, such as the circular arc model proposed by Enstad [11], parabolic arc model by Walker [9], and Matchett’s two-dimensional arch and rotationally symmetrical system [25,26] to predict arch shape and arch stresses. Further investigation and validation are needed to achieve good agreement, while technological advances enable improvement of the methods of measurement.
Berry et al. [27] measured the shapes of arches under a range of hopper geometry by using a scanning laser-ranging device, while also investigating the critical slot width. They concluded that the approach of the slice of stacked arches—or so-called ‘onion ring’—does not apply and Jenike’s design method is over-designed. For silo design, in addition to the arching phenomenon causing flow blockage in the operation, it is necessary to know the loads on silo walls to ensure the silo construction is safe and reliable. Jenike performed much of the research work on bin loads [3,7,28].

Based on studies mainly from Jenike, Roberts [16,29] considered surcharge pressures and conducted an intensive study on the influence of surcharge loads by comparing the pressure ratios under passive state of stress equilibrium condition for the circular arch method and horizontal slice method. In addition, Han et al. [30] adopted a plane slice method for the bin section and spherical slice method for the hopper section to investigate the loads on bin walls. They observed that the wall stress and discharge rates during lowering of the upper surface were affected only at the beginning of flow, if compared with the steady-state solution. However, these results were highly dependent on the assumptions of K value (Janssen’s ratio of horizontal to vertical pressures) for both the bin and hopper sections, and no agreement has been achieved on this subject thus far.

1.4 Funnel Flow in Silos and Stockpiles
1.4.1 Formation and Concepts of Rathole

Apart from mass flow, another principal flow mode is funnel flow. When the hopper section is too flat or the friction of the hopper wall is too high, funnel flow occurs. The bulk solid moves out through a vertical flow channel that forms within the material just above the opening, where a stable piping is developed—called a ‘rathole’. This flow mode is schematically shown in Figure 1.2. The flow continues until a drop height of \( h_D \) is reached in the bin—called ‘draw-down head’. Therefore, there are dead zones that cannot be reclaimed in funnel-flow bins, which is the major disadvantage of funnel flow.
The rathole is dominated by the outlet dimension $B$, and expands slightly upwards to the level of draw-down head. As a result, the outlet dimension $B$ significantly affects the discharge rate, and there should be a critical opening dimension $B_{R}$ at which the rathole is on the verge of collapsing [14]. However, funnel flow has the advantage of providing wear protection of the bin walls, since the material flows along the flow channel instead of the bin walls. Thus, funnel-flow bins are usually used for storing hard, abrasive, lumpy solids.

Another flow mode is expanded flow, as shown in Figure 1.3. This flow mode incorporates the wall protection of funnel flow with the reliable discharge of mass flow. In a broad definition, the stockpile is a special case for expanded flow with no constraining walls around the funnel-flow section. Its height can vary between 10 to 40 metres; thus, it is widely used in the mining and mineral processing industries to store large quantities of materials that are generally not affected by environmental conditions.
Most of the fundamentals of bulk solids have been established by professional societies in the last century, since the German engineer, HA Janssen, laid the foundations for understanding the behaviour of particulate solids in relation to the loads on silo walls under storage conditions. Great improvements have been made based on observation, experiment and theoretical analysis, while the technological development in computing science enables the performance of numerical simulation research focusing on bulk solids behaviour. Discrete Element Method (DEM) is a major approach adopted in the numerical investigation of bulk solids handling. However, despite all that has been accomplished, the field of funnel flow in silos and stockpiles is still ripe for innovation. A number of common problems occur when information about the handling of bulk solids is limited, and unusual bulk solids that do not follow traditional methods must also be handled. No agreement from academics or industry has been reached regarding the best ways to address the challenges faced in this field, and there is a desperate need for better solutions.

1.4.2.1 Loads on Bin Walls

Generally, the pressures exerted by bin walls on solids determine the flow of bulk solids from storage bins. Jenike and his co-workers considered that flow pressures are
larger than the initial Janssen pressures [33] in tall funnel-flow bins; thus, flow pressures were employed for design purposes. However, in Johanson’s study [34], the flow factors for rathole during initial filling without any withdrawal could be several times greater than those previously given by Jenike [3,4,7]. This means that the initial pressures may cause ratholes previously not predicted, based on steady flow pressures. This is in good agreement with the theory of Carson and Jenkyn [35]. The research work of Johanson [34] recommended that initial filling loads on funnel-flow bins can be determined based on Janssen’s formula; however, Janssen’s equations were adapted to suit converging channels by Carson et al. [35] to calculate the loads on conical hopper section.

For the flow case, both Carson et al. [35] and Jenike et al. [36] agreed that there is a peak pressure occurring at the level of the effective transition if the flow channel intersects the cylinder wall, and a Janssen’s stress field can be assumed above the effective transition. Some research work on the pressures in the flow channel was undertaken by Carson et al. [35], who suggested that the pressures could be treated similarly to in a mass-flow hopper, by using the flow channel angle as the hopper angle and the internal friction angle of particles as the wall friction angle. However, Carson et al.’s research did not explore how this pressure distribution is transmitted to the vertical wall of the cylinder. All these studies reveal that there is still no comprehensive theory about funnel-flow bin loads.

1.4.2.2 Stability of Ratholes

It is assumed that stable ratholes can be formed in funnel-flow silos or gravity reclaim stockpiles when the strength of the stored bulk solid exceeds the circumferential stress in the surface of the rathole formed. As for the rathole stability prediction, an unstable rathole equation and piping flow factors were proposed in Jenike’s original theory described in Bulletin No. 108 and Bulletin No. 123 [3,7] based on the assumption of a semi-infinite plane of uniformly compacted bulk solids, with a uniform and constant unconfined yield strength. Later, Jenike’s [7] corrected theory revised the oversight that the pressure causing the unconfined yield strength is proportional to the bin diameter, instead of the rathole diameter; however, this still leads to impractical solutions.
Johanson [37] assessed these two theories and presented a new rathole analysis using the initial solids contact stress associated with the initial bin filling for determining the unconfined yield strength. This obtained more reasonable critical rathole diameters for flow on flat conical hopper walls. Meanwhile, Hill et al. [38] also re-examined the classical rathole theory of Jenike in both analytical and numerical ways. They believed that Jenike’s ‘stable rathole equation’ does not accurately reflect actual material behaviour. Later, Matchett [39] proposed a two-dimensional model, and developed a rotated, circular arc approach to stress analysis in cohesive bulk solids in a vertical, cylindrical vessel. He assumed that the spherical dome surface formed by principal stresses within the vessel makes a constant angle with the wall, normal with moving arc centre and varying element thickness. Matchett’s analysis enables deep and stable ratholes to exist, thereby allowing for the curvature of the rathole—in contrast to the conventional stability criteria, which only allows a shallow rathole to be stable. However, this model has great sensitivity to assumptions of the state of stress.

1.4.2.3 Anisotropic Behaviour of Bulk Solids

Most prior research work has been based on the assumption that powder behaviours like an isotropic continuum and hence two-dimensional stress field was assumed according to axial symmetry, which sometimes gives unsatisfactory applications. Therefore, several related measurements using a Jenike shear tester and biaxial tester were conducted by Saraber et al. [40] to compare the results. The results showed that the anisotropy considerably influenced the yield behaviour of a cohesive solid when the directions of the principal stresses at steady state and incipient failure were different.

In addition, determining the critical outlet width for funnel-flow bins according to Jenike’s method can be significantly affected by anisotropic character. This conclusion was verified by Ittershagen et al. [41] and Schwedes et al. [42]. Schwedes et al. [42] compared the existing two design criteria for rathole stability from Jenike [3] and Johanson [34] by using a silo centrifuge, and an anisotropy-factor was suggested to be incorporated for Johanson’s criterion. To further investigate the anisotropic character of
bulk solids, a new Anisotropy-Tester was developed by Ittershagen et al., enabling quantitative measurement of the anisotropic behaviour. However, the anisotropic character is highly material dependent, and a better understanding of the complexity of this behaviour has not been achieved.

1.4.2.4 New Hoop Stress Theory

In funnel flow, the discharge rate is largely dependent on the outlet diameter of the silo, and, for complete discharge, the bin opening (for stockpiles, the dimension for reclaim mass-flow hopper) needs to be at least equal to the critical rathole dimension. The existing theories for funnel flow are empirical. The original Jenike solution derived from flow channel arch is based on the equivalent mass-flow analysis, in which the critical rathole dimensions were calculated from pipe flow factors, and the surcharge pressure was completely ignored. The consequence of this is that the critical rathole dimensions were underestimated, which would lead to an overestimated draw-down head and live capacity for a given funnel-flow bin outlet. This is referred to as Jenike’s original solution, or ‘Lower Bound Solution’. The work originally published in Bulletin No. 108 [3] and Bulletin No. 123 [7] was subsequently reviewed to take into account the consolidation conditions with an assumption of two-dimensional axis-symmetrical stress field, which overestimated the rathole dimension, thereby leading to underestimated draw-down and live capacity. This method is the ‘Upper Bound Solution’, which has been found to be too conservative.

With an extensive background in industrial stockpile design, Roberts [14,15,43] investigated the mechanism of rathole formation occurring in funnel-flow bins and stockpiles. He proposed a new, more realistic hoop stress theory [15,44] based on the three-dimensional stress state occurring in the rathole, and the anisotropy was considered. This theory assumes that the hoop strength of the bulk solid in the vicinity of the free surface of central flowing channel governs the rathole stability. According to the assumption of rathole expansion angles, he modified the existing two main bound solutions of critical rathole diameter: Jenike’s original Lower Bound Solution, which underestimates rathole diameters due to neglecting surcharge pressure, and the
subsequent modified Upper Bound Solution, which is too conservative, based on time FF.

A new concept, the FF for the third or circumferential direction, was proposed in Roberts’s hoop stress theory, defining the unconfined yield stress in the circumferential direction as a function of the major consolidation stress. Roberts also showed that the unconfined yield strength in the circumferential direction can be estimated by the mean stress of the major and minor principal stresses from the conventional two-dimensional direct shear test. However, this FF in circumferential direction can also be obtained using an experimental procedure by turning Jenike shear cell through 90° after normal consolidation. Comparisons from experiments show good agreement between the measured and estimated circumferential FF.

The hoop stress theory provides far better predictions of rathole geometry and stress conditions than do other existing theories, especially in predicting bin critical diameter and reclaim capacity. Despite this, further research is required to identify more precisely the equilibrium conditions defining rathole stability and geometry in order to achieve maximum reliability and efficiency in industrial applications.

1.4.2.5 Numerical Simulation

The complex constitutive behaviour of bulk solids makes accurate bulk solids flow measurements difficult; however, advances in technology enable computer-aided simulation. Consequently, there has been an increase in the use of computer modelling to simulate granular flow. To date, there are two main types of numerical algorithms for modelling: the DEM and Finite Element Analysis (FEA). DEM is based on individual particles, which is different from the traditional FEA. While much earlier research on the application of discrete particle models was undertaken to explore the behaviour of bulk materials, the DEM is usually attributed to Cundall [45], who applied it to problems in rock mechanics in 1971. This method was named by Cundall and Strack in 1979 publications [46,47].
Many simulations and relative experiments [48-51] have already shown the capability of DEM to model the dynamic behaviour of granular flow systems. For example, Ketterhagen et al. [49] achieved strong agreement between their simulation results for flow mode prediction in silos and the design charts originally developed by Jenike. In addition, research performed by Faqih et al. [50] proved that DEM algorithms are device independent. Further, the method coupling DEM with FEM has been applied to investigate macroscopic dynamic granular flow performance in conjunction with microscopic stress conditions [52-54]. However, these simulations are highly dependent on the assumptions of the particle and contact properties. Therefore, a contact model exactly reflecting reality is a prerequisite to successfully apply the DEM algorithm. Nevertheless, this is a major obstacle that requires a better solution.

1.5 Loads on Support Structures Buried in Stockpiles

Large bulk storage facilities—such as gravity reclaim stockpiles, bulk sheds and bins—are commonly used in industrial applications to store and handle bulk solids in large quantities. These storage facilities often incorporate structural members, such as trestle legs to support load-out conveyors of open stockpiles, or columns to support roof structures in large bulk storage sheds. These support structures often stand on the ground and are partly buried in the stored bulk materials. As a result, they are subject to remarkable loads by the surrounding bulk solids. These loads exerted on the support structures are quite complex and can vary significantly during the filling and emptying of the bulk storage facility, making them difficult to predict. Since Janssen began investigating bulk solids handling more than 100 years ago, great effort has been invested on the subject of wall loads for mass-flow and funnel-flow bins by many researchers, such as Jenike et al. [28,32,36] and Walters [10]. However, the subject of loads on buried support structures has received little or no attention. For this reason, Roberts’s theoretical approach to predict the loads exerted on buried structures [55] is considered an important contribution.

Roberts investigated the loads exerted on support structural elements, such as trestle legs and support columns buried in stockpiles. He proposed a load analysis model based on
the classical continuum approach to predict the loads on structural elements, and obtained a correlation between the type of stress states developed in the handled bulk materials in the immediate region of the buried structures and the load distribution on these support elements. The stress states developed in bulk materials can vary significantly and are closely related to parameters [55] such as the manner of loading and reclaim, loading and unloading history, length of undisturbed storage time in the stockpile, rigidity of the stockpile floor, type of bulk materials, and variations in the flow properties of the handling material. Some preliminary experimental studies were undertaken by Roberts, such as an industrial case study concerning the loads on vertical support columns in a large bulk fertiliser shed, and relevant tests on a bench-top scale experimental model.

The subsequent numerical simulation work undertaken by Katterfeld [56] involved the application of the DEM to calculate the pressure distributions on the faces of buried structures. The findings of all previous studies show reasonable agreement with the theoretical predictions. However, additional research is required to further validate and improve Roberts’s design equations to determine the loads on support columns. Therefore, the current research built a laboratory scale test rig to measure the pressure distributions on support columns partly buried in gravity reclaim stockpiles, and a numerical simulation study corresponding to the experimental setup was performed using DEM.

Based on the pioneering work, some experimental and simulation research was carried out to further analyse the stress states developed in the stored bulk material surrounding the buried structures. The stress state can vary significantly during the handling processes of loading, storage and discharge, and can be an active stress state, passive stress state, or combination of both. Each stress state can introduce different load distributions on the buried support structure, and the dimensions of support structures in current industrial designs are often overestimated to make them safe and reliable. Therefore, an in-depth exploration to better understand these load conditions is required for the correct design of support structures in gravity reclaim stockpiles and bulk storage sheds, and this was the motivation for this study.
1.6 Existing Standards

Many structural failures occurred in silos and stockpile support structures around the world during the last two decades. Therefore, it is essential to understand the relevant standards as the first step in design. There are a number of codes of practice dealing with the subject of bin loads (BS 5061, 1974; ITBTP No. 189, 1975; ACI 313-77, 1984; BMHB, 1985; DIN 1055, 1986; Gorenc et al., 1986; JIS, 1987; AFNOR P22-630, 1992; ISO DIS 11697, 1992; EC1 Part 4, 1993; AS 3774-1996, 1996) [57]. These codes all address the problems of design pressures on silo walls under the most obvious granular solid loading conditions, such as concentric filling and concentric discharge in either funnel-flow or mass-flow mode. Among all these design codes, both ACI 313-77 [58] and AS 3774-1996 [59] use the Janssen method to predict the wall loads in funnel-flow bins, as described in the work of Jenike et al. [36]. The Australian Standard AS 3774-1996 presents a very comprehensive review of the loads acting on bin and silo walls under a full range of operating conditions that are likely to occur in practice, and is widely used in the designs for bulk solids handling storage facilities.

1.7 Scope

The goal of this thesis was to investigate the stress conditions developed in bulk materials during the handling processes of loading, storage and discharge in funnel-flow silos and gravity reclaim stockpiles. By applying experimental and numerical simulation methodologies, different studies were undertaken to further verify and improve the existing theories. The aims were specifically as follows:

- To verify and improve Roberts’s hoop stress theory on the prediction of rathole mechanism. To accomplish this, experimental work with different test materials and varied geometrical dimensions was undertaken to analyse the bin wall loads on funnel-flow bins and rathole geometries in both funnel-flow bins and gravity reclaim stockpiles. Following this, a comparison between Jenike’s theory and Roberts’s theory can be made in order to determine which one is more suitable to
define rathole stability and geometry, and then a more rigorous approach to
designs of funnel-flow silos and stockpiles can be obtained.

- To verify and improve Roberts’s theory about loads exerted on support structures
  buried in stockpiles of bulk materials. Based on Roberts’s prediction model,
  numerical simulation and experiments with a laboratory scale test rig were
  undertaken to investigate the load conditions on these support structures. Two
  different column setups as the support structures buried in stockpiles were tested:
  one is a fixed column and the other one is a laterally moveable column. Different
  materials were tested with regard to the influence of the flow properties on the
  stress states developed in the stored material surrounding the buried structures.
  Stockpile tests under different conditions of loading, storage and discharge were
  conducted, and both the measured results and numerical data were analysed to
  verify and improve Roberts’s theory.

1.8 Thesis Structure

The aim of the research project was to further explore the stress conditions developed in
the bulk materials during loading, storage and discharge, based on which, a safe, reliable
and economic method can be achieved to provide a guide to the designs of silo and
stockpiles. The study is divided into two subprojects: rathole mechanism research and
investigation of loads on support structures. These projects are described in detail in the
seven chapters of this thesis, which are presented as follows.

Chapter 1 generally reviews two subprojects of bulk solids handling: the introduction of
funnel flow in silos and gravity reclaim stockpiles, and buried support structures in
stockpiles and bulk storage sheds. It also briefly outlines the development and problems
in research and design. It introduces the notions of improving the rathole study and
filling the research gap in the buried support structures, which were both presented in
Roberts’s new theories. In addition, existing standards of design pressures on bin or silo
walls are presented. Chapter 1 also outlines the scope and detailed structure of the thesis.
Chapter 2 presents a laboratory study of the properties of the test bulk solids. It discusses the experimental work undertaken to determine the flow properties of the different bulk solids employed in all experimental investigations, and they are cohesive iron ore and free-flowing beach sand. Test procedures are described, and the FF (Flow Function) in circumferential direction FF₀ (Flow Function in the circumferential direction), examined with a Jenike shear tester is also discussed.

Chapter 3 explores the mechanism of ratholes occurring in funnel-flow silos and gravity reclaim stockpiles. Experimental work was conducted to measure rathole profiles using laser devices, and the experimental results by a probe-profile gauge obtained in 1987 are discussed. A comparison is made between the experimental data and theoretical predictions to verify and improve the existing hoop stress theory. In addition, this chapter explores the measurement of the pressure distributions on silo walls and hopper walls in order to compare Janssen’s equation with Roberts’s load analysis regarding the load on cylindrical walls, and to study the stress field established within the hopper section.

Chapter 4 discusses the loads on a fixed support column buried in stockpiles—a fixed column that was restrained with no degree of freedom. A series of stockpile tests under different conditions—such as loading, storage and discharge—is investigated. All experimental results showed reasonable agreement with the theoretical predictions in regard to the normal pressures on the column front and rear faces, while the stress state developed in the stored bulk material during all handling processes varied more than it remained constant.

Chapter 5 investigates the loads on an unfixed support column buried in stockpiles. This unfixed column was supported with the necessary degrees of freedom, which meant it could only move laterally in the direction of flow rill. Both pressure sensors and load cells were applied, and this chapter presents a comparison between the pressure distributions on column faces from pressure sensors and the forces acting on the column measured by load cells. All of these comparisons will help to verify the capability of Tekscan tactile pressure sensors in bulk materials measurement, importantly to verify
Roberts’s analysis of the vertical and lateral shear forces on the column, and improves the theory.

Chapter 6 introduces the related simulation study about the loads on support columns based on the experimental setups described in Chapters 4 and 5; however, the simulation scale was reduced due to a limitation in the quantity of particles allowed in the existing computer configuration. Both normal loads and shear loads exerted on column faces are investigated, and found to be in reasonable agreement with the related experimental results. The applicability of the simulation method for the pilot-scale model is confirmed, and some existing deficiencies are discussed.

Chapter 7 summarises the findings achieved in this investigation regarding the analysis and improvement of the theories on stress conditions developed within bulk materials during handling processes in funnel-flow silos and gravity reclaim stockpiles. This includes investigating the rathole mechanism for funnel-flow silos and gravity reclaim stockpiles, and the load conditions occurring on the buried support structures. A brief discussion of each conclusion is presented, as are some recommendations for further work that can be undertaken as an extension of this research.
Chapter 2: Properties of the Test Bulk Materials

2.1 Introduction

To design reliable devices to handle bulk solids, knowledge of the strength and flow properties of these bulk solids is important, since these property parameters can help predict the flow behaviour of handled bulk solids. A bulk solid consists of an enormous number of individual particles that encompass a broad range of shapes and particle sizes. Although, in principle, it is possible to describe the behaviour of a bulk solid by considering the particle-particle interactions—which is currently realised by DEM calculations—it is difficult to implement an experimental procedure due to the vast number of particles, irregular particle shapes and sizes, and varied adhesive forces between individual particles. Therefore, it is reasonable to regard the bulk solid as a continuum [1], and the measurement for properties can be undertaken in a representative volume element. As long as the volume element is sufficiently large with respect to the particle size, and represents the characteristic of the test bulk material, the properties of the bulk solids measured from tests can be considered accurate.

Measuring the properties data for bulk solids has been examined for the past 50 years, and a number of methods and testers are subsequently available. Among the existing testers, the Jenike shear tester—as a translational shear tester—is capable of simulating the effect of consolidation at rest for a bulk solid, has been proven to be a versatile shear tester for design applications, and has become an industrial standard [19]. However, the Jenike shear tester was developed for testing fine particles; thus, solids containing particles with particle sizes larger than 6 mm cannot be tested reliably in the standard size shear cell. According to the studies of Jenike [7] and Arnold et al. [20], fine particles—rather than large particles—in a bulk solid play an important role in the development of strength within the bulk solid. Therefore, the strength and flow properties measured by the Jenike shear tester can approximately represent the actual situations.
Bulk solids can be divided into two main categories: cohesive bulk solids and free-flowing, non-cohesive bulk solids. Cohesive bulk solids can gain bulk strength under the action of consolidating pressures and retain this strength when the consolidating pressures are removed. Non-cohesive bulk solids, such as dry sand and grains, are a special case because they gain zero unconfined yield strength, while the shear strength increases with the increase in consolidating pressures, but loses this shear strength once the pressures are released. Therefore, a bulk material with a high percentage of fine particles and high moisture content can produce high strength within the bulk solid, which can impede the flow of the bulk material. Nevertheless, cohesive bulk solids represent the majority of handling materials that occur in practice.

2.2 Flow Properties of Test Bulk Materials

The flow properties of bulk materials are determined by testing a representative sample in order to design storage bins and associated handling systems. There are many terms used to describe the properties of bulk materials. Those generally adopted when determining a bulk solid in research and design [1,3,6,7,18,28,32,36,60,61] are discussed in this chapter, including:

- angle of repose $\theta_R$
- PSD
- bulk density $\rho$ as a function of consolidation
- effective angle of internal friction $\delta$ produced by the effective yield locus
- wall friction angle defined by wall yield locus
- instantaneous yield loci defining the instantaneous FF
- time yield loci defining the time FF ($FF_t$).

All the properties listed above are obtained by carrying out a series of tests based on the test procedures described in Jenike’s approach [7]. For fine powders, flow rate predictions may be critical. For determining flow rates, it is necessary to measure solids’ density and permeability as a function of consolidation. This is beyond the scope of the investigation in this thesis because only coarse mineral solids are applied.
Two different bulk materials were employed in all experimental investigations presented in this thesis:

- cohesive iron ore from different mining sites with different moisture content and different PSDs
- free-flowing beach sand.

### 2.2.1 Moisture Content

Moisture content can greatly contribute to the cohesion character of a bulk solid, thereby influencing the strength and flow behaviour of the bulk material. In reality, the moisture content of a test material is difficult to keep constant and can vary greatly due to changes in the surrounding environment, such as the ambient temperature, air velocity and humidity, as well as the exposure surface area of the test material. Therefore, it is essential to know the moisture content of the material in each test.

In laboratory tests, moisture content is usually examined based on the mass difference divided by the original mass, with this mass difference caused by heating the moist bulk material in an oven. Before flow properties tests are undertaken, the moisture content for each test bulk solid must be specified. The moisture content for all bulk materials samples is described in Table 2.1. Since there is less than 500kg Iron Ore A in the laboratory and this cannot form a stockpile with expected buried heights, Iron Ore A is only used in the funnel flow tests.

<table>
<thead>
<tr>
<th>Test categories</th>
<th>Materials</th>
<th>Iron Ore A (%)</th>
<th>Iron Ore B (%)</th>
<th>Beach Sand (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funnel flow</td>
<td></td>
<td>2.7</td>
<td>6.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Buried support columns</td>
<td></td>
<td>6.3</td>
<td>7.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### 2.2.2 Angle of Repose

The angle of repose is the angle created by the sloping surface of the pile that is formed when a loose, unconsolidated bulk solid is deposited on a horizontal surface. This angle
can vary with the moisture content, internal friction, shape, cohesion, size distribution and compaction of the material. The fact that material can be piled in this manner is mainly due to shear Coulomb frictional properties and cohesion, and demonstrates the major difference between materials and liquids. However, the angle of repose does not provide a measure of flowability and—as indicated by Roberts [43]—its use should be restricted to determining the contours of piles. All results of angle of repose are shown in Table 2.2.

Table 2.2: Summary of Angle of Repose for All Test Materials

<table>
<thead>
<tr>
<th>Iron Ore A</th>
<th>Iron Ore B</th>
<th>Beach Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC = 2.7%</td>
<td>MC = 6.3%</td>
<td>MC = 6.3%</td>
</tr>
<tr>
<td>34.3°</td>
<td>39.5°</td>
<td>38.5°</td>
</tr>
</tbody>
</table>

2.2.3 PSD

A wide range of particle sizes usually exists in any bulk material. As indicated in Jenike’s [7] research, fine particles play an important role in the development of cohesive strength in the bulk material. Thus, it is necessary to know the percentage of fine particles. PSDs are usually determined by sifting using sieve series with different mesh apertures in order to separate particles of different sizes for a bulk solids sample. The results for all test materials are illustrated in Figure 2.1.
2.2.4 Bulk Density

Bulk density is another important parameter that plays a critical role in the strength and flow characteristics of a bulk solid. It varies significantly among different bulk materials, and strongly depends on the moisture content and consolidation pressures. It is defined as the bulk mass of a certain amount of particles of the material divided by the total volume they occupy. The total volume includes particle volume, inter-particle void volume and internal pore volume. During this study’s tests, a range of vertical loads—which were regarded the major consolidation stresses—were applied on the material.
sample so that the relationship between bulk density $\rho$ and the major consolidation stress $\sigma_1$ could be determined.

Bulk density is not an intrinsic property of a bulk solid—it can change depending on how the material is handled. A bulk solid after a compaction process or settling process can result in a higher bulk density. Therefore, the bulk density of bulk solids is often determined as a function of major consolidation pressure. This is achieved by using a compressibility tester [20] in a laboratory study. However, according to vast numbers of flow properties tests performed by the TUNRA Bulk Solids Laboratory in the University of Newcastle, the behaviour of bulk density $\rho$, effective angle of internal friction $\delta$ and wall friction angle $\phi_w$ can be approximated by using a two-term exponential model as a function of the major consolidation stress. Figure 2.2 presents the bulk density results for the two test materials.

![Figure 2.2: Bulk Density Results for Iron Ore A](image_url)
2.2.5 Flow Properties from Jenike Direct Shear Tester

The pioneering work of Jenike [3,7] led to the development of the well-known and widely used Jenike shear tester. With this tester, it is possible to determine the FF that is traditionally used to characterise the mechanical properties of a bulk solid with respect to its behaviour in the handling process. The strength and flow properties of bulk solids are based on the various yield loci obtained from laboratory tests. These tests involve a consolidation phase in which the sample is brought to the critical state condition in terms
of voidage, followed by a shear phase. This shear versus consolidation characteristic is obtained by direct shear using the Jenike type shear tester.

These tests are applicable to bulk solids over a wide range of moisture contents up to and beyond the saturated moisture content. This type of shear tester enables measurement of the relevant inner bulk solid parameters, as well the friction between a bulk solid and a wall or boundary. Several factors influence the strength—and hence the FF—of bulk solids, including the moisture content, temperature, storage time and PSD [43]. It is usual to test a particle size fraction below 4 mm (-4 mm fraction) when performing shear tests on bulk solids. Although a bulk material is usually comprised of particles of a broad range of sizes—from coarse to fine particles—it is the fine particles that contribute to the material’s cohesive strength, while the coarse particles are generally free flowing. In addition, the coarse particles tend to roll on the outside, while the fine particles congregate in the centre—particularly in the region of the outlet, when a bulk material is loaded into a storage bin or onto the ground to form a stockpile. Therefore, it is reasonable to believe the test results from Jenike’s direct shear tester, where a sample with a -4 mm size faction is examined to represent the strength and flow properties of a bulk solid. The test procedure for using Jenike’s direct shear tester was detailed by Jenike in Bulletin No. 123 [7] and Arnold et al. [20]. This thesis only presents the test results for bulk materials.

2.2.5.1 Flow Function (FF)

The ability of a bulk solid to flow is highly dependent on the strength developed by the bulk material due to consolidation from the self-settling process or compaction from external loads. Arches or ratholes formed in the stored bulk solids after discharge in silos or gravity reclaim stockpiles principally result from this strength. The unconfined yield stress is a measure of the material’s strength at a free surface, and is a function of the major consolidating stress. A yield locus represents the shear situation when a sample of a bulk solid is first pre-consolidated and then consolidated under shear, in which the material is made to flow under specified consolidation stresses until a steady state is reached or approached. A consolidation stress and its corresponding unconfined yield
stress can be obtained from each yield locus by Mohr stress semi-circle method. The unconfined yield stresses from the yield loci under different consolidation stresses reveal a function of the major consolidation stresses of the solid—the FF. The test procedures to obtain FF were summarized in Jenike’s research [7].

The instantaneous FF is obtained from a series of instantaneous yield loci when the bulk material sample is sheared immediately after the normal consolidation. If the shear test is undertaken after the sample is stored for the requisite period under the major consolidating stress, a time yield locus can be obtained. The time FF ($FF_t$) is based on a series of time yield loci. Only the instantaneous FF is presented in this chapter because most experiments performed in this research were under instantaneous circumstances. Figure 2.3 shows the results of the FF for all the test bulk materials.
2.2.5.2 Effective Angle of Internal Friction

As defined by Jenike in Bulletin No. 123 [7], there is a straight line through the origin, tangential to the Mohr circle, defining the major and minor principal stresses. This is the effective yield locus, and the slope of this locus is defined as the effective angle of internal friction. This angle exhibits the relationship between the major and minor principal stresses [7] as follows:

\[
\frac{\sigma_1}{\sigma_2} = \frac{1 + \sin \delta}{1 - \sin \delta}
\]  

(2-1)

The effective angle of internal friction \( \delta \) usually decreases with increasing consolidation stress for cohesive bulk materials. For free-flowing bulk solids, this angle is not a function of the consolidation stress. In this case, all effective yield loci coincide with the steady-state yield locus and go through the origin. The relevant results for all test materials are described in Figure 2.4.
Figure 2.4: Effective Angles of Internal Friction for All Test Materials

(a) Effective Angle of Internal Friction: Iron Ore A

(b) Effective Angle of Internal Friction: Iron Ore B

(c) Effective Angle of Internal Friction: Beach Sand
2.2.5.3 Wall Friction Angle

The wall friction angle represents the friction characteristic at the contact zone of the bulk solid and wall or boundary surface. This parameter can significantly influence the gravity flow performance of bulk solids in hoppers and chutes, and low wall friction can permit better gravity flow in hoppers. Wall friction primarily depends on the interaction between three groups of variables [43]:

- bulk solids characteristics, including particle size, size distribution, shape, density and hardness; moisture content; bulk density; temperature; surface chemistry characteristics and undisturbed storage time
- wall surface characteristics, including surface roughness, hardness and chemical composition
- loading and environmental factors involving normal pressure between bulk solids and wall surface, relative rubbing or sliding velocity, temperature and humidity conditions, and wall vibrations.

Wall friction angle can be measured by Jenike’s direct shear tester, and the detailed procedure is described in the research work [7,20,43]. Varied normal loads were applied on the material sample as the major consolidation stress on the sample, followed by the shear process, to achieve wall yield loci. Different wall surface materials were employed in the experiments, and the relevant configurations are summarised in Table 2.3.

<table>
<thead>
<tr>
<th>Test categories</th>
<th>Materials</th>
<th>Iron Ore A (2.7%)</th>
<th>Iron Ore A (6.3%)</th>
<th>Iron Ore B (6.3%)</th>
<th>Iron Ore B (7.5%)</th>
<th>Beach Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funnel-flow silos</td>
<td>3 mm Perspex</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buried support columns</td>
<td>Steel + two Teflon sheets</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

The application of two thin Teflon sheets attached over the buried support columns sought to measure the wall friction angles under the same condition as the experiments for buried support columns, where two thin Teflon sheets were applied on the test
column surface. This changed the contact to that between the bulk solids and two Teflon sheets. Figure 2.5 illustrates the wall friction angles for all test materials.
2.3 90 Degree Shear Tests: Flow Function in Circumferential Direction \( \text{FF}_0 \)

Roberts proposed a new theory on rathole analysis—hoop stress theory [15,44], which is based on the three-dimensional stress analysis model. The basis of hoop stress theory involves determining the unconfined yield strength of the bulk solid in the circumferential direction, which is orthogonal to the two-dimensional stress state of Jenike theory. For the critical condition, when flow has just ceased, the hoop stress is just equal to the unconfined yield strength in the circumferential direction. Therefore, this unconfined yield strength is critical for stress conditions and the stability of ratholes occurring in funnel flow, and dominates the diameter of the rathole. In Roberts’s approach, this strength can be estimated by the average of the major and minor consolidation stresses from the two-dimensional axi-symmetric analysis. Alternatively, as indicated in Roberts’s theory, this unconfined yield strength in the circumferential direction can also be measured using Jenike’s direct shear tester, via which the FFs for the conventional two-dimensional cases are determined.

The \( \text{FF}_0 \) for the third or circumferential direction is required to determine the critical rathole diameter. It defines the unconfined yield strength \( \sigma_{c \theta} \) in the circumferential
direction as a function of the major consolidation stress $\sigma_1$. This FF$_0$ can be obtained by consolidating each test sample in the shear cell in the normal manner to establish the two-dimensional consolidation stress state for axi-symmetry, followed by a shear process with the shear cell turned through $90^\circ$ and relocated in the shear tester. FF$_0$ tests were conducted with all bulk solids applied in funnel-flow experimental investigations, and the results are presented in Figure 2.6. Figure 2.7 shows the relevant effective angle of internal friction.

![Graph of direction as a function of the major consolidation stress $\sigma_1$. This FF$_0$ can be obtained by consolidating each test sample in the shear cell in the normal manner to establish the two-dimensional consolidation stress state for axi-symmetry, followed by a shear process with the shear cell turned through $90^\circ$ and relocated in the shear tester. FF$_0$ tests were conducted with all bulk solids applied in funnel-flow experimental investigations, and the results are presented in Figure 2.6. Figure 2.7 shows the relevant effective angle of internal friction.](image-url)
(c) FF: Beach Sand

Figure 2.6: FF in the Circumferential Direction

(a) Effective Angle of Internal Friction: Iron Ore A

(b) Effective Angle of Internal Friction: Iron Ore B
2.4 Conclusions

This chapter has outlined the principal properties that are used to determine the strength and flow behaviour of a bulk solid. The traditional method takes the constant values of bulk solids’ properties into account, which correspond to the worst condition likely to occur in real applications. Unlike the traditional method, the properties determined by a series of tests in this research were processed by curve fitting as functions of the major consolidation stress. More accurate bulk solids’ properties were achieved by considering those variations. These varied properties, instead of constant values, were used in the theoretical calculations based on the existing theories, as will be described in detail in the following chapters.
Chapter 3: Experimental Investigation of Rathole in Funnel-flow Silos and Gravity Reclaim Stockpiles

3.1 Introduction

Funnel flow is one of the two principal flow modes, as defined in Jenike’s publications [3,4,7]. It can occur in both silos and gravity reclaim stockpiles. Funnel-flow silos are often used to store hard, abrasive, lumpy solids because the material flows along the flow channel (namely, piping or rathole) formed just above the opening, instead of the bin walls, which provides wear protection for the bin walls. The gravity reclaim stockpile is a typical case of expanded flow because it incorporates funnel flow in the upper section, with the reliable discharge of mass flow at the bottom reclaim hopper section.

The consolidation stress fields generated in a storage bin or stockpile—whether initial or active, flow or passive or a combination of both—depend on the flow pattern developed under the constraining conditions imposed by the walls of a bin or surrounding stored product, in the case of a stockpile. In the case of funnel-flow bins, there is some uncertainty regarding the role of flow pressures and initial pressures for design purposes. Jenike et al. [36] considered the flow pressures for the designs of tall funnel-flow bins, while Johanson [34] stated that the initial pressures may cause ratholes previously not predicted, based on the steady flow pressures given by Jenike [7], which means the initial pressures must be considered in designs. This agrees with the proposition of Carson and Jenkyn [35]. For the flow case, both Carson et al. [35] and Jenike et al. [36] stated that there is a peak pressure occurring at the level of the effective transition if the flow channel intersects the cylinder wall. However, practical field observation is needed to verify these assumptions.

In the case of funnel-flow silos and stockpiles, the discharge capacity or reclaim efficiency is highly dependent on the geometry of the rathole and draw-down head. For
complete discharge, the bin opening (for stockpiles, the dimensions for reclaim mass-flow hopper) needs to be at least equal to the critical rathole diameter. In terms of rathole stability and rathole geometry, some theoretical studies [3,7,37-39] about rathole forming lead to the assumption of the two-dimensional axis-symmetrical stress field, which is quite empirical. Jenike’s original method, the Lower Bound Solution, derived from flow channel arch analysis, leads to underestimation of the critical rathole dimension. His later Upper Bound Solution, in which the consolidation conditions were taken into account, is too conservative and overestimates the rathole dimension.

From this perspective, based on Jenike’s fundamental work [3,7], a new realistic solution has recently been proposed by Roberts [15], in which a hoop stress theory is applied based on the three-dimensional stress state occurring in the rathole. This hoop stress theory assumes that the hoop strength of the bulk solid in the vicinity of the free surface of central flowing channel governs the rathole stability, and the FF for the third or circumferential direction defines the unconfined yield stress in the circumferential direction as a function of the major consolidation stress. This unconfined yield strength can be estimated by using the mean stress of the major and minor principal stresses from the conventional two-dimensional direct shear test. In Roberts’s theory [15], a comprehensive analysis to explore the nature of rathole is provided on the subject of rathole wall expansion angles and the effective head. Roberts’s pioneering work showed that the hoop stress theory can provide more realistic predictions of rathole geometry than existing theories. However, further experimental research is needed to verify this theory.

Technological advances enable improvement of the methods of measurement to be developed. Berry et al. [27] measured the shapes of arches under a range of hopper geometry by using a scanning laser-ranging device. In addition, a crossed laser measurement apparatus was employed in McBride’s [62] experiments to map the shape of the ratholes. The work described here is an experimental study of the rathole geometries in a series of tests performed in gravity reclaim stockpiles and funnel-flow bins with different geometrical configurations and different bulk materials. Laser devices were employed to depict rathole profiles. The primary goal of the work was to
compare the experimental results with the theoretical predictions from both Roberts’s hoop stress theory and Jenike’s theory, regarding the rathole geometries, effective draw-down heads and reclaim efficiencies. The findings of the experiments help explore the nature of funnel flow and verify the existing theories.

3.2 Review of Roberts’s Hoop Stress Theory

The hoop stress theory proposed by Roberts [15] concerns funnel-flow performance in bins and gravity reclaim stockpiles. Funnel flow is characterised by material sloughing off the top free surface and flowing down the central flow channel formed just above the opening. The flow continues until the bulk solid in the bin or stockpile drops to a level of the draw-down h_D and the material below this level represents ‘dead’ storage. After a stable rathole is formed, as shown in Figure 3.1, the sloughing angle of rathole ψ during the sloughing action from the free surface is not normally constant, and this angle increases to β close to the region of the flow channel.

Figure 3.1: Hoop Stress Model [15]

Roberts summarised the characteristics of the two angles as follows:

\[ \phi_t < \psi \leq \delta \]  

(3-1)

where \( \phi_t \) = static angle of internal friction.

\[ \beta = \frac{\pi}{4} + \frac{\phi_t}{2} \]  

(3-2)
For the critical condition when flow has just ceased, the hoop stress is equal to the unconfined yield strength \( \sigma_{c0} \) in the circumferential direction—that is, \( \sigma_h = \sigma_{c0} \). The critical rathole dimension is given by:

\[
D_f = \frac{\sigma_{c0} G(\beta)}{\gamma}
\]  

(3-3)

where \( G(\beta) = 2\tan \beta \); \( \gamma = \rho g \) = bulk-specific weight; \( \rho = \) bulk density; and \( \beta \) is defined by Equation (3-2).

In Roberts’s hoop stress theory, the unconfined yield stress \( \sigma_{c0} \) in the circumferential direction is defined by \( FF_0 \) for the circumferential direction as a function of the major consolidation stress \( \sigma_1 \). According to the comparison between the predicted and measured \( FF_0 \) undertaken by Roberts [15], this circumferential unconfined yield stress \( \sigma_{c0} \) can be estimated from the normal \( FF \) of the bulk material by using the average of the major and minor consolidation stresses. This average consolidation stress \( \sigma_{av} \) can be given by:

\[
\sigma_{av} = \frac{\sigma_1}{1 + \sin \delta}
\]  

(3-4)

Figure 3.2: Flow Properties and Design Lines [15]
The circumferential unconfined yield stress $\sigma_{c\theta}$ can be obtained as shown in Figure 3.2. By substituting Equation (3-2) and $\sigma_{c\theta}$ into Equation (3-3), the critical rathole diameter $D_c$ can be calculated.

3.3 Experimental Methodology

Two laboratory scale test rigs were built to investigate the flow of bulk material in funnel-flow bins and gravity reclaim stockpiles. Laser technology was applied to measure rathole profiles, such as rathole sloughing angles, rathole diameters and draw-down heads occurring in funnel flow. Material profiles before and after the flow were obtained by laser scanning. Direct manual measurements of filling levels, draw-down heads and expansion angles were also undertaken for some tests in order to verify the capability of the laser measuring technique. Considering the characteristics of bulk materials, at least three repetitions were conducted for all measurements.

3.3.1 Experimental Setup for Funnel-flow Bin Tests

A series of tests were performed using small-scale funnel-flow bins with different dimensions of outlets and different bin diameters. The experimental rig is shown in Figure 3.3. The laser scanner (Acuity AR 4000-LV, two-dimensional) was positioned directly above the bin, which was filled with test materials. A uniaxial drive system consisting of a stepper motor, controller and drive was coupled with the laser enclosure in order to realise equal spacing scanning lines automatically. Different Tekscan tactile pressure sensors (type 5315, 5051, 9901) were placed against the vertical bin wall and the base of a flat-bottom bin or hopper wall to measure the pressure distributions, as shown in the central section of Figure 3.3 (type 5315 on the wall, type 5051 on the base).

Four load cells were mounted under the four corners of the rectangular steel table in order to record the instantaneous loads that remained on the table during the dynamic processes. This allowed the discharge time and discharge rate to be calculated, which enabled comparison of the instantaneous unloading mass between the experimental results and subsequent simulation outcomes. The test rig was designed to accommodate...
a range of model funnel-flow bins with varying bin diameters (0.3, 0.4 and 0.5 m) and different outlet diameters (0.075, 0.10 and 0.15 m). In addition to the flat-bottom bins, bins with a diameter of 0.3 m, combined with two different hoppers, were also tested. The two hoppers each had a hopper half angle of 40° and 60°, and an outlet diameter of 0.08 m for both. These flat-bottom bins and hoppers were made of Perspex with a thickness of 3 mm. The filling hopper shown in the figure was used to load test material centrically into the flat-bottom bin. This filling hopper was 0.8 m above the table surface. Iron Ore A with two moisture contents (2.7 and 6.3%) and Iron Ore B with a moisture content of 6.3% were tested.

![Figure 3.3: Experimental Setup for Funnel-flow Bin Tests](image)

3.3.2 Experimental Setup for Stockpile Tests

3.3.2.1 Rathole Profiles Measured by a Probe-profile Gauge in 1987

This study was undertaken in 1987 in TUNRA Bulk Solids Laboratory as an industrial research project [63], and the relevant research was published in a subsequent paper [64]. It concerned the modelling of the primary stockpile for an iron ore project in Western Australia. The objective of the study was to examine the gravity reclaim efficiency and
rathole shapes for several possible configurations of single and double apron feeders, with the aim of obtaining the optimum solution within the limitations of the available space. A model stockpile scale was built to a scale of 1:50 to facilitate the tests in the laboratory. Based on this scale factor, the model stockpile had a nominal height of 0.62 m, corresponding to the real stockpile height of 32 m. The base of the model was 2.4 m² and was mounted 1.2 m above the ground on a steel frame, as indicated in Figure 3.4. Provision was made for ready change of the reclaim hopper opening configuration to meet the specifications required for each test. A slide gate was used to initiate discharge of the test material—iron ore. A reclaim belt conveyor was installed beneath the stockpile to move the discharging iron ore.

The profile gauge illustrated in Figure 3.4 was assembled with a series of small long probes, and used to measure the external profile of each model stockpile, as shown in this figure, as well as the interior shape of the rathole formed after discharge, as shown in Figure 3.5. A total of six different slot arrangements were examined using this setup, as displayed in Figure 3.6. The test material was blended by the samples of iron ore with a moisture content of 2.35%. The mixed iron ore were prepared by sieving to a size range of below 5.6 mm.

![Figure 3.4: Experimental Setup with Probes Applied](image)
Figure 3.5: Illustration of the Rathole Profile Measurement by Probes [63]

Figure 3.6: Percentage Draw-down and Surface Profiles of Stockpiles—Six Slot Configurations
3.3.2.2 Three-Dimensional Laser Device

Some other stockpile tests were performed using the same table illustrated in Figure 3.4, but only a single slot with an outlet dimension of $0.15 \times 0.41$ m was applied. Iron Ore B was tested, and the moisture content varied within the range of 4.1 to 8.2% during all tests. Nearly three tonnes of the test bulk materials used in all tests generated a height of the model stockpile of up to 0.95 m. Since the large exposed surface of a stockpile can accelerate the drying process and rapidly decrease the moisture content, profile measuring tests should be performed in as short a time as possible. Therefore, a three-dimensional laser scanner (the FARO® Laser Scanner Focus3D) was employed, giving the advantages of larger scanning area, high speed and high accuracy. This laser scanner is capable of capturing detailed three-dimensional measurements and photorealistic three-dimensional colour scans due to the integrated colour camera. As shown in Figure 3.7, the scanner was placed above the stockpile during the measurements.

![Three-dimensional Laser Scanner](image)

Figure 3.7: Three-dimensional Laser Scanner

3.4 Experimental Results

3.4.1 Experimental Results for Funnel-flow Bin Tests

The two-dimensional laser scanner performed scanning work once the funnel-flow bin was filled with the test material, and when a stable rathole was formed after flow ceased.
The information from the scan of the material surface after filling was used to obtain the filling heights, and the comparison of the two scans was used to calculate draw-down heads and rathole profiles. During the tests, the laser scanner recorded 32,000 data points per scan, and the spacing distance after each scan was adjusted to 6 mm by the drive system. The laser device collected the range data with a rotating mirror, and the raw data were saved in polar coordinates (in distance and angle), as shown in Figure 3.8(a). Therefore, data filtering, smoothing and coordinate conversion were performed for all single two-dimensional line scans, and the related data processing procedures were undertaken with software such as MATLAB. By filtering the noise adhering to the raw signal, only the data within a specified range were extracted and converted into Cartesian coordinates, as shown in Figure 3.8(b). A three-dimensional material profile could be obtained by interpolating all related processed two-dimensional data for each test, and then the rathole geometry for each test could be obtained.

(a) Raw Data of One Scan in Polar Coordinates   (b) Two-dimensional Rathole Profile in Cartesian Coordinates

Figure 3.8: Two-dimensional Laser Scanning Data

3.4.1.1 Pressure Distributions on Bin Wall and Base or Hopper Wall

Tekscan pressure sensors were placed against the vertical bin wall and the base of the model bin (for the flat-bottom bin) or the hopper wall (if a hopper section was combined). Two different group configurations of Tekscan tactile pressure sensors in the
pressure measurements were applied: one group configuration was sensor type 5315 (matrix height × matrix width = 426.7 × 487.7 mm) on the cylinder wall and 5051 (matrix height × matrix width = 55.9 × 55.9 mm) on the base or hopper wall when the filling heights were smaller than 0.5 m and the bin diameter was the smallest size of 0.3 m. Another was sensor type 9901 (matrix height × matrix width = 609.6 × 9.7 mm) at the two measurement locations for the higher filling heights and bigger bin diameters.

After the filling processes, the pressure distributions on the two measurement locations were measured. For example, the pressure results for initial filling were analysed, as presented in Figures 3.10 and 3.11 for the flat-bottom bin tests, and Figures 3.12 and 3.13 for the tests using bins combined with hoppers. In each case, two predicted pressure distributions on the silo wall were drawn for comparison: one calculated based on Janssen’s equation [33], and the other calculated from a variation of the refined Roberts’s load analysis (detailed in Chapter 5) and adapted to the current analysis. Appendix A provides the equations upon which these Roberts’s pressure curves on the cylinder part in all figures were based. The reason for the application of Roberts’s load analysis was that the stress condition developed near the front face of a support column buried in a stockpile is similar to that near a silo wall, owing to the similar material stacking patterns, as illustrated in Figure 3.9.

![Figure 3.9: Illustration of Ribbon-shaped Silo Wall Approximately Equivalent to the Column Front Face Buried in a Stockpile](image)

Figure 3.9: Illustration of Ribbon-shaped Silo Wall Approximately Equivalent to the Column Front Face Buried in a Stockpile
All measured pressure distributions showed some uneveness, which could have resulted from the heterogeneity of the bulk materials and possible limited resolution of the sensors applied. As shown in Figure 3.10(a), the pressures on the bin wall with smaller filling heights agreed more with Janssen’s predicted results in terms of the magnitudes, in comparison with the prediction from Roberts’s load theory. However, there was a ‘constant pressure’ zone from the top filling surface to a buried height of nearly 0.1 m, with the pressure remaining at approximately 1,000 Pa within this zone, where the pressure was supposed to change contineously when increasing the filling height from zero to 1,000 Pa. This may have resulted from the low sensitivity of the sensor when applied in the low-pressure range.

For the pressure distributions on the base of the flat-bottom bin, Figure 3.10(b) indicates that the pressures on the horizontal base in the central region coincided with Janssen’s theory; however, Roberts’s approach was more reasonable if peak pressures were considered. As depicted in the measured base pressure distributions for all repetition tests, there were pressure increases in the locations near the bottom of the bin wall and in the vicinity of the outlet, as would be expected—especially for the former, where more drastic pressure increases were observed. This was possibly caused by the sudden change of the stress field within the stored bulk material in these regions due to the changed structure and rigidity, and the change of the stress field was more dramatic in the structural discontinuity area, such as the junction vicinity of vertical bin wall and horizontal base. However, in the theoretical calculations, the pressure distributions on the base were uniform, according to the assumption of the isotropical behaviour of the bulk solid and the uniform rigidity of the base that no outlet was present.
When the filling height increased, another type of Tekscan tactile pressure sensors with longer measuring length (type 9901) was applied. The pressure distributions on the bin wall and base are shown in Figure 3.11(a) and (b), respectively. It was observed that the trend and magnitude of pressures for both the bin wall and base were more likely to match the pressures calculated from Roberts’s load analysis model when the scale of tests increased. Two possible explanations can be given for this. First, the PSD of Iron Ore B, which was employed in the tests presented in Figure 3.11, is different and has a larger proportion of coarse particles than does the Iron Ore A in Figure 3.10. These coarse particles may produce many discrete high pressure spots on the bin wall, thereby
making the pressure distribution higher than Janssen’s prediction and closer to Roberts’s prediction. Second, when the diameter of a flat-bottom bin becomes larger, the stress condition near the bin wall is more approximate to that near the front face of a support structure buried in a stockpile, if a ribbon-shaped bin wall is taken into account, as illustrated in Figure 3.9. Therefore, compared with the pressures from the bin with a diameter of 0.3 m, the pressure distribution along the bin wall with a diameter of 0.4 m—as shown in Figure 3.11(a)—was similar to the pressure distribution along the front face of a buried structure, and hence could be estimated by Roberts’s load analysis model.

Figure 3.11: Normal Pressure Distributions—Flat-bottom Bin Diameter = 0.4 m, Iron Ore B (MC = 6.3%)
In terms of the pressure distributions on the bins combined with the two different hoppers, the same sensor types as the tests for the flat-bottom bin with a diameter of 0.3m were applied: sensor type 5315 was against the cylindrical wall, and sensor type 5051 was placed on the hopper wall. The pressure results after filling for the silo with the hopper half angle of 40° and 60° are exhibited in Figures 3.12 and 3.13, respectively. A funnel-flow pattern was observed in the two hoppers (hopper half angle = 40° and 60°) by using Iron Ore A at a moisture content of 2.7%. As suggested in Jenike’s research
[32,36], the initial loading on funnel-flow bins can be represented by Janssen’s equation all the way down to the hopper outlet. For the pressures on the two funnel-flow bins with a diameter of 0.3 m, in addition to the employment of Janssen’s equation for both the cylinder and hopper parts, Roberts’s load theory was also applied to calculate the theoretical pressures on the cylindrical part. The same outcomes as those in Figure 3.8(a) were achieved—the pressure distributions on the cylinders of both funnel-flow bins agreed slightly more with Janssen’s equation. The reason for this has already been discussed in the foregoing paragraph.

For the pressures on a funnel-flow hopper wall, the existing commonly used theory was from Jenike [32,36]. Two different cases for the pressures on hopper walls are considered in Jenike’s theory: filling case and flow case. In this study, only the filling case was investigated in the pressure measurements. As shown in Figures 3.12 and 3.13, the pressure distributions on the walls of both funnel-flow hoppers did not agree well with Janssen’s equation, where the pressure was supposed to increase towards the opening. Rather, the pressure decreased towards the outlet and was considerably lower than that calculated by Janssen’s equation. This probably resulted from local arching of solids in the region approaching to the outlet, and this situation was more prominent for the steeper hopper in Figure 3.12. In addition, there was a pressure peak in the vicinity of the transition of cylinder and hopper. It may be because the stress field created by the stored bulk material experienced a sudden change from the cylinder to a converging hopper, and this has to be considered if Janssen’s equation is applied in the design of a funnel-flow hopper.

In summary, the pressures on bin walls, on the base of a flat-bottom bin, and on a hopper wall are complex, and the heterogeneity in bulk materials always causes variations in the pressure distributions. However, based on the pressure measurements discussed above, it was more appropriate for Janssen’s equation to predict the pressure distribution when the diameter of the storage bin was 0.3m. When the diameter of the bin increases to 0.4m, the pressure distributions could be estimated by applying Roberts’s load theory. However, Roberts’s load theory could be used as a conservative tool for designing these storage bins. Regarding the pressure distribution on a funnel-flow hopper wall, the
commonly-used Janssen’s equation is proper for the hopper designs as long as the sudden pressure change near the transition is taken into account.

3.4.1.2 Rathole Geometry

Figure 3.14 shows the comparison between the three-dimensional material profiles from laser scanning and the pictures taken by a digital camera. Visually, strong alignment was noted between the observed experimental profiles and the laser scanning profiles both before and after flow in the flat-bottom funnel-flow bin. Some distortion occurred when the laser beam scanned the bin wall surface, which may have resulted from the transparency of Perspex, which interfered with the reflection of the laser beam. Overall, the profile results confirmed the capability of the laser scanning method in a qualitative manner.

In order to determine the rathole geometry from the scanning results, six sample locations exactly corresponding to three cross sections were chosen, and uniformly distributed across the centre axis of the bin, as shown in Figure 3.15. In the subsequent data processing of all laser results, these three section planes were adopted as typical reference planes to determine the rathole geometry. One cutting plane example is displayed in Figure 3.16, which clearly illustrates how the rathole parameters were calculated. The results of the rathole parameters (refer to Figure 3.1) from the six locations were averaged for each test, such as the rathole sloughing angle $\psi$ and $\beta$, rathole expansion angle ($\epsilon_c$ for conical wall and $\epsilon_p$ for plane-flow side wall), draw-down head $h_D$ and rathole diameter $D_f$.

Since all bulk material that flowed out in the silo combined with the steeper hopper (hopper half angle = 40°) for all chosen test materials during all repetition tests when the outlet was opened, ratholes for hopper tests were only developed in the silo combined with the flatter hopper (hopper half angle = 60°). Thus, only the results from this hopper were analysed. All the processed results from the tests with all flat-bottom bins and the flatter hopper are presented in Figures 3.17 to 3.21. To facilitate a comparison between the experimental and theoretical results, the related predicted results were also plotted in
all corresponding figures. By doing so, a comparison of rathole dimensions between the manual measurement and laser scanning could be made to verify the accuracy of the laser device applied.

Figure 3.14: Two-dimensional Laser Scanning Profiles for Funnel-flow Bins

Figure 3.15: Calculated Section Planes and Sample Locations
Figure 3.16: One of the Three Section Planes of Rathole

- Iron Ore A, MC=2.7%, bin diameter=0.3m
- Iron Ore A, MC=2.7%, bin diameter=0.4m
- Iron Ore A, MC=2.7%, bin diameter=0.3m, hopper $\alpha=60^\circ$
- Iron Ore A, MC=6.3%, bin diameter=0.3m
- Iron Ore B, MC=6.3%, bin diameter=0.3m
- Iron Ore B, MC=6.3%, bin diameter=0.4m
- Iron Ore B, MC=6.3%, bin diameter=0.5m
- Effective angle of internal friction $\delta$
- Static angle of internal friction $\phi_1$
- $0.35\phi_1+0.65\delta$

Figure 3.17: Rathole Sloughing Angle $\psi$—Silos
As illustrated in Figure 3.1, the rathole angle $\psi$ is the sloughing angle that forms near the top free surface of the funnel-flow crater. Figure 3.17 shows this angle for all tests with different test materials, different bin diameters and different outlet diameters. In the prediction of Roberts’s hoop stress theory, the sloughing angle $\psi$ is within the range of the two internal friction angles $\phi_i$ and $\delta$, and this was confirmed by the experimental results. In addition, the results showed that angle $\psi$ is more likely to approach the effective angle of internal friction $\delta$.

![Figure 3.18: Rathole Sloughing Angle $\beta$—Silos](image)

The rathole sloughing angle $\psi$ is not constant, but develops to a steeper angle $\beta$ towards the central flow channel near the level where the draw-down head $h_D$ and critical rathole diameter $D_f$ are defined. According to the comparison of angle $\beta$ between the predicted results and experimental results in Figure 3.18, most tests verified Roberts’s assumption that $\beta$, for incipient flow, can be approximated to be $45^\circ + 0.5\phi_i$ (refer to Equation (3-2)).
However, in the tests performed with Iron Ore A with a moisture content of 2.7% in the silo with a diameter of 0.4 m for all chosen outlet diameters (0.075, 0.1 and 0.15 m), the rathole sloughing angles $\psi$ and $\beta$ did not align with the theoretical calculations—they were slightly smaller. This may have been because the filling heights for all these tests were too small, and even smaller than the silo diameter, which made the funnel flow incomplete and meant that the two rathole sloughing angles could not fully develop.

The rathole walls’ expansion angles are the sloughing angles of the flow channel, and can represent the funnel-flow behaviour occurring in silos or stockpiles. These angles develop above the outlet then expand towards the level of draw-down head $h_D$. The existing theory about this is Jenike’s approach, in which the mass-flow limits apply. In this approach, mass flow is assumed to occur along the flow channel, and the flow channel acts as a boundary surface, such as mass flow along a hopper wall. Therefore,
the limits for the hopper half angle apply to the rathole wall expansion angle. The wall friction angle at the flow channel is defined by:

$$\phi_{ws} = \tan^{-1}(\sin\delta)$$  \hspace{1cm} (3-5)

This is based on the assumption that the failure by internal shear at the boundary occurs when the shear stress reaches a maximum. According to the calculated wall friction angles at flow channel $\phi_{ws}$, by applying Equation (3-5) and effective angle of internal friction $\delta$, corresponding to the different test materials, the rathole expansion angles, $\varepsilon_c$ (circular outlet or conical end wall for long rectangular outlet) and $\varepsilon_p$ for plane-flow side wall angle, can be derived by applying Jenike’s mass-flow limits in hoppers. Both the measured and predicted $\varepsilon_c$ are presented in Figure 3.19, which shows that the measured results were slightly larger than the predicted ones. There are two possible reasons for this. First, most funnel-flow storage facilities—such as silos and stockpiles—are designed based on the time storage criteria, which are slightly conservative; hence, steeper hopper half angles (corresponding to expansion angle $\varepsilon_c$ in rathole) than the real angles are introduced to avoid funnel flow or flow blockage. Second, it is more likely that the consolidation stresses in the model were lower than the predictions, which led to a larger $\varepsilon_c$ than the calculated angles.

When the material in the silo drops down to a level of draw-down head $h_D$ the flow ceases. Rathole parameters $h_D$ and the critical rathole diameter $D_t$—which will be discussed in the following figure—are critical for predicting funnel-flow characteristics and reclaiming efficiency. Since the rathole expands from the diameter of silo opening $B$ with the expansion angle $\varepsilon_c$ to the critical rathole diameter $D_t$ at the level of $h_D$, the theoretical $h_D$ is obtained from the following equations:

Based on Jenike’s theory\(^1\), an equation describing rathole expansion can be obtained:

$$B + 2(H - h_D)\tan\varepsilon_c = \frac{\sigma_c A \tan \phi_t}{\gamma}$$ \hspace{1cm} (3-6)

---

\(^1\) The phrase ‘Jenike’s theory’ refers to the Upper Bound theory as described in Roberts’ comparison [14] throughout this chapter.
\[
\sigma_1 = \frac{\gamma R}{k_1 \tan \phi_w} \left(1 - e^{-k_1 \tan \phi_w \frac{h_D}{R}}\right) [3,7] \tag{3-7}
\]

where \( B \) = diameter of silo opening
\( H \) = filling height
\( h_D \) = actual draw-down head of solids
\( k_1 \) = ratio of horizontal to vertical pressure in the bin, normally assumed to be 0.4
\( R \) = characteristic radius
\( \gamma = \rho g \) bulk-specific weight
\( \phi_w \) = wall friction angle

Based on Equation (3-1) to (3-4), Roberts’ theory describing rathole expansion is given:

\[
B + 2(H - h_D) \tan \phi_c = \frac{\sigma_c \beta^2 \tan (\frac{\pi}{4} + \frac{\phi_c}{2})}{\gamma} \tag{3-8}
\]

\[
\sigma_{av} = \frac{\sigma_1}{1 + \sin \delta} = \frac{\gamma R}{k_1 \tan \phi_w (1 + \sin \delta)} \left(1 - e^{-k_1 \tan \phi_w \frac{h_D}{R}}\right) [14] \tag{3-9}
\]

*Figure 3.20: Draw-down Head \( h_D \)—Silos*
By substituting Equation (3-7) into Equation (3-6), and Equation (3-9) into Equation (3-8), the theoretical draw-down head \( h_D \) for silos from Jenike’s and Roberts’s theories can be derived, respectively. Since the filling height of test materials in silos has a direct effect on the draw-down head, both silo filling height and the two calculated \( h_D \) are plotted in Figure 3.20 to better illustrate the relationship between the measured and predicted \( h_D \) and the filling heights. This figure shows that Roberts’s approach gave slightly higher \( h_D \) than Jenike’s, and both reasonably coincided with the measured results in the low filling height range. However, when the filling height increased to 0.7 m, the predicted draw-down heads from both theories were much smaller than the measured ones.

Figure 3.21 shows the measured critical rathole diameter \( D_f \) and the calculated results from the two approaches for all tests performed with different silos. In order to investigate how the draw-down head affected the critical rathole diameter, the measured \( h_D \) was also used in the theoretical calculation of \( D_f \) for comparison purposes. In the high filling height range, all results showed good agreement between the measured rathole diameters and the predicted ones. However, in the low filling height range, the measured results did not coincide well with the theoretical ones. In addition, by applying the measured draw-down heads based on Roberts’s approach, the calculated critical rathole diameters did not align with the other two predicted results. This may have been because there were unavoidable variations in the rathole parameters during all tests due to the characteristics of the bulk materials, and these were not all included in the existing theories.
The four load cells supporting the table not only enable variable loads to be traced during the entire dynamic process, but also allowed the relative performance parameters—such as discharge time, discharge rate, live capacity and discharge rate—to be calculated from the measured results. The load variables in all three processes are demonstrated in Figure 3.22. As shown in this figure, the loading period and total loading mass could be obtained in the results from the filling process, while the discharge process facilitated calculation of the discharge time, live capacity and discharge rate. These data could be applied to the subsequent simulations.

**3.4.1.3 Reclaim Efficiency**

![Figure 3.21: Critical Rathole Diameter $D_r$—Silos](image)
During the dynamic filling and emptying processes, the filling flow rate and discharge rate were nearly unchanged. For all tests performed in different bins with different bulk materials, the reclaim efficiencies were calculated and are summarised in Figure 3.23. The measured draw-down head $h_D$ and critical rathole diameter $D_f$ were also involved in calculating the reclaim efficiencies, and the results matched well with the measured reclaim efficiencies. This verified the calculation method for the reclaim volume of bulk materials. However, the comparison in the figure indicated some degree of deviation in the two theoretical reclaim efficiencies from the measured results. This occurred due to the aforementioned inconsistency between the theoretical $h_D$ and $D_f$ and the corresponding measured results. Further, the results presented in the figure revealed that an increase in filling height or increase in outlet diameter could lead to an increase in reclaim efficiency, while an increase in bin diameter reduced the reclaim efficiency.
To determine the accuracy of the measured results from the laser and load cells, an error analysis was undertaken and summarised in Table 3.1 [65]. For the laser measurements, both the total loading mass before initiation of the flow and residual mass after the flow were determined by the calculated material volume multiplying the bulk density. The material volume was computed using MATLAB software based on the interpolated laser data within the range of the bin diameter. The bulk density was a mean value attained by averaging the bulk density curve within the measuring consolidation range.
Table 3.1: Error Analysis of Laser and Load Cells [65]

<table>
<thead>
<tr>
<th>Variables</th>
<th>Test no.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>Relative errors (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total loading mass (kg)</td>
<td>Scale</td>
<td>66.935</td>
<td>66.855</td>
<td>66.810</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Load cells</td>
<td>67.10</td>
<td>67.10</td>
<td>67.70</td>
<td>0.648</td>
</tr>
<tr>
<td></td>
<td>Laser</td>
<td>62.22</td>
<td>61.96</td>
<td>61.92</td>
<td>7.228</td>
</tr>
<tr>
<td>Residual mass on the table (kg)</td>
<td>Scale</td>
<td>24.725</td>
<td>26.590</td>
<td>25.50</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Load cells</td>
<td>24.95</td>
<td>25.85</td>
<td>25.80</td>
<td>1.623</td>
</tr>
<tr>
<td></td>
<td>Laser</td>
<td>25.776</td>
<td>26.646</td>
<td>25.677</td>
<td>1.716</td>
</tr>
<tr>
<td>Expansion angle (°)</td>
<td>Manual</td>
<td>15.16</td>
<td>14.82</td>
<td>15.87</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Laser</td>
<td>15.72</td>
<td>15.54</td>
<td>15.74</td>
<td>3.12</td>
</tr>
</tbody>
</table>

Table 3.1 indicates that the four load cells provided high accuracy in obtaining the instantaneous loads on the table. It also shows that the laser scanner was capable of mapping rathole profiles, despite possible influencing factors, such as the assumption of constant bulk density and the data processing approach. This error was more noticeable in the results of total loading mass. However, the comparison of the expansion angles between the direct measurements and the laser detection verified the capability and reasonable accuracy of this two-dimensional laser measuring technique, and proved the laser scanner to be an efficient instrument to obtain nearly realistic rathole profiles. Therefore, the rathole geometries measured by this laser device are credible to apply in determining rathole prediction theory.

3.4.2 Experimental Results for Stockpile Tests

3.4.2.1 Measured Results from the Probe-profile Gauge (1987)

To investigate the rathole mechanism in bulk stockpiles, some relevant tests were performed by Roberts [63] in 1987 using a probe-profile gauge, and two-dimensional rathole profiles were obtained for the six different outlet configurations, as detailed in Figure 3.6. An example of the two-dimensional rathole profiles from a single slot (0.290 × 0.082 m) configuration is shown in Figure 3.24 for the side view and end view. From the two views, parameters such as angle of repose, stockpile filling height, draw-down head, critical rathole diameter, rathole sloughing angles and expansion angles can be obtained. The reclaim efficiencies for each slot configuration were calculated based on these measured parameters.
Figure 3.24: Instantaneous Test—Single Slot, 290 × 82 mm [63]

3.4.2.2 Measured Results from Three-dimensional Laser Scanner

Since the accuracy of the traditional profile measuring instruments, such as the probe-profile gauge, is normally not ideal, a three-dimensional laser scanner with higher accuracy was applied to depict the rathole geometry for further investigations, with details of the setup described in Section 3.3.2.1. An example of the measured profile results are presented in Figure 3.25. Several section planes were taken to determine the related rathole parameters.
3.4.2.3 Rathole Geometry

The following figures plot and compare the results of the rathole geometries, such as the rathole sloughing angle $\psi$ and $\beta$, rathole expansion angle $\varepsilon_c$ and $\varepsilon_p$, draw-down head $h_D$, critical rathole diameter $D_f$ and reclaim efficiencies from both the probe-profile gauge and three-dimensional laser scanner. Since a number of tests were performed by applying the laser technique using more than two tonnages of bulk material, it was difficult to keep the moisture content steady during all tests. As a result, two different moisture contents were considered as the references for the application of the tested flow properties in the related theoretical calculations: 6.3 and 7.5%.

In Figure 3.26, the measured rathole sloughing angle $\psi$ from both probe-profile gauge and laser scanner agreed well with the predictions that ranged between the static angle of internal friction $\phi_s$ and the effective angle of internal friction $\delta$. The results from the laser scanning for the single outlet indicated that angle $\psi$ was normally closer to $\delta$. This was also found in the results of the silo tests discussed in Section 3.4.1.2.
(a) Rathole Sloughing Angle $\psi$ from Probe-profile Gauge: Six Slot Configurations [63]

Figure 3.26: Rathole Sloughing Angle $\psi$—Stockpiles

(Slot Configurations refer to Figure 3.6)
In addition to the situation in the silo tests, the Rathole sloughing angle $\beta$ from the stockpile tests—by using the two different measuring instruments—also confirmed...
Roberts’s hoop stress theory on the assumption of the sloughing angle $\beta$. Due to the less accurate measuring method of the probe-profile gauge, the results measured by this gauge were slightly smaller than the theoretical $\beta$. In addition, there were some observable fluctuations in the measured results from laser scanning, which was most likely caused by the varying moisture content in all tests (ranging from 4.1 to 8.2%) and the nature of the bulk materials. Nevertheless, overall, all these test results are close to the predictions.

(a) Rathole Expansion Angle $\varepsilon_c$ and $\varepsilon_p$ from Probe-profile Gauge: Six Slot Configurations [63]
The rathole expansion angles presented in Figure 3.28 confirm the general rule that, for long rectangular outlets, the end wall expansion angle $\varepsilon_c$ is normally much smaller than the side wall expansion angle $\varepsilon_p$, where plane-flow is likely to occur. In Figure 3.28(a), both $\varepsilon_c$ and $\varepsilon_p$ are obviously larger than the theoretical angles, especially for the slot configurations 3 and 4 with two long rectangular openings and small separation distances. Besides, the stockpile filling heights for the tests using the two slot configurations in response to the slot dimensions were relatively small. In this case, after the stockpiles were discharged by gravity, the strength developed in the relatively small amount of the residual bulk material on the stockpile base was not enough to sustain the expected fully developed flow channel; thus, the rathole expansion angles $\varepsilon_c$ and $\varepsilon_p$ were slightly bigger than the angles for the other slot configurations. Moreover, the accuracy of the probe-profile gauge was limited; thus, the overall measured rathole expansion angles were bigger than the predictions.

The results from the laser measurements coincided more with the calculated figures due to the much higher accuracy of the three-dimensional laser scanning technique. Figure
3.28(b) also indicates that the expansion angle $\varepsilon_c$ and $\varepsilon_p$ increased with the increase of moisture content. This was because higher moisture content in bulk materials normally results in higher internal friction angle, which helps develop higher strength within the bulk material, and a much steeper flow channel can be created.

Draw-down head $h_D$ is an important rathole parameter, like the critical rathole diameter $D_f$, which is directly related to the performance of the reclaim for stockpiles. A similar calculation method can be used for the draw-down head $h_D$ as the one adopted for the silo case. In this, the rathole above the stockpile outlet, with a diameter of the diagonal $D_R$, expands to the critical rathole diameter $D_f$ at the level of $h_D$, with the expansion angles $\varepsilon_c$ on the narrow side (the end wall) and $\varepsilon_p$ on the long side (the side wall). Once the rathole becomes circular in the cross-section, the angle $\varepsilon_c$ dominates the diverging sloughing of the rathole. Therefore, the theoretical $h_D$ for stockpile is derived from the following equations:

Jenike’s theory describing rathole expansion in stockpiles:

$$D_R + 2(H - h_D)\tan\varepsilon_c = \frac{\sigma_c 4.3 \tan \phi_t}{\gamma}$$  \hspace{1cm} (3-10)

$$\sigma_1 = \gamma h_D \cos \theta_R \text{ [14]}$$  \hspace{1cm} (3-11)

$$D_R = \sqrt{L^2 + W^2} \text{ [14]}$$  \hspace{1cm} (3-12)

where  
$W$ = width of the stockpile outlet/reclaim hopper transition  
$D_R$ = diagonal of the rectangular stockpile outlet/reclaim hopper transition  
$L$ = length of the stockpile outlet/reclaim hopper transition  
$\theta_R$ = angle of repose for the stockpile

Roberts’ theory about rathole expansion in stockpiles (based on Equation (3-1) to (3-4)):

$$D_R + 2(H - h_D)\tan\varepsilon_c = \frac{\sigma_c 2 \tan \left(\frac{\pi \phi_t}{2}\right)}{\gamma}$$  \hspace{1cm} (3-13)

$$\sigma_{av} = \frac{\sigma_1}{1 + \sin \delta} = \frac{\gamma h_D \cos \theta_R}{1 + \sin \delta} \text{ [14]}$$  \hspace{1cm} (3-14)
By substituting Equations (3-11) and (3-12) into Equation (3-10), and Equations (3-12) and (3-14) into Equation (3-13), the theoretical draw-down head $h_D$ for stockpiles from Jenike’s and Roberts’s theories can be derived, respectively.

(a) Draw-down Head $h_D$ from Probe-profile Gauge: Six Slot Configurations [63]

(b) Draw-down Head $h_D$ from Three-dimensional Laser: Single Outlet

Figure 3.29: Draw-down Head $h_D$—Stockpiles
The results from Figure 3.29 demonstrate that both Jenike and Roberts’s theories are capable of predicting the draw-down head for a given bulk material with known flow properties and a specified stockpile outlet configuration and filling height. In the comparison, it was observed that Roberts’s theory gave slightly higher draw-down head for both measurements, while Jenike’s approach was more conservative. The difference in Jenike and Roberts’s methods was not significant for the small-scale laboratory tests; however, for larger stockpiles of 30 to 40 m in height, the Jenike method could be far too conservative as proven in some case studies in Roberts’ research [14] and some relevant consulting projects carried out in TUNRA Bulk Solids. In the stockpile tests with six different outlet configurations, as shown in Figure 3.29(a), for a single opening, the two measured results agreed well with the two theoretical calculations. However, for the remaining four double outlet configurations, both Jenike and Robert’s theories underestimated the draw-down heads. This was because the flow channels were not fully developed above these long rectangular openings with small separation distance when the stockpile filling height was not high enough. In this case, the draw-down head dropped to the level in the flow channel, while \( h_D \) in the theoretical calculations was based on the assumption that the flow channel was fully developed. All of these issues may have caused the differences between the measured and predicted draw-down heads.

As discussed before, the filling height is closely related to the draw-down head, as demonstrated in Figure 3.29(b). The stockpile tests were performed using Iron Ore B with varying moisture content and slightly different filling heights, but the trend of the measured \( h_D \) was more similar to the trend of the filling heights than the moisture content.
Critical Rathole Diameter $D_f$ from Probe-profile Gauge: Six Slot Configurations [63]  

Critical Rathole Diameter $D_f$ from Three-dimensional Laser: Single Outlet

Figure 3.30: Critical Rathole Diameter $D_f$—Stockpiles

In the theoretical calculation, the critical rathole diameter $D_f$ and draw-down head $h_D$ are dependent on each other. Figures 3.30(a) and (b) show good agreement between the
measured critical rathole diameter and theoretical calculations, and reveal that Jenike’s theory produced a slightly higher $D_f$ than Roberts’s. This was because the draw-down head $h_D$ from Jenike’s calculations was slightly smaller than Roberts’s, as indicated in Figure 3.29(a) and (b); thus, the solved $D_f$ from the related equations was higher than Roberts’s. Since draw-down head $h_D$ plays an important role in the theoretical calculation of critical rathole diameter $D_f$, the measured $h_D$ for the two different measurements was also involved in the theoretical calculation of $D_f$. As shown in both figures, the calculated critical rathole diameters applying the measured draw-down heads in the related equations were more consistent with the measured critical rathole diameters. This verified the calculation approach for $D_f$ and indicated that a more realistic $D_f$ can be derived from a more realistic $h_D$.

![Diagram](image)

(a) Reclaim Efficiencies from Probe-profile Gauge: Six Slot Configurations [63]
Reclaim efficiency is an important parameter to evaluate the reclaim capacity of a stockpile. The reclaim efficiencies for all tests from the two different measuring methodologies were analysed based on Roberts’s calculation method for the reclaim volume [64], and the results are exhibited in Figure 3.31. There was reasonable agreement between the two predicted reclaim efficiencies and the measured results for the most slot configurations, while the theoretical calculations were obviously lower than the measured reclaim efficiencies for the double outlet configurations 4 and 5. The trend of reclaim efficiencies for all slot configurations was quite similar to that for the critical rathole diameter \( D_t \), as displayed in Figure 3.30(a). This explains why the reclaim efficiencies for slot 190 × 80 mm with separation distances of 110 and 150 mm did not coincide with the predictions.

In the reclaim efficiencies of stockpiles with a single outlet from the three-dimensional laser, both Jenike and Roberts’s predictions were higher than the measured results. This can be explained by the fact that the scale of the laboratory stockpiles was far smaller.
than that in practice, which can have a significant influence on the applications of these theories. The theoretical calculations from Roberts’s approach were slightly higher than Jenike’s due to its larger draw-down heads. In addition, there was a large difference between the theoretical and measured results when the moisture contents were within the range of 7.6 to 7.9%. First, this may have been because the moisture content for the flow properties applied in the theoretical calculations was 7.5%, which was slightly lower than that in the experiments, and hence a higher reclaim efficiency than the practice was introduced. Second, and more importantly, these lower reclaim efficiencies than the predictions could have resulted from the extra strength generated within the material due to loading mechanism when the stockpile was filled with a bulk material of higher moisture. This extra strength could make the bulk material in the stockpile high consolidated, and lead to a smaller live capacity when the stockpile was discharged under gravity.

Moreover, it was observed that the trend of the reclaim efficiencies was similar to the trend of the filling heights. This revealed that the reclaim efficiency for a stockpile changed with the change of the stockpile filling height, with a higher filling height producing a higher reclaim efficiency for a specified outlet dimension. By comparing the reclaim efficiencies in Figure 3.31 with the draw-down heads in Figure 3.29, it was found that the stockpile filling height significantly influenced both the draw-down head and reclaim efficiency.

### 3.5 Discussion and Conclusions

This chapter has illustrated the application of Roberts’ new hoop stress theory based on a three-dimensional consolidation stress field to describe the critical stability of ratholes in funnel flow. With this background, the focus of this chapter was the experimental studies performed on laboratory scale funnel-flow bins and stockpiles, in which special instrumentation—notably, probe-profile gauge, laser profile scanning, pressure sensing pads and load cells—were employed. It has been proven that load cells and pressure sensing pads can greatly contribute to determining funnel-flow performance parameters. The heterogeneity in bulk materials always creates uncertainty in the geometry of
ratholes, which produces difficulties in measuring pressure distributions and rathole geometries. However, according to the results presented here, it can be concluded that both the two-dimensional and three-dimensional laser scanners and associated data analysis procedures clearly demonstrate the capability of laser technology in determining rathole geometries.

The pressure distributions on cylindrical bin walls were not consistent for all tests applying different bin diameters. It was more likely for Janssen’s pressure to occur when the diameter of the storage bin was small, while the pressure distributions could be more appropriately estimated by Roberts’s load analysis model when the diameter of the bin was large. This may be because when the diameter of a flat-bottom bin grows larger, the stress condition near a chosen narrow ribbon-shaped bin wall—created by the stored bulk material—is more approximate to the stress condition near the front face of a support structure buried in a stockpile. However, it is very difficult to define the boundary to determine which theory is more appropriate, and more research about this subject needs to be undertaken. Nevertheless, Roberts’s approach is slightly more conservative than Janssen’s method, and can be adopted as a conservative approach for determining a cylinder wall for the purpose of a safe design. However, for the pressures on a funnel-flow hopper after filling, the pressure close to the outlet is significant lower than the pressure calculated by Janssen’s equation. This is because of local arching of solids in the region approaching to the opening.

Based on the experimental results of the rathole geometries in funnel-flow bins and gravity reclaim stockpiles from both the traditional methodology of a probe-profile gauge and the new laser scanning technique, the two existing theories about the mechanism of funnel flow—which was originally defined by Jenike and then modified by Roberts—were verified. The difference between the two theories regarding all rathole parameters was not significant for the small-scale laboratory tests, while, for larger stockpiles of 30 to 40 m in height in industrial practice, Jenike’s method was proven to be too conservative in many practical field observations as illustrated in Roberts’ studies and some associated consulting projects accomplished in TUNRA Bulk Solids. Roberts’s theory provided more specific predictions regarding the rathole sloughing
angles and rathole expansion angles. In addition, it produced a more realistic estimation of the critical rathole diameter $D_t$, and provided a better guide for the design of the opening dimensions for funnel-flow silos or stockpiles, while the calculations of the critical rathole diameter from Jenike’s approach were slightly more conservative.

The experimental results revealed that both Roberts’s hoop stress theory and Jenike’s Upper Bound theory are appropriate for predicting funnel-flow behaviour for gravity reclaim stockpiles with single outlet or double outlet configurations when the filling height and separation distance are enough for ratholes to be fully developed. Predicting reclaim efficiency strongly depends on the loading manner, silo or stockpile filling height, outlet dimensions and configurations, and bin diameter if silos are considered. The reclaim efficiency can increase with an increase in silo or stockpile outlet dimensions, but will decrease if the silo diameter increases for the silo case. Nevertheless, the fundamental prerequisite to apply these theories is to achieve realistic flow properties for a given handling bulk material because these properties play a vital role in determining the performance of funnel flow in funnel-flow silos or stockpiles.
Chapter 4: Experimental Investigation of Loads on a Fixed Support Column Buried in Iron Ore Stockpiles

4.1 Introduction

It is a common practice to apply support structures in open bulk stockpiles or bulk storage sheds. In these storage facilities, support structures are necessarily employed, such as trestle legs to support load-out conveyors of open stockpiles—as illustrated in Figure 4.1(a)—or columns to support roof structures and load-out conveyors in large bulk storage sheds—as illustrated in Figure 4.1(b).

However, these support structures are often substantially submerged in the handled bulk materials and subjected to varying loads occurring during the handling processes of bulk solids, such as loading, storage and reclaim. The loads exerted on the support structures by the surrounding bulk solids are complex and thus difficult to predict. While the subject of wall loads has been dealt with in some detail for mass-flow and funnel-flow bins by many researchers—such as Jenike et al. [28,32,36] and Walters [10]—the subject of loads on buried support structures has received little or no attention. For this reason, Roberts’s [55] theoretical approach to predict the loads exerted on buried structures is seen as an important contribution to fill this research gap. A new continuum approach with design equations to analyse the loads on structural elements was established, and the correlation between the type of stress states developed in the handling of bulk
materials, as well as the load distribution on these support elements, were investigated. The stress states developed within bulk materials are closely related to the following parameters [55]:

- manner and mechanisms of loading and reclaim
- loading and unloading history
- length of undisturbed storage time in the stockpile
- rigidity of the stockpile floor
- type of bulk materials
- variations in the flow properties of the handling material.

An industrial case study concerning the loads on vertical support columns in a large bulk fertiliser shed was conducted by Roberts, and some relevant tests were performed on a small-scale experimental model. All these results correlated reasonably with Roberts’s analytical predictions. In addition, Roberts’s theory was applied to predict the loads on trestle legs supporting a load-out conveyor buried to a maximum depth of 35 m in a coal stockpile (with trestle legs that had functioned properly since being built). Follow-up numerical simulations were undertaken by Katterfeld involving applying DEM to model the interacting forces between particles and buried support columns from a macroscopic perspective. The results of these simulations, as published by Katterfeld and Roberts [56], showed reasonable agreement with the theoretical calculation of the pressure distribution over the buried column height.

Since support structural elements employed in large bulk storage facilities are commonly employed in industrial applications, the need for reliable and effective design criteria for these structures is increasing. Although Roberts’s theory has already extended understandings of the loads on buried structures, additional research is required to further validate and improve the continuum approach. For this reason, the current study constructed a laboratory scale test rig to determine experimentally the load conditions on support structures partly buried in stockpiles. Tekscan tactile pressure sensors (Tekscan, Inc., Boston, MA) were employed to measure the pressure distributions on the faces of a buried support column. This thin-film type of sensor consists of a matrix of patented semi-conductive ink coating on two paper-thin carrier foils, and allows real-time
continuous data collection. This sensor type has already been applied in several studies [66-69] for pressure measurement in granular materials and soils. It shows a good degree of accuracy, especially when compared to the other available means of pressure measurements in bulk materials.

It is well known that the flow properties of bulk solids vary with consolidation pressures, moisture content and time under undisturbed storage. For the design of bulk storage systems, such as the determination of bin wall loads [6,8,28,32,36,70-73], a conservative approach is adopted in which the particular properties are chosen to maximise the loads determined. This procedure is normally prescribed in bin load standards. However, there are applications where the actual variability cannot be ignored. An example in which the changing consolidation stresses must be taken into account was described by Roberts et al. [74] for design equations for mass-flow hoppers using stringy and compressible bulk materials. A similar method was adopted in the current study by considering the variations in bulk solids’ properties, with a few refinements made to Roberts’s approach [55], as described in Section 4.2.2.

Two different column setups were used as the support structures buried in stockpiles, and one column is fixed and the other is a laterally moveable column. In this chapter, the detailed Roberts’s load theory and both static and dynamic normal pressure distributions on the fixed column with the usage of cohesive Iron Ore B are discussed. The experimental results from the other column will be discussed in Chapter 5.

4.2 Loads on Buried Structures

4.2.1 Continuum Load Analysis for Buried Structural Elements

Due to its relevance to the present study, the theory proposed by Roberts [55] is now reviewed. The calculation model proposed by Roberts uses a classical continuum approach to analyse the varying load conditions exerted on the surfaces of a structural element buried in bulk materials. It is based on the application of the wall load theories for bin and silo design, as described, for example, by Jenike et al. [28,32,36] and Walters [10,68]. During the loading process of a stockpile, a bulk solid is usually discharged
from a belt conveyor, and comes to rest on the pile surface under the bulk material’s angle of repose $\theta_R$. As necessary, active stress state, passive stress state or combination of both is assumed to develop during the processes of loading, storage and reclaim. Therefore, two separate load models were proposed by Roberts to determine the loads on the column faces.

Loads are exerted on the column faces by the surrounding bulk solid, as illustrated in Figure 4.2. Figure 4.2(a) shows that the lateral force $F_u$ dominates the load condition on the leading face of the column—also defined as the front or upper face. Frictional drag forces $F_s$ act along the sides of the column and a back-filling force $F_l$ is introduced by a void created behind the rear or lower face. As shown in Figure 4.2(b), the load situation for cohesive bulk solids results in an increase of effective area on the column surfaces due to material build-up, and is most likely to occur on the front face. This chapter focuses on the investigation of $F_u$ and $F_l$ in the case of low-cohesive material, as depicted in Figure 4.2(a).

Roberts’s continuum analysis model describes the characteristics of the loads exerted on the buried support column, which are shown schematically in Figure 4.3. In this approach, the failure surface for internal shear inclines with an angle $\phi$ relative to the
vertical direction, producing a wedge-shaped loading condition. Angles $\alpha_c$ and $\alpha_s$ depend on the friction condition at the column upper surface and the internal shear surface, respectively, and the two angles define the centre line of the wedge. A differential equation deduced from the force equilibrium at a slice element in Roberts’s theory is given as follows:

$$\frac{dp_y}{dy} + \frac{j p_y}{y_h - y} = \gamma_y$$  \hspace{1cm} (4-1)

where:

$$j = \frac{K_c (\tan \alpha_c + \tan \phi_w) + K_s (\tan \alpha_s + \sin \delta)}{\tan \alpha_c + \tan \alpha_s} - 1$$  \hspace{1cm} (4-2)

where $K_c = \frac{p_{nc}}{p_y}$; $K_s = \frac{p_{ns}}{p_y}$; $\gamma_y = \rho g \cos \alpha_c$; $y = z / \cos \alpha_c$.

### 4.2.2 Refinements of Roberts’s Load Prediction Theory

The profile of a wedge-shaped load condition is defined by the inclination of the failure surface for internal shear. The flow properties of bulk solids have a significant effect on the load conditions. In a great number of previous research works about the handling of bulk materials [6,8,28,32,36,70-73], the flow properties were assumed to be constant in predicting the pressures on bin walls or flow patterns in silos to facilitate the calculations. In reality, the flow properties can vary significantly with changing major consolidation stress, especially in the low-pressure range, which has already been considered by Roberts et al. [74] for design equations for mass-flow hoppers using stringy and compressible bulk materials. In this load prediction theory, appropriate constant values for the flow property characteristics of bulk materials were applied by Roberts to predict the likely worst case pressure distribution [55,56]. However, refinements of the theory regarding flow properties variations are deemed necessary.

As described in Chapter 2, bulk density $\rho$, effective angle of internal friction $\delta$ and wall friction angle $\phi_w$ change significantly with a change in the major consolidation stress. In
order to consider these variations in Roberts’s theoretical model, appropriate expressions to fit the experimental flow property data are necessary. During all available fitting methods, the two-term exponential fitting model can successfully express these flow properties as functions of the major consolidation stress $\sigma_1$, with the highest goodness of fit. Therefore, the following assumptions were made to solve Equation (4-1):

\begin{align*}
\rho &= b_1 \exp(b_2 \sigma_1) + b_3 \exp(b_4 \sigma_1) \\
\delta &= c_1 \exp(c_2 \sigma_1) + c_3 \exp(c_4 \sigma_1) \\
\phi_w &= d_1 \exp(d_2 \sigma_1) + d_3 \exp(d_4 \sigma_1)
\end{align*}

where $b_i, c_i, d_i \ (i = 1 \ldots 4)$ are constants determined by fitting the experimental data, and $\sigma_1$ is the major consolidation stress.

### 4.2.2.1 Load Conditions on Column Faces

The load conditions along buried columns strongly depend on the stress states developed in the handled bulk solid in the immediate region of the buried columns. Two separate load conditions were defined by Roberts based on the wedge-shaped load analysis model (refer to Figure 4.3) to analyse the loads on column faces for each stress state, as illustrated in Figure 4.4. The refinements of the load analysis model [75] have been previously published, and this chapter only investigates normal loads on column front face and rear face. Chapter 5 will discuss normal loads on column side faces, as well as shear loads.

#### 4.2.2.1.1 Active Stress State

When a bulk solid is discharged from the load-out conveyor, it slides down the sloping surface of the growing stockpile and comes to rest under the inclined angle of repose $\theta_R$. However, the bulk solid continues to move by internal shear at a steeper angle, and this angle approaches the effective angle of internal friction $\delta$ within the bulk solid. In this case, the slope angle $\phi = (\pi/2 - \delta_D)$, where $\delta_D$ defines the effective angle of internal friction near the base of the buried column. The corresponding major consolidation
stress $\sigma_{1D}$ acts predominantly in the steeper inclined direction; thus, the loading condition is defined as active stress state, as shown in Figure 4.4(a). Therefore, it is reasonable to consider pressure $p_y$ as major consolidation stress in the load analysis model described in Figure 4.3.

Therefore:

$$p_y = \sigma_1 \quad (4-6)$$

The dimensions of the wedge shape can be defined as follows:

$$\alpha_c = \alpha_s = \frac{\pi}{4} - \frac{c_1 \exp(c_2\sigma_{1D}) + c_3 \exp(c_4\sigma_{1D})}{2} \quad \text{where} \quad \sigma_{1D} = \lim_{z \to D} \sigma_1(z) \quad (4-7)$$

$$y_h = \frac{D}{\cos(\alpha_c)} + D \sin\left(\frac{\delta - \phi_w}{2}\right) \quad (4-8)$$

Based on this assumption, the pressure ratio at the column surface $K_c = K_{ca}$, and the pressure ratio at the failure surface of the bulk material $K_s = K_{sa}$ is defined in Equations (4-9) and (4-10):
For the load analysis, it is assumed that the coefficient of internal friction on the slip plane corresponds to the maximum shear stress; hence, \( \mu_i = \sin \delta \). Equations (4-9) and (4-10) were substituted in Equation (4-2) and \( j \) is then defined by:

\[
(4-11) \quad j = \frac{\tan \alpha_c (\sin \delta + 1) + \tan \alpha_s (\sin \delta + 1)}{\tan \alpha_c + \tan \alpha_s} - 1 = \sin \delta
\]

The initial condition for the definition of the pressure in Equation (4-1), \( p_y(0) \), is very small and thus can be ignored to facilitate the calculation:

\[
(4-12) \quad p_y(0) = p_{y0} = 0
\]

By inserting the parameters defined by Equations (4-3) to (4-8) and Equations (4-10) to (4-12) into Equation (4-1), a numerical solution for the pressure \( p_y \) can be obtained and the normal pressure \( p_{nc} \) can be calculated by:

\[
(4-13) \quad p_{nc} = (\sin \delta + 1) \left( \frac{\tan \alpha_c}{\tan \alpha_c + \tan \phi_w} \right) p_y
\]

### 4.2.2.1.2 Passive Stress State

The stress field for the passive stress state in Roberts’ theory is defined as shown in Figure 4.4(b) with the slope angle of the failure plane \( \phi = (2\theta - \delta) \), while the height of the wedge \( y_h \) and angles \( \alpha_c \) and \( \alpha_s \) are given by:

\[
(4-14) \quad y_h = \frac{D}{\cos \alpha_c}
\]

\[
(4-15) \quad \alpha_c = \alpha_s = \frac{\phi}{2} = \theta_R - \frac{[c_1 \exp(c_2 \sigma_{1D}) + c_3 \exp(c_4 \sigma_{1D})]}{2} \text{ where } \sigma_{1D} = \lim_{z \to D} \sigma_1(z)
\]
In the passive stress state, the major consolidation stress $\sigma_1$ is acting more in a lateral direction. In this case, Equation (4-2) applies with the pressure ratio $K_c = K_{cp}$ and $K_s = K_{sp}$ given by:

$$K_{cp} = \frac{1 + \sin \delta \cos (2 \eta_c)}{1 - \sin \delta \cos [2(\alpha_c + \eta_c)]}$$  \hspace{1cm} (4-16)$$

$$K_{sp} = \frac{1 + \sin \delta \cos (2 \eta_s)}{1 - \sin \delta \cos [2(\alpha_s + \eta_s)]}$$  \hspace{1cm} (4-17)$$

where:

$$\eta_c = 0.5 \left[ \phi_w + \sin^{-1} \left( \frac{\sin \phi_w}{\sin \delta} \right) \right]$$  \hspace{1cm} (4-18)$$

and:

$$\eta_s = 0.5 \left[ \phi_{ws} + \sin^{-1} \left( \frac{\sin \phi_{ws}}{\sin \delta} \right) \right]$$  \hspace{1cm} (4-19)$$

It is assumed that the failure on the slip plane occurs under maximum shear stress, for which the friction angle is defined by:

$$\phi_{ws} = \tan^{-1} (\sin \delta)$$  \hspace{1cm} (4-20)$$

Based on the load analysis defined by Jenike et al. \cite{2,5,7} and Johanson \cite{6}, the following equation can be achieved to describe the relationship between major consolidating stress $\sigma_1$ and minor consolidating stress $\sigma_2$:

$$\frac{\sigma_1}{\sigma_2} = \frac{1 + \sin \delta}{1 - \sin \delta}$$  \hspace{1cm} (4-21)$$

The relationship between major consolidation stress $\sigma_1$ and the normal pressure at the failure surface for internal shear $p_{ns}$ can thus be defined using Mohr's stress diagram \cite{16}:

$$\sigma_1 = \frac{(1 + \sin \delta) p_{ns}}{1 + \sin \delta \cos 2\eta_s}$$  \hspace{1cm} (4-22)$$
where \( p_{ns} = K_s p_y \).

By substituting the parameters defined by Equations (4-14) to (4-20) and (4-22) into Equation (4-1), a numerical solution for \( \sigma_1 \) and \( p_y \) can be achieved. Consequently, the normal pressure \( p_{nc} = K_{cp} p_y \).

4.2.2.2 Normal Loads on Column Front Face

Normal loads on the column front face often vary more significantly than those on the rear face during handling processes, which may be because the flow of a bulk solid is the first impeded by the front face during loading, and the first to initiate during reclaim. This can result in great variations in the stress conditions on the front face. The stress state occurring on the front face can be only an active stress state, only a passive stress state, or a combination of both stress states, when a bulk solid is being loaded, stored or discharged. A combination of both stress states means that one stress state occurs at the upper part of the buried structure and another stress state dominates the load condition at the lower part. This combination tends to be a passive stress state at the upper part and an active stress state at the lower part of the buried structure. Since there is uncertainty about where the passive stress state finishes and the active stress state begins, there could be multiple formations in this combination of both stress states. The theoretical calculation of the pressure distributions normal to column front face for active stress state, passive stress state and an upper bound combination of both stress states is illustrated in Figure 4.5. The calculation was undertaken with the assumption of 0.7 m as the buried depth for Iron Ore B.
The comparison shows that the pressure distribution on the front face can vary significantly if a different stress state is developed. In the passive stress state, the pressure varies in a parabola-like trend, which first increases with the buried depth until it approaches a peak pressure at the mean height of the buried depth, then decreases to zero at the bottom. Compared with the pressure in the passive stress state, the pressure in the active case behaves differently—it increases until a peak pressure is reached near the bottom, then decreases slightly.

4.2.2.3 Normal Loads on Column Rear Face

When a stockpile is being loaded, some back-filling occurs on the rear side of the column. The pressure introduced by this back-filling behaviour can reduce the lateral deflection of the buried column, with this back-filling effect prone to be weakened when cohesive bulk materials are handled. The pressure on the rear face of a buried column exerted by a cohesive bulk material is generally smaller than the pressure on the leading front face. However, it cannot be neglected, especially for free-flowing materials that can produce nearly similar magnitudes of pressure on the front and rear faces. The stress state developed within the bulk solid near the column rear face is similar to the load.
conditions for the column front face, as depicted in Figure 4.6. The load analysis is similar to that for the front face, as described in detail in Section 4.2.2.1.1 for the active stress state, and Section 4.2.2.1.2 for the passive stress state.

![Diagram of load conditions on the column front face](image)

(a) Active Stress State  (b) Passive Stress State

Figure 4.6: Load Conditions on Rear Face

4.3 Experimental Methodology

4.3.1 Experimental Setup

Due to the impossibility of duplicating a bulk material stockpile at a real industrial size, a laboratory scale test rig was constructed to investigate the load conditions on support columns buried in gravity reclaim stockpiles. Figure 4.7 shows schematically the experimental setup, which incorporated a rectangular stockpile base surface of approximately 5.76 m². The test rig had a capacity of approximately 1.5 m³ when the angle of repose of the bulk material was within the range of 36 to 42°, without disrupting the naturally-buried stockpile profile. The base surface was enclosed by four skirt plates.
to avoid spillage, and a steel frame extension above the base enabled positioning of the vertical test column that was buried in the test material.

The load distributions on the column faces were determined with the test column fixed at the two ends. A slide gate, mounted underneath the reclaim hopper, was operated to initiate the discharge of the bulk materials from the stockpile. To facilitate the loading and discharging process, load-out and reclaim belt conveyors were used, with the former aimed at mimicking the loading process in real industrial applications. The load-out belt conveyor was connected to a variable speed drive to control belt velocity, and hence the flow onto the table. The reclaim belt conveyor was installed below the hopper outlet underneath the stockpile base to remove the discharging bulk material.

![Experimental Setup](image)

**Figure 4.7: Experimental Setup**

The test rig was designed to accommodate different sized model stockpiles using different test materials and with different buried depths. In this chapter, the results are presented from pressure measurements performed for the front and rear face of the buried column by using Iron Ore B only. The results from free-flowing sand will be discussed in Chapter 5. The moisture content for Iron Ore B varied in the range of 6.9 to 7.7% during all tests, and 80% of particles were below 5.6 mm. The test column was made of carbon steel tubing featuring a rectangular cross-section (0.035 x 0.065 m). Three different test series were conducted to determine the instantaneous pressure
distribution after loading and after discharge, as well as the pressure increase due to time-consolidation. All tests were undertaken using around 1.3 m$^3$ of free-flowing sand and cohesive iron ore, with similar buried depths for each of the front and rear faces of the support column.

4.3.2 Tactile Sensors Applied

Common methods to evaluate pressure distributions in bulk materials involve the use of load cells or strain gauges, which restrict the amount of possible measuring points. Tekscan tactile pressure sensors (Tekscan, Inc., Boston, MA) provide the ability to map the pressure distribution over the entire sensor area, as well as the ability to capture stress variations in-situ during the testing process. Two Tekscan tactile pressure sensors (type 9901) with a pressure range of zero to 69 kPa were applied to measure the pressure distributions on the column front and rear faces over the full buried depth. The two long, strip-type tactile sensors each featured an array of 96 sensing units (called sensels), as shown in Figure 4.8, and had the ability to measure the magnitude and distribution of stresses normal to the sheet surface over a length of 0.6096 m.

![Figure 4.8: Tekscan Tactile Pressure Sensor—Type 9901](image)

4.3.2.1 Calibration for Tekscan Tactile Pressure Sensors

Experimental investigation of the load conditions on buried columns involves shear stresses that cannot be ignored during loading, storage and reclaim processes. Tactile pressure sensors are designed to depict normal stress only; thus, methods of minimising shear stress effects should be adopted to achieve reliable results. Laboratory tests were undertaken by Palmer et al. [68] to evaluate shear effects, and demonstrated that the method of placing two layers of Teflon sheets above the tactile sensor can reduce shear
stress effects significantly, without affecting the normal stress measurement. Therefore, based on the findings from Palmer et al., two thin Teflon sheets were applied in both the calibration and experiment tests to create a low-friction sliding plane.

Before the calibration process, conditioning and equilibration were performed. Due to the difficulty of applying a composite of normal and shear stress, similar to the real situation in experiments, only normal force was applied on the sensor in all calibration tests. The calibration process was conducted for each experiment based on the changes in the surrounding environment and the changes of the pressure sensor itself. Two different approaches were employed to produce this normal load on sensors: dead loads and water.

4.3.2.1.1 Calibration with Dead Loads

It is suggested that the best possible calibration results for sensors can be achieved by mimicking the real situations in experiments to the greatest extent, such as the time of loading, manner of loading, contact situation between bulk solids and sensor surface. To achieve this in the laboratory tests, the test column was placed in a shallow U-shaped trough with open ends, and the test sensor was placed above the column face and covered with two thin Teflon sheets. This shallow U-shaped trough was only around one centimetre higher than the Teflon sheet surface, which enabled a thin layer of test material to be laid evenly above the Teflon sheet surface to create a similar material interface and similar stiffness of the interacting surface to the experimental application. This column was the same as that buried in the material during experiments. Following this, dead loads were evenly piled above the test material to produce a known normal force on the sensor. The calibration setup is shown in Figure 4.9.
Several different calibration types are available for Tekscan I-Scan software, such as linear calibration, 2-point power law calibration and multi-point calibration. Linear calibration is comparatively the simplest method, but has lower accuracy because only one linear interpolation is performed between zero load and one known calibration load. Due to the heterogeneous characteristics of bulk solids, it is difficult to apply multiple known loads evenly above the sensor to conduct the calibration process in accordance with power law. In addition, the measurement loads likely to occur in experiments can vary considerably, which results from the significantly different contact situations between particles and sensels during testing. Therefore, 2-point power law calibration was applied, with this setup a compromise between the low accuracy of linear calibration, and failures in the performance of multi-point calibration.

In 2-point power law calibration, two different known loads are exerted on the sensor, and two points of approximately 20 and 80% of the expected maximum test load occurring in experiments are recommended. However, the interface between the thin layer of test bulk material and sensels is not homogeneous, even if all procedures are performed evenly in actual operation, and this causes uneven pressure distribution over
the sensor length, as presented in Figure 4.9. In addition, lower pressure usually results in more sensels in an inactive state that does not correspond with the load exerted, which can make the pressure distribution for calibration more uneven. Therefore, 40 and 80% of the expected maximum test load (less than 20 kPa) were applied with this calibration setup. A power law interpolation was performed based on zero load and the two known calibration loads by applying the equation \( y = ax^b \).

### 4.3.2.1.2 Calibration with Water

Another approach for calibration tests is the application of water. Water does not produce the same contact situation as experiments, but can produce an ideally even pressure distribution on the test sensor, which is a big advantage compared with the dead loads calibration setup. Figure 4.10 shows the calibration setup. A Perspex tube with a diameter of 0.06 m was filled with water at different heights to generate the required pressures, and the bottom of the tube was sealed with a layer of very thin elastic rubber to facilitate the pressure transmission. The test sensor was covered with two thin Teflon sheets, and the tube was placed vertically above them. As the length of the sensor was much bigger than the tube diameter, equilibration had to be conducted before calibration to compensate for these slight differences among all individual sensels. All 96 sensels could be assumed identical after equilibration, in terms of the sensitivity responding to pressures. Therefore, the calibrated sensels within the tube diameter could represent all sensels for the test sensor.
Multi-point calibration method was applied, also based on the power low fit algorithm similar to the 2-point power law calibration. This method takes advantage of additional reference points to further finetune the two-point calibration process. Based on the studies of Tekscan sensor behaviour, the power law curve best matches the steady-state non-linear physical behaviour of the sensor [76]. More than 10 different calibrated loads—ranging from about one to 15 kPa—were applied by filling water into the tube at specified levels in order to find the best power low fit of the calibration points. The maximum pressure likely to occur in experiments of this scale is approximately between 15 to 20 kPa; thus, this setup had the advantage of a wide calibrated pressure range.

Based on the calibration results, the two calibration methods—each with their own strengths and weaknesses—produced similar results for the three bulk solids employed. In terms of applying the two methods for other bulk solids, the dead loads method is more appropriate for bulk materials with a wide range of particle sizes, especially with a certain percentage of coarse particles, which can better simulate the real interface between particles and sensels. However, the water method can be used for fine particles such as the Beach Sand (the experimental results using Beach Sand will be discussed in Chapter 5), whose interface with sensels can be approximated by the interface between water and sensels.
4.3.2.2 *Time-dependent Effect*

Tactile sensors exhibit drift or creep even when a constant load is applied on them. It is widely proven that the specified force increases slightly after a time. This drift usually lies between 0 to 3% per log time. Palmer et al. [66-68] reported that the drift starts at approximately 120 seconds after loading, which means that reasonably accurate pressures can be obtained if both calibration and measurement are taken at 120 seconds. However, in the case of sensor response for longer durations of measurement, this notable pressure creep should be subtracted over time from the measured pressure to reduce the effect of sensor drift, or to mimic the duration of time of the application when sensors are calibrated. For all experimental results with the application of Tekscan tactile pressure sensors, the time-dependent effect was taken into account by subtracting the pressure drift over time. This drift was determined with relevant calibration tests.

4.4 Experimental Results and Discussion

4.4.1 Flow Properties Tests

The flow properties of Iron Ore B were determined for a moisture content of 7.5%. The results for bulk density $\rho$, effective angle of internal friction $\delta$, wall friction angle $\phi_w$ and PSD for Iron Ore B were discussed in Chapter 2. Parameters $\rho$, $\delta$ and $\phi_w$ were approximated as two-term exponential functions of the major consolidation stress $\sigma_1$ to best describe these varying properties and facilitate the subsequent theoretical calculations.

4.4.2 Experimental Pressures on Buried Column Faces: Static Results

Pressure measurements were performed on both column front and rear faces under three different conditions: (i) instantaneous pressure distributions immediately after loading; (ii) pressure distributions when a static state was approached after the test material was discharged from the stockpile under gravity and (iii) pressures after an undisturbed storage time. Six tests were conducted for each condition under similar operation circumstances. A single frame of Tekscan recorded data was analysed for each test when
each condition approached its static state macroscopically. Therefore, the results presented in this section are defined as the ‘static results’.

The interaction surface between particles and Tekscan individual sensels is unpredictable, especially for coarse particles. One possible illustration of the interface is presented in Figure 4.11, where the size of Iron Ore B is up to 8 mm. As a result, the test results varied considerably and were usually slightly scattered. An example of these Tekscan raw pressure distributions over the sensor length is shown in Figure 4.12. To minimise the pressure variations resulting from the general heterogeneous nature of the interface, as well as the influence of the small size of sensels compared with the largest size of test materials, an average mean was adopted for each test by averaging the value of each six sequential sensels with an overlap of three sequential sensels. The improvement induced by the averaging method is demonstrated in Figure 4.13.

Two curve-fitting methods were also applied in order to smooth the discrete measured pressures and facilitate the comparison between the test results and theoretical pressures, and they are exponential fitting and polynomial fitting methods. Due to the large material surface and varying ambient conditions, it was difficult to keep the moisture content of the test material constant, and the moisture content during all tests varied between 6.9 to 7.7%. Nevertheless, it was still feasible to take the flow properties of Iron Ore B at a moisture content of 7.5% into the theoretical calculations for the pressure distributions on the column.

![Image: Illustration of the Interface of Coarse Particles and Tekscan Sensels](image-url)
4.4.2.1 Condition I: Instantaneous Pressures on Buried Column Faces After Loading

The instantaneous pressures on column faces were measured immediately after the loading process was completed. Six typical tests with similar stockpile heights and buried depths for the front and rear faces of the column were analysed, and the results are shown in Figures 4.14 and 4.15, respectively. Table 4.1 summarises the test parameters for conditions (I) and (II).
Table 4.1: Test Parameters for Cases I to VI

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case no.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of repose $\theta_R$ (°)</td>
<td></td>
<td>40.5</td>
<td>40.1</td>
<td>40.3</td>
<td>40.2</td>
<td>40.1</td>
<td>40.2</td>
</tr>
<tr>
<td>Moisture content (%)</td>
<td></td>
<td>7.7</td>
<td>7.0</td>
<td>7.1</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>Average height of stockpiles (m)</td>
<td></td>
<td>0.839</td>
<td>0.868</td>
<td>0.947</td>
<td>0.925</td>
<td>0.905</td>
<td>0.920</td>
</tr>
<tr>
<td>Average buried depth—front face (m)</td>
<td></td>
<td>0.704</td>
<td>0.691</td>
<td>0.763</td>
<td>0.760</td>
<td>0.760</td>
<td>0.770</td>
</tr>
<tr>
<td>Average buried depth—rear face (m)</td>
<td></td>
<td>0.632</td>
<td>0.538</td>
<td>0.667</td>
<td>0.680</td>
<td>0.670</td>
<td>0.690</td>
</tr>
<tr>
<td>Buried depth of discharge—front face (m)</td>
<td></td>
<td>0.424</td>
<td>0.578</td>
<td>0.578</td>
<td>0.350</td>
<td>0.310</td>
<td>0.280</td>
</tr>
<tr>
<td>Buried depth of discharge—rear face (m)</td>
<td></td>
<td>0.578</td>
<td>0.578</td>
<td>0.578</td>
<td>0.680</td>
<td>0.650</td>
<td>0.590</td>
</tr>
</tbody>
</table>

The pressure distributions on the front face in Figure 4.14 presented significant differences in both the magnitude and tendency in all six cases, although all tests were conducted under similar operation conditions and similar buried depths. Theoretical pressure distributions for both the front face and rear face of the buried column were calculated using the design equations discussed in Section 4.2.2. The pressure distribution observed in Case I indicated that a passive stress state was established surrounding the front face over the full buried height of the column within the bulk material. The results of Case IV presented an active stress state for the column front face, and matched well with the theoretical curve of the active case. The other pressure distributions in Cases II, III, V and VI all indicated a passive profile in the upper section of the buried height, while, for the lower part of the column, the pressure increased nearly linearly, which was characteristic for the active stress state. This allowed the assumption that a combination of passive and active stress states was developed for these four cases, with a passive stress state forming in the upper section of buried column, and an active stress state occurring at the lower part.
Figure 4.14: Instantaneous Pressures on Column Front Face Immediately After Loading for Cases I to VI

The instantaneous pressures measured on the rear face are shown in Figure 4.15. A general pressure increase with increasing buried depth was observed for all six cases, which agreed well with the predictions. This proved that an active stress state dominated the load condition on the rear face of the buried column. However, the pressure magnitudes varied slightly, which could be explained by the general heterogeneous behaviour of bulk materials. Compared with the measured pressures on the front face, the pressure distributions on the rear face showed a totally different pattern, although the pressure maxima in both stress states were of the same magnitude. This indicated that
the pressure on the column rear face could not be neglected, although a slightly lower buried depth was obtained on the rear face of the buried column.

![Figure 4.15](image)

**Figure 4.15: Instantaneous Pressures on Column Rear Face Immediately After Loading for Cases I to VI**

4.4.2.2 Condition II: Comparison of Pressures After Loading and After Reclaim

After the required amount of test material was loaded on the table, the material rested for few minutes. The gate was then opened to discharge the material by gravity through a rectangular outlet in the base surface with dimensions of $0.410 \times 0.150$ m. A stable
rathole was formed when the discharge ceased, and the support column was still partly buried in the remaining material. The pressures on the front and rear faces were recorded immediately after the flow ceased. Six test cases are described in this section, and shown in Figure 4.16 for the front face and Figure 4.17 for the rear face.

The pressures on the front face changed significantly after the material was discharged. In all six tests, it was reasonable that the pressures on the front face in the upper buried section of the column decreased sharply, while the pressures on the lower part reduced slightly, especially for Cases I, III, IV and V, which appeared to present a pure active stress state after reclaim. This phenomenon may have arisen from the flow behaviour of the material. When the gate was opened, the material surrounding the front face of the column faced towards the outlet and began to flow first. The bulk material in the upper section of the pile featured a higher porosity than in the higher consolidated base section. Therefore, there was a higher tendency for the material in the upper part of the pile to collapse and initiate the flow; thus, the pressure reduced more noticeably at the upper part of the column front face than at the lower part. However, a significant pressure increase at the lower part of the column front face was observed in Case I. This may have resulted from a sudden change of the stress state from a pure passive stress state before flow to a pure active stress state after flow was initiated. The other two tests, Cases II and VI, exhibited a combination of two stress states before flow, as discussed in condition (I), and this pressure trend remained similar after the discharge process. Overall, the stress state dominating the load conditions for the column front face after discharge was more likely to be an active stress state or, if a combination of both stress states occurs, it was more likely to be the same stress state as before the flow.
The corresponding six tests of the pressures on the column rear face after reclaim are presented in Figure 4.17. These six test results showed three different patterns of pressure distributions after reclaim. Cases I and II presented similar magnitudes and pressure trends as those before flow, which indicated that the stress state at the rear face of the buried column did not change when flow was initiated, and remained an active stress state. The pressures on the column rear face after reclaim illustrated in Cases IV and V exhibited an apparent pressure increase in the upper part of the buried column, and pressure decrease near the bottom. The two pressure trends showed that the active stress state before discharge changed to a passive stress state after reclaim. The third
pattern was observed in Cases III and VI, where a combination of both stress states was established after reclaim from the active stress state before discharge.

Figure 4.17: Pressures on Column Rear Face Before and After Reclaim for Cases I to VI

Moreover, it was interesting to observe that all six pressure trends of the front face before reclaim were similar to the ones for the rear face after the flow ceased. Since the change of pressure highly depends on the change of stress state, it can be assumed that, once the gate was opened, there was a tendency for all the material near the opening to flow towards the opening. This resulted in the load condition on the column rear face
after reclaim being similar to the load condition on the front face before reclaim. This meant that the stress state developed in the material surrounding the column rear face after reclaim could also be one of the three situations, similar to the front face after loading: active stress state, passive stress state or a combination of both.

Based on the analysis of all test cases, it can be concluded that the stress state on the column rear face after flow ceased was the same as the stress state on the column front face before reclaim. In contrast, the stress state on the front face after the material was discharged tended to change to an active stress state, or remained the same if it was a combination of both stress states before flow.

4.4.2.3 Condition III: Experimental Pressures on Buried Column Faces After Time-consolidation

The third test series was conducted to investigate the effect of time-consolidation. During an undisturbed storage time, the load settlement may continue for several days until a critical consolidation condition is approached asymptotically. The amount of load settlement depends on the stockpile dimensions, properties of the handled bulk material and manner of loading. Six tests are studied in this section, three of which were conducted with a short storage time of slightly more than 10 minutes, and the rest were performed with a longer storage time of up to four days. The test parameters for all six cases are listed in Table 4.2. Figures 4.18 and 4.19 show the measured pressures on the column front and rear faces of the buried column after consolidation of 17 minutes, 11 minutes, 14 minutes, 37 hours, 91 hours and 67 hours, respectively, with the relevant instantaneous pressures plotted for comparison. The pressure increase due to the sensor drift itself was subtracted from all measured pressures.

The pressure trends on both the front and rear faces were most likely to remain nearly unchanged after consolidation for both the short and long storage tests. In the short storage tests, the effect of internal creep on the consolidation of bulk materials is illustrated by Cases IV, V and VI in the two figures for the column front face and rear
face. Although the period from completing loading to before initiation of reclaim was only 10 to 20 minutes, the pressure increases were very small, but could still be observed.

For long periods of consolidation, noticeable pressure increases were observed in Cases VII and IX for both the front and rear faces, as shown in Figures 4.18 and 4.19, respectively. This behaviour is a typical example of internal creep due to shear in the stored bulk material over an extended period of storage. It is reasonable that the pressure increase in Case IX was more than that in Case VII due to its longer storage time. However, the pressures in Case VIII for both front and rear faces exhibited little change after time-consolidation of over 91 hours. This indicated that the effect of the time-consolidation within the stored material was very weak in this particular case. This is uncommon for the storage of bulk materials, but does occur. This uncommon phenomenon may have been because there was variation in the loading pattern from the other two cases, which can result in changes in many parameters, such as particle segregation, material porosity, impact stresses and the manner of particle packing. All of these influences may have made it is possible for Case VIII to approach its critical consolidation condition directly after loading, so the pressure nearly did not change even after 91 hours of storage. Further, it is also possible for a change of stress state to occur, such as in Case VI, from a combination mode to a passive stress state, even though only stored for 14 minutes.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case no.</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of repose $\theta_{R}$ (°)</td>
<td></td>
<td>40.2</td>
<td>40.1</td>
<td>40.2</td>
<td>40.5</td>
<td>40.1</td>
<td>40.3</td>
</tr>
<tr>
<td>Moisture content (%)</td>
<td></td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.0</td>
<td>7.1</td>
<td>6.9</td>
</tr>
<tr>
<td>Stored time (m and h)</td>
<td></td>
<td>17 m</td>
<td>11 m</td>
<td>14 m</td>
<td>37 h</td>
<td>91 h</td>
<td>67 h</td>
</tr>
<tr>
<td>Height of stockpiles—instantaneous (m)</td>
<td></td>
<td>0.925</td>
<td>0.905</td>
<td>0.920</td>
<td>0.897</td>
<td>0.986</td>
<td>0.952</td>
</tr>
<tr>
<td>Height of stockpiles—after storage (m)</td>
<td></td>
<td>0.922</td>
<td>0.904</td>
<td>0.918</td>
<td>0.887</td>
<td>0.960</td>
<td>0.922</td>
</tr>
<tr>
<td>Buried depth—front face—instantaneous (m)</td>
<td></td>
<td>0.760</td>
<td>0.760</td>
<td>0.770</td>
<td>0.657</td>
<td>0.762</td>
<td>0.762</td>
</tr>
<tr>
<td>Buried depth—front face—after storage (m)</td>
<td></td>
<td>0.760</td>
<td>0.760</td>
<td>0.770</td>
<td>0.670</td>
<td>0.762</td>
<td>0.747</td>
</tr>
<tr>
<td>Buried depth—rear face—instantaneous (m)</td>
<td></td>
<td>0.680</td>
<td>0.670</td>
<td>0.690</td>
<td>0.532</td>
<td>0.632</td>
<td>0.632</td>
</tr>
<tr>
<td>Buried depth—rear face—after storage (m)</td>
<td></td>
<td>0.680</td>
<td>0.670</td>
<td>0.690</td>
<td>0.590</td>
<td>0.637</td>
<td>0.637</td>
</tr>
</tbody>
</table>
Figure 4.18: Pressure Drift Over Storage on Column Front Face for Cases VI to IX

The results of all test cases for the time-consolidation effect also revealed three different stress states for the front face in the state of storage, while a common pressure trend for the rear face indicated that an active stress state developed, as shown in Figure 4.19. Although the pressure increase during storage was found to be highly dependent on the manner of loading, the pressure results confirmed that time-consolidation due to creep of the bulk material can continue for several days after a stockpile is formed. Therefore, the influence of the storage time on the pressures exerted on the buried column must be considered in the design of support structures.
4.4.3 Experimental Pressures on Buried Column Faces: Dynamic Results

Since Tekscan tactile pressure sensors allow real-time continuous data collection, the dynamic changes of pressure distributions on column faces during the loading and discharge processes were captured in the experimental measurement. This greatly helped obtain a better understanding of the formation of the stress conditions within the bulk materials during the dynamic handling processes. The relevant data processing was conducted by including some typical discrete pressure distribution frames in the calculation from the continuous recorded Tekscan movies. The averaging method described in the section of static results was also adopted. In considering the
unavoidable variations resulting from the characteristics of bulk solids and operation conditions, two tests for each column face were analysed both for dynamic loading and discharging, as discussed in the following sections.

4.4.3.1 Dynamic Changes of Pressure Distributions on Column Front Face

In addition to the static pressure results on the front face of the buried column after loading, as discussed previously, the dynamic changes of pressure distributions on the front face during the loading process were investigated, and the results for the two tests are shown in Figure 4.20. Five typical discrete Tekscan frames were calculated from the beginning of loading until a full stockpile was formed. The theoretical pressure distributions for the full size of stockpile under the two stress states are presented for comparison.

![Dynamic Changes of Pressure Distributions on Front Face—Loading Process](image)

The results presented in Figure 4.20 revealed that, during the dynamic loading process, the loads exerted on the column front face increased with increasing stockpile size. The five discrete Tekscan frames for each test described the development of the pressure distribution with the stepwise increase in stockpile size. According to the comparison of these stepwise increasing pressure distribution trends, the stress state on the column
front face could be an active stress state from a small size stockpile, and remained the same until the loading process was finished, as demonstrated in test 1. In addition, the stress state could develop from an active stress state at the initial loading stage to a combination of both active and passive stress states when a fully loaded stockpile was obtained, as in test 2.

The results from the two test examples confirmed that the stress state can change within the bulk material during the loading process, which means that the pressure distribution on the column front face can vary and experience different stress states until the loading is finished, when a final pressure distribution is achieved. Moreover, the results of the two tests indicated that an active stress state or a combination mode can possibly occur on the front face after loading, which are two of the three states discussed in the section on static results Section 4.4.2.1, while the third state is the passive stress state.

The dynamic resulting force acting on the column front face for each test was also calculated, and both tests showed a gradually increasing trend during the loading process, as shown in Figure 4.21. There was a slight difference in the increasing trends of the results force for the two tests. This resulted from variations in loading conditions, such as the speed of the material flow, flow rate, particle sizes and shapes contacting the sensor during the loading process, and—most importantly—variation in the stress state. The different stress states that developed after loading—that is, the active stress state for test 1 and the combination state for test 2—explained why the resulting force for test 2 was larger than for test 1, even with a similar buried depth for the column front face.
When the gate underneath the stockpile base was opened, the flow was initiated. Since the time for discharge under gravity through the small opening (0.15 × 0.41 m) was very short and the pressure could change very quickly, more recorded pressure frames were examined to look into the detailed dynamic pressure changes occurring on the column front face during the reclaim process. Twelve typical discrete pressure frames were analysed within the time range between the start and termination of the flow, as shown in Figure 4.22. In the results of test 1 for the active stress state before flow, the pressure decreased gradually and the same stress state was kept at the beginning of reclaim. This active stress state changed into a combination mode afterwards, which remained for several seconds then finally evolved into an active stress state until the end. However, the dynamic pressure distributions in test 2 experienced a stress state change from the combination of both stress states before the initiation of the flow, to a passive stress state when being discharged halfway, and to an active stress state when flow ceased. Some avalanches occurred during discharge in test 2, which led to irregular flow rates and made the pressure drop drastically within the time range of 58 to 60 seconds.

All these results proved that an active stress state is most likely to occur as the dominant stress state on the column front face after reclaim. The evolution of the stress state began when flow was initiated, and this active stress state in the end of reclaim can evolve from the active stress state or other stress states. In all cases, the stress state always experienced more than one change, rather than direct change, before reaching its final
state. This confirmed that the stress state usually undergoes one or more intermediate stress states during the dynamic process of reclaim before an active stress state is reached at the end.

![Figure 4.22: Dynamic Changes of Pressure Distributions on Front Face—Reclaim Process](image1)

![Figure 4.23: Dynamic Resulting Force on Column Front Face—Reclaim Process](image2)

During the reclaim process, the dynamic resulting forces acting on the column front face for the two tests were calculated, as shown in Figure 4.23. The resulting force in test 1 experienced a generally gradual decrease before reaching a static state, while test 2 remained nearly unchanged before the force dropped, which verified that some avalanches occurred in test 2 during reclaim. However, a dramatic force jump was
observed in each test, which increased steeply to a peak and decreased to the bottom immediately. This may have been because the stress state during the reclaim process changed to another stress state mode and made the force jump occur. This usually happens when a stress state changes to an intermediate stress state, or an intermediate stress state changes to the final stress state near the end of the discharge.

According to the residual resulting forces of the two tests, there was still some test material surrounding the column that exerted the loads on the front face of the support column. This coincided with the corresponding pressure distribution results at the end of reclaim for the two tests, as described in Figure 4.22. Moreover, the results of the two tests verified that an active stress state may occur on the front face after reclaim, which was one of the two states discussed in Section 4.4.2.2. The second state discussed was the combination of both stress states after reclaim, if a combination mode is developed after loading.

4.4.3.2 Dynamic Changes of Pressure Distributions on Column Rear Face

To investigate the dynamic changes of the stress state developed on the column rear face during the handling processes, the same analysis procedures as the column front face were conducted. As shown in Figure 4.24, for dynamic pressure changes on the column rear face during the loading process, both tests demonstrated that the stress state remained the same with a small buried depth at the initial loading stage, to a full buried depth when loading finished. It may be reasoned that the unchanged stress state (which differed from the front face) resulted from the back-filling effect, which caused the mechanics in the bulk material to remain unchanged when the material piled up during loading.
Corresponding to the unchanged stress state on the column rear face during loading, the resulting forces for the two tests presented a steady increasing trend, as shown in Figure 4.25. Compared with the front face, the fluctuations of the resulting forces on the rear face during loading were smaller because the impact effect acting on the rear face during loading was smaller.
Figures 4.26 and 4.27 show the dynamic pressure distributions and resulting forces on the column rear face, respectively, during the process of reclaim. The buried depth for the column rear face did not change much after the material was discharged under gravity, which is why the magnitude of pressure did not decrease much after reclaim. According to the 12 trends of pressure distributions, it was observed that the pressure at column upper part increased considerably, and decreased considerably at the bottom. This indicated that an active stress state had evolved into a passive stress state in test 1, and a combination of both stress states in test 2, after reclaim was completed. Moreover, the results of the two tests showed that a passive stress state or a combination mode could occur after reclaim, which are two of the three states discussed in Section 4.4.2.2 on static results related to the static pressures on the column rear face before and after reclaim. The third state is the active stress state.

Different to the front face, the resulting forces for both tests experienced a steady increase after a sharp force drop occurred when the flow ceased. This may have resulted from the change of stress state in the remaining bulk material from an active to other stress state. The fluctuations in the resulting forces on the rear face were bigger for the reclaim case than for loading because the load condition on the column rear face during reclaim can be considered that on the column front face during loading, which also exhibited this level of fluctuations due to the impact effect. As shown in Figure 4.27(a),

![Figure 4.26: Dynamic Changes of Pressure Distributions on Rear Face—Reclaim Process](image)
the step-like change of the force in test 1 was caused by the avalanches occurring during the flow of the bulk solid, which is a common phenomenon when handling bulk solids.

![Graph showing force over time for Test 1 and Test 2](image)

**Figure 4.27: Dynamic Resulting Force on Column Rear Face—Reclaim Process**

### 4.5 Discussion

Although significant variations were observed for all measured pressures, based on the analysis of the static and dynamic results, the measured pressure distributions matched qualitatively with the theoretical calculations on column two faces in both magnitudes and pressure trends, particularly for the rear face. A number of possible explanations can be given for the pressure variations observed:

- the low spatial resolution of the applied sensor type (1.6 sensing elements per cm)
- the low sensitivity of the used tactile sensor type in the pressure range below 10 kPa
- the general heterogeneous behaviour of granular material induced by the irregular distribution of particle shape and particle size
- variations in the flow rate during loading.

The pressure distribution on the front face and the resulting stress state varied noticeably in all tests conducted, although the same test parameters and same test material were employed. When a passive stress state was developed in the handling material, the wedge-shaped load condition on a structure buried in a stockpile was comparable to the
stress state developed in a hopper in the flow case, as illustrated in Figure 4.4(b). Therefore, the pressure on the front face of a vertical buried column decreased with increasing depth after reaching a pressure maximum close to the half-height of the buried depth.

The pressure in the active stress state behaved differently to the pressure distribution in the passive stress state. The pressure increased until a peak pressure was reached near the bottom, then decreased gently. When an active stress field developed, the major consolidating stress acted predominantly in a steeply inclined direction. This load state approximated the stress field in the cylindrical section of a storage bin for the initial filling case [28,36], where the pressure on the bin wall increased with increasing depth. Close to the stockpile base, the material interface changed from particle-to-particle contact to particle-to-wall-surface contact, thereby inducing a gentle decrease of pressure near the bottom region. During all test results, the pressure trends with the feature of a lesser passive-shaped trend in the upper part of the column and an approximately linearly increasing trend in the bottom part—such as Cases II and VI (in Figure 4.14)—indicated a combination of active and passive stress states.

Among all possible combinations, only those with a passive stress state at the upper part and an active stress state at the lower part of the buried column were observed in all measured results. However, it was difficult to predict where the transition of the two stress states was located. As a result, multiple combinations of active and passive stress states could develop within the bulk material, which explains why the pressure distributions on the column faces under the action of combinations of both stress states varied greatly. According to all experimental tests conducted, the situations for the stress state dominating the pressure distributions on column faces are summarised as shown in the flow chart in Figure 4.28.
The pressure distributions on the two column faces from both the static and dynamic results during all handling processes were investigated. Based on the analysis of these results, the stress state dominating the load condition on the front face of the buried support column after loading process can be one of the three stress states (active, passive or a combination of both). During the dynamic process of loading, an active stress state was developed on the front face at the initial stage of loading, and remained the same or changed to one of the other two at the final stage. The resulting force exerted on the front face after loading highly depended on which stress state developed.

After the bulk material was discharged under gravity, the stress state was most likely to be active, or remain as a combination if a combination mode occurred before reclaim. However, the change of stress state was not direct, and usually experienced one or more intermediate stress states during the dynamic process of reclaim, until an active stress state or combination mode developed at the final stage. During this dynamic process, the resulting force underwent a dramatic force jump that increased steeply to a peak and dropped to the bottom immediately when the stress state changed to either an intermediate stress state or the final stress state. This must be considered in the design of these support structures.

Figure 4.28: Flow Chart for Load Conditions on Column Front and Rear Faces
For the column rear face, an active stress state played a dominant role in the load condition for the loading process, and the stress state remained unchanged until the loading was completed. This resulted from the back-filling effect occurring on the rear face during loading, which introduced nearly unchanged mechanics in the bulk material. When the reclaim process began, this active stress state either remained the same or changed into a passive or combination state. This confirmed that the mechanics during the reclaim process for the column rear face were equivalent to those occurring during the loading process for the front face, so three stress states were able to occur. In addition, due to the change of stress state near the termination of discharge, the resulting force acting on the column rear face experienced a short steady increase just after the force dropped to the bottom.

Further, the pressure trends after an undisturbed storage over a period largely remained the same for both the front and rear faces, regardless of the length of time. Nevertheless, the pressure magnitudes on both faces of the buried column exhibited a significant increase due to internal shear in the stored material, if the bulk materials were stored for a long time.

4.6 Conclusions

The loads exerted on support columns partly buried in bulk material stockpiles are complex and difficult to measure in an experimental manner. These loads are not consistent because numerous variations exist, such as the strength and flow properties of bulk solids, manner of loading and discharging, and rigidity and geometrical dimensions of the support structure buried in the bulk materials.

Pressure measurements were undertaken for three different conditions on both the front and rear face of the buried column with Tekscan tactile sensors, and both static and dynamic results were analysed. Despite the obvious fluctuations in the measured results due to the existing limitations in the tactile sensors employed, and the heterogenic characteristics of granular materials, it was noted that the experimental pressure distributions showed reasonable agreement with the theoretical approach of the refined
Roberts’s theory. In addition to the static results, the analysis of the dynamic pressures provided a thorough understanding of how the stress state changed during the handling processes by evaluating these dynamically changing pressure distributions.

During the dynamic process of loading, the final stress state developed surrounding the front face of the buried column could be one of any of the three stress states. The final stress state usually evolved from an active mode at the initial loading stage, while the active stress state consistently dominated the load condition on the rear face. After the material was discharged under gravity, the stress state on the front face was most likely to be active, or remain a combination, if a combination mode occurred before reclaim. In contrast, any of the three stress states could occur on the rear face because the reclaim process for column rear face was equivalent to the loading for the column front face. However, the change of the stress state was not direct, and usually experienced one or more intermediate stress states until the final stress state was achieved.

The jumps or drops in the resulting forces on the column faces when the stress states change must be considered for design purposes. In addition, the pressure increase due to load settlement during storage must be taken into account for determining loads on buried support structures if the handling material is stored for a long period. Due to uncertainty in the possible variations in the loading conditions in each application, it is recommended to use a conservative approach for the design of support structures buried in bulk materials. By using an envelope of both active and passive pressure curves as the upper pressure limit, as illustrated in Figure 4.5(c), all possible combined stress states on the column faces will be enclosed within this upper bound curve. The pressure maximum of each stress state must be considered in this approach in order to achieve maximum reliability of the support structures buried in stockpiles.

Therefore, the analysis presented here clearly demonstrates that the refined Roberts’s calculation method is a valuable tool to estimate loads on support columns. It is believed that this research work has contributed important findings to the development of a better understanding of the load conditions exerted upon buried support structures in bulk material stockpiles and storage sheds.
Chapter 5: Further Investigation of Loads on Fixed and Moveable Support Columns Buried in Stockpiles with Two Materials

5.1 Introduction

Chapter 4’s experimental investigation of the load conditions exerted on a fixed support column with Iron Ore B helped modify Roberts’s existing continuum theory [55]. These modifications were verified by a number of tests in the application of pressure prediction for cohesive handling materials. As investigated in Roberts’s theory and several previous studies [18,40,77-82], the flow properties of bulk materials can significantly contribute to the variation of the stress state developed in stored material, or flow patterns occurring in the containing vessels. For this reason, in addition to the cohesive Iron Ore B, a different test material was investigated in this study: free-flowing Beach Sand. Both materials were employed to demonstrate how load conditions behave on support columns buried in bulk materials, and to further verify the refined Roberts’s approach.

The flow properties of the two test materials are significantly different, which facilitated a more comprehensive understanding of the load performance on buried columns that can occur in practice. Tekscan tactile thin-film pressure sensors (Tekscan, Inc., Boston, MA) were employed in this study. Two different column setups were tested. A ‘fixed’ column was restrained with no degree of freedom, with a focus on determining the loads exerted on the front and rear face of the buried column. Although Tekscan tactile pressure sensors can provide accurate results for things such as pressure measurements in granular matter or soils (in previous studies) [66-69] and applications with Iron Ore B (discussed in Chapter 4), it will be more than convincing to confirm this quantitatively regarding the reliability of this type of sensor. For this reason, an alternative arrangement was used—a ‘laterally moveable’ column setup.
This column setup incorporated both load cells and pressure sensors, and thus enabled comparison of the test data from the load cells and Tekscan tactile sensors. More importantly, this setup facilitated investigation of the lateral and vertical shear forces occurring on the column faces, which could be conducted with the fixed column setup with only pressure sensors employed. Since the flow rate \([69]\) and anisotropic behaviour of bulk materials \([40,82]\) can contribute to different stress conditions, this experimental study involved two different locations for the fixed column setup. The normal vector of the column front face was parallel or perpendicular to the flow direction, with the aim of investigating the effect of flow rill\(^2\) on the load conditions developed.

Chapter 4 discussed the pressure distributions on the front and rear faces of the support column buried in iron ore stockpiles, and a few modifications were made to Roberts’s approach based on the test results. The objective of this chapter is to further verify the refined Roberts’s approach via an experimental study with another column setup and two test materials. It also seeks to investigate the influence of material cohesion on the pressure distributions over column faces based on the comparison of experimental results from cohesive Iron Ore B and free-flowing Beach Sand. Moreover, this chapter looks into the shear stresses acting on all column faces by analysing the test data from tactile sensors and load cells.

### 5.2 Continuum Load Analysis for Buried Structural Elements

The calculation model proposed by Roberts uses the classical continuum theory to analyse the varying load conditions exerted on the surfaces of a structural element buried in bulk materials. Chapter 4 discussed the details of the load analysis model and the constitutive differential equation deduced from this model. Load conditions along buried columns highly depend on the stress states developed in the stored bulk solid in the immediate region of the buried columns. Two separate load conditions were defined by Roberts based on the wedge-shaped load analysis model. The analysis for the two separate stress states—active and passive stress states—was refined according to the

\(^2\) Here ‘rill’ refers to the flow over the surface as a stockpile is being filled. This terminology is adapted throughout this chapter.
previous experimental investigations elaborated in Chapter 4. Variations in the flow properties of the test materials were also taken into account in this study, and the same method as presented in Chapter 4 was adopted by using the two-term exponential fitting in the flow properties. Based on Roberts’s approach [55], some of the major modifications made in Chapter 4 are reviewed in this section. In addition, the normal loads acting on the column side faces and shear loads are analysed based on Roberts’s pioneering research.

5.2.1 Active Stress State Model

The modifications for the dimensions of the wedge shape and pressure ratios are shown as follows:

\[
\alpha_c = \alpha_s = \frac{\pi}{4} - \frac{c_4 \exp(c_2 \sigma_{1D}) + c_3 \exp(c_4 \sigma_{1D})}{2} \quad \text{where} \quad \sigma_{1D} = \lim_{z \to D} \sigma_1(z)
\]

\[
y_h = \frac{D}{\cos(\alpha_c)} + D \sin \left( \frac{\delta - \phi_w}{2} \right)
\]

\[
K_{ca} = (\sin \delta + 1) \left[ \frac{\tan \alpha_c}{\tan \alpha_c + \tan \phi_w} \right]
\]

\[
K_{sa} = (\sin \delta + 1) \left[ \frac{\tan \alpha_s}{\tan \alpha_s + \sin \delta} \right]
\]

The initial condition for the definition of the pressure is:

\[
p_y(0) = p_{y0} = 0
\]

5.2.2 Passive Stress State Model

The modifications of the dimensions of the wedge shape for passive stress state are reviewed in this section:

\[
y_h = \frac{D}{\cos \alpha_c}
\]

\[
\alpha_c = \alpha_s = \frac{\theta}{2} = \theta_R - \frac{c_4 \exp(c_2 \sigma_{1D}) + c_3 \exp(c_4 \sigma_{1D})}{2} \quad \text{where} \quad \sigma_{1D} = \lim_{z \to D} \sigma_1(z)
\]
\[ \sigma_1 = \frac{(1 + \sin \delta) p_{ns}}{1 + \sin \delta \cos \eta_4} \]  (5-8)

5.2.3 Normal Loads on Column Faces

5.2.3.1 Normal Pressures on Column Front and Rear Faces

As stated in Chapter 4 and shown in Figure 5.1, the normal pressure \( p_{nu} \) acting on the column front face is complex and highly dependent on the loading procedure and properties of the handling bulk materials. Hence, the stress state occurring on the front face can be an active stress state, passive stress state or combination of both during the process of loading for the two test materials. Some back-filling occurs on the rear side of the column simultaneously during loading. The pressure \( p_{nl} \) induced by the back-filling acts against the column rear face and helps reduce the lateral deflection of the column. An active stress state is mostly likely to develop on the column rear face due to the back-filling, which introduces fewer variations and less impact stress on the rear face compared with those on the front face in the loading. The degree of back-filling effect is highly dependent on the degree of material cohesion, and this effect is prone to be weakened by cohesive bulk materials. However, during the reclaim process, the flow of the material in the region of the column rear face is similar to the situation occurring near the front face for loading. Therefore, the load condition on the column rear face for the reclaim case can also be as complex as the front face for the loading case.

![Figure 5.1: Cross-section Loading Situation](image)

Note: Refer to Figure 4.2(a)

The theoretical normal pressure distributions under the two stress states were calculated for both Iron Ore B and the free-flowing Beach Sand for comparison. As shown in Figure 5.2, since there were great differences in the strength and flow properties of the two materials, both the magnitudes and pressure trends presented a significant difference
for each stress state with a buried depth of 0.7 m. The smaller bulk density of the free-flowing Beach Sand was the major cause of the lower peak pressures for both stress states. The location of the peak pressure under passive stress state was lower for the free-flowing Beach Sand, which may have resulted from its higher flowability than the Iron Ore B, which is cohesive and has a higher effective angle of internal friction.

(a) Theoretical Pressures for Iron Ore B   (b) Theoretical Pressures for Beach Sand

Figure 5.2: Theoretical Normal Pressure Distributions Under the Two Stress States

5.2.3.2 Normal Pressures on Column Side Faces

The side walls of a buried column are also subject to significant loads during the handling process. The normal pressures on the column’s two side faces \( p_{ns1} \) and \( p_{ns2} \) are assumed to be dominated by the active stress state. In the active stress state, the major consolidation stress \( \sigma_1 \) is in stockpile radial direction, which is parallel to the two side faces. However, the stress \( p_y \) contributing the normal pressures on the two side faces in the active stress state model directs to the side faces. That is, the stress \( p_y \) acting on the column side faces is in the circumferential direction, but with an inclined angle \( \varphi \) in respect to each side face. This situation is illustrated in Figure 5.3. According to Roberts’s hoop stress theory [15], this stress \( p_y \) in the circumferential direction can be correlated to the major consolidation stress \( \sigma_1 \) as follows:

\[
p_y = \frac{\sigma_1}{1 + \sin \delta}
\]

(5-9)
5.2.4 Shear Loads on Column Faces

During all material handling processes, buried structures are not only subject to normal loads on column faces, but also withstand significant shear loads, including vertical compressive shear loads and lateral shear loads.

5.2.4.1 Vertical Compressive Shear Force

The vertical shear stresses acting along the four column faces producing compressive loads give rise to a buckling effect on the column. The compressive force is calculated as follows, according to Roberts’s approach [55], not including the loads from the supported objects, such as the roof of a shed or a load-out belt conveyor:
Lateral shear forces exerted on the column front and rear faces can be ignored due to little movement or potential movement in lateral direction on these two faces. However, the lateral shear forces acting along the two side faces—denoted by $F_{s1}$ and $F_{s2}$ in Figure 5.1—must be considered because internal creep due to shear occurs in the stored bulk material. This internal creep is proportional to the stress condition on the column front face, as given below (per unit depth):

$$F_{s1} = F_{s2} = C_S p_{nu} d \tan \phi_w$$

(5-11)

$C_S$ is the coefficient for shear (normally $0 \leq C_S \leq 1.0$). $C_S$ is highly dependent on material cohesion, and cohesive bulk materials introduce smaller $C_S$ than free-flowing materials. Therefore, the total lateral force per unit depth on the column is given by:

$$F_t = F_u + F_{s1} + F_{s2} - F_1 = C_A p_{nu} b + 2 C_S p_{nu} d \tan \phi_w - p_{n1} b$$

(5-12)

where $C_A$ is the coefficient for material build-up on the column front leading face (normally $1.0 \leq C_A \leq 2.0$). $C_A$ is dependent on the flow property of test materials and width of the buried column.

5.3 Experimental Results

5.3.1 Experimental Setup

The same stockpile test rig as described in Chapter 4 was used in this study to investigate the load conditions on the buried columns. Two different buried columns were examined: a fixed column setup and a laterally moveable column setup. The fixed...
The laterally moveable column featured a square cross-plane with a dimension of 0.065 × 0.065 m, and four tactile sensors were equipped on all column faces. Since Tekscan tactile pressure sensors are designed to measure pressures normal to the sensor sheet, three load cells were installed on the column to facilitate the measurement of the shear loads exerted on the column, in addition to applying the pressure sensors. Moreover, the results from these load cells enabled checking the accuracy of the pressure sensor. The three load cells were installed at different locations, as indicated in Figure 5.5. The
column was suspended by the load cell denoted as ① to measure the total vertical draw-down force, and load cells ② and ③ were installed to allow the reaction forces parallel to the direction of flow rill at the top and bottom ends of the column, during all processes to be measured.

This column was supported with the necessary degrees of freedom in the direction of flow rill; however, the rotational movement and translational movement in the direction perpendicular to the flow rill were prevented by the flexure wire restraints that were applied at both the top and bottom of the column, as shown in Figure 5.5. A comparison could be made between the forces obtained from these load cells and pressure sensors in order to confirm the pressure results from the Tekscan sensors and analyse the shear stresses on the column faces. The resulting forces from all sensors in the comparison were calculated from the pressure distributions, according to the force and torque equilibrium for the laterally moveable column.

![Figure 5.5: Experimental Setup of Laterally Moveable Column with Load Cells Installed](image)

Note: ① Measures the total vertical draw-down force; ② measures the top lateral reaction force and ③ measures the bottom lateral reaction force

Calibration tests for all sensors were conducted by applying the water method for both Iron Ore B and Beach Sand, and the relevant procedures were detailed in Chapter 4.
Two layers of Teflon sheet were also applied on the sensor surface for both calibration and the experiment to minimise shear effects.

5.3.2 Flow Properties Results

The relevant tests for the flow properties of the cohesive Iron Ore B at a moisture content of 7.5% and the free-flowing Beach Sand were performed in the laboratory. As discussed in Chapter 2, a two-term exponential fitting method was applied in order to describe the correlation between the flow properties of the test materials and the major consolidation stress $\sigma_1$. For comparison of the two different test materials, bulk density $\rho$, effective angle of internal friction $\delta$, wall friction angle $\phi_w$ and PSDs were drawn in one graph for each parameter, as described in Figures 5.6 to 5.9. Significant differences in the bulk density, effective angle of internal friction and PSD between the two materials were observed. The cohesion character of Iron Ore B was reflected in the higher effective angle of internal friction. Due to the application of Teflon sheets, the wall friction angles for the two materials were similar.

![Figure 5.6: Bulk Density](image1)

![Figure 5.7: Effective Angle of Internal Friction](image2)
5.3.3 Comparison of the Pressures on the Fixed Buried Column between Two Materials: Static Results

For the fixed column setup, pressure measurements for the comparison between the two test materials were conducted immediately after the stockpile was fully loaded. The measurements were undertaken with the column fixed at the two different positions, as shown in Figure 5.4. Two tests producing similar stockpile heights and buried depths for the column faces at each column position for each test material are presented, regarding pressure distributions on the front and rear face of the buried column. All test parameters for the fixed column are summarised in Table 5.1. The same averaging method as described in Chapter 4 was adopted for each test to improve the visible scattering behaviour in the pressure distribution. This approach was to average the value of each six sequential sensels, with an overlap of three sequential sensels for each test. In addition, to facilitate comparison between the test results and theoretical pressures, curve-fitting tools were applied to make the discrete measured pressure curves smooth.
### Table 5.1: Test Parameters for Tests I to IV for the Fixed Column

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Test no.</th>
<th>Angle of repose (°)</th>
<th>Moisture content (%)</th>
<th>Height of stockpile (m)</th>
<th>Buried depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-flowing Beach Sand</td>
<td>Test I</td>
<td>31.10</td>
<td>-</td>
<td>0.737</td>
<td>Front 0.672</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rear 0.622</td>
</tr>
<tr>
<td></td>
<td>Test II</td>
<td>31.50</td>
<td>-</td>
<td>0.740</td>
<td>Front 0.670</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rear 0.597</td>
</tr>
<tr>
<td></td>
<td>Test III</td>
<td>31.65</td>
<td>-</td>
<td>0.730</td>
<td>Front 0.643</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rear 0.570</td>
</tr>
<tr>
<td></td>
<td>Test IV</td>
<td>31.20</td>
<td>-</td>
<td>0.735</td>
<td>Front 0.650</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rear 0.567</td>
</tr>
<tr>
<td>Cohesive Iron Ore B</td>
<td>Test I</td>
<td>41.30</td>
<td>7.85</td>
<td>0.842</td>
<td>Front 0.662</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rear 0.475</td>
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<tr>
<td></td>
<td>Test II</td>
<td>39.75</td>
<td>7.60</td>
<td>0.842</td>
<td>Front 0.666</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rear 0.589</td>
</tr>
<tr>
<td></td>
<td>Test III</td>
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<td>6.95</td>
<td>0.866</td>
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<td>0.950</td>
<td>Front 0.766</td>
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<tr>
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<td></td>
<td>Rear 0.666</td>
</tr>
</tbody>
</table>

5.3.3.1 Pressure Distributions on the Column Front Face with Two Materials at Two Column Positions

The results of the pressure measurements on the front face of the column are depicted in Figures 5.10 and 5.11 for both test materials at column positions 1 and 2, respectively. Although the experimental results showed apparent differences in both the magnitude and distribution under similar operating conditions and buried depths, reasonable agreement was found with the theoretical predictions that were also drawn in each test graph.
In addition, the pressures on the front face were slightly higher for the Iron Ore B than for the free-flowing Beach Sand. This could be explained by the higher bulk density and cohesive character of Iron Ore B, which was consistent with the theoretical predictions. According to the comparison between the measured and calculated pressures, the approximate parabolic-shaped pressure distribution observed in Cohesive Iron Ore B Test I and free-flowing Beach Sand Test III can be considered a result of a passive stress state established over the whole height of the bulk material surrounding the front face of the buried column. The other pressure trends in a parabolic-type shape in the upper section of the total buried height indicated a passive stress state, while, for the lower part of the column, the pressure increased nearly linearly, which was characteristic of the active stress state. This indicated the occurrence of a combination of passive and active stress states, where a passive stress state was formed at the upper part of the buried
column, and an active stress state occurred at the lower part. Overall, there was no observable difference caused by the column position for either of the two test materials.

![Graphs showing pressure distributions for different materials and positions](image)

**Figure 5.11: Pressure on the Front Face—Position 2**

### 5.3.3.2 Pressure Distributions on the Column Rear Face with Two Materials at Two Column Positions

The instantaneous pressures measured at the rear face of the buried column at the two different positions for both test materials are shown in Figures 5.12 and 5.13. All measured pressure distributions showed a general pressure increase with increasing buried depth, which agreed well with the predictions. This showed that the active stress state dominated the load conditions surrounding the rear face of the buried column. The magnitude of pressure on the rear face was found to be very similar for both free-flowing Beach Sand and Iron Ore B, which indicated that the effect of material cohesion...
on the pressure for the column rear face was not very significant. The nature of the free-flowing Beach Sand allowed quicker back-filling than for the cohesive Iron Ore B. Thus, similar pressures on the rear face of the column, due to the back-filling effect, were reached, despite the lower bulk density for the free-flowing Beach Sand. This was expected. Moreover, all these pressure results for the two column positions were similar for each test material. This indicated that the stress state that developed with the bulk materials was symmetrical, and the influence of the direction of flow rill on the stress state was not observable.

Figure 5.12: Pressure on the Rear Face—Position 1
5.3.4 Results of the Laterally Moveable Buried Column from Pressure Sensors and Load Cells with Two Materials—Dynamic Results

A number of tests were undertaken by using the laterally moveable support column with free-flowing Beach Sand and cohesive Iron Ore B. During all processes of loading, settling and reclaim, the pressure distributions on the four sides of the column, and total vertical draw-down force and lateral reaction forces at the top and bottom parts of the column were measured simultaneously with tactile pressure sensors and load cells, respectively.

5.3.4.1 Pressure Distributions on All Faces from Tekscan Sensors

To investigate the load conditions around all faces of the buried column, the pressure measurements were conducted during all undisturbed processes by using four tactile
pressure sensors. One test for each material involving pressure measurements on all column faces was chosen as an example to investigate the load conditions in a static state. The pressure results for each face with each material were from a single pressure frame immediately after the stockpile was fully loaded, and are displayed in Figures 5.14 and 5.15. Theoretical normal pressure distributions were drawn for each face based on the analysis discussed in Section 5.2.4, which enabled a comparison of the measured and predicted pressure trends. All these pressure trends matched reasonably with the corresponding theoretical curves.

The stress states that formed on the front and rear faces at the static state for both materials were similar to those for the fixed column (described in Section 5.3.3). In these, the pressure on the front face of the laterally moveable column exhibited a combination of both active and passive stress states, and the pressure on the rear face showed a pure active stress state. Both coincided well with the outcomes discussed in Chapter 4—that, at a static state, an active stress state, passive stress state or combination occurs on the column front face, while a pure active stress state dominates the load condition on the rear face. As indicated in the two figures, the stress states developed on the left and right faces of the column were shown to both be the active mode, which agreed well with the predictions. However, the magnitudes of the measured pressure distributions on all faces were slightly lower than the theoretical predictions. This may have resulted from the low sensitivity of this type of sensor applied in the low-pressure range.
Figure 5.14: Pressures on Faces of the Laterally Moveable Column for Iron Ore

B—Test 1
The dynamic changes of the stress states that developed on the front and rear faces of the fixed column were analysed in Chapter 4, and similar outcomes were obtained from this moveable column. Thus, only the results for the column two side faces are presented here, regarding the dynamic changes of the stress states occurring during the processes of loading and discharge. Two tests were chosen for each side face and each test material. The same data processing method as in the study on the column front and rear faces in Chapter 4 was applied for the two side faces. As shown in Figures 5.16 to 5.19, all these chosen discrete pressure distributions on the column left face and right face for the cohesive Iron Ore B and free-flowing Beach Sand indicated an active stress state at initial loading stage. This stress state remained constant until the loading process was finished. Theoretical curves based on the calculation with the fully loaded buried depths after loading were drawn in all these figures. This showed that the measured pressure
results on the column side faces had a high degree of coincidence with the related predicted pressure distributions.

Figure 5.16: Dynamic Pressures on Column Left Face During Loading Process for Iron Ore B—Two Tests

Figure 5.17: Dynamic Pressures on Column Right Face During Loading Process for Iron Ore B—Two Tests
Figure 5.18: Dynamic Pressures on Column Left Face During Loading Process for Beach Sand—Two Tests

(a) Test 1

(b) Test 2

Figure 5.19: Dynamic Pressures on Column Right Face During Loading Process for Beach Sand—Two Tests

(a) Test 1

(b) Test 2

For the reclaim process, the results are shown in Figures 5.20 to 5.23. For the cohesive Iron Ore B, there were observable pressure increases near the middle of the buried depths for the column two side walls during reclaim, as shown in the two tests in Figure 5.20 (column left face) and test 1 in Figure 5.21 (column right face). This can be considered an indication of a combination stress state mode. Therefore, the active stress state established on the column side faces before reclaim, as displayed in the two figures, was most likely to be transformed into a combination mode after reclaim, or remain the same, as indicated in test 2 in Figure 5.21.
For the free-flowing Beach Sand, due to its high flowability, the residual material surrounding the column was minimal after the discharge was completed, which resulted in nearly no pressure on the column side faces after reclaim. As shown in Figures 5.22 and 5.23, all pressures eventually tended to zero after reclaim, but all pressure trends showed one common stress state and remained unchanged during reclaim—this was the active stress state.
Figure 5.22: Dynamic Pressures on Column Left Face During Reclaim Process for Beach Sand—Two Tests

(a) Test 1  
(b) Test 2

Figure 5.23: Dynamic Pressures on Column Right Face During Reclaim Process for Beach Sand—Two Tests

(a) Test 1  
(b) Test 2

Overall, all the outcomes from the column side faces regarding the dynamic changes of stress state during loading and reclaim were similar to the situation for the rear face, as analysed in Chapter 4. An active stress state dominated the load condition during the entire loading process, and changed to one of the three stress state cases—active stress state, passive stress state or a combination of both. It can be reasoned that the dynamic flow mechanics were similar for the rear face and two side faces. During the loading process, some material slid against the column side faces when the material was discharged from the load-out conveyor, and slid down the sloping surface of the growing
stockpile. Some back-filling also occurred and flowed towards the two side faces simultaneously. All these were similar to the situation for the rear face during loading. For the reclaim process, some material flowed towards the two side faces after the flow was initiated, which was similar to the situation occurring at the rear face.

5.3.4.2 Comparison of Resulting Forces Calculated by Tekscan Sensors and Directly Measured by Load Cells

Lateral shear forces exerted on column faces are difficult to measure directly. In order to investigate these shear forces, a comparison was made between the three forces measured directly by the three load cells and the resulting forces calculated from the pressure distributions obtained by the pressure sensors. The capability of the real-time continuous data collection for both the load cells and pressure sensors allowed this comparison of the forces between the load cells and pressure sensors to be conducted during all undisturbed processes by considering all continuous recorded data. The calculated total vertical draw-down force (or vertical compressive shear force) was obtained by applying Equation (5-10) based on the continuously recorded pressure distributions on the column all faces from these pressure sensors. The results presented in Figures 5.14 and 5.15 for the two test materials are just a single frame of these pressure distributions.

Another two calculated forces from the tactile pressure sensors were obtained based on the analysis of the force and torque equilibrium for the laterally moveable column by processing pressure distributions on all faces for the two test materials. There were few or no lateral shear forces acting on the column front and rear faces. As a result, they were ignored in the equilibrium analysis, and the two calculated forces could be considered to exactly correspond to the two measured lateral reaction forces from the load cells at the top and bottom of the column. The comparison between the load cells and pressure sensors regarding the three forces during all undisturbed processes are shown in Figures 5.24 and 5.25 for the two test materials. There was an approximately linear increase on all forces during the loading process for both load cells and pressure sensors in the test with free-flowing Beach Sand, while the variable flow rates due to
avalanches occurring during the filling process resulted in fluctuating load increase in the test with cohesive Iron Ore B. All forces from the load cells remained nearly unchanged during a short settling process in both tests, and the same situation occurred with the forces from the pressure sensors after the pressure drifts were deducted from the raw data. The pressure drift is a result of the characteristic of this sensor type, and was determined in the calibration tests.

Significant changes happened to all forces during the reclaim process, especially for the top lateral force and bottom lateral force, which both showed a sudden change of force direction from compressive to tensile. This situation was worse in the test with Iron Ore B, where the discharge under gravity finished immediately after the gate was opened. This was unlike the smaller discharge rate, but longer discharge times, in the Beach Sand test. The sudden change of force indicated that the resulting lateral force changed from a direction that was the same as the flow rill, to a direction opposite to the flow rill. There were two reasons for this. First, a sharp decrease of force occurred on the front face, while the force on the rear face of the laterally moveable column decreased more slightly at the moment of opening the gate, thereby reducing the total compressive lateral force and even producing a tensile lateral force. Second, there was a sudden change of stress state when the discharge was initiated, as described in Chapter 4—that is, from the three possible stress states to an active or a combination stress state on the column front face, and from the active stress state to the three possible stress states on the rear face during the reclaim process. The explanation for this is that the force transmission from the stress states before reclaim to the stress states during reclaim usually leads to an immediate load enhancement, but with fluctuations on the buried column. Both these reasons contribute to the change of direction of the total lateral forces acting on the column. This dynamic response may cause a fatal collapse in the column structure. Therefore, a comprehensive study combining safety factor is required if a design of support structures is involved.

Overall, the vertical draw-down forces acting on all column faces for both materials showed strong agreement between the vertical load cell and four tactile pressure sensors. This confirmed the accuracy of the measured continuous normal pressures, such as in
the one frame shown in Figures 5.14 and 5.15, and verified the capability of Tekscan tactile pressure sensors for the pressure measurement of bulk materials. According to the calculations from the results for the forces and pressure distributions, the lateral shear forces acting along the left and right column faces can be extrapolated. The comparison on the forces between the other two load cells measuring the top and bottom lateral reaction forces and the calculated forces from pressure sensors are illustrated in Figures 5.24 for the Iron Ore B test.

This proved that there was little or no lateral shear forces acting on the left and right side walls during all undisturbed processes for cohesive Iron Ore B—that is, $F_{s1} = F_{s2} = 0$. This makes sense because flow was impeded due to the cohesive nature of Iron Ore B, and consequently introduced little or no lateral shear forces on the two side faces. However, the comparison illustrated in Figures 5.25 for the Beach Sand test revealed that the lateral shear forces occurring on the column side walls were between the range of the full lateral shear case ($C_S = 1$) and zero lateral shear case ($C_S = 0$). For the free-flowing Beach Sand employed, $C_S$ (refer to Equation (5-11)) approximately equals 0.6. Moreover, the experimental results confirmed that the coefficient for build-up $C_A$ (refer to Equation (5-12)) is 1.0 for the free-flowing Beach Sand, and approximately equal to 1.2 for the cohesive Iron Ore B.
**Figure 5.24:** Comparison of the Calculated Forces from Pressure Sensors and the Directly Measured Forces from Load Cells During Entire Undisturbed Processes for Iron Ore B

Note: Lateral shear forces on left and right sides are ignored: $F_{s1} = F_{s2} = 0$

**Figure 5.25:** Comparison of the Calculated Forces from Pressure Sensors and the Directly Measured Forces from Load Cells During Entire Undisturbed Processes for Beach Sand

Note: Lateral shear forces acting along two sides $F_{s1}$ and $F_{s2}$ between zero shear case and full shear case with $C_S = 0.6$
5.4 Discussion

The pressure distribution and resulting stress state on the front face varied noticeably in all tests, although they were under similar operating circumstances and the same test materials were employed. The significant variations observed in all measured pressures both for the fixed column and moveable column possibly resulted from the following factors, as discussed in Chapter 4:

- the low spatial resolution of the applied sensor type
- the low sensitivity of the used tactile sensor type in the pressure range below 10 kPa
- the general heterogeneous behaviour of the bulk material induced
- variations in the flow rate during loading.

Overall, the instantaneous pressures in the static state on all column faces for both cohesive Iron Ore B and free-flowing Beach Sand aligned qualitatively with the theoretical predictions.

The comparison of the pressure results measured at the two different column positions showed that the variation of the column position did not result in significant pressure differences for the column front or rear faces for the two test materials. This indicated that the influence on the stress state introduced by the direction of the flow rill in respect to the column front and rear faces was negligible, and the stress field formed within the stored material was nearly symmetrical in the radial direction. In addition, the situations for the stress states formed within the two test materials at the static state were similar, and all agreed well with the findings discussed in Chapter 4. However, the pressure trends for the two test materials were different, even under the same stress state, which resulted from the large differences in the strength and flow properties of the two materials.

The lower peak pressures for the free-flowing Beach Sand for both stress states may have resulted from its smaller bulk density. In addition, its higher flowability always enables a lower strength to be developed within the material, which leads to a lower location for the peak pressure under the passive stress state, compared to the cohesive
Iron Ore B. Moreover, the influence of the material cohesion on the pressures measured on the front face was more observable than that on the rear face, which may have been because material build-up is more likely to occur on the column front face for cohesive materials. This material build-up can increase the effective area on the column surface and, most importantly, introduce strong influence on the stress state developed on the front face.

As detailed before, the wedge-shaped load condition on a structure buried in a stockpile for the passive stress state (refer to Figure 4.4(b)) can be comparable to the stress state developed in a hopper in the flow case. This explains the pressure trends in Cohesive Iron Ore B Test I in Figure 5.10 and free-flowing Beach Sand Test III in Figure 5.11, in which the pressure on the front face of the buried column decreased with increasing depth after reaching a pressure maximum close to the half-height of the buried depth. In the active case, the pressure on the bin wall increased with increasing depth, and the load state was similar to the stress field in the cylindrical section of a storage bin for the initial filling case. All pressure results for the rear face of the column in Figures 5.12 and 5.13, and the rear, left and right faces in Figures 5.14 and 5.15 well demonstrate the feature of active stress state.

As defined in Chapter 4, the pressure trends with the feature of a lesser passive-shaped trend in the upper section and a linearly increasing trend in the bottom section indicated a combination of both active and passive stress states. Although this type of pressure distributions appeared in several tests on the column front face for both test materials—as shown in Figures 5.10 and 5.11—the trends for all of them varied. This was because multiple combination formations can develop due to the variations in the mechanics in the stored material near the front face. Compared with the pressure on the front face, the pressure distributions measured on the rear face showed good agreement with the theoretical calculations in terms of the pressure trends and magnitudes for both the cohesive Iron Ore B and free-flowing Beach Sand.
The pressure measurements conducted on all faces of the laterally moveable column not only confirmed the pressure distributions on the front face and rear face once again, but also revealed the load conditions on the left and right faces. According to the analysis of the dynamic pressure distributions on the two side faces, the dynamic changes of the stress states for the two side faces during loading and reclaim were similar to the situation for the rear face that was discussed in Chapter 4. As indicated in the flow chart in Figure 5.26, an active stress state provided the dominant effect and remained unchanged during the entire loading process. This could then change to one of the three possible stress states after reclaim, usually undergoing one or more intermediate stress states during the dynamic process of reclaim until the final stress state developed.

In addition, according to the findings in Figures 5.24 and 5.25, and the outcomes from Chapter 4, the resulting forces on the column side walls most likely undergo a dramatic force jump at the moment when the stress state changes during the reclaim process. This drastic change of the force exerted on the column must be taken into account in the design of relevant buried structures. Moreover, the good agreement on the forces determined for the laterally moveable column by the load cells and calculations from Tekscan pressure data for cohesive Iron Ore B indicated that the lateral shear forces
along the left and right faces were minimal and could be ignored during all processes for the cohesive material. However, these lateral shear forces could not be neglected for the free-flowing Beach Sand, which was proportional to the force exerted on the front face with a shear coefficient $C_S$ of about 0.6. Material build-up occurred on the leading front face of the buried column and the coefficient for this $C_A$ was approximately 1.2 for the cohesive Iron Ore B, while $C_A$ was 1.0 for the free-flowing Beach Sand. Further, the vertical shear stresses acting along the column faces were shown to be proportional to the normal stresses acting on the corresponding column faces, with a friction coefficient of $\tan \phi_w$.

The research presented in this chapter was a further investigation of the loads exerted on column faces by using two different materials, based on what was achieved in Chapter 4. It also involves an investigation of the load conditions on the two side faces. All this chapter’s findings not only prove that the Tekscan tactile pressure sensor is a good tool for pressure measurement in bulk materials, but also verify the capability of Roberts’s theory in determining the loads on support buried structures.

### 5.5 Conclusion

Support structures in stockpiles or storage sheds are partly buried in the stored material, and the loads exerted on them are complex and variable. Tekscan tactile pressure sensors provide good accuracy for determining pressure distributions on column faces for both cohesive Iron Ore B and free-flowing Beach Sand on two column setups, despite the fluctuations in measured results due to the existing limitations of the tactile sensors employed and the heterogenic characteristics of the granular materials. Overall, the experimental results agree reasonably with the refined Roberts’s approach, as follows:

- The column position did not influence the stress state occurring on the column faces, which indicated that the stress state developed with the bulk materials was nearly symmetrical, and the influence of the direction of flow rill on the stress state was not observable.
• The cohesion of the test materials did not influence the pressures on the rear face significantly, but did affect the loads on the front face regarding the pressure trends and magnitudes.

• This research further confirms the findings in Chapter 4 that, in a static state, the stress state appeared to be one of the three possible stress states dominating the normal pressures on the column front face, while the active stress state dominated the load condition for all other faces.

• During the dynamic processes of loading and reclaim, the change of stress states occurring on the column two side faces were similar to the situation for the column rear face, which changed from the active stress state to one of the three possible stress states (active, passive or a combination). The sudden changes of the resulting forces on the column faces must be considered in designs, and often occur when the stress state changes to an intermediate stress state or to the final stress state.

• The lateral shear stresses along two sides were negligible for the cohesive Iron Ore B, but proportional to the force acting on the front face for the free-flowing Beach Sand with an additional shear coefficient applied. In addition, the vertical shear stresses along all column faces for both materials were proportional to their corresponding normal pressures.

Considering the uncertainty of the possible variations in the loading conditions in each application, a conservative approach is recommended for the design of support structures buried in stored bulk materials, using the envelope of both active and passive pressure curves as the upper pressure limit. This allows for all possible combined appearances of both stress states on the column front face, and the pressure maxima of each stress state is considered in this approach. Moreover, the drastic change of the force acting on columns due to the sudden change of stress state when discharge is initiated can cause a sudden steep change in the column loads, and hence must be considered when determining the loads on buried support structures. As a result, a safety factor must be involved in this conservative envelope approach to ensure the reliable and efficient design of support structures buried in stockpiles. Therefore, this analysis further verifies the capability of the refined Roberts’s method in determining the loads on
support columns buried in stockpiles, and the important outcomes from this research work provide a more thorough understanding of both the normal and shear loads exerted on support structures buried in bulk material stockpiles and storage sheds.
Chapter 6: Simulation Investigation of Loads on Support Columns Buried in Stockpiles with Two Materials

6.1 Introduction

Simulation techniques have been widely used in academia and industries as a cost-effective and flexible tool for research analysis and industrial conceptual design. DEM is a commonly used simulation tool in numerical studies of the complex constitutive behaviour of bulk solids in storage bins, containing vessels, stockpiles, transfer chutes and transfer conveyors. By applying DEM, a number of researchers have undertaken successful simulation work on the subject of bulk solids flow and stress field in hoppers [48,50,83-87], which has greatly helped understand the macroscopic and microscopic interactive behaviour between a bulk material and the solid hopper walls. Since Roberts [55] proposed the load theory for support structures buried in stockpiles, the fundamentals on this subject have been established. To further verify the load analysis theory on the basis of Roberts’s pioneering experimental studies, DEM was applied in subsequent simulation investigations undertaken by Katterfeld et al. [56].

This DEM simulation work involved a complex user-defined contact model for individual particles, and the contact properties were detailed by Groger and Katterfeld [88]. A similar contact model was employed in several publications by Groger and Katterfeld [89-91], and was clearly validated in its application to particulate systems. However, there are wide variations in the properties of discrete particles among different bulk solids, and these parameters can have a significant influence on the flow behaviour of particles [78,92]. These parameters have been studied in a number of publications in terms of factors such as particle shape [51,93-98], porosity [87,99,100], particle lump and breakage [52,101], friction effect [102], rolling resistance [103,104] and the cohesion characteristics between particles [50,77,105]. There are difficulties reproducing the great variety of particle shapes in simulation; hence, particles are commonly assumed to be spherical, but a proper rolling friction coefficient is adopted to
compensate the assumption of the spherical particles, as proposed by Wensrich et al. [103].

In applying DEM, the calibration of bulk material parameters is essential. An accurate characterisation of the material properties employed in DEM can help achieve realistic simulation results. The existing commonly used calibration parameters [103,106-110] are based on the standard flow property tests, such as particle shape, PSD, particle density, particle stiffness, angle of repose, effective angle of internal friction, and wall friction angle. It is difficult to apply all the properties of a bulk material in the DEM simulation because the existing computers and software cannot meet the requirements to fulfil this purpose. Therefore, some simplifications are made, such as Wensrich’s assumption that rolling friction is applied to the discrete spherical particles as a technique to model the arbitrarily shaped particles of a bulk material [103].

Two advanced DEM software packages, PFC3D (ITASCA Consulting Group, Inc.) and Rocky (Granular Dynamic International, LLC.), were available in the research centre and employed in this simulation study to compute the contact forces between the walls of a support column buried in a stockpile and the particles that were in contact with the column walls. Both PFC3D and Rocky are commercial software packages, and PFC3D has several contact models itself but it also can run by applying a user-defined contact model; Rocky can only operate with its own contact models, and for cohesive materials it is more reasonable to run “rolling resistance model: type 3 – elastic-plastic spring-dashpot resistance model”. To further verify the refined Roberts’s load analysis model with a simulation approach, the computed normal and shear loads on the column walls were compared with the experimental results and the theoretical calculations from Chapters 4 and 5.

6.2 Simulation Modelling
6.2.1 Geometry of the Simulation Modelling

To facilitate the comparison between the simulation results and the experimental results from Chapter 5, a simulation setup similar to the experimental rig was developed. The
geometry of the simulation rig is shown in Figure 6.1 for PFC3D and Figure 6.2 for Rocky. Since it is difficult to compute the simulation system with the same scale as the experimental setup, the geometry of the simulation system was scaled down to the one with which the simulation work could be perform with computers, within a reasonable computational time. In this study, 60,000 spherical particles were generated in PFC3D as test particles and about 200,000 spherical particles were created in Rocky to achieve similar scale to that in PFC3D. The diameters of these particles were within 4 mm and were generated randomly using PFC3D software because the practical PSDs will significantly increase the computational time, and the fine fraction of all particles (≤4 mm) plays a critical role in the formation of the strength within the material. However, PSDs in Rocky were divided into three main groups without significant increase in the computational time.

Stockpiles were formed with a maximum height of approximately 0.10 m. Therefore, the scale of the simulation model to experimental setup was around 1:7.5. The dimension of the support column was 10 × 10 × 100 mm, and the widths of all column faces were 10 mm instead of 8.7 mm, according to the scale, because the particle sizes were not reduced as much as the scale required, considering the computational time. The support column was placed on the stockpile base 30 mm away from the opening of the stockpile, according to the scale. Each face of the column consisted of 10 measuring elements with a dimension of 10 × 10 mm for each, as the measuring walls behaved like individual sensel elements for the Tekscan sensors in the experiments.

All particles were generated within the silo on a nearly discrete incompact condition. The gate of the silo was closed at the initial stage and the particles were discharged downwards, being consolidated under gravity for around one second of computational time until a macroscopic steady state was achieved and the unbalanced forces were negligible. The gate was then opened to initiate the flow of particles, and the belt conveyor underneath the outlet was activated at the same time. The velocity of the belt conveyor was adjusted to be 1.2 m/s in order to make the vertex of a formed stockpile coincide with the centre of the stockpile outlet. The computing results of both normal
loads and shear loads on all elements of the four column faces were obtained when all particles in the silo flowed out and a full stockpile was developed.

![Figure 6.1: Geometry of Simulation Rig (PFC3D Software)](image1)

![Figure 6.2: Geometry of Simulation Rig (Rocky Software)](image2)

6.2.2 Contact Constitutive Model of the Simulation Modelling

DEM comprises a variety of computational modelling techniques and can simulate the dynamic mechanical behaviour of a system comprised of a collection of a large number
of arbitrarily shaped particles that are subject to continuously varying contact constraints. The overall constitutive behaviour of a material is simulated in PFC3D by associating a proper constitutive model with each contact. There are a number of alternative contact models available in PFC3D. However, to simulate the behaviour of cohesive particles, a user-defined contact model was established by Wensrich and applied to PFC3D in this simulation study. The C++ codes document for this cohesive contact model was added to this thesis, as detailed in Appendix B. The simulation conducted by Rocky applied its own cohesive contact model. For the two different DEM software packages employed in the simulation, the principles are similar, hence only the general principles for PFC3D [111] are summarised herein. In the simulation modelling applied with PFC3D, the generated particles were simplified as rigid spherical bodies for both. These particles collide with one another, and new contacts are established, while old contacts may be released, which produces changes in the contact status and contact interaction forces, including particle-to-particle and particle-to-boundary situations. This influences the subsequent movement of particles. In the calculation cycle of PFC3D, a time-stepping algorithm is applied that requires the repeated application of the law of motion to each particle, a force-displacement law to each contact and a constant updating of wall positions [111]. The contacts between two particles or between a particle and a wall are formed and broken automatically during the process of a simulation. Figure 6.3 illustrates the calculation cycle in PFC3D.

Figure 6.3: Calculation Cycle in PFC3D [111]
The force-displacement law relates the relative displacement between two entities at a contact to the contact force acting on the entities. It is described for both particle-particle and particle-wall contacts. For particle-particle contact, the relevant equations are presented for the case of two spherical particles, denoted as A and B in Figure 6.4. For particle-wall contact, the relevant equations are presented for the case of a spherical particle and a wall, denoted as b and w, respectively, in Figure 6.5. $U^n$ denotes the overlap in both cases. For particle-particle contact, the unit normal, $n_i$, that defines the contact plane is given by:

$$n_i = \frac{x_i^{[B]} - x_i^{[A]}}{d} \quad \text{ (particle – particle)} \quad (6-1)$$

where $x_i^{[A]}$ and $x_i^{[B]}$ are the position vectors of the centres of particles A and B, and $d$ is the distance between the particle centres:

$$d = \left| x_i^{[B]} - x_i^{[A]} \right| = \sqrt{(x_i^{[B]} - x_i^{[A]})^T(x_i^{[B]} - x_i^{[A]})} \quad \text{ (particle – particle)} \quad (6-2)$$

![Figure 6.4: Particle-particle Contact [111]](image-url)
For particle-wall contact, $n_i$ is directed along the line defining the shortest distance $d$ between the particle centre and the wall. This direction is found by mapping the particle centre onto a relevant portion of space defined by the wall. The idea is illustrated in Figure 6.6 for a two-dimensional wall composed of two line segments, AB and BC. All space on the active side of this wall can be decomposed into five regions by extending a line normal to each wall segment at its endpoints. If the particle centre lies in regions 2 or 4, it will contact the wall along its length, and $n_i$ will be normal to the corresponding wall segment. However, if the particle centre lies in regions 1, 3 or 5, it will contact the wall at one of its endpoints, and $n_i$ will lie along the line joining the endpoint and the particle centre. (For the three-dimensional convex polygonal walls in PFC3D, the above idea is extended such that the particle may contact the wall at a vertex, along an edge joining two wall segments, or on a face.)
More details about the general formulation of contacts are described in the *PFC3D 4.0 Help Manual* in the chapters on general formulation [111] and contact constitutive models [112].

6.3 Simulation Calibration

The calibration tests for the simulation study were performed in the laboratory for both cohesive Iron Ore B with a moisture content of 7.5% and dry Beach Sand, and the test setup and results are illustrated in Figures 6.7 and 6.8. Since there was a limitation on the scale of simulation modelling, the scale of the calibration test setup was reduced to a reasonable size. The calibration setup has two cuboid chambers: the top one with a dimension of 0.50m×0.50m×0.10m, and the bottom one with a dimension of 0.60m×0.60m×0.10m. There is a swing gate at the bottom of the small top chamber and its dimension is 0.15m×0.10m. The test material was filled into the top cuboid chamber through a hopper above the chamber, and the flow was initiated when the swing gate was opened. The length of the gate was smaller than the length of the chamber; hence, there was residual material in the top chamber that produced inclined angles similar to the rathole expansion angle and rathole sloughing angle in the funnel flow case. The two angles were two of the calibrated parameters. The material that was discharged and dropped to the cuboid chamber underneath formed a thin stockpile, and the angle of repose of this stockpile was another parameter for calibration. Moreover, the discharge time for the flow could be obtained and was also a calibration parameter. Nevertheless, the simulation results in this study were mainly based on the angle of repose of the stockpile, and the calibration parameters of rathole sloughing angles and discharge time are still ongoing.
6.4 Simulation Results

Before the relevant simulation computations were performed, simulation calibrations were conducted. This was a trial-and-error process that involved adjusting the simulation parameters until all the calibration parameters obtained from the laboratory calibration tests were reproduced. The final parameters applied in the simulation in both PFC3D and Rocky are summarised in Table 6.1.
Table 6.1: Parameters for Simulation in PFC3D

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</tbody>
</table>

By applying these simulation parameters, a series of simulation tests were conducted to compute both the normal and shear loads exerted on the four faces of a column buried in a stockpile comprised of spherical particles. The very long computation time in DEM simulation cannot allow a great number of simulation cases and repetitions to be performed, and thus four simulation cases and three repetitions were carried out. One of the four simulation cases whose simulation results are closest to the experimental and theoretical outcomes for each material was selected and its two representative repetitions were chosen in this thesis. Figures 6.9 and 6.10 show the load conditions from the DEM simulation for the cohesive Iron Ore B and dry Beach Sand, respectively. The loads exerted on the test column in the simulation were analysed after the flow ceased and the particles came to rest. As displayed in the two figures, the contact force chains within the two stockpiles were different, owing to the different parameter settings applied to simulate the particle behaviours for the two different test materials. This led to two
different angles of repose and different stockpile heights from a macroscopic perspective. The analysis of the loads consisted of normal loads, lateral shear loads and vertical compressive loads on all faces of the column, which was comprised of a number of small wall elements (10 × 10 mm).

Figure 6.9: Load Conditions—Cohesive Iron Ore B
Figure 6.10: Load Conditions—Dry Beach Sand
6.4.1 Normal Pressure Distributions on Column Faces

In order to compare the pressure distributions normal to all faces of the buried column among the simulation results, relevant experimental results and theoretical calculations, all these results were non-dimensionalised because the geometric scales of the experimental and simulation setups were different. The maximum theoretical pressure for each column face in an active stress state based on the simulation scale was chosen as the normalising factor for the simulation pressure result of the corresponding column face. The maximum theoretical pressure for each column face in an active stress state, based on the experimental scale, was adopted as the normalising factor for both the relevant experimental pressure results and the corresponding theoretical pressure calculations. The normalising factor for buried depth was the maximum buried depth on each column face for all results. The non-dimensional pressure distributions normal to the four column faces are shown in Figures 6.1 to 6.14 for both the cohesive Iron Ore B and dry Beach Sand.

6.4.1.1 Normal Pressure Distributions on the Front Face of the Buried Column

![Graphs showing pressure distributions](image)

Test 1

Test 2

(a) Cohesive Iron Ore B
The pressure distributions on the column front face showed observable variations in the trends and magnitudes for both the simulation and experimental results. The possible explanations for variations in trends were discussed in Chapters 4 and 5. The magnitudes of the pressures from simulation varied significantly, which could have resulted from the larger proportion of coarse particles in simulation than that for the test materials, especially for the free-flow Beach Sand, which had a large percentage of fine particles. These coarse particles usually cause huge impact loads when contacting column faces. In addition, it was noticeable that the pressure distributions from the simulation were higher than the measured pressures for both the cohesive iron ore and free-flowing sand, especially for the results from PFC3D, the magnitudes of which agreed more with Roberts’s theory. This may have been because the calculation principles for the contact forces between the particles and the contacting wall elements in the simulation were more accurate than the applied measuring technique, such as the Tekscan tactile pressure sensors, which had low spatial resolution and low sensitivity in the pressure range below 10 kPa. Overall, the normal pressures on the column front face from the simulation, experiments and theoretical calculations showed reasonable agreement.
6.4.1.2 Normal Pressure Distributions on the Rear Face of the Buried Column

![Graphs showing pressure distributions](image)

(a) Cohesive Iron Ore B

(b) Dry Beach Sand

**Figure 6.12: Normal Pressure Distributions on the Column Rear Face**

The pressure distributions on the column rear face from the simulation for the dry Beach Sand were more consistent with the relevant experimental results than those for the cohesive Iron Ore B, as illustrated in Figure 6.12. For cohesive materials such as iron ore with a moisture content of 7.5%, the back-filling effect during the loading process is much weaker than that for free-flowing materials. This weaker back-filling effect always introduces lower consolidation within the bulk material in the vicinity of the rear face of a buried column, thereby introducing lower pressures on the column rear face. The cohesive contact model applied in this simulation was based on the principles of Liquid Bridge between the contacting particles, and this contact model may introduce a weaker
back-filling effect than that for cohesive materials. This could explain why the simulation pressures on the column rear face for the cohesive Iron Ore B were lower than the experimental results, while the simulation pressures for the free-flowing Beach Sand were consistent with the measured pressures. Another potential reason for the lower pressures on the rear face could be that the simulation modelling with current small scale cannot well simulate the back-filling behaviour of particles. However, all simulation results presented in Figure 6.12 confirmed that an active stress state developed on the rear face of the buried column during loading or after a stockpile was loaded, as discussed in Chapter 4 (refer to Figure 4.28).

6.4.1.3 Normal Pressure Distributions on the Side Faces of the Buried Column

![Normal Pressure Distributions on the Side Faces of the Buried Column](image)

Figure 6.13: Normal Pressure Distributions on the Column Left Face
The normal pressure distributions on the two side faces of the buried column using the two materials are exhibited in Figures 6.13 (left face) and 6.14 (right face). The pressures on both the left and right face of the column were almost identical between the simulation results, measured results and theoretical predictions for the free-flowing Beach Sand regarding both the trends and magnitudes. Since the cohesive contact model employed in the simulation of the cohesive Iron Ore B may better facilitate the movement or potential movement of particles across the two side faces than that in practice, the interactions between particles and the two side faces in the normal direction were weakened during the loading process. As a result, the pressure distributions normal to both side faces for the cohesive Iron Ore B were slightly lower than the experimental...
and theoretical results. Overall, all the simulation results showed good agreement with the experimental and theoretical outcomes. They also confirmed the stress state that developed on the two side faces of a buried column, as discussed in Chapter 5 (see Figure 5.26), which was the active stress state during and after loading.

6.4.2 Lateral Shear Pressure Distributions

Lateral shear loads are the traction stresses due to the shear across a column’s four faces. These lateral shear stresses can occur during all processes of loading, settling (consolidation due to storage) and discharge, and it is difficult to measure them in experiments. Based on the simulation and predictions, this section only analyses the lateral shear stresses immediately after a stockpile was filled.

6.4.2.1 Lateral Shear Pressure Distributions on the Column Front Face and Rear Face

In an ideally symmetrical system, the lateral shear stresses acting across the column front and rear face after a stockpile is loaded are very small due to little movement or potential movement in the lateral direction, and can be ignored for either cohesive or free-flowing bulk materials. Figures 6.15 and 6.16 show the lateral shear pressures from the two materials exerted on the column front and rear faces, respectively. Despite the variations in the simulation results with changing pressure directions (in positive or negative), the lateral shear pressures on both the column front and rear faces from PFC3D and Rocky fluctuated near the line of zero for both the cohesive Iron Ore B and dry Beach Sand. Therefore, the lateral shear loads across the front and rear face of a buried column could be negligible.
Figure 6.15: Lateral Shear Pressure Distributions on the Column Front

(a) Cohesive Iron Ore B

(b) Dry Beach Sand
As denoted by $F_{s1}$ and $F_{s1}$ in Figure 5.1, the lateral shear stresses acting across the column left and right faces were introduced by the internal creep due to shear within the stored bulk material. Figures 6.17 and 6.18 illustrate the comparison of the lateral shear pressure distributions on the column left and right faces between the simulation and the theoretical calculations, which were detailed in Chapter 5 (see Equation (5-11)). The simulation results for free-flowing Beach Sand coincided well with the theoretical predictions by applying a shear coefficient $C_S$ of 0.6 either in the active stress state, passive stress state or a combination of both. This confirmed the deduction that $C_S$ is equal to 0.6 for free-flowing Beach Sand, as stated in the discussion in Section 5.3.4.2 (see Figure 5.25). Nevertheless, for cohesive Iron Ore B, the lateral shear stresses on the two side faces agreed with the theoretical predictions when a shear coefficient $C_S$ equal to 0.6 was applied to all the related simulation results from PFC3D, rather than approximate to zero. The simulation results from Rocky were even smaller. This was inconsistent with the deduction in Figure 5.24 where $C_S$ was extrapolated to be zero. This may be because the movement or potential movement of particles across the two side faces was greater than that in practice by applying the cohesive contact model in the simulation, as discussed in Section 6.4.1.3. This enhanced the lateral shear action on the side faces and led to the observable lateral shear pressures on both the left and right face.
of the buried column for cohesive Iron Ore B. This could also be a result of the small scale of the simulation modelling.

![Diagram](image1)

Test 1  
(a) Cohesive Iron Ore B

Test 2

![Diagram](image2)

Test 1  
(b) Dry Beach Sand

Test 2

Figure 6.17: Lateral Shear Pressure Distributions on the Column Left Face

![Diagram](image3)

Test 1  
(a) Cohesive Iron Ore B
6.4.3 Vertical Shear Pressure Distributions

During loading and setting processes, the bulk material in a stockpile is consolidated by the weight of material in the upper layer, and vertical shear stresses occur along the faces of a buried column. The vertical shear stresses acting along the column faces introduce compressive loads to the column. Since there are difficulties in measuring these vertical shear loads in experiments, the simulation methodology provides a good opportunity to investigate these loads and compare them with the corresponding theoretical calculations. The vertical compressive shear stresses exerted on all column faces from the simulation and theoretical prediction are shown in Figures 6.19 to 6.22 for both materials. The theoretical vertical shear pressures were calculated based on Roberts’s assumption [55] that they are proportional to their corresponding normal pressures with a friction coefficient of \( \tan \phi_w \).
Figure 6.19: Vertical Shear Pressure Distributions on the Column Front Face
Figure 6.20: Vertical Shear Pressure Distributions on the Column Rear Face

(b) Dry Beach Sand

Figure 6.21: Vertical Shear Pressure Distributions on the Column Left Face

(a) Cohesive Iron Ore B
(a) Cohesive Iron Ore B

(b) Dry Beach Sand

Figure 6.22: Vertical Shear Pressure Distributions on the Column Right Face

Overall, these four figures illustrated qualitative agreement of vertical shear pressures between the simulation and theoretical calculations on all column faces for both materials. The vertical shear pressures on the column front face from the simulation in Figure 6.19 showed similar trends to the normal pressures on the column front face presented in Figure 6.11. The vertical shear pressure distributions on the left and right faces of the buried column from the simulations in Figures 6.21 and 6.22 were consistent with the corresponding theoretical results. In addition, the trends were very similar to those in Figures 6.13 and 6.14 for the normal pressures on the column side faces. However, among all the simulation results, those computed by Rocky for cohesive Iron Ore B were smaller than those from PFC3D. This may be because the cohesive contact
model applied in PFC3D was closer to the contact mechanism of the bulk materials applied in the tests than that in Rocky regarding the computation of shear loads.

All these findings from the comparison of normal and vertical pressures in Figure 6.13, 6.14 and Figure 6.19 to 6.22 confirmed the assumption that the vertical shear pressure distributions on the faces of the buried column were proportional to the corresponding normal pressures. However, the difference was slightly larger between the simulation and theoretical results for the column rear face, as shown in Figure 6.20, especially for the cohesive Iron Ore B. This was because the normal pressure results on the column rear face were lower than the predicted results for both materials, which led to lower vertical shear pressures. The potential reasons for the lower normal pressure distributions were discussed in Section 6.4.1.2.

6.4.4 Total Forces Exerted on the Column

To investigate the loads acting on the buried column and verify the accuracy of the simulation results, comparisons of the total lateral forces and total vertical compressive shear forces from the simulation, experiments and theoretical results were conducted. All these force results were non-dimensionalised. The normalising factors for total lateral force/vertical compressive force were the maximum theoretical total lateral force/vertical compressive force in an active stress state, based on the simulation scale for the simulation results. The maximum theoretical total lateral force/vertical compressive force in the active stress state was based on the experimental scale for both the experimental and corresponding theoretical results.

6.4.4.1 Total Lateral Force

The total lateral forces on the buried column for both the cohesive Iron Ore B and free-flowing Beach Sand are presented in Figure 6.23(a) and (b), respectively. The total lateral force was highly dependent on the stress state developed on the column front face, which determined three components of the theoretical total lateral force (lateral force on the front, left and right faces) based on the calculation formula in Equation (5-12).
the stress state for the column front face was not constant, two theoretical total lateral forces for each test were calculated based on the two pure stress states occurring on the column front face: active and passive stress state. In the foregoing discussion of the normal pressure distributions on the column front face, and lateral shear pressures on the column side faces from the simulation, the stress states for most of the chosen tests were a combination stress state or passive stress state for the two materials. This meant that the simulation total lateral forces were supposed to be much closer to the theoretical results of the case of the passive stress state than that of the active stress state.

The simulation results for the free-flowing Beach Sand from both DEM software packages showed agreement with this, but the simulation total lateral forces for the cohesive Iron Ore B from PFC3D were much larger than what were expected. This was because the lateral shear pressures acting across the two side faces were deducted to be zero (the coefficient for shear $C_S = 0$) for cohesive Iron Ore B, based on the related experimental results, as discussed in Chapter 5; however, the lateral shear results on the two side faces from the PFC3D simulation revealed that $C_S$ equals 0.6. This certainly led to the significantly large total lateral shear forces for cohesive Iron Ore B, as shown in Figure 6.23(a). In addition, the normal pressures on the column rear face introduced by the weaker back-filling effect in the PFC3D simulation were much lower than the experiments for cohesive Iron Ore B, which also moved the total lateral forces for the two tests in Figure 6.23(a) into a higher range than that in Figure 6.23(b).

Moreover, in the foregoing discussions concerning the normal pressure distributions on the column front face for the two test materials, the stress states for most of the chosen tests were an active stress state, but the total lateral forces from the experiments were much smaller than the theoretical calculations in the active case for both test materials. This was because all the related pressure distributions from the experiments were lower than the theoretical results. In general, all these results showed reasonable agreement with the total lateral shear forces between the simulation results, experimental results and theoretical predictions for free-flowing Beach Sand. However, for the cohesive Iron Ore B, the simulation results from PFC3D were much larger than expected due to the
significant lateral shear loads on the two side faces, which were negligible during the experiments.

![Graphs showing lateral force comparison](image)

(a) Cohesive Iron Ore B

(b) Dry Beach Sand

**Figure 6.23: Total Lateral Force on the Buried Column**

### 6.4.4.2 Total Vertical Compressive Shear Force

The total vertical compressive shear forces from the simulation and theoretical predictions were obtained by applying Equation (5-10), excluding the loads from the supported objects, such as the roof of a shed or a load-out belt conveyor. The results of the total vertical compressive shear forces for cohesive Iron Ore B and free-flowing Beach Sand are exhibited in Figure 6.24(a) and (b), respectively. Same as for the total
lateral shear forces, the two theoretical total vertical compressive shear forces for each test were calculated based on the two pure stress states that can occur on the column front face. Figure (a) and (b) show the same situation—that the experimental total vertical compressive shear force for each test and each material was smaller than the two theoretical results (for both the active and passive case), but much larger than the related simulation results. This was because the experimental normal pressure distribution on each column face was slightly lower than the theoretical pressure for each test material, which led to a difference in the total vertical compressive shear force between the experimental and theoretical outcomes. In addition, the vertical compressive pressure distributions from the simulation on all column faces, except the front face, for the two materials were lower than the corresponding theoretical predictions, especially for the rear face, on which the difference was significantly large. For the simulation results from Rocky, the vertical compressive pressures were much lower. Consequently, the total vertical compressive shear force for each test and each material from the simulation was much smaller than the corresponding experimental and theoretical results, especially for that from Rocky.

Test 1

Test 2

(a) Cohesive Iron Ore B
6.5 Conclusions

In summary, all the non-dimensional simulation results concerning the normal pressure distributions, lateral shear pressures, vertical shear pressures, total lateral force and total vertical compressive shear force exerted on the column buried in the stockpile agreed reasonably with the relevant non-dimensional experimental and theoretical results for the two test materials. In the two DEM software packages, PFC3D is more capable to simulate the shear loads. This shows the capability of the cohesive contact mode applied in the PFC3D simulation and the relevant calibration methodology to simulate the loads on buried columns. Moreover, the theory about both normal and shear loads exerted on a support column buried in a stockpile was verified by simulation.

The normal pressure distributions on all faces of the buried column from the simulation of the two materials verified the experimental outcomes that the stress state dominating the load condition on a column front face can be an active stress state, passive stress state or combination of both after a stockpile is fully loaded, and that the active stress state is dominant for the column rear, left and right face. The magnitudes of these normal pressures agreed well with the experimental results regarding the column left and right faces for both materials. For the column front face, the magnitudes of the simulation normal pressures agreed well with the theoretical predictions, but were
slightly higher than the experimental results, which may have been caused by the low sensitivity of the Tekscan tactile pressure sensors applied in the experiments in the low-pressure range. However, the pressure distributions on the column rear face from the simulation for cohesive Iron Ore B were observably lower than the experimental results, which possibly arose from the small scale of the simulation modelling and the weaker back-filling effect than that in practice for cohesive materials introduced by the cohesive contact model in the simulation.

DEM simulation provides opportunities to obtain the lateral shear pressures and vertical shear pressures exerted on all faces of the buried column, which are difficult to realise in experiments. This study confirmed that the lateral shear pressures across the column front and rear faces could be ignored for either cohesive or free-flowing bulk materials because there was little movement or potential movement in the lateral direction in a nearly symmetrical system. For the lateral shear stresses on the column side faces, they were dominated by the stress state occurring on the column front face, and a shear coefficient $C_S$ equal to 0.6 for free-flow Beach Sand was verified by the simulation results, which was consistent with the experimental outcomes. However, a shear coefficient $C_S$ equal to 0.6 for the lateral shear pressures across the two side faces for cohesive Iron Ore B was also obtained from the simulation, instead of the zero extrapolated from the relevant experimental results. This may have been because the movement or potential movement of particles across the two side faces was greater than that in practice by applying the cohesive contact model in the simulation, which made the lateral shear action on the side faces more intensive, and introduced the observable lateral shear pressures for the cohesive Iron Ore B.

In addition, the scale of the simulation modelling may have been too small to suitably simulate the behaviour of particles. The vertical shear pressures along all column faces from the simulation showed similar pressure trends to their corresponding normal pressures, and both the trends and magnitudes agreed reasonably with the theoretical calculations. This verified that the vertical shear pressures were proportional to their corresponding normal pressures with a friction coefficient of $\tan\phi_w$. 

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The total lateral forces and total vertical compressive shear forces acting on the buried column from the simulation, experiments and theoretical calculations were non-dimensionalised to facilitate comparison between all of them. The total lateral forces from the experimental results were smaller than the theoretical calculations for both test materials since the pressure distributions on the column front face from the experiments were lower than the predicted ones. For the simulation results, the total lateral forces for cohesive Iron Ore B were much larger than the experimental results due to the significant lateral shear pressures ($C_S = 0.6$) acting on the two side faces from the simulation, which were negligible during experiments, which led to the larger total lateral shear forces in the simulation. In addition, the normal pressure distribution on the column rear face was much lower than the measured pressures for cohesive Iron Ore B, which certainly increased the total lateral shear forces in the simulation.

However, the results from the free-flowing Beach Sand showed reasonable agreement with the total lateral forces between the simulation, experiments and theoretical predictions. In the analysis of the total vertical compressive shear forces, the experimental normal pressure distribution on each column face was slightly lower than the theoretical pressure for each test material, and the compressive pressure distribution on the column rear face for each material from the simulation was significantly lower than the corresponding theoretical predictions. Therefore, for the total vertical compressive shear force, the experimental results for both materials were smaller than the theoretical results (both active and passive case), but much larger than the related simulation results.

Overall, the DEM simulation provided good confirmation of Roberts’s load analysis theory to some extent, but there were still some deficiencies, such as the smaller loads on the column rear face than that in practice for cohesive bulk materials. This situation may be improved in future research by increasing the scale of the simulation modelling, if it is facilitated by using more powerful computers, or by improving the current cohesive contact model applied in this DEM simulation. However, for the purposes of designing support columns buried in stockpiles, the theoretical calculations based on the
active stress state for the column front face can be adopted as a conservative and safe approach.
Chapter 7: Conclusions and Recommendations

This thesis has investigated the mechanics of funnel flow in relation to draw-down in silos and stockpiles and loads exerted on the support structures buried in stockpiles. The outcomes from the experiments and simulation successfully verified the related theories, and can provide good criteria for design practice. Based on the work presented in this thesis, the following conclusions are made and some recommendations are proposed regarding further research that may be undertaken.

7.1 Conclusions

Two different research topics were investigated in this thesis: (i) the mechanism of funnel flow and (ii) the loads on support structures buried in stockpiles. The conclusions for each topic are as follows.

7.1.1 The Mechanism of Rathole in Silos and Stockpiles

The performance of a funnel-flow silo or stockpile is highly dependent on the bulk material handled and the geometry of the facility. The properties of a bulk material that determine its strength and flow character in the handling process were obtained by shear testers. The dimensions of a funnel-flow silo or stockpile (filling height and outlet dimensions) played a critical role in the geometry of the rathole formed and draw-down head, as well as reclaim efficiency. For complete discharge, the silo opening (for stockpiles, the dimensions for reclaim mass-flow hopper) needed to be at least equal to the critical rathole diameter, which made the dimensions of the outlet unrealistic. To investigate the rathole mechanism, a comparison was made of the relevant results between the experimental outcomes (from both laser scanning and probe-profile gauge detection) and the theoretical calculations from both Roberts’s hoop stress theory [15] and Jenike’s Upper Bound theory [14].
Despite the heterogeneity in bulk materials, which can create uncertainty in the geometry of ratholes, the comparison provided good confirmation of the two existing theories concerning the mechanism of funnel flow. Moreover, it was observed that both Roberts’s hoop stress theory and Jenike’s Upper Bound theory were more appropriate for predicting the rathole mechanism for gravity reclaim stockpiles with a single outlet or double outlet configuration when the filling height and separation distance were enough for ratholes to fully develop. However, a more realistic prediction of the critical rathole diameter $D_r$ was obtained from Roberts’s hoop stress theory, while calculations of the critical rathole diameter from Jenike’s Upper Bound theory were slightly more conservative. The hoop stress theory also provided more specific predictions regarding the rathole sloughing angles and rathole expansion angles. Therefore, Roberts’s hoop stress theory can provide a better guide for designing the opening dimensions for funnel-flow silos and gravity reclaim stockpiles. In addition, it was confirmed that the laser technology was capable of measuring rathole profiles using bulk materials.

Moreover, the pressure distributions on cylindrical bin walls from the experimental measurements showed that Janssen’s prediction was more appropriate for small bins, while Roberts’s load analysis model (for support columns buried in stockpiles) was more suitable to predict the pressure distributions for bins with larger diameters. Overall, Roberts’s load analysis model could be used as a conservative approach to guide designs. This can be explained as, when the bin diameter grows larger, the stress condition near a chosen narrow ribbon-shaped bin wall can be approximated to that near a support structure buried in a stockpile. For the pressures on a funnel-flow hopper wall after filling, the pressure close to the outlet is significant lower than the pressure calculated by Janssen’s equation. This is because of local arching of solids in the region approaching to the opening.

### 7.1.2 Loads on Support Structural Elements Buried in Stockpiles

Loads exerted on support structures buried in open bulk stockpiles or bulk storage sheds are complex and closely related to many factors, such as the manner and mechanism of
loading and discharge, handing bulk material, length of undisturbed storage time and so forth. The design of the dimensions of a support structure highly depends on the load conditions acting on all faces of the structure. The original load analysis model proposed by Roberts [55] is a new continuum approach based on the chosen particular properties of a bulk material. In the current study, this continuum approach was refined in terms of the wedge shape of the load analysis model and the properties of a bulk solid whose varying character was considered. Two different column setups buried in laboratory scale stockpiles as the support structures were tested: a fixed column and laterally moveable column. Tekscan tactile pressure sensors were applied to determine the pressure distributions on column faces for each column setup, while load cells were installed on the laterally moveable column to verify the accuracy of the pressure sensors and investigate the shear loads. Although significant variations were observed owing to the low sensitivity of the used pressure sensors in the low-pressure range and the general heterogeneous behaviour of bulk materials, the experimental outcomes confirmed that Tekscan tactile pressure sensors can provide good accuracy for determining pressure distributions on column faces buried in bulk materials. In addition, the measured pressure distributions on all column faces agreed reasonably with the theoretical calculations from the refined Roberts’s theory.

The pressure distributions on the column faces during the static and dynamic state were obtained from the experiments. The pressure distributions on the column front face varied more significantly than that on the other column faces, even though the same test parameters and same test material were employed. The stress state dominating the load condition on the front face of a buried column after loading process can be one of the three stress states: active stress state, passive stress state or combination of both. During the loading process, an active stress state developed at the initial stage of the loading, and either remained the same or changed to one of the other two stress states at the final stage. When the bulk material was discharged, the stress state changed to an active stress state, or remained a combination if a combination mode occurred before the discharge. This change of the stress state usually experienced one or more intermediate stress states during the dynamic reclaim process, until an active or a combination stress state developed when the discharge finished.
On the subject of the combination stress state of active and passive, it was difficult to predict where the transition of the two stress states was located (where the active/passive stress state on the upper section of the column ended, and the passive/active stress state on the lower section of the column began). Thus, multiple combinations occurred within the bulk material. This resulted in varying pressure distributions under the action of combinations of both stress states on the column front face. Moreover, the resulting force acting on the front face during the dynamic processes of loading, settling and reclaim was highly dependent on which stress state developed, and, during reclaim, a dramatic force jump occurred that increased steeply to a peak and dropped to the bottom when the change of the stress state happened. This force jump must be taken into account in designs, for safety purposes.

For the column rear, left and right faces, an active stress state was the dominant stress mode after loading, and this stress state remained unchanged during the entire dynamic loading process. When the material was discharged, the active stress state either remained the same or changed to a passive stress state or a combination, and this change usually underwent one or more intermediate stress states during the dynamic reclaim process until the final stress state was developed. On the subject of the force conditions, the resulting force acting on the column rear face experienced a short steady increase just after the force dropped to the bottom due to the change of the stress state near the termination of the discharge. The resulting forces on the column side faces were most likely to undergo a dramatic force jump at the moment when the stress state changed during the reclaim process, and this must be considered to ensure the reliable and efficient designs of support structures.

In addition, the experimental results measured at the two different column positions proved that there were no significant pressure differences for column faces caused by the variation of the position of the buried column. The influence of material cohesion on the pressures measured on the column front face was more observable than that on the rear face, which was a result of the occurrence of material build-up, which was more likely to form on the column leading front face. The coefficient for material build-up was proved
to be 1.0 (no material build-up) for free-flowing materials, such as the dry Beach Sand, and was larger than 1.0 for cohesive materials, such as approximately 1.2 for the cohesive Iron Ore B. In terms of the shear loads, the vertical compressive shear stresses acting along all column faces were proved to be proportional to the normal stresses on the corresponding column faces with a friction coefficient of $\tan \phi_w$. The lateral shear forces along the column front and rear faces were negligible for all test materials because there was little movement or potential movement in the lateral direction in a nearly symmetrical system. The lateral shear forces along the left and right faces could be ignored for cohesive material, but were proportional to the force exerted on the column front face with a shear coefficient of about 0.6 for the dry Beach Sand employed.

Further, when the handling material is stored for a long time, the increase of pressure during storage due to load settlement must be taken into account when determining loads on buried support structures. For design purposes, a conservative approach for the calculation of loads on buried support structures is recommended because there are many existing uncertainties of the possible variations in the loading conditions. This approach involves using an envelope of both active and passive pressure curves as the upper pressure limit, as shown in Figure 4.5(c). All possible combined appearances of both stress states on the column faces are enclosed in this upper bound curve, which introduces maximum reliability in designs. More to the point, a safety factor must be involved in the conservative envelope approach since a drastic change of the force on the column can occur due to the sudden change of the stress state when discharge is initiated.

In the DEM simulation, the non-dimensional simulation results from the PFC3D and Rocky software packages concerning the normal loads, lateral shear loads and vertical shear loads exerted on the buried column after loading agreed more or less with the relevant non-dimensional experimental and theoretical results for the two test materials. It was again verified that the dominant stress state for the column front face can be one of the three possible stress states after a stockpile is fully loaded, and the active stress state dominated the load conditions on the column rear, left and right faces. Both the magnitudes and trends of the normal pressure distributions and vertical shear pressure
distributions on the column front and two side faces from the simulation agreed reasonably with the non-dimensional experimental and theoretical results. However, the user-defined cohesive contact model employed in PFC3D software package is more capable to simulate the shear load conditions.

The normal pressure distribution and vertical shear pressure distribution on the column rear face for both materials from the simulation were lower than the corresponding experimental and predicted results, especially for the cohesive Iron Ore B. This may have been a result of the small scale of the simulation modelling and the cohesive contact model applied in the simulation, which introduced a weaker back-filling effect than that in practice for cohesive materials. On the subject of the lateral shear loads, the simulation results confirmed that the lateral shear pressures across the column front and rear faces could be ignored for both materials. The lateral shear pressures on the column side faces from the simulation confirmed the shear coefficient equal to 0.6 for free-flowing Beach Sand; however, the shear coefficient was also 0.6 for the cohesive Iron Ore B, which was expected to be zero. This may also have resulted from the small simulation scale and the cohesive contact model employed. Overall, despite some deficiencies, the DEM simulation provided a confirmation of Roberts’s load analysis theory to some extent.

In conclusion, both the experimental and simulation methods applied in this thesis verified the capability of the refined Roberts’s load analysis model in determining loads on support columns. The outcomes from this research work provide a more thorough understanding of both normal and shear loads exerted on support structures buried in bulk material stockpiles and storage sheds. Therefore, it can be a criterion for the designs of those support structures.
7.2 Recommendations for Future Work

Based on the research described in this thesis, a number of areas can be further investigated. There are four main directions in which this study believes that further research should be undertaken.

7.2.1 Constitutive Equations to Describe the Stress Conditions Within the Bulk Materials During Storage and Flow in a Silo or Stockpile

The capability of Roberts’s hoop stress theory in predicting the rathole geometry and the refined Roberts’s load analysis model in estimating the loads on buried support structures has been confirmed by the present work. However, there is a need to develop constitutive equations that can describe the stress conditions in bulk materials during storage and flow in a silo or gravity reclaim stockpile. These constitutive equations are deduced from the force and torque equilibriums at a chosen element with a three-dimensional analysis involved on the basis of the classic continuum theory. The loads on silo walls or on the support structures buried in a stockpile can be obtained according to the constitutive equations.

Consolidation or settling effect must be taken into account in the constitutive equations when the handling bulk material is stored for a certain period. When the flow is initiated, all the stored material (for mass flow silo case) or a part of the material (for funnel-flow silo and stockpile case) will flow out under gravity. The constitutive equations are also required to describe the change in stress conditions due to flow, and predict the rathole geometry after flow ceases, if the storage facility is a funnel-flow silo or gravity reclaim stockpile. Therefore, these constitutive equations can relate the loads on bin walls or loads on support structures to the formation of ratholes, which can be very beneficial to achieving a thorough understanding of the behaviour of bulk materials during handling processes, according to the principle analysis.
7.2.2 Computer Programs to Simulate Funnel Flow in a Silo or Gravity Reclaim Stockpile

An experimental investigation of funnel flow in a silo or gravity reclaim stockpile was undertaken in this thesis, and the results verified Roberts’s hoop stress theory. However, there is a research gap concerning simulation investigations of the formation of ratholes in funnel-flow silos or gravity reclaim stockpiles. This simulation work can be performed by computer programs, and there are two possible approaches to fulfil the task. One is to simply apply DEM to simulate ratholes similar to the experimental ones by adopting proper simulation parameters through a trial-and-error process. The calibration tests for the simulation can be improved in future research. DEM simulation can provide the geometry of the rathole and the stress circumstances in the vicinity of the rathole, which will be beneficial for verifying the hoop stress in Roberts’s hoop stress theory, or the stresses in the constitutive equations discussed in Section 7.2.1. The second approach is to couple DEM and FEA to investigate the interaction between the formation and geometry of the rathole and the stress field in the vicinity of the rathole.

7.2.3 Prediction of the Load Conditions for Different Configurations and Shapes of the Support Structures Buried in Stockpiles

The support column buried in stockpiles from different bulk materials in both the experimental and simulation investigations presented in this thesis was rectangular steel tubing, with only one column employed in each test. However, the configurations regarding the number and positions of the support structures buried in a stockpile can vary in practice. In addition, the shapes of these structures can be different. In this case, the stress conditions developed within the stored bulk material will be dissimilar to the foregoing discussion; hence, the loads exerted on these support structures can be different. Therefore, it is necessary to conduct further experimental and simulation studies to predict these loads by involving different configurations and shapes of support structures.
7.2.4 Prediction of the Deformation of the Buried Support Structures

In this study, only the loads acting on the buried support column were investigated, and the deformation that may occur was not considered. In industrial applications, stockpiles can be dozens of metres high, and the buried depths of the support structures can be considerable. Consequently, the resulting significant total lateral force and vertical compressive force exerted on each of these support structures can deform the structure, and collapse may result if the structure is not strong enough. This deformation would be more sizeable under one or more conditions, such as the bulk material being stored for a long time, the ambient moisture content increasing due to rain and the stockpile being reloaded. To achieve efficient and reliable designs of these support structures, a combination of DEM and FEA can be adopted. The normal loads, lateral shear loads and vertical shear loads can be obtained from DEM, and these results can be applied to the simulation modelling of the support structure in the FEA simulation program to compute the deformation of the structure. Therefore, the combination of both DEM and FEA can provide a guide for the geometry design of the support structure for a chosen structure material.
References


[58] A.C. Institute, Recommended practice for design and construction of concrete bins, silos, and bunkers for storing granular materials (ACI 313–77), reported by ACI Committee 313, American Concrete Institute, Detroit, Michigan, 1984.


[79] M.S.A. Bradley, G. Lee, M.S. Bingley, Effects of bulk solids properties on relative performance of polyethylene and stainless steel wall lining materials in mass flow


Appendix A: The Deduction of Roberts’s Load Analysis Model in the Application of Bin Loads

Roberts’s load analysis model was adapted to the analysis of pressure distributions on a cylindrical bin wall. The modified model is illustrated in Figure A.1. A differential equation can be deduced from the force equilibrium as a slice element in y direction.

\[
p_y s_1 - (p_y + dp_y) s_2 + \gamma_y V - p_{nc} s_3 - p_{nsy} s_4 - \tau_{cy} s_3 - \tau_{sy} s_4 = 0 \quad (A-1)
\]

where:
\( s_1 = \) the area of the top surface of the slice element:

\[
s_1 = \frac{R_0}{\cos \alpha_c} (y_h - y)(\tan \alpha_c + \tan \alpha_s) - \frac{1}{2} \theta (y_h - y)^2 (\tan \alpha_c + \tan \alpha_s)^2 \quad (A-2)
\]

\( s_2 = \) the area of the bottom surface of the slice element:

\[
s_2 = \frac{\theta}{\cos \alpha_c} (y_h - y - dy)(\tan \alpha_c + \tan \alpha_s) - \frac{1}{2} \theta (y_h - y - dy)^2 (\tan \alpha_c + \tan \alpha_s)^2
\]

\( s_3 = \) the area of the left surface of the slice element:

\[
s_3 = \frac{\theta}{(\cos \alpha_c)^2} \ dy \quad (A-4)
\]

\( s_4 = \) the area of the right inclined surface of the slice element:

\[
s_4 = \frac{\theta}{\cos \alpha_c \cos \alpha_s} \ dy - \frac{\theta (y_h - y)(\tan \alpha_c + \tan \alpha_s)dy}{\cos \alpha_s}
\]

\( V = \) the volume of the slice element:

\[
V = \frac{\theta}{\cos \alpha_c} (y_h - y - 0.5dy)(\tan \alpha_c + \tan \alpha_s)dy - \frac{1}{4} \theta [(y_h - y)^2 + (y_h - y - dy)^2](\tan \alpha_c + \tan \alpha_s)^2dy
\]

\( p_{ncy} = \) the \( y \) component of the normal pressure on column surface:

\[
p_{ncy} = \sin \alpha_c p_{nc} = \sin \alpha_c K_c p_y
\]

\( p_{nzy} = \) the \( y \) component of the normal pressure on the failure surface:

\[
p_{nzy} = \sin \alpha_s p_{ns} = \sin \alpha_s K_s p_y
\]
\( \tau_{cy} \) is the \( y \) component of the shear stress at the column surface:

\[
\tau_{cy} = \cos\alpha_c \tau_c = \cos\alpha_c \tan\phi_w K_c p_y \quad (A-9)
\]

\( \tau_{sy} \) is the \( y \) component of the shear stress at the failure surface:

\[
\tau_{sy} = \cos\alpha_s \tau_s = \cos\alpha_s \sin\delta K_s p_y \quad (A-10)
\]

\( \theta \) is the radial angle of the slice element in the direction perpendicular to the height \( dy \), and is in radians. Therefore, Equation (A-1) can be simplified as follows:

\[
\frac{dp_y}{dy} \left[ \frac{R}{\cos\alpha_c} (\tan\alpha_c + \tan\alpha_s) (y_h - y) - \frac{1}{2} (\tan\alpha_c + \tan\alpha_s)^2 (y_h - y)^2 \right] - \\
\frac{p_y R}{\cos\alpha_c} (\tan\alpha_c + \tan\alpha_s) - (\tan\alpha_c + \tan\alpha_s)^2 (y_h - y) - \frac{K_c R}{(\cos\alpha_c)^2} (\sin\alpha_c + cos\alpha_c \tan\phi_w) - \frac{K_c R}{(\cos\alpha_s \cos\alpha_s)} (\sin\alpha_s + \cos\alpha_s \sin\delta) + \frac{K_s (\tan\alpha_c + tan\alpha_s)(\sin\alpha_s + \cos\alpha_s \sin\delta)}{\cos\alpha_s} (y_h - y)
\]

\[
y = \gamma \frac{R}{\cos\alpha_c} (\tan\alpha_c + \tan\alpha_s) (y_h - y) - \frac{1}{2} (\tan\alpha_c + \tan\alpha_s)^2 (y_h - y)^2 \right] \quad (A-11)
\]

By assuming the parameters \( ii, jj, kk, mm, nn \) as follows:

\[
ii = \frac{R}{\cos\alpha_c} (\tan\alpha_c + \tan\alpha_s) \quad (A-12)
\]

\[
jj = (\tan\alpha_c + \tan\alpha_s)^2 \quad (A-13)
\]

\[
kk = \frac{K_c R}{(\cos\alpha_c)^2} (\sin\alpha_c + \cos\alpha_c \tan\phi_w) \quad (A-14)
\]

\[
mm = \frac{K_c R}{\cos\alpha_c \cos\alpha_s} (\sin\alpha_s + \cos\alpha_s \sin\delta) \quad (A-15)
\]

\[
mm = \frac{K_s (\tan\alpha_c + \tan\alpha_s)(\sin\alpha_s + \cos\alpha_s \sin\delta)}{\cos\alpha_s} \quad (A-16)
\]

The force equilibrium analysis in the \( y \) direction leads to the following differential equation:
\[
\frac{dp_y}{dy} - \frac{(nn-j)(y_h-y)+ii-kk-mm}{i(y_h-y)-0.5j(y_h-y)^2} p_y = \gamma_y
\]  

(A-17)

The definitions of the other parameters of Roberts’s load theory were described in Chapter 4.
Appendix B: C++ Codes for the Cohesive Contact Model in the Simulation

C++ codes for the cohesive contact model (hertz model):

hertz.h file

```cpp
#ifndef __CMODEL_HERTZ_H
#define __CMODEL_HERTZ_H

#ifndef __CMODEL_H
#include "cmodel.h"
#endif

// User-defined contact model "udm_hertz"
const unsigned long ulCM_hertz = 101;
// User-written models should use type numbers >= 100, and each model
// should have a unique number. These numbers are used in save/restore.
//#define PI 3.141592

class CM_hertz : public ContactModel {
private:
    double meff_;        // effective mass
    double hn_;          // Hertz normal constant
    double hs_;          // Hertz shear constant
    bool   hssd_;        // scale down tangential force when overlap decreases ?
    double fric_;        // friction coefficient
    // AS Declare all the variables!!!
    double un;
    double kn;
    double hnforce_old;
    double ks_old;
    double ks;
    double kst;
    double max_s_force;
    double fsMag;
    double rat;
    double cn;
    double cs;
    double dF;
    double kr_;          // AK 11.03.2011
    double mom_mag_;      // AK 11.03.2011

```
double rfric_;  // rolling friction
double Rr_;   // rolling radius
double rmom_;  // rolling resistance moment
double vol_;   // Volume in water bridge
bool Brid_;   // Water bridge active flag
double Sten_;  // Surface Tension of the liquid
double PI;
double dist;
double maxdist;
double k;
double Rone;
double Rtwo;
double Requ;
double maxratio;
double Rgeo;
double lamb;
double Cap_forceini;
double Cap_forceexp;
double Cap_force;

//
double vn_;     // viscous damping normal critical ratio
double vs_;     // viscous damping shear critical ratio
bool    nt_;    // viscous no-tension flag
//Read only:
double   hnforce_; // Hertz Elastic contribution to the Hertz normal force (read-only)
UMdvect3 hsforce_; // Hertz Elastic contribution to the shear force (read-only - 2D uses 1st comp.)
double   vnforce_; // Viscous contribution to the normal force (read-only)
UMdvect3 vsforce_; // Viscous contribution to the shear force (read-only - 2D uses 1st comp.)

public:
EXPORT CM_hertz( bool bRegister=false, ICodeFunc *cf=0 );
EXPORT ~CM_hertz(void) { }
EXPORT ContactModel *Clone( ICodeFunc *cf=0 ) const
{ return new CM_hertz(false, cf); }

//
EXPORT const char   *Name(void) const;
EXPORT const char  **PropNames(void) const;
EXPORT double ReturnProp(int n) const;
EXPORT void AcceptProp(int n, double v);
EXPORT double KnEstimate(void) const { return hn_; }
EXPORT double KsEstimate(void) const { return hs_; }
EXPORT unsigned long Version(void) const { return 1; }
EXPORT const char   *PreCycle(ICodeFunc &) { return 0; }
EXPORT void FDLaw(FdBlock &fb, ICodeFunc &cf);
EXPORT const char   *SaveRestore(ModelSaveObject *mso);
EXPORT bool TransferProps(const ContactModel &cm, bool reverse=false);
```cpp
hertz.cpp file

#include "hertz.h"
#include <math.h>

inline double mag(UMdvect3 v) { return sqrt( v.x*v.x + v.y*v.y + v.z*v.z );}

static CM_hertz CM_hertz(true);

CM_hertz::CM_hertz( bool bRegister, ICodeFunc *cf ) : ContactModel(ulCM_hertz, bRegister),
  meff_(0.0),hn_(0.0),hs_(0.0),hssd_(false),
  fric_(0.0),vn_(0.0),vs_(0.0),nt_(false),
  hnforce_(0.0),vnforce_(0.0),rfric_(0.0),Rr_(0.0), rmom_(0.0),
  vol_(0.0),Brid_(false),Sten_(0.0)

  // Add any extra variables that need to be accessible/definable by fish here
  (to be initialised)
  {
    hsforce_.Fill(0.0);
    vsforce_.Fill(0.0);
    MustOverlap( false );
  }

const char *CM_hertz::Name(void) const {
  return("udm_hertz");
}

const char **CM_hertz::PropNames(void) const {
  static const char *strKey[] =
  {
    "hertz_meff", // 0
    "hertz_hn",  // 1
    "hertz_hs",  // 2
    "hertz_hssd", // 3
    "hertz_fric", // 4
    "hertz_vn",  // 5
    "hertz_vs",  // 6
    "hertz_nt",  // 7
    "hertz_hnforce", // 8
    "hertz_hsxforce",// 9
    "hertz_hsyforce",// 10
    "hertz_hszforce",// 11
    "hertz_vnforce", // 12
    "hertz_vsxforce",// 13
    "hertz_vsyforce",// 14
```
"hertz_vsforce", // 15
    // AK 11.03.2011
"hertz_rfric", // 16
"hertz_Rr",    // 17 CMW 23.5.2011
"hertz_rmom",    // 18
0
};
return(strKey);
// Add any extra variables here that need to be accessible/definable by fish.
// These contact property names correspond to the names given in the ~setup~.dat file
}

double CM_hertz::ReturnProp(int n) const {
    switch (n) {
        case 0: return meff_; break;
        case 1: return hn_; break;
        case 2: return hs_; break;
        case 3: return hssd_; break;
        case 4: return fric_; break;
        case 5: return vn_; break;
        case 6: return vs_; break;
        case 7: return (nt_ ? 1.0 : 0.0); break;
        case 8: return hnforce_; break;
            case 9: return hsforce_.x; break;
            case 10: return hsforce_.y; break;
            case 11: return hsforce_.z; break;
            case 12: return vnforce_; break;
            case 13: return vsforce_.x; break;
            case 14: return vsforce_.y; break;
            case 15: return vsforce_.z; break;
                // AK 11.03.2011
        case 16: return rfric_; break;
        case 17: return Rr_; // CMW 23.5.2011
        case 18: return rmom_; break;
        case 20: return (Brid_ ? 1.0 : 0.0); // AS 13.12.2011
        default: return 0.0;
            // These are the cpp variables that correspond to the names given above
    }
}

void CM_hertz::AcceptProp(int n, double v) {
    switch (n) {
        case 0: meff_ = v; break;
        case 1: hn_ = v; break;

case 2: hs_ = v; break;
case 3: hssd_ = (v == 0 ? false : true); break;
case 4: fric_ = v; break;
case 5: vn_ = v; break;
case 6: vs_ = v; break;
case 7: nt_ = (v == 0 ? false : true); break;
  case 8: hnforce_ = v; break;
  case 9: hsforce_.x = v; break;
  case 10: hsforce_.y = v; break;
  case 11: hsforce_.z = v; break;
  case 12: vnforce_ = v; break;
  case 13: vsforce_.x = v; break;
  case 14: vsforce_.y = v; break;
  case 15: vsforce_.z = v; break;
  // AK 11.03.2011
  case 16: rfric_ = v; break;
  case 17: Rr_ = v; break; // CMW 23.5.2011
  case 18: rmom_ = v; break;
  case 19: vol_ = v; break; // AS 12.12.2011
  case 20: Brid_ = (v == 0 ? false : true); break; // AS 13.12.2011
  case 21: Sten_ = v; break; // AS 13.12.2011
  // Add any extra variable here (note the cpp variable name is used - not the fish name)
  default: break;
}
}

void CM_hertz::FDLaw( FdBlock& fb, ICodeFunc &cf ) {
  if ( cf.dim() == 2 ) { // PFC2D
    if (fb.u_n <= 0.0) {
      fb.n_force = 0.0;
      fb.s_force = 0.0;
      fb.knest = 0.0;
      fb.ksest = 0.0;
      hnforce_ = 0.0;
      hsforce_.Fill(0.0);
      vnforce_ = 0.0;
      vsforce_.Fill(0.0);
      fb.active = false;
    } else {
      // Hertz part
      un = fb.u_n;
      if (!cf.c_state().Obj2IsBall) un *=2.0;
      kn = 1.5 * hn_ * sqrt(un);
      hnforce_old = hnforce_;
      ks_old = hs_ * pow(hnforce_old,(1.0/3.0));
      hnforce_ = hn_ * pow(un,1.5);
      ks = hs_ * pow(hnforce_,(1.0/3.0));
      kst = ks * fb.tdel;
if (hssd_ && (hnforce_ < hnforce_old)) {
    hsforce_.x *= (ks / ks_old);
}

hsforce_.x -= fb.u_dot_s * kst;
// Coulomb friction criterion (on elastic contributions only)
    max_s_force = fric_*hnforce_;  
    fsMag = abs(hsforce_.x);
    if (max_s_force < fsMag) {
        rat = max_s_force / fsMag;
        hsforce_.x *= rat;
        fb.bsliding = true;
    }
// Viscous part
    cn = 2.0 * sqrt(meff_ * kn) * vn_;  
    vnforce_ = - cn * fb.u_dot_n;
    if (nt_ && ((hnforce_ + vnforce_) < 0.0)) vnforce_ = - hnforce_;  
    vsforce_.x = 0.0;
    if (!fb.bsliding) {
        cs = 2.0 * sqrt(meff_ * ks) * vs_;  
        vsforce_.x = fb.u_dot_s * (-1.0 * cs);
    }
// Accumulate contributions
    fb.n_force = hnforce_ + vnforce_;  
    fb.s_force = hsforce_.x + vsforce_.x;
// Update stiffness estimates (corrected if viscous damping)
    fb.knest = kn;
    fb.ksest = ks;
    if (vn_ != 0.0) {
        dF = sqrt(1.0 + vn_*vn_) - vn_;  
        fb.knest /= (dF*dF);
    }
    if (vs_ != 0.0) {
        dF = sqrt(1.0 + vs_*vs_) - vs_;  
        fb.ksest /= (dF*dF);
    }
}  
else { // PFC3D
    if (fb.u_n <= 0.0) {
        fb.n_force = 0.0;
        fb.s_force.Fill(0.0);
        fb.knest = 0.0;
        fb.ksest = 0.0;
        hnforce_ = 0.0;
        hsforce_.Fill(0.0);
        vnforce_ = 0.0;
        vsforce_.Fill(0.0);
        //SetDelete( false ); //AS Doesn't allow deletion as waterbridge is still present
if (Sten_ == 0.0) { //AS if no surface tension allows contact to be deleted and to
skip rest of logic
    //SetDelete( true );
    fb.active = false;
}
else {
    // Hertz part
    un = fb.u_n;
    if (!cf.c_state().Obj2IsBall) un *= 2.0;
        kn = 1.5 * hn_ * sqrt(un);
    hnforce_old = hnforce_;  
    ks_old = hs_ * pow(hnforce_old,(1.0/3.0));
    hnforce_ = hn_ * pow(un,1.5);
    ks = hs_ * pow(hnforce_,(1.0/3.0));
    kst = ks * fb.tdel;
    UMdvect3 hsforce_old = hsforce_ + (fb_srot & hsforce_); // Accomodate objects relative
rotation to achieve proper accumulation
    UMdvect3 vec = fb_u_dot_s * kst;
    if (hsd_ && (hnforce_ < hnforce_old)) {
        rat = ks / ks_old;
        hsforce_old *= rat;
    }
    hsforce_ = hsforce_old - vec;
    // Coulomb friction criterion (on elastic contributions only)
    max_s_force = fric_*hnforce_;  
    fsMag = mag(hsforce_);  
    if (max_s_force < fsMag) {
        rat = max_s_force / fsMag;
        hsforce_ *= rat;
        fb.bsliding = true;
    }
    // Viscous part
    cn = 2.0 * sqrt(meff_*kn) * vn_;  
    vnforce_ = - cn * fb.u_dot_n;
    if (nt_ && ((hnforce_ + vnforce_) < 0.0)) vnforce_ = - hnforce_;  
    vsforce_.Fill(0.0);
    cs = 2.0 * sqrt(meff_*ks) * vs_;  
    if (!fb.bsliding) {
        vsforce_ = fb_u_dot_s * (-1.0*cs);
    }
    // Accumulate contributions
    fb.n_force = hnforce_ + vnforce_;  
    fb.s_force = hsforce_ + vsforce_;  
    // Update stiffness estimates (corrected if viscous damping)
    fb.knest = kn;
    fb.ksest = ks;
if (vn_ != 0.0) {
    dF = sqrt(1.0+vn_*vn_) - vn_;
    fb.knest /= (dF*dF);
}
if (vs_ != 0.0) {
    dF = sqrt(1.0+vs_*vs_ - vs_);
    fb.ksest /= (dF*dF);
}

// Model C rolling friction - CMW 23.5.2011
if (fric_ > 0){
    kr_ = pow(Rr_,2)*ks; //Rolling stiffness is defined from the shear stiffness as
                     //per Iwashita and Oda
    fb.moment.x = kr_ * fb.t_dot_rel.x * fb.tdel; //subtract kr*dTheta_rel from
    fb.moment.y = kr_ * fb.t_dot_rel.y * fb.tdel;
    fb.moment.z = kr_ * fb.t_dot_rel.z * fb.tdel;
    mom_mag_ = mag(fb.moment);
    if (mom_mag_ > fric_*Rr_*fb.n_force) { // If it is above
        fb.moment *= fric_*Rr_*fb.n_force / mom_mag_; // scale it back so that it is on the limit
    }
    rmom_ = fb.moment.y; // A variable used to monitor the rolling moment
}

// Model C Liquid Bridge - AJS 1/02/2012
if (Sten_ > 0.0) {
    PI = 3.141592;
    dist = -fb.u_n;
    maxdist = pow(vol_,(1.0/3.0));
    if (dist > maxdist) { //AS checks if distance is greater than max liquid bridge
        Brid_ = false; //AS Liquid bridge is not present
        //SetDelete( true ); //AS Allows contact to be deleted
        fb.active = false; //AS skips remainder of contact logic
    }
    if (fb.u_n > 0.0) { //SetDelete( false ); //AS Doesn't allowed to be deleted. For the case
        Brid_ = true; // Contact is made liquid bridge is formed
    }
}
if (Brid_) { //AS if liquid bridge
    k = 2*PI*Sten_;
    Rone = cf.c_state().rad_obj1; // Radius of the two balls in contact
    Rtwo = cf.c_state().rad_obj2;
    if (!cf.c_state().Obj2IsBall) Rtwo = Rone; // If ball-wall contact Rtwo = Rone
    }
\( \text{Requ} = \text{Rone} \times \text{Rtwo}; \)
\( \text{Requ} = \sqrt{\text{Requ}}; \)
\( \text{Cap\_forceini} = -k \times \text{Requ}; \)

\[
\text{if (fb\_u\_n} \leq 0.0) \{ // \text{if there is a gap exponential drop off of tension and only Capillary forces}
\]
\[
\quad \text{maxratio} = \frac{(\text{Rone}/\text{Rtwo})>(\text{Rtwo}/\text{Rone})?(\text{Rone}/\text{Rtwo})(\text{Rtwo}/\text{Rone});}
\]
\[
\quad \text{Rgeo} = (1.0/\text{Rone})+(1.0/\text{Rtwo});
\]
\[
\quad \text{lamb} = 0.9*(\sqrt{\text{vol}_{\text{}}})^\text{pow}(\text{maxratio},-0.5)\times\sqrt{\text{Rgeo}})/1.414213562;
\]
\[
\quad \text{Cap\_forceexp} = \exp(\text{fb\_u\_n}/\text{lamb});
\]
\[
\quad \text{Cap\_force} = \text{Cap\_forceini}\times\text{Cap\_forceexp};
\]
\[
\quad \text{fb\_n\_force} += \text{Cap\_force};
\]
\[
\}
\]
\[
\quad \text{else} \{ // \text{Overlap and Capillary force is added on to other forces}
\]
\[
\quad \quad \text{Cap\_force} = \text{Cap\_forceini};
\]
\[
\quad \quad \text{fb\_n\_force} += \text{Cap\_force};
\]
\[
\}
\]

\}

const char *CM_hertz::SaveRestore(ModelSaveObject *mso) {
    const char *str = ContactModel::SaveRestore(mso);
    if (str) return str;
    mso->Initialize(19, 0, 5); // Update the first number to the number of variables (doubles
    // doubles
    mso->Save(0, meff_);
    mso->Save(1, hn_);
    mso->Save(2, hs_);
    mso->Save(3, fric_);
    mso->Save(4, vn_);
    mso->Save(5, vs_);
    mso->Save(6, hnforce_);
    mso->Save(7, hsforce_x_);
    mso->Save(8, hsforce_y_);
    mso->Save(9, hsforce_z_);
    mso->Save(10, vnforce_);
    mso->Save(11, vsforce_x_);
    mso->Save(12, vsforce_y_);
    mso->Save(13, vsforce_z_);
    mso->Save(14, rfric_);
    mso->Save(15, Rr_);
    mso->Save(16, rmom_);
    mso->Save(17, vol_);
    mso->Save(18, Sten_);
    // New variables need to be specified here in order for them to be included in the "save" process
// bools
mso->Save( 0, delete_flag);
mso->Save( 1, must_overlap);
mso->Save( 2, hssd_);
mso->Save( 3, nt_);
mso->Save( 4, Brid_);
return 0;
}

bool CM_hertz::TransferProps(const ContactModel &cm, bool reverse)
{
    must_overlap = cm.MustOverlap();
    delete_flag  = cm.OKtoDelete();
    for (int i=0; i<22; i++) // AK 11.03.2011: i<16
        //The counter must be incremented with the addition of any extra variable
        AcceptProp(i,cm.ReturnProp(i));
    // change shear direction if reverse is true
    if (reverse)
    {
        hsforce_* *= -1.0;
        vsforce_* *= -1.0;
    }
    return true;
}

/** EOF */

Configure the c++ user defined model:

; fname: udm_hertz_setup.p3dat
; Configure the CPPUDM option and load the udm_hertz library
; Define functions:
;  - "catch_contact"  -- to register with #FC_CONT_CREATE
;  - "modify_contacts" -- to modify existing contacts
;  - "liquidbridge"   -- to reallocate the volume of the liquid bridges when one is created or breaks
;
; config cppudm
model load udm_Hertz_wet_rfric_64.dll

; def catch_contact     ; Called whenever new contact made.
    cp = fc_arg(0)
    modify_cp
end

; def modify_contacts
realocate_water_dist
cp = contact_head
loop while cp # null
    modify_cp
    cp = c_next(cp)
endloop
end

;================================================================================

def modify_cp

c_model(cp) = 'udm_hertz'
bp1 = c_ball1(cp)
bp2 = c_ball2(cp)
_fricset = _fric

if pointer_type(bp2) = 100 ; Ball-Ball contact
    _meff = b_realmass(bp1)*b_realmass(bp2)/(b_realmass(bp1)+b_realmass(bp2))
    _rbar = 2.0 * b_rad(bp1) * b_rad(bp2) / (b_rad(bp1) + b_rad(bp2))
    _Rr = b_rad(bp1)*b_rad(bp2)/(b_rad(bp1)+b_rad(bp2)) ; CMW 23.5.2011
else    ; Ball-Wall contact
    _meff = b_realmass(bp1)
    _rbar = b_rad(bp1)
    _fricset = _wfric
    _Rr=b_rad(bp1) ; CMW 23.5.2011
end_if

_hn = 2.0 * sqrt(2.0*_rbar)*_G / (3.0*(1.0-7poi));
_temp = _G*_G*3.0*(1.0-7poi)*_rbar;
_hs = 2.0 * exp((1.0/3.0)*ln(temp)) / (2.0-7poi);
if pointer_type(bp2) = 100 ; Ball-Ball contact
    _cos_theta1 = ((b_rad(bp1)+b_rad(bp2))^2+(b_rad(bp1)+_waterthickness)^2
        - b_rad(bp2)^2)/(2*(b_rad(bp1)+b_rad(bp2))*(_waterthickness))
    volcon1 = 2*(3*b_rad(bp1)^2*_waterthickness + 
            3*b_rad(bp1)_waterthickness^2+_waterthickness^3)*pi*(1-7cos_theta1)/3
    _cos_theta2 = ((b_rad(bp2)+b_rad(bp1))^2+(b_rad(bp2)+_waterthickness)^2
        - b_rad(bp1)^2)/(2*(b_rad(bp2)+b_rad(bp1))*(_waterthickness))
    volcon2 = 2*(3*b_rad(bp2)^2*_waterthickness + 
            3*b_rad(bp2)_waterthickness^2+_waterthickness^3)*pi*(1-7cos_theta2)/3
    _vol = volcon1 + volcon2
else
    _cos_theta1 = (b_rad(bp1))/(b_rad(bp1)+_waterthickness)
    volcon1 = 2*(3*b_rad(bp1)^2*_waterthickness + 
            3*b_rad(bp1)_waterthickness^2+_waterthickness^3)*pi*(1-7cos_theta1)/3
    _vol = volcon1
end_if

c_prop(cp,'hertz_meff') = _meff
c_prop(cp,'hertz_hn') = _hn
c_prop(cp,'hertz_hs') = _hs
c_prop(cp,'hertz_fric') = _fricset
  c_prop(cp,'hertz_vn') = _vn
  c_prop(cp,'hertz_vs') = _vs
  c_prop(cp,'hertz_nt') = _nt
  c_prop(cp,'hertz_hssd') = _hssd
  c_prop(cp,'hertz_rfric') = _rfric ; AK 11.03.2011
  c_prop(cp,'hertz_Rr') = _Rr ; CMW 23.5.2011
  c_prop(cp,'hertz_Sten') = _Sten ;AS 13.12.2011
  c_prop(cp,'hertz_vol') = _vol ;AS 30.3.2012
  c_prop(cp,'hertz_Brid') = 0 ;AS 13.1.2012
end

;==============================================================================================

def reallocate_water_dist ;This function recalculates the total mass and the total surface area ;Run this if enlarging particles after they have been created

  Total_surf = 0.0
  Total_Mass = 0.0
  bp_1 = ball_head
loop while bp_1 # null
    Total_Mass = Total_Mass + b_realmass(bp_1)
    Total_surf = Total_surf + 4*pi*b_rad(bp_1)*b_rad(bp_1)
    bp_1 = b_next(bp_1)
end_loop

  Total_mliquid = Total_Mass*_Moistcontent ;Calculating the thickness of the water
  Total_vol = Total_mliquid/liqdensity
  _waterthickness = Total_vol/Total_surf
end

;===============================================================================================

def Add_surf_mass ;Keeping track of total surface area and mass for new balls being created
  bp_a = fc_arg(0)
  Total_Mass = Total_Mass + b_realmass(bp_a)
  Total_surf = Total_surf + 4*pi*b_rad(bp_a)*b_rad(bp_a)
  Total_mliquid = Total_Mass*_Moistcontent ;Calculating the thickness of the water
  Total_vol = Total_mliquid/liqdensity
  _waterthickness = Total_vol/Total_surf
end

;===============================================================================================

def Sub_surf_mass ;Keeping track of total surface area and mass for balls being deleted
  bp_a = fc_arg(0)
  Total_Mass = Total_Mass - b_realmass(bp_a)
  Total_surf = Total_surf - 4*pi*b_rad(bp_a)*b_rad(bp_a)
  Total_mliquid = Total_Mass*_Moistcontent ;Calculating the thickness of the water
  Total_vol = Total_mliquid/liqdensity
  _waterthickness = Total_vol/Total_surf
end
def report
    contactdel = fc_arg(0)
    forceincontact = c_nforce(contactdel)
    if forceincontact # 0
        ;_s = out("************************************************")
        ;_s = out("********* contact deleted with force************")
        ;_s = out("* '+string(forceincontact)+' *")
        ;_s = out("************************************************")
        nocondeleted = nocondeleted + 1
    end_if
end

set fishcall 4 Add_surf_mass ; Called whenever a new ball is created
set fishcall 5 Sub_surf_mass ; Called whenever a ball is deleted
set fishcall 8 catch_contact ; Called whenever a new contact is detected
set fishcall 9 report
    nocondeleted = 0
;=================================================================================
return
;=================================================================================
;EoF: udm_hertz_setup.p3dat