

Accessed from: http://hdl.handle.net/1959.13/1056237
Applications of Graph Theory in Architectural Analysis: Past, Present and Future Research.

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Abstract

In the 1970s architectural scholars adopted graph theory to support several major analytical approaches to interior and urban design. While the basis for graph theory in architecture is identical to that in mathematics, architects developed several discipline-specific methods for mapping nodes and edges to various spatial and formal features, and then set out to interpret these maps through a combination of mathematical analysis and observations of human behaviour, social structures and building types. This chapter undertakes a review of architectural applications of graph theory, spanning from its initial use for solving pedestrian circulation problems, through to more recent applications providing insights into access planning, design psychology and way-finding. Through this critical review, three mapping or abstraction techniques, which are a precursor to graph theory analysis, are described and demonstrated. Finally, the chapter identifies three areas that could be the subject of future applications of graph theory in design.

Introduction

Few authors have contributed more to the application of graph theory in architectural analysis than Bill Hillier and Julienne Hanson. Their canonical 1984 work, the Social Logic of Space, establishes a link between social phenomena and spatial configuration in such a way that understanding one contributes an insight into the other. The opening line of the Social Logic of Space sets this agenda with the claim that, “[h]owever much we may prefer to discuss architecture in terms of visual styles, its most far-reaching practical effects are not at the level of appearances at all, but at the level of space” (Hillier and Hanson 1984; p. ix). Because this division between space and form is so pivotal to the rise of graph theory applications in architecture it is important to differentiate between the two.

Hillier observes that “[b]uildings and cities exist for us in two ways: as the physical forms that we build and see, and as the spaces that we use and move through” (Hillier 2005; p. 97.). Architecture has traditionally been understood in terms of its shape, materiality and texture. These properties, commonly grouped under the heading “architectural form”, describe the combined geometry and dimensionality of a building (Gelernter 1995). In contrast, space is that which is either enclosed by, or shaped by, form. Thus, a building delineates both the
space it contains, its interior, and the space it is contained within, its site or context (Fig. 1). Ching describes the relationship between form and space as a “unity of opposites” (Ching 2007; p. 96). The role of form in architecture is to structure and define the spaces we live in. However, we cannot inhabit form, we can only inhabit the voids that are framed or delineated by form.

![Diagram of differentiating space and form](image)

Figure 1: Differentiating space and form: forms exist in space and can, in turn, define space. A building contains internal space (left), has form and dimensions (centre) and is itself contained within space (right).

Hillier’s and Hanson’s research encouraged a paradigm shift in architecture by suggesting that the study of the structure of space should be divorced from the more innately subjective study of architectural form. They argue that space may be empty, invisible and amorphous, but it does have two critical qualities, depreciable difference and permeability. The first of these relates to the capacity to differentiate one space from any other, and the second refers to the way in which spaces are connected or configured. Drawing on a common linguistic metaphor for conceptualising architecture, they defined the study of the arrangement of space as “space syntax”. This title is appropriate because the set of rules that govern the generation of form is typically described as the “grammar” of architecture, thus, the pattern of arrangement of spaces could be thought of as the “syntax” of architecture.

The theory of space syntax is constructed around a reciprocal arrangement between two ideas. First, it proposes that “a spatial layout can reflect and embody a social pattern” (Hillier 2005; p. 104.). Such a pattern serves to enshrine the collective social structure and values of a group in the spatial configuration of the buildings designed to accommodate that society. Second, “space can also shape a social pattern” (p. 104.); because the way it is structured places certain areas in more central positions and locates others to the periphery. Thus, when
moving between any two spaces occupants will pass through the central ones more frequently. In this way such spaces offer greater potential for co-presence of inhabitants and subsequent heightened social interaction (Montello 2007). This means that adjusting the spatial structure also alters the potential for social interaction. Peponis and Wineman (2002) summarise this two-way dependency between space and social structure with the observation,

[...] that it is possible to identify certain underlying structures of space that are linked to observable patterns of behavior and that these patterns, in turn, create social function, whether generative or reproductive. (p. 272.)

Bafna (2003) offers a similar account of this reciprocal dependency, as being “that social structure is inherently spatial and inversely that the configuration of inhabited space has a fundamentally social logic” (p. 18). Space syntax is therefore a way of studying the relationship between the configurational patterns in the built environment and the social and psychological properties of spatial experience. While this is its goal, in an operational sense, space syntax works by first reducing or abstracting an environment, typically an architectural or urban plan, into a series of differentiated and connected spaces, and second, by examining the topological relationships between these spaces using graph theory. Consequently, the connections which are formed between space and social structure are all reliant on graph theoretic measures.

This chapter provides an overview of the relationship between architecture and graph theory, focussing on applications and innovations but without referring to the mathematical methods which are common to most applications of graph theory (Ostwald 2011a). It commences with a brief review of the origins of graph theory and its foundational role in the development of the theory of space syntax. The logic of using graph theoretic measures is then addressed along with the interpretation of these results. This is followed by a detailed discussion of the three common abstraction or mapping models used in space syntax analysis to translate complex environments into graphs. This is the central purpose of the chapter, to describe these models of graphing space, their relative strengths and weaknesses, variations and applications. Thereafter the chapter reviews several underutilised variants of graph theory in architecture. These include applications for understanding spatial preference, a move to the analysis of buildings and urban spaces in three dimensions, and the inclusion of previously excluded contextual information.

**Graph Theory and Space Syntax**
The origins of graph theory are conventionally traced to a particular arrangement of seven bridges over the Pregel river in the Prussian town of Konigsberg. Historic accounts suggest that the bridges were a source of a popular local conundrum. The local populace would attempt to walk a circuit of these bridges, visiting each of the town’s four landmasses in turn, by crossing each bridge only once, before returning to their point of departure (Hopkins and Wilson 2004). In 1736, Leonhard Euler simplified the problem into a set of abstract relationships and thereby proved that it was an impossible task. Euler also developed a general theorem to address similar problems of topological relationships including alternate configurations of landmasses and bridges. These advances remained largely undeveloped until the mid 19th Century when modern node and edge diagrams emerged and the study of graph theory began to be formalised. Node and edge diagrams offer a simplified representation of complex spatial relationships. In the case of Konigsberg, the landmasses could be abstracted to become graph nodes and the bridges to become graph edges, producing a diagrammatic representation which could be used to analyse the relationship between the two (Fig. 2).

![Figure 2: The spatial arrangement of bridges and landmasses in Konigsberg and a graph of these topological relationships expressed as a node and edge diagram.](image)

Early spatial applications of graph theory include analyses of accessibility and land use (Hansen 1959), transport networks (Kansky 1963; Taaffe, Gauthier et al. 1973) and facility planning (Seppanen and Moore 1970). In one of the first architectural examples, March and Steadman (1971) created a graph representing the topological relationships between rooms in a building and used this to demonstrate the use of graph theory for design development and evaluation. However, despite their proposal, more than a decade passed before it was realised that such a graph of spatial relationships also offered a means of understanding the underlying social structure of the building. It was at this point that the isolation of space from
form, or topology (understood as “relationship without measure”) from geography (“measure without relationship”), became crucial for architectural research. This shift also posed a controversial challenge for architectural scholars; the realisation that the appearance or form of a building may be far less important than its underlying spatial configuration.

Consider an example of three villas, each positioned on adjacent sites and designed in different architectural styles; respectively Classical, Modernist and Post-Modernist (Fig. 3). A conventional architectural analysis of these villas – judging them in terms of stylistic details, building shape and materiality – would conclude that the three share little in common. The first has Doric columns beneath a Greek pediment, the second has a flat roof and an asymmetrical rectilinear geometry and the third features a raked and modelled silhouette, with a bifurcated gable framing a dominant chimney. However, despite these apparent differences, the three villas share an identical internal spatial structure and are thus, in a social sense, the same (Fig. 4). Conversely, it is possible that three identical looking villas could contain radically different spatial structures. Thus, space and form do not exist in a fixed or predictable relationship.

Figure 3: Three villas in different architectural styles: the Classical, Modernist and Post-Modernist.
In order to study spatial topology there must be a repeatable and logical way to “abstract” or “map” a graph from a plan of a building and then allow for the mathematical analysis of this graph, and in turn use these results to interpret various properties of the original plan. Three methods for abstracting the environment are common, and while all of these are described in detail over the following sections of this chapter, an overview is provided here (Fig. 5). The first technique, “convex space analysis”, abstracts the environment into the fewest number of visually coherent spaces. When derived from an architectural plan these spaces are convex in shape so that their entire perimeter is visible from any point within. This form of analysis investigates the configurational relationship between spaces as defined by the capacity to pass between them. Architectural interiors and similar well-defined spaces are the most frequent subject of this approach (Bafna 1999; Major and Sarris 1999; Ostwald 2011b; 2011c).

The second method, axial line analysis, maps the environment to the fewest number of straight lines that surveil all spaces in the environment. Axial lines represent idealised paths through space and the analysis of the topological relationships between axial lines is effectively an analysis of the movement potential of an environment. This form of abstraction
is ideal for producing graphs for the analysis of urban street networks or passage through complex buildings (Hanson 1998; Hillier and Tzortzi 2006; Ermal and Peponis 2008; Dawes and Ostwald 2012).

The third abstraction procedure, visibility graph analysis, converts the plan of an environment into a series of isovists located at a regular spacing. An isovist represents the portion of the environment that is visible from a particular location. In practice though, a regular grid of squares is imposed on the environment and the centres of the squares are linked to determine which observation points are visible from each square. Depending on the input data, the analysis of these relationships reveals the visible (sight-related) or traversable (movement-related) properties of the space. Visibility graph analysis is best suited to well-defined environments such as those within buildings, but it can also be used at the urban scale (Turner, Doxa et al. 2001; Antonakaki 2007; Choudhary, Heo et al. 2007; Dawes and Ostwald 2013b). Each of these methods possesses unique strengths and weaknesses that are examined in more detail in the following section. However, before progressing, we will return to the conceptual features of space syntax.

Using convex space mapping, the underlying spatial configuration of a plan can be reconceptualised topologically as a graph of rooms (nodes) and doors between them (edges). By including the exterior world as an additional node (signified by a crossed circle) the set of spatial relationships embodied in the plan can be visually examined. However, it must be remembered that in an experiential sense, the “spatial layout not only looks but is different when seen from different points of view in the layout” (Hillier 2005; p. 101). Accordingly, returning to the villas described previously (Fig. 4), the plan can be represented in different ways depending on whether a person is approaching from the exterior, is based in a public antechamber to the right of the entry hall, or is situated in the most isolated and private space at the rear of the building. Each of these relationships can be represented by justifying the

![Figure 5: Three abstraction models use three different units of space. Adapted from (Hillier 2005)](image-url)
underlying graph in different ways to illustrate the perceived spatial structure, from the point of view of these different people; respectively a visitor, and two types of occupants. Redrawing or justifying the graph in this way maintains the topological structure while supporting alternative intuitive readings of the spatial properties of the plan. In contemporary practice, visual graph analyses of this type may be coupled with colour coding to allow for rapid identification of functional and social patterns. However, as graph size increases it quickly becomes impossible to ascertain useful information using visual analysis alone that is why mathematical analysis is widespread.

Figure 6: Graphs of a villa plan justified to reflect alternative spatial positions: (left) visitor, (centre) occupant in a more public space, (right) occupant in a more private space.

Mathematical analysis of an architectural graph can commence with the derivation of simple summative measures including the number of nodes, edges or types of social spaces (for example, classified as “public” or “private”). As a proportion of the total, these can be compared with the results of other, similarly constructed graphs. More commonly, the centrality or closeness of each graph node in relation to all other nodes, provides the basis for comparison in the majority of space syntax analyses. The Total Depth ($TD$) or Mean Depth ($MD$) of nodes are typically determined along with measures for Relative Asymmetry ($RA$), Control Value ($C$) and integration ($i$) (Ostwald 2011a). Nevertheless, to be useful, several of these measures need to be normalised in some way to allow for comparisons to be constructed in graphs of differing size. To counter this problem Hillier and Hanson propose an alternate measure, Real Relative Asymmetry ($RRA$); a formula which normalises relative asymmetry values against those of an idealised diamond-shaped graph. Although it is still
widely used, the justification for the use of a diamond-shaped graph is not compelling and many other graph shapes could equally serve as a normalising benchmark (Teklenburg, Timmermans et al. 1993). Despite this concern, the mathematical basis for architectural graph analysis remains consistent with that in other fields, but it is how these results are interpreted that is more controversial.

Osman and Suliman (1994) argue that the “interpretation process of the numerical results remains complex [and] subjective” (p. 190). This final stage in the space syntax approach, wherein the results of the graph theory analysis are used to interpret a building plan is, even after four decades of research, a tendentious issue. Most often numerical results are reported and used to sequence or compare the values derived from various nodes, before the overall properties are described qualitatively in terms of spaces that are “shallow” and “integrated”, or “deep” and “segregated”. However, derived values can also be “related to psychological variables such as memorability” (Montello 2007; p. iv-04). The following sections focus on the three major abstraction methods, how they operate and how they have been used to interpret the mathematical results of the graph analysis.

**Method 1: Convex space analysis**

Convex space analysis is one of the two original space syntax methods described in *The Social Logic of Space*. This method maps or abstracts the environment into the minimum number of visually coherent areas known as convex spaces. These are often described as an environment’s “fewest and fattest spaces that cover the entire plan, the former always prevailing over the latter” (Markus 1993; p. 14). A convex space is a psychologically self-contained unit of space where every point of the perimeter is visible from every point within. Hillier and Hanson (1984) describe a convex space more precisely as one wherein “no line drawn between any two points in the space goes outside the space” (p. 98). Thus, an ‘L-shaped’ space is not convex and must be divided into two smaller spaces for it to comply with the rule. Convex spaces are visually coherent locations of social interaction. Peponis and Bellal (2010) state that “[t]he convex map represents the maximal units of potential reciprocal coawareness that are implied by a given disposition of boundaries” (p. 984). Architectural interiors are the most common subjects of convex space analysis, as these environments tend to contain defined two-dimensional spaces, as opposed to urban scale spaces that are typically dominated by long streets which have a lower level of defined visual coherence.

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Figure 7: Convex maps of three simple villas, and their representation as graphs justified to an external carrier.

Convex space analysis functions by mapping an environment to a set of connected convex spaces and analysing these visually and mathematically. Figure 7 shows three simple villa plans that have been abstracted into a graph of connected convex spaces. In a mathematical analysis, each convex space becomes a graph node with connections to other spaces where it is possible for an occupant to move to these spaces without passing through a third intermediary space. When the graph is drawn it is a simple procedure to calculate graph theoretic measures for each space (Ostwald 2011a). Undertaking a visual analysis is even simpler; here the graph is drawn justified to a particular space allowing interpretation of the shape of the graph. A graph may be dominated by particular topological relationships resulting in a linear (enfilade), branching (arborescent) or looping (rhizomorphous) structure. Linear graphs suggest a high level of control over an occupant’s spatial experience, Branching graphs are hierarchical and looping graphs provide the occupant with multiple alternate routes through an environment, suggesting a flexible experience. It is also possible for a graph to contain different sub-structures where, for example, public spaces are located on flexible looping parts of the graph and private spaces are located within a more controlled hierarchical part of the structure. As described in the previous section, it is also possible to justify the graph to a particular space in order to represent different perceptions of the environment.
Convex space analysis is used to identify structural genotypes (Hanson, Hillier et al. 1987; Conroy Dalton and Kirsan 2008) and to track spatial manifestations of socio-cultural trends (Hanson 1998). It has been used in an analysis of power relations within a range of institutional buildings (Markus 1987; 1993; Dovey 1999; 2010) or historic spaces (Ferguson 1996; Cooper 1997; Bustard 1999; Dawson 2001) and to provide insights into the way architects think about spatial and social structures (Bafna 1999; Major and Sarris 1999; Ostwald 2011a). “A structural genotype is a socially authorised, ideal spatial configuration, for a particular programmatic type” (Ostwald 2011a; p. 464). Identifying these genotypes requires a consistent analysis of large numbers of functionally similar buildings within a given socio-cultural context. Dovey (1999) argues that the “great achievement of spatial syntax analysis has been […] to reveal a social ideology embedded in structural genotypes” (p. 24).

The main challenge in implementing convex space analysis is the lack of clarity in the original description of the method provided by Hillier and Hanson (1984). The authors use a range of ambiguous terminology when describing the abstraction procedure leading Dovey (1999) to claim that the “work is at times highly difficult to understand” (p. 24). This can lead to significant frustration for novice researchers attempting to develop the “correct” convex map for a given environment. Hillier’s and Hanson’s more algorithmic definition of a convex map – relying on the use of circular geometry to determine the largest circle that can be traced in a plan without breaking the convexity of the space – has proven to be insufficient to reproduce their convex maps (Yoon 2009) or determine the fewest number of convex spaces (Peponis, Wineman et al. 1997c). Desyllas and Duxbury (2001) reiterate the lessons of deBerg, et al. (1997) when they claim that “a unique set of convex polygons cannot be computed with only this [original] minimal rule set.” (p. 27.6). Hillier and Hanson also suggest that it is possible to intuitively develop convex maps, however this approach is innately subjective and non-repeatable, leading to a situation where a single spatial configuration will produce multiple, equally correct, abstractions. Nevertheless, the ambiguity in the abstraction techniques is not the Achilles heel it may initially appear to be.

Within the space syntax community, the intuitive approach is accepted, in large part due to its simplicity and flexibility which allows a degree of tailoring the abstraction process to best suit a particular avenue of inquiry. One popular variation is “functional space analysis”, where the convexity rule is broken and room functions guide the abstraction procedure (Markus 1993; Hanson 1998; Dovey 1999; 2010). This variation allows single function
rooms which contain several convex spaces to be regarded as a single graph node and thereby provides a solution for the criticism that space syntax produces unrealistic abstractions of articulated spaces (Osman and Suliman 1994). This flexibility also allows for smaller convex spaces, such as those created within doorways, to be incorporated into larger adjacent convex spaces and serves to simplify the graph. Any time a convex map is created using this intuitive abstraction procedure, good practice dictates publishing the map with the research to ensure it is clear exactly where the spaces have been delineated.

A further critique of this method is that it fails to account for some important aspects of the geometry of the environment. For example, Osman and Suliman (1994) argue that space syntax will record identical social structures for the traditional single room communal dwellings of the North African Berber, the Amazonian Bari or the Madagascan Betsilo peoples. This is because many traditional or primitive communities construct large single-room structures, to house a diverse range of functions and social groups. In reality though, “each of these cultural groups has a distinct set of rules which is used to divide the internal space and regulate the relationship among its members” (Osman and Suliman 1994; p. 200). Thus, within a single convex space, important practical and symbolic divisions may exist and thus the strict topological structure of space, as defined by visually coherent convex zones, may be irrelevant.

This criticism, and others like it, were no doubt instrumental in leading Hillier (1999) to argue that selecting the correct abstraction model is critical for producing an insightful analysis. In the case of the primitive long house, dividing the single room into adjacent functional zones might prove more informative. Osman and Suliman’s critique highlights the fact that space syntax analysis provides a single set of measure for each graph node. Thus a room is abstracted into a single node which is then described by a single mean depth, or integration value, regardless of the room size or any sub-functions within it. A final criticism offered by Osman and Suliman is that convex space analysis fails to model non-visual connectivity. They allege that olfactory and auditory connections between spaces may differ from those identified by convex space analysis; it may be possible to hear but not move through a wall. Osman and Suliman suggest a non-binary, fuzzy, value for topological relationships that changes with varying types of connectivity, however, this perceived limitation is again solved by turning to, or developing, a more appropriate abstraction model.

**Method 2: Axial line analysis**
Axial line analysis is the second of the original space syntax analytical methods. It abstracts
the built environment into the minimum number of connected straight lines that surveil all
non-trivial spatial features (Ostwald and Dawes 2011). The technical description of the axial
map can be paraphrased as the set of fewest and longest lines that can get everywhere and see
everything. An axial line is a straight line of movement and/or sight. Hillier (2005) notes that
people move in straight lines and that the axial line represents the idealised maximum
extension of one of these movement paths. “Unlike metric distance, axial distance is about
changes in direction. This is why it corresponds to our sense of intelligibility of spatial
patterns and our sense of orientation within them” (Peponis, Wineman et al. 1997b; p. 15).
Axial lines are ideally suited to the analysis of urban environments where long, straight,
movement-oriented, streets dominate the spatial structure. However, axial lines may be
applied, equally successfully, at the architectural scale when research is called to identify and
analyse an efficient network of paths. In practice the axial map looks like a set of angled
lines, all of which intersect at least one other line, and is the primary representation of this
type of analysis; its node and edge graph is rarely depicted (Fig. 8).

Figure 8: Axial line maps for three simple villas and their representation as a graph.

Axial line analysis functions by abstracting the built environment into a set of lines (nodes)
and their connections (edges) and analysing these visually and mathematically. An analysis
of an urban scale environment may yield hundreds, if not thousands, of axial lines and due to
this level of complexity; the primary form of analysis is mathematical. Mathematical analysis
of the topological relations of the axial lines produces a range of measures of which
“[e]xtensive research has shown that in most urban layouts the best predictor of movement is
integration” (Hillier, Brudett et al. 1987; p. 237). This means that there is a high level of
correlation between the graph theory value integration (i) for an axial line and the volume of movement observed or recorded along the equivalent urban street or building corridor (Hillier, Penn et al. 1993; Hillier 1996).

Visual analysis of an axial line map is typically limited to an identification of the longest lines in a system and noting the location of groups of short lines. Visual analysis is more effective when the lines in the axial map are colour coded to represent their mathematically derived properties. In these representations high values are often shown in red and low values in blue or violet, with intermediate values distributed along a colour gradient. Colouring the axial map in this way provides researchers with an opportunity to intuitively seek patterns in the data before using mathematical analysis to examine the pattern less subjectivity.

Applications of axial line analysis are broadly divided into two categories. The first utilises correlations between calculated measures and observational data to explain sociological phenomena such as pedestrian traffic and co-presence (Hillier, Penn et al. 1993; Peponis, Ross et al. 1997a; Desyllas and Duxbury 2001). The syntactic measures have also been shown to correlate to levels of criminal activity (Hillier and Shu 2000; Friedrich, Hillier et al. 2009), whereas commercial property values and rental returns correlate with visual and spatial prominence within an urban environment (Desyllas 2000). At an architectural scale, axial line analysis utilises movement potential to predict the social encounters in office buildings (Ermal and Peponis 2008), or identify spatial structures causing navigation problems in hospitals (Haq and Girotto 2003; Haq and Zimring 2003). It has also been used to compare the work of different architects (Hanson 1998) or patterns in design thinking present in the work of a single architect (Dawes and Ostwald 2012). The second category of analysis, which is less well developed, focuses on utilising mathematical measures to articulate spatial differences arising from different socio-cultural traditions (Hillier 2005).

Like convex space analysis, the main challenge in implementing axial line analysis can be traced to the methodological description provided in the *Social Logic of Space*. The original abstraction procedure defines the axial map as the minimal set of “straight lines which pass through each convex space” (Hillier and Hanson 1984; p. 92). Therefore, the confusion implicit in the convex space abstraction method results in attenuated ambiguity in the axial line method. During the last two decades the axial mapping abstraction procedure has undergone a series of refinements and reinventions in an attempt to standardise and automate the generation of axial maps. Moreover, development in this area has been much more intense than it has for refining the convex mapping procedure. This shift in emphasis is likely
due to the resource requirements inherent in the larger scale analysis of urban environments. As one of the first steps in this development process researchers abandoned the convex map stage of the abstraction procedure in favour of methods that approximated such conditions using an extension of surface vertices or visible areas (Peponis, Wineman et al. 1997b; Turner, Penn et al. 2005). Other researchers rejected the process of dividing the space entirely, utilising isovists to generate long sight lines (Batty and Rana 2004) or generating all possible axial lines computationally and then reducing these to a minimal set using a protocol (Penn, Conroy et al. 1997). Further alternatives proposed combining space syntax and GIS data (Jiang, Claramunt et al. 2000; Jiang, Claramunt et al. 2000; Jiang and Claramunt 2002) or simply street names to define axial line locations (Jiang and Claramunt 2004). Abandoning convex spaces as an analytical precursor eventually led to the growth of more stable procedures for generating axial maps (Penn, Conroy et al. 1997; Ostwald and Dawes 2011).

Unlike convex maps, the inherent ambiguity in the abstraction process of axial lines did not translate into flexibility for researchers. Only two variations in the abstraction procedure are feasible. The first relates to the decision of whether the line represents a line of movement or of sight; a decision which has an impact on the inclusion or exclusion of barriers, such as furniture or glazing. The second variation allows researchers to include or exclude particular spaces based on theoretical criteria such as a focus on habitable space, by excluding dedicated storage areas in a building.

Another challenge for implementing axial line analysis is the “edge effect”. Undertaking a space syntax analysis requires a decision on which information to include in the abstraction procedure. For axial line analysis, a demarcation at the edge of the study area is required.

In building interiors, this is often simple because they are usefully thought of as (literally) bounded spatial systems. Measures of mean depth always create an ‘edge effect’ and that is the whole point: the interesting thing about them is the way that they define what is central and what is edge. (Desyllas and Duxbury 2001; p. 27.9.)

At the urban scale such neat demarcations are often unachievable and will necessarily exclude topological links that fall beyond the periphery of the study area. Here, the edge effect is significant because nodes that represent lines close to the demarcation become artificially segregated relative to those closer to the centre. Early solutions to this problem included expanding the axial map beyond the area of interest (Hillier, Penn et al. 1993), leading researchers to map a 3 by 3 grid of space while only interested in syntactic measures
for the central square (Turner 2003). Later developments adjusted the mathematical formulas to minimise the impact of the edge effect (Hillier 1996). The consequence of this change is that, rather than calculating global integration as in traditional analyses, researchers now calculate local integration. Where global integration determines the depth of one node relative to all other nodes, local integration calculates the depth of a node in respect of other nodes at a predefined depth. For example, some initial applications of this variation calculate depth based on the number of nodes within three graph steps, while later work suggests calculating local integration using the mean depth of the system (Hillier and Penn 2004). Local integration serves to minimise, but not eliminate, edge effects and a combination of this measure and an enlarged study area appear to be the best option currently available.

Desyllas and Duxbury (2001) offer an alternative solution to the problem of the edge effect, being to “use local measures that are not dependent on any relations to the entire graph, such as the visual connectivity of a point […] or the clustering co-efficient” (p.27.9.). However, utilising non-topological measures in this way undermines the fundamental basis of space syntax theory, which advocates focusing on the relation of every space to every other space. Nevertheless, selecting the study boundary is doubly important as Eisenberg (2007) demonstrates, patterns of integration also change with the scale of the study and “[i]ntegration values are not independent of the size of urban areas. Consequently it is difficult to compare areas of different size” (Teklenburg, Timmermans et al. 1993; p. 347.). While acknowledging this position, integration measures calculated from real relative asymmetry values offer an opportunity to compare graphs of different size, provided one accepts the diamond graph normalisation logic that is the basis for their calculation.

More serious critiques of the axial line approach to graph analysis focus on the limitations inherent in abstracting an entire environment into a set of lines. Probably the most famous of these criticisms is from Carlo Ratti (2004a; 2004b) who argues that, for example, axial line analysis records an identical distance for a New Yorker walking around the corner of a city block to a similar resident walking the entire length and breadth of central park. Hillier’s and Penn’s (2004) response is that axial line analysis “deals only with observed flows and thus only with aggregate statistical effects” (p. 504); meaning that it does not account for the actions of a single person, but rather for the pattern of actions of many thousands of people. Axial line analysis models only consider the aggregate movement potential of every location relative to every other location and is therefore incapable of describing the spatial experience of individual journeys, except for suggesting locations along the trip that are more or less
likely to be shared with others. To model actual journeys requires some form of agent based model with specific origins and destinations (Batty, Jiang et al. 1998). Alternatively, Michael Batty (2004a) explores methods for introducing metric distances into the axial line map, however these have not been adopted by the space syntax community.

Ratti (2004a; 2004b) further highlights the importance of boundary conditions in his critique of the axial map’s inability to handle regular grids. Ratti demonstrates that in cities like New York it is possible to select a study area where every East-West street intersects every North-South street (and vice versa) so that every street in the analysis shares an identical total depth. This necessitates that subsequent space syntax measures, like integration, will be identical for all streets. This problem is largely theoretical because in real world environments some streets in the grid will connect with distant areas while others do not. The solution is that the researcher must expand the area of study so that at least one street possesses a variable connection to create differentiated results throughout the entire grid.

An apparently more significant problem arises where a regular grid is slightly deformed (Ratti 2004a; 2004b). Here Ratti demonstrates a critical juncture where an infinitely small rotation of the city block requires multiple axial lines be produced in the graph abstraction process rather than the original single line, even though very little else has changed spatially, socially or experientially. This critique shares similarities with the discussion about the inability of axial lines to navigate around corners, which produce multiple lines for almost straight streets that may be encountered as a single psychological unit of space (Thomson 2004). Ratti argues that this adjustment of the city blocks would not significantly alter movement patterns but provides no data to support this. “The question then is: are such marginally produced discontinuities – or continuities – important in urban space? Two kinds of evidence, morphological and behavioural, suggest they are” (Hillier and Penn 2004; p501). Hillier and Penn explain this by identifying differences in average line connectivity in a range of cultural settings and suggest that these variations are key to different spatial cultures (Hillier 2002). They also point to behavioural evidence presented by Conroy (2001) which found that people exhibit superior abilities to navigate a diagonal line through regular grids compared to distorted grids. Ultimately, resolving this issue may require a dedicated experiment, similar to those undertaken by Conroy, and designed to test Hillier and Penn’s response.

Ratti’s final criticism is derived from the methodological assumption, implicit in the axial mapping technique, that within a spatial system there is an even distribution of populations
and addresses, along with starting points and end points of journeys. The problem is that urban space is almost never distributed in such a uniform way, and taller buildings are more significant generators and attractors of movement than shorter buildings. This position is confirmed by significant correlations that have been found between movement and building height. For example, Penn et al (1998a; 1998b) show that “building height was a significant variable in pedestrian movement at the level of the area, though not at the level of the individual road segment” (Hillier and Penn 2004; p. 504.). Contradicting an earlier (Hillier 1999), and rather unsatisfactory suggestion of simply adding additional lines (graph nodes) to the map where social attractors are located; Hillier’s response to Ratti’s position is that, “for research purposes we prefer not to obscure the effects of spatial configuration by compounding it with other variables” (p. 504). This statement in particular, along with the previous critique of the method, illustrates that it is important to remember that axial line analysis is not a model of actual movement patterns within the city. It is a means of determining movement potential based on spatial configuration alone, and while often demonstrating significant correlations to observational data, ultimately fails to account for important variables that will affect actual movement patterns. This means that, while the method is open to reasonable criticism, it still allows for rapid and repeatable analysis of large and complex environments. Hillier (2005) does, however, leave open the potential for additional variables when he talks about a system, that he calls Space Syntax Mark II, which “works at the level of the line segment, rather than the whole line, and connections between segments can be weighted for metric distance, or the angle of change, as well as for complexity distance” (p. 111.). This suggestion also addresses another critique of axial line analysis; the inability of the method to articulate spatial differences along the length of a single line.

In the traditional graph approach to axial lines, one line will provide only one calculated measure, despite potentially crossing a variety of spatial experiences (Desyllas and Duxbury 2001). The possibility of segmenting the axial line circumvents this problem without resorting to more exotic variations of axial line analysis including intersection point analysis (Batty 2004b; Ostwald and Dawes 2012; 2013a; Dawes and Ostwald 2013b), angular segment analysis (Turner 2007), multiple centrality assessment (Crucitti, Latora et al. 2006), or reverting to the final mode of abstraction considered in this chapter, visibility graph analysis (Turner, Doxa et al. 2001). Focusing on the segments of lines between intersection points also has the advantage that it mirrors the data held in many GIS databases. This would
allow researchers to forego the process of creating an axial map, along with complex interpretations of GIS data, allowing them to focus on the analysis of the environment (Jiang, Claramunt and Klarqvist. 2000; Jiang, Claramunt and Batty. 2000; Jiang and Claramunt 2002). Ultimately, like all forms of syntactic analysis, it is important to understand the limitations of the axial line method and not blindly rely on mathematical measures for an absolute description of social phenomena.

**Method 3: Visibility graph analysis**

The third common abstraction procedure in space syntax research is the visibility graph. Unlike convex space and axial line analyses, visibility graph analysis was not part of Hillier and Hanson’s original theory. Visibility graph analysis abstracts the environment into a series of polygons representing the space visible from a series of defined observation locations. The method’s origins lie within the work of environmental psychologist James Gibson (1947). Gibson proposed that visible space could be represented as a polygon, called an optic array, and illustrated the way in which the properties of these polygons changed as the observation location changed position. Michael Benedikt (1979) was the first to call these polygons ‘isovists’ and to develop mathematical measures to describe their properties. Benedikt (1979) defined an isovist as “the set of all points visible from a single vantage point in space with respect to an environment” (p. 47) and, working with Larry Davies, developed stable, repeatable algorithmic procedures for generating and measuring isovists (Davis and Benedikt 1979). Benedikt and Davies also developed the “isovist field” where a regular grid is superimposed on an environment’s plan and an isovist is generated at the centre of each grid square, representing the measures of each isovist on a scalar field (similar to a synoptic chart or topographic map). At this stage in the development of the visibility graph all measures are local and based on metric values of the isovist. Some 15 years later Alasdair Turner and colleagues revisited the isovist field concept. Following the lead of De Floriani et al (De Floriani, Marzano et al. 1994) Turner et al treat each observation point as a graph node, linking any two mutually visible observation points with a graph edge thus enabling the calculation of topological measures (Turner and Penn 1999; Turner, Doxa et al. 2001; Turner 2003). The isovist is the third unit of space in space syntax (Fig. 9), Wiener and Franz (2005) argue that

isovists describe spatial properties from an inside beholder-centered perspective, and there is […] empirical evidence that they capture environmental properties of space that are relevant for spatial behavior and experience. (p. 44)
Visibility graph analysis is less reliant on the shape of spaces than either convex space or axial line analyses, and potentially provides useful insights into the analysis of both architectural interiors and urban environments.

The appeal of the concept is that isovists are an intuitively attractive way of thinking about a spatial environment, because they provide a description of the space ‘from inside’, from the point of view of individuals, as they perceive it, interact with it, and move through it” (Turner, Doxa et al. 2001; p. 103.).

Figure 9: Three villas showing: regular grid locating observation positions (top), the isovists at three selected positions (second row), graph edges to other visible observation positions for the three selected positions and grid squares shaded for
Visibility graph analysis functions by abstracting an environment into a set of connected
isovists and analysing these visually or mathematically. A mathematical analysis
superimposes a regular grid over the environment and locates an isovist observation point at
the centre of each grid square. These observation points become graph nodes and each is
linked to every other node it is possible to draw a straight node to, thus creating the graph.
Calculation of measures follows standard graph theory procedures and the results are
typically analysed using statistical software. Like axial line analysis, the visibility graph may
contain thousands of graph nodes necessitating computational calculation of measures and a
graphical representation of results. The most common graphical representation of results
colours each grid square to represent the measures of the isovist observation point contained
within.

In addition to topological measures, it is possible to measure a range of metric properties of
the isovist polygon and to derive further normalised or statistical measures from these
(Benedikt 1979; Batty 2001; Stamps III 2005; Dawes and Ostwald 2013a). A further
graphical presentation method for results depicts each isovist in miniature at the centre of its
grid square (Christenson 2010). An alternative to the grid square representation is to select a
significant path through an environment and plot the values of each grid square along this path
sequentially. This produces a pseudo timeline of changing spatial experiences for the path,
assuming an equal time for an occupant to pass from one grid square to the next (Conroy
Dalton 2001; Ostwald and Dawes 2013b).

After some early isolated applications of visibility graph analysis (Braaksma and Cook 1980)
the primary use has been in the identification of regions of space that are more or less central
to the entire environment and more or less likely to be location from which it is possible to
see other occupants. This information can be used to predict rates of spatial occupation and
social encounters, though like all space syntax analyses, the prediction is limited to relative
distributions rather than absolute figures (Desyllas and Duxbury 2001; Ueno, Nakazawa et al.
2009). Visibility graph analysis is also useful for understanding the structure of space through
its visual properties. This includes providing insights into social interaction and the rationale
for the positioning of manager’s workstations within office buildings (Steen and Markhede
2010) and spatial use in urban plazas (Bada and Farhi 2009). Visibility graph analysis also
contributes to an understanding of navigation, behaviour and spatial experience (Conroy
Dalton 2001; Wiener and Franz 2005; Hölscher, Brösamle et al. 2006). Identifying the visual properties of an environment also allows for the comparison of visual properties within a series of buildings by the same architect, (Choudhary, Heo et al. 2007) or a greater understanding of single canonical residences (Peponis and Bellal 2010).

The major strength of the visibility graph method is its stability and repeatability as an analytical procedure. This is largely due to the clear, algorithmic descriptions of the procedures that commenced with Benedikt and Davies (1979) and appeared at every significant stage in the refinement of the technique. This clarity is likely reinforced by the scale of this form of analysis and the requirement to automate mundane tasks through computational analyses. However, this stability does not eliminate flexibility from the method. One source of flexibility is the size of the grid used to locate isovist observation points. Turner et al (2001) adopt “the pragmatic approach of using ‘human-scale’ grid spacing of around one meter” (p. 106.). In contrast, a smaller spacing will assist in locating observation points in constricted locations although this will be at the cost of greater numbers of graph nodes and subsequently greater resources required for the analysis. Fine scale grids may be appropriate at an architectural scale but may be unmanageable for urban analysis; Desyllas and Duxbury (2001) use a five meter grid at the urban scale. Adjusting the actual location of the grid offers an additional small degree of flexibility in that it is possible to locate a critical isovist observation point while allowing the grid to locate the remaining points. Turner et al (2001) suggest that if the number of graph nodes becomes too large it may be possible to identify a

[...] minimal covering set of isovists for the space [...] by choosing the most ‘strategic’ location in the environment, then continue by selecting additional locations which maximise the area viewable from the set as each location is added. However, although finding a sufficient set by this method is possible, it is not guaranteed to be a minimal covering set [...] is likely to provide only relatively obvious information about the environment, because locations at the end of long corridors or at prominent street junctions will tend to be favoured. (p. 107.).

One advantage of the large number of graph nodes required for visibility graph analysis is the potential density of measures generated. Whereas convex space analysis will produce a single integration value for an entire room, and axial line for a long vista or path, visibility graph analysis can provide a measure of integration for every grid position in an environment. Therefore, because there are potentially numerous isovists located in the same space as a
single axial line, visibility graph analysis offers the potential of a greater articulation of measures.

A second source of flexibility arises from altering the height of the isovist plane. The isovists used in visibility graph analysis are two-dimensional and usually located at the eye level of a standing observer, thereby recording the visual perception of that individual. A useful variation is the permeability graph: “the special case of a visibility graph constructed at floor level” (Turner, Doxa et al. 2001; p. 108.). This variation models the movement perception of the occupant. However, this may require designating areas as “inaccessible” to avoid the confusion arising from being able to “see under” an object, a table for instance, that does not allow for movement. Designating inaccessibility in a map offers additional flexibility through excluding particular areas from the analysis. This may be useful where the research focus is on the habitable areas of a building and services areas, or spaces hidden behind closed doors, are of no interest. The decisions made about grid spacing, location, and accessibility will not affect the reproducibility of the visibility graph analyses, if properly defined in the research.

A further strength of visibility graph analysis is the variety of measures available. In addition to standard graph theory measures, software such as UCL Depthmap routinely produces a number of metric and statistical measures of isovists. This leads to further opportunities to calculate hybrid measures normally unavailable to space syntax. One weakness arising from this potential is that there is relatively little evidence as to which of these measures correlate to particular socio-spatial concepts. The relative newness of visibility graph analysis may be an advantage here; there appears to be a greater openness to including metric and non-standard space syntax measures in comparison to the two original abstraction models which remained fiercely topological in their framing.

The edge effect, discussed previously, also has an impact on the results derived from visibility graphs. Like axial maps (and convex maps for that matter) the edge effect is a desirable characteristic where a complete, bounded, system is the subject of analysis, but becomes problematic where the analysis includes only a portion of a system.

Whereas axial maps have often been used to calculate mean depth measures, the very low mean depths of visibility graphs make any depth analysis subject to extreme edge effect which has to be controlled. (Desyllas and Duxbury 2001; p. 27.9.).

The solutions for visibility graphs are identical to those of axial line analyses; expand the field of study beyond the area of interest and utilise local integration measures.
The final variation of visibility graph analysis chooses to forego the simple two-dimensional area-based isovist mapping of space in favour of a three-dimensional volumetric model (Yang, Putra et al. 2007; Morello and Ratti 2009). Currently this variation is still, for practical purposes, in its infancy, but represents a significant new direction in visibility graph analysis that is more capable of modelling the actual spatio-visual experience of occupants. This is discussed in further detail in the next section.

Discussion

From the preceding sections it is clear that space syntax is, to a certain extent, a victim of its own success. The opening line of the Social Logic of Space establishes the need to study space without form and achieves this through topological analyses of the configuration of spaces relative to other spaces. This decision has been at the core of criticisms from the beginning including that the method excessively excludes metric or phenomenological data, and is capable of predicting only relative rates of spatial occupation in lieu of absolute predictions. Space syntax literature addresses these concerns, to various levels of conviction, by suggesting alternate abstraction methods or models of analysis appropriate to the environmental description that is sought. However, in 2005 Hillier suggested the need for a Space Syntax Mark II, which includes a consideration of line segmentation, and potentially paves the way for speculation on a new version of space syntax, incorporating metric measures, which might silence the critics. The focus of this final section, grounded in current methodological developments, explores possible developments of space syntax and draws attention to approaches that current practitioners may have overlooked.

One rarely considered variation of space syntax inverts the axial line graph in order to examine the topological relationships between their intersection points (Batty 2004b; Dawes and Ostwald 2013b). Research using this method in parallel with convex space analysis (a later chapter in this book by (Ostwald and Dawes 2013a)) indicates that a small number of spaces in Mies van der Rohe’s residential work are highly important to navigation. This is a result that standard convex analysis alone completely fails to identify, and suggests that this method is worthy of further exploration. An alternative method of analysis is to create a dual of the convex space graph to topologically analyse the walls of a building and possibly address the issue of the absence of form that has plagued space syntax analysis from the start. Jupp and Gero (Jupp and Gero 2010) present a more useful alternative to this by abstracting a building such that each wall becomes a graph edge linking nodes at wall vertices. In this way,
they demonstrate the possibility of constructing an automated tool for the qualitative analysis and categorisation of architectural styles.

Ratti’s (2004a) attack on the assumption that the potential of each node is equally loaded prior to its configuration in a graph, could be mitigated by incorporating additional data into the space syntax model using of Digital Elevation Models (DEMs) that record building heights (a better indicator of population and destination potential). Ratti and Richens (Ratti and Richens 2004; Ratti 2005) have developed procedures to utilise DEMs in the calculation of various measures of spatial experience. DEMs offer a superior, and far more scientific, method of weighting axial line segments in a graph than Hillier’s (1999) proposal of artificially raising integration values by simply adding axial lines to the map at the location of shopping areas. Penn et al (1998a; 1998b) demonstrate the significance of building height on volume or pedestrian traffic and the DEM may potentially form the basis of a multiplier of spatial occupation rates predicted by configurational analysis. It may even be possible to begin construction of such a multiplier without undertaking new observational studies; if building heights in a particular area have remained unchanged following an observational study a DEM can modify the space syntax values in the search for superior correlations to previously observed pedestrian movement. Ratti (2005) argues that there are further benefits of using DEMs as they are becoming increasing available and are in common usage in the geosciences. The use of DEMs offers the potential for greater correlations between calculated measures and observational data, however this is still only capable of calculating relative occupation of space. Predicting actual spatial occupation will require additional data defining absolute numbers of occupants. The difficulty of achieving this at the urban scale suggests that interior analyses, such as of office buildings where specific numbers of staff are available, might offer the most advantageous starting position for modifying or weighting a graph to analyse actual space use.

A second option for incorporating the third dimension into space syntax is the development of the three-dimensional isovist and the opportunity this offers visibility graph analysis (Penn, Conroy et al. 1997; Yang, Putra et al. 2007). Traditional visibility graph analysis abstracts the environment into a horizontal plane incapable of differentiation between the spatial experience of standing under a low and claustrophobic roof, or a high and agoraphobic one, and is unable to document visibility up or down staircases. Three-dimensional isovists may be either a full 360° modelling of space or a partial modelling of space which more closely approximates a human cone of vision (180°). Initially, three-dimensional isovists
have been primarily utilised to study the local properties, such as spatial openness (Fisher-Gewirtzman, Burt et al. 2003; Fisher-Gewirtzman and Wagner 2003) and the effects of changing urban forms (Yang, Putra et al. 2007; Wong, Chalup et al. 2012). However, the third dimension contains significant information used in way-finding also called measures of spatial salience (Bhatia, Chalup et al. 2012), such as a distant view to a church bell tower used to orientate oneself within the global environment of a traditional town. Morello and Ratti (2009) approach this concept of building visibility by developing a three-dimensional visibility graph capable of identifying the frequency with which building surfaces are visible from any location in an urban environment. This approach can theoretically identify the importance of specific buildings in urban navigation and builds on Conroy-Dalton and Bafna’s (2003) work to quantitatively reinterrogate navigation concepts originally described in Kevin Lynch’s (1960) *The Image of the City.*

Quantitative re-evaluation of a famous psycho-spatial paradigm, prospect-refuge theory, presents a further opportunity for the application of three-dimensional isovists, as Hildebrand (1991) argues that the height and articulation of the roof level in the architecture of Frank Lloyd Wright strongly influences concepts of psychological well-being. Yet, quantitative analyses using two-dimensional isovist methodologies has been unable to account for this property of spatial experience (Dawes and Ostwald 2013a; Ostwald and Dawes 2013b). Combining the measures derived from three-dimensional isovists with a permeability graph abstraction of space could offer insights into the relationship between local visual properties of form and global properties of configuration.

One challenge with quantitatively interrogating architectural properties using graph theory is determining appropriate ways to measure different spatial configurational patterns and correlate them to observed behaviour (Stamps III 2005; Montello 2007; Ostwald and Dawes 2013b). Virtual environments offer researchers the opportunity to test and systematically evaluate the effects of making changes to an environment. It is also possible to use such settings to examine the navigability of an environment with or without directional signage (or directional cues including changes in texture, colour or light levels) or to alter the dimensions of windows and subsequently isovist properties when evaluating psychological theories of space and form. Montello argues that such environments allow for greater potential for future research because they limit problems of “causal circularity or ambiguity” which have plagued the space syntax analysis of real locations since the early days of the theory.
Manipulating actual built environments is possible, but quite difficult and expensive […], so the possibility of studying space syntax and other aspects of environmental psychology with virtual environments has great appeal. (Montello 2007; p. iv-08.).

Finally, virtual environments may provide researchers with opportunities to conduct large-scale studies at reduced financial costs. Early observational research using virtual environments limited participants to modestly sized groups (Conroy 2001; Franz, Von der heyde et al. 2004). However, the last decade has seen the rise of online software distribution services, such as the Steam online-only video game store, and in 2013 a daily peak number of over 4.5 million Steam users were online. Such digitally connected users, most with access to high powered hardware, offer new opportunities for examining large scale spatial experiences which can be mapped and analysed using graph theory. One advantage of this approach, compared to using established virtual environments such as Second Life, is that researchers can design the virtual environment to record the coordinates of an occupant and their view direction while navigating through a virtual environment. This could provide useful data on way-finding, and only a minute proportion to the total Steam users would need to participate in order to generate datasets orders of magnitude larger than those of previous studies. The obvious disadvantage of this approach is that it may not yield a sample representative of the general population, though this could provide a useful benchmark for comparisons of the general population to ‘experts’ in virtual environment navigation.

Conclusion

One of the peculiarities in the relationship between architecture and graph theory is that many architectural researchers are unaware that such a connection exists at all. Because the primary architectural application of graph theory has been subsumed under the rubric “space syntax” for more than three decades, and its centrality to this field has been further masked by a focus on developing topological extraction techniques, the basic connection between architecture and graph theory has tended to be either understated or obscured. Furthermore, while graph theory proponents in mathematics have developed new techniques for analysing factorisation and connectivity in networks, along with dual, non-separable and automorphic graphs (Bondy and Murty 2008; Naimzada, Stefani et al. 2009), architecture’s efforts have been directed more to the application of classical principles in new ways. There are certainly exceptions to this (Batty 2004b), but the core space syntax application of graph theory has not been
substantially revised since the 1990s, even though many hundreds of minor refinements have occurred in its application and interpretation.

It is against this backdrop that the present chapter has sought to provide a critical overview of the three most common approaches developed by architectural researchers to use graph theory to analyse the relationship between space and social behaviour. The majority of these examples use standard graph theoretic measures and formulas, and the true innovations proposed by architects and urban designers have been focussed on the development of mapping or abstraction procedures along with attempts to correlate graph values with observed behaviour. For the three major abstraction processes described herein, key applications, findings and criticisms have all been discussed. In addition, in the penultimate section of the chapter several new opportunities for graph theory developments in design have been considered.

Acknowledgments

An ARC Fellowship (FT0991309) and an ARC Discovery Grant (DP1094154) supported this research.

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