Uncertainty Issues in Deterministic and Stochastic Nonlinear Systems

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I hereby certify that the work embodied in this thesis contains published papers of which I am a joint author. I have included as part of the thesis a written statement, endorsed by my supervisor, attesting to my contribution to the joint publications.

__________________________
Diego S. Carrasco Yáñez

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Robustness issues arise in every real world control problem. The objective of any robust control strategy is to preserve closed-loop stability in situations where the real plant differs from the model used to design the controller, i.e. the real system is, in some sense, unknown. There are different ways to quantify, or describe, the uncertainty of a model. It is the amount of uncertainty, or lack of confidence in the model, that ultimately determines, and constrains, what the closed-loop can achieve.

In this thesis we address particular issues concerned with how to quantify and reduce the impact of uncertainty. To this end, the present thesis is divided in two parts:

The first part is aimed at linear systems. We propose two ideas on how to improve closed-loop performance in the face of general uncertainty, namely, (i) augmenting the control architecture with a feedforward component and (ii) augmenting the observer architecture by using the more general class of unbiased observers. We then illustrate the first strategy applied to an Artificial Pancreas problem.

The second part is aimed at nonlinear systems. A common source of uncertainty in this area is the use of approximate sampled-data models of continuous time systems, be it for control design or system identification. This is due to the fact that, contrary to the linear case, exact discretisations are not generally possible in the nonlinear case. In particular, we deal with the sampled-data scenario in both deterministic and stochastic cases and focus our attention on accuracy and related properties of sampled-data models.

We first study the accuracy properties, or error dynamics, of a particular deterministic sampled-data model, and show that it possesses an improved order of accuracy when compared to the usual Euler approximation. We then demonstrate the usefulness of having such a quantification via several applications, namely, (i) obtaining better bias-variance tradeoffs in the parameter estimation of continuous-time systems from sampled-data, (ii) obtaining a sampled-data model that depends only on input-output data that retains the improved order of accuracy, and (iii) obtaining better performance in high-gain sampled-data feedback control of nonlinear systems, via feedback
In addition, we extend the analysis to stochastic sampled-data nonlinear systems. In this case, we show that the error dynamics are tightly intertwined with other system properties that arise due to the sampling process. In particular, we show the existence of stochastic sampling zero dynamics that are closely related to the sampling zero dynamics associated with the deterministic case.
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