INVESTIGATION OF ARCHING BEHAVIOUR UNDER SURCHARGE PRESSURE IN MASS-FLOW BINS AND STRESS STATES AT HOPPER/FEEDER INTERFACE

Jie Guo
BE (Mech) Central South University, China

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School of Engineering
The University of Newcastle

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Declaration

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______________________________
Jie Guo
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Abstract

Since the era of industrialisation, the handling of materials in bulk form has become a necessary process for a range of industries throughout the world. Bulk handling and storage facilities should be designed and operated to obtain both maximum reliability and efficiency as well as encourage economy. A better understanding of the stress field and arching behaviour and an accurate assessment of the bin wall pressure are essential steps to achieving reliable design of the storage bins or the feeder system.

To date, the determination of the critical mass-flow hopper opening dimensions to prevent blockages due to the formation of stable, cohesive arches has been based on the radial stress field generated in the lower region of the hopper. The surcharge loads on the hopper have not been taken into account. Non-linear effects caused by the hopper surcharge loads in mass-flow analysis need to be addressed. In this study, a large number of experiments were conducted to investigate the influence of the surcharge pressure and the filling level on the hopper wall pressure during arching. The failure mechanism of the arches is discussed by means of studying the arch profile in the mass-flow hopper.

In this study, to ensure efficient feeding, the hopper and feeder geometry are designed as an integral unit. The stress field at the hopper/feeder interface is of particular interest due to the role that it plays in estimating the feeder loads. In reality, the two-dimensional stress field assumed in the mass-flow hopper is influenced by the shear force due to the feeder operation. The combination of the active stress field and the passive stress field in the mass-flow hopper is suggested. This study also attempts to provide an accurate estimation of the feeder loads by investigating the feeder loads given by a combination of two methods.

Apart from the comparisons between the experimental and theoretical results, numerical simulations using the discrete element method were carried out additionally to assist in the investigation of the hopper wall pressures and feeder loads. The influences of the various parameters on the hopper wall pressure, such as filling heights and outlet dimensions, are presented in this thesis. For the feeder system, the effects of the clearance between the hopper and feeder, the belt speed and other areas are also studied.
Nomenclature

A(x) = cross-sectional area [m²]

B = hopper outlet width or diameter [m]

B = hopper outlet width at the front [m]

B = average hopper outlet width [m]

B'av = average width between skirt plates [m]

C₁ = factor defined by Equation 2.30

C₂ = factor defined by Equation 2.18

cc = 1 for axisymmetric flow \( c_c = 1.2 \) for plane-flow

Cₙ = factor defined by Equation 2.26

D = width of the cylindrical section [m]

Fₜ = the force applied for the torque [N]

fₜ = constant part of the flow function

g = acceleration due to gravity [m/s²]

H = the head of bulk solid in cylinder [m]

H = distance between the front end of the hopper outlet and belt [m]

hₕ = hₙ + hₜ = total effective height of stored solid, referring to Figure 2-1

hₙ = surcharge head acting at transition of cylinder and hopper [Pa·m³/N]

hₜ = distance from apex to the transition of hopper [m]

hₕ = filling height [m]

hₙ = bin surcharge head [m]

hₜ = effective surcharge [m]
Hs = actual surcharge head
K = ratio of pnv for hopper
k = pnhf/pni = lateral pressure ratio for initial condition in the cylinder
k0 = coefficient in the linear flow function
khf = pressure ratio in the flow case
khi = pressure ratio in the initial filling condition
Kj = pressure ratio in Janssen equation

Kv = ratio of lateral to vertical pressure at skirt plates
L = hopper outlet length [m]
Llever = the lever arm [m]
Lₙ = total length of belt ≥ 2(Lₙ+Le+xₜ)+1.5 m [m]
Le = length of skirt plates for extended section [m]
Lₙ = length of skirt plates form hopper section [m]

m = symmetry factor, m=0 for plane-flow hopper, m=1 for axisymmetric or conical hopper

mₛ = 0 for triangular surcharge; mₛ = 1 for conical surcharge

n = the idler revolution [r/min]

P₁ = the total output power from motor [w]

pn = normal wall pressure [Pa]

pnhf = normal wall pressure at hopper wall for the flow condition [Pa]

pnhf = normal wall pressure in the hopper for the initial filling condition [Pa]

pₙi = Janssen pressure given by Equation 2.8 [Pa]

pₛ = surcharge pressure from passive stress field in the hopper at distance zₙ or transition [Pa]
$p_{s0} =$ surcharge pressure at datum transition

$p_v =$ average vertical pressure [Pa]

$p_{s0} =$ average surcharge pressure due to natural surcharge of material caused by filling [Pa]

$p_{vft}$ = average vertical pressure for the flow condition [Pa]

$p_{vhi}$ = average vertical pressure in the hopper for the initial filling condition [Pa]

$p_{vi}$ = average vertical pressure in the cylinder for initial condition [Pa]

$p_{vx}$ = vertical pressure at position $x$ [Pa]

$q =$ non-dimensional surcharge factor

$Q =$ mass-flow rate [kg/s]

$r =$ radial coordinate along wall [m]

$R =$ distance from vertex along the hopper wall to the transition [m]

$r_c =$ characteristic radius of container [m]

$r_m =$ distance from vertex to the critical outlet width $B_c$ [m]

$r_{m0} =$ distance from vertex to the critical outlet width $B_{c0}$ [m]

$T =$ the output torque for motor [N*m]

$V =$ vertical load on the shear plane [N]

$V_b =$ velocity of the belt or apron [m/s]

$V_f(x) =$ average feeder velocity at location $x$ [m/s]

$w_{b} =$ belt or apron weight per unit length [m]

$W_c =$ weight of material in extended skirt plate zone [N]

$W_h =$ weight of material in skirt plate zone of hopper [N]

$x =$ distance from the axis of symmetry [m]
\( y = \text{height from the apex of the hopper [m]} \)

\( y_e = \text{average height of material against skirt plates for extended section [m]} \)

\( y_h = \text{average height of material against skirt plates for hopper section [m]} \)

\( z = \text{depth coordinates from hopper transition [m]} \)

\( z_g = \text{height of the hopper [m]} \)

\( z_h = \text{distance from transition at which the passive stress field switches into active stress field [m]} \)

\( z_h = \text{distance from the transition [m]} \)

\( \alpha = \text{hopper half angle [radians]} \)

\( \gamma = \text{bulk specific weight [N/m}^3\text{]} \)

\( \gamma = \text{unit weight of bulk solid } \gamma = \rho g [\text{kg/m}^2\cdot\text{s}^2] \)

\( \delta = \text{effective angle of internal friction [radians]} \)

\( \eta = \text{angle between major consolidation stress at wall and the normal to the wall [radians]} \)

\( \eta' = \text{angle between major consolidation stress at wall and the horizontal [radians]} \)

\( \eta_v(L) = \text{volumetric efficiency at exit} \)

\( \theta = \text{slope angle [°]} \)

\( \lambda = \text{divergence angle [°]} \)

\( \lambda = \text{half divergence angle of skirt plates [°]} \)

\( \mu = \tan \phi_w = \text{coefficient of wall friction [-]} \)

\( \mu_b = \text{idler friction [-]} \)

\( \mu_E = \text{equivalent friction coefficient [-]} \)

\( \mu_{sp} = \text{friction coefficient for skirt plates [-]} \)

\( \mu_{sph} = \text{equivalent skirt plates friction coefficient [-]} \)
\( \rho \) = bulk density \([\text{kg/m}^3]\)

\( \rho \) = bulk density of the material in shear zone \([\text{kg/m}^3]\)

\( \sigma \) = mean stress \([\text{Pa}]\)

\( \sigma_1 \) = major consolidation stress \([\text{Pa}]\)

\( \sigma_2 \) = minor consolidation stress \([\text{Pa}]\)

\( \sigma_R \) = mean surcharge stress \([\text{Pa}]\)

\( \bar{\sigma}_2(R) \) = minor consolidation stress in the critical condition at the transition \([\text{Pa}]\)

\( \phi_w \) = wall friction angle \([\text{radians}]\)

\( \psi \) = release angle \([\degree]\)

\( \omega \) = idler angular velocity \([\text{rad/s}]\)