An equivalent stress implementation of Barcelona Basic Model

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Abstract
The paper presents a novel implementation of the Barcelona Basic Model based on the equivalent stress concept. The Barcelona Basic Model may be regarded as an extension of the Modified Cam Clay model for use with partially saturated soils. This extension is achieved by changing the size of the yield locus in line with changes of suction. The paper presents an alternative way of implementing the Barcelona Basic Model, where the main yield surface remains unaffected by changes of suction, but the stress (alongside the stress-strain relationship) is modified instead. This approach is therefore called an equivalent stress approach. The presented model based on the equivalent stress approach offers the same capabilities and predicts the same soil behaviour as the Barcelona Basic Model; the only difference lies in the implementation of the model, though the end-user would not recognise that. However, equivalent stress offers a new interpretation of unsaturated soil behaviour. It follows that, with the help of the equivalent stress technique, a number of existing models for saturated soils can be easily enhanced to allow for modelling of unsaturated soils.

1 CHALLENGES IN CONSTITUTIVE MODELLING OF UNSATURATED SOILS

In saturated soils, it is assumed that all the pores between the soil grains are filled with porous fluid and no gas phase is present. Such an assumption is often valid, especially for soils below the water table. However, soils above water table level are frequently only partially saturated, which means that the pores between the soil grains are filled not only with porous fluid, but also some air bubbles or even continuous gas phase is present. In such soils, commonly referred to as unsaturated soils, additional effects due to unsaturation are present, mostly due to capillary effects in the porous fluid. Those soils exhibit a suction, defined as the difference between pore air and water pressures, which replaces the pore water pressure of saturated soils. Generally, for aggregated soils, the presence and increase of suction leads to the soil becoming stiffer and more resistant to shearing; however when suction is reduced, this additional stiffness is also lowered correspondingly.

One of the most important features of aggregated unsaturated soil behaviour is the possibility of collapsible behaviour. Generally, for collapse to occur, a soil with high suction needs to be loaded with a mean net stress beyond the plastic limit of the saturated soil. During loading such soil will exhibit much lower deformation as compared with saturated soil. However, if the soil is than wetted without reduction in load, the soil will collapse—reducing its volume even though the external load is constant. Finally, in the fully saturated state, the soil will generally achieve a specific volume corresponding to that of a saturated soil. Capturing this behaviour is one of the major challenges for constitutive models for unsaturated soils.

Generally, constitutive models, to cope with the problem of collapse, assume that yield locus enlarges with increase of suction and contracts when suction is reduced. Unlike saturated soils, this enlargement/contraction of the yield locus is not associated with corresponding plastic deformations; the plastic deformations may occur only when the stress state fulfils the suction dependent yield locus equation.

The models for unsaturated soils either use traditional net stress as stress variables or a new stress variable being combination of net stress and suction. Models based on the net stress approach have been proposed, among many others, by Alonso et al. (1990), Cui & Delage (1996), Sun et al. (2007), Sheng et al. (2008) and Kohler & Hofstetter (2008) whereas models with complex stress variables were given, among others, by Bolzon et al. (1996), Jommi (2000), Wheeler et al. (2003), Gallipoli et al. (2003) and Russel & Khalili (2006).

In the course of the paper it is shown that the well known Barcelona Basic Model proposed by Alonso et al. (1990) can be reformulated with the use of the equivalent stress concept (a more general procedure is given in Sołowski & Sloan 2011). The obtained formulation of the Barcelona Basic Model has the main yield locus independent from suction—its size changes only when plastic deformation changes, similar to models for saturated soils. It must be stressed that the introduction of equivalent stresses does not lead to any change of model prediction—the reworked

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model is exactly the same as the original Barcelona
Basic Model, though the equations written with use
of equivalent stresses are different.

2 INITIAL OBSERVATIONS

The Barcelona Basic Model describes the elliptic yield
locus $F$ in terms of total stresses and suction $s$
\[ F = q^2 - M^2(p + p_s)(p_0 - p) = 0 \]  
(1)
where
\[ p_s = ks \quad p_0 = p_0 \]
\[ \lambda(s) = \lambda(0) \left[ (1 - \gamma)e^{-\beta s} + r \right] \]
\[ \lambda(0), \kappa, M, p', k, r, \beta \] are model parameters and $p_0^*$ is
the hardening variable (for detailed description see
Alonso et al. 1990). The Barcelona Basic Model yield
locus is of similar shape to that of Modified Cam
Clay. Therefore, following Solowski & Gallipoli
(2010), the yield locus can be scaled by dividing it by
\[ \left( \frac{p_0 + p_s}{p_0} \right)^2 - M^2 \left( \frac{p + p_s}{p_0 + p} \right) = 0 \]  
(2)
After introducing new equivalent stress variables
\[ q' = \frac{p^*}{p_0 + p} \quad p' = \frac{p_0 + p_s}{p_0 + p} \]  
(4)
the new yield locus equation reads
\[ F_n = q'^2 - M^2 p'(p_0 + p) = 0 \]  
(5)
This resulting yield locus is that of Modified Cam
Clay where the hardening mean stress is denoted by
$p_0^*$ (which is the hardening parameter of BBM at zero
suction). After exchanging the mean net stress $p$ and
shear stress $q$ by the equivalent stress variables $p'$ and $q'$, the Barcelona Basic Model yield locus can be
written in exactly the same way as the Modified Cam
Clay yield locus. Of course, this leads to a number of
alterations in all the remaining equations so the
equivalent stress can be used uniformly. Suction is
still necessary in the formulation due to the nature
of the Barcelona Basic Model where some deforma­
tions depend on suction only. Also, as the Barcelona
Basic Model was not created with constitutive stress
in mind, some of the remaining equations are more
elegant and descriptive when left their original
forms using net stresses $p$ and $q$. However, the for­
mulation based purely on constitutive stresses and
suction is feasible as shown subsequently.

3 CONSTITUTIVE MODEL FORMULATION
BASED ON EQUIVALENT STRESS

The net stresses can be recovered from equivalent stresses as:
\[ q = q' \frac{p_0 + p_s}{p_0^*} \quad p = p' \frac{p_0 + p_s}{p_0^*} - p_s \]  
(6)
As such, all the equations of the original model need
to have the net stresses replaced in line with Equation
(6). The specific volume is calculated as
\[ \nu = N(s) - \lambda(s) \ln \left( \frac{p_0}{p'} \right) + \kappa \ln \left( \frac{p_0}{p} \right) \]  
\[ = N(s) - \lambda(s) \ln \left( \frac{p_0}{p'} \right) + \kappa \ln \left( \frac{p_0 p_0^*}{p'(p_0 + p_s) - p_s p_0^*} \right) \]  
(7)
with the volumetric strain increment following as
\[ \Delta e_v = \ln \frac{\nu}{\nu_0} \]  
(8)
The increment of the volumetric plastic strain is the
same as in original formulation
\[ de_{vp}^{pl} = \frac{\lambda(0) - \kappa}{\nu} \frac{d p_0^*}{p_0^*} \]  
(9)
which under the standard assumption of a constant
specific volume in a given increment can be inte­
grated as
\[ de_{vp}^{pl} \equiv \frac{\lambda(0) - \kappa}{\nu} \ln \frac{p_0^* + \Delta p_0^*}{p_0^*} \]  
(10)
The change of hardening parameter $\Delta p_0^*$ is determined
by the change of yield locus size. Should the suction
yield locus be considered (in many formulations it is
omitted, e.g. Solowski & Gallipoli 2010), the volumet­
ric plastic strain change associated with infinitesimal
increment of suction $ds = ds_0$ above the yield limit $s_0$ is
\[ de_{vp}^{ps} = \frac{\lambda_0 - \kappa}{\nu} \frac{ds_0}{s_0 + p_{aim}} \]  
(11)
which under the assumption of no change of specific
volume in a given increment $\Delta s = \Delta s_0$, can be inte­
grated as
\[ \Delta e_{vp}^{ps} \equiv \frac{\lambda_0 - \kappa}{\nu} \ln \frac{s_0 + \Delta s_0 + p_{aim}}{s_0 + p_{aim}} \]  
(12)
The total plastic volumetric change can be com­
puted as sum of volumetric plastic strains
\[ de_{vp} = de_{vp}^{pl} + de_{vp}^{ps} \]
\[ \Delta e_{vp} = \Delta e_{vp}^{pl} + \Delta e_{vp}^{ps} \]  
(13)
In the Barcelona Basic Model, a non-associated flow
rule is postulated with parameter $\alpha$ being equal to
\[ \alpha = \frac{M(M - 9)(M - 3)}{9(6 - M)} \frac{1}{1 - \kappa / \lambda(0)} \]  
(14)
The value of parameter $\alpha$ has been adopted in the Barcelona Basic Model after Ohmaki (1982) such that the flow rule predicts zero lateral strain for stress states corresponding to Jaky's (1948) equation for ratio of vertical and horizontal in-situ stress
\[ K_0 = 1 - \sin \phi = \frac{6 - 2M}{6 + M} \]  
(15)
The shear plastic strain is related to volumetric plastic strain as
\[ \frac{d\varepsilon_p^s}{d\alpha} = -\frac{1}{\alpha} \left( \frac{dF_p}{dp} + \frac{dF_p}{dq} \right) \]
(16)
Equation (16) may seem a little complex, but all the differentials are very straightforward to calculate as
\[ \frac{dF_p}{dp} = M^2 (2p' - p_0^e) \]
\[ \frac{dF_p}{dq} = 2q' \]
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\[ \frac{dF_p}{dq} = 2q' \]
\[ \frac{dF_p}{dq} = 0 \]
\[ \frac{dF_p}{dq} = 0 \]
which allows for simplifying (16) into
\[ \frac{d\varepsilon_p^s}{d\alpha} = \frac{1}{\alpha} \left( \frac{M^2 (2p' - p_0^e)}{2q'} \right) \]
(18)
This is easy to implement and no more complex than the initial formulation
\[ \frac{d\varepsilon_p^s}{d\alpha} = \frac{1}{\alpha} \left( \frac{M^2 (2p' - p_0^e)}{2q'} \right) \]
(19)
given by Alonso et al. (1990). Note also that (18) can be obtained directly from (19) by simple substitution of net stresses with equivalent stresses.
The infinitesimal change of volumetric elastic strain is computed from two parts, connected to mean net stress change and suction change:
\[ \varepsilon_{ve} = \frac{\kappa}{\nu} d\sigma \]
\[ \varepsilon_{ve} = \frac{\kappa}{\nu} d\psi \]
(20)
Again, under the usual assumption of constant specific volume in the increment, the elastic strain increment can be integrated as:
\[ \Delta \varepsilon_{ve} = \frac{\kappa}{\nu} \ln \left( \frac{p + \Delta p}{p} \right) \]
\[ \Delta \varepsilon_{ve} = \frac{\kappa}{\nu} \ln \left( \frac{s + \Delta s + p_{sat}}{s + p_{sat}} \right) \]
(21)
The above equation can be written in terms of equivalent stress as
\[ \Delta \varepsilon_{ve} = \frac{\kappa}{\nu} \ln \left( \frac{p'p_0^e + p_0}{p_0^e} - p_s + \Delta \left( \frac{p'p_0^e + p_0}{p_0^e} - p_s \right) \right) \]
(22)
Assuming no suction change in a given increment, this can be simplified to
\[ \Delta \varepsilon_{ve} = \frac{\kappa}{\nu} \ln \left( \frac{p'p_0^e + p_0}{p_0^e} - p_s \right) \]
(23)
The elastic shear strain increment is calculated as
\[ \varepsilon_{qe} = \frac{1}{3} G d\psi \]
\[ \Delta \varepsilon_{qe} = \frac{1}{3} G \Delta \psi \]
(24)
which in equivalent stresses can be computed as
\[ \Delta \varepsilon_{qe} = \frac{1}{3} G \left( \frac{p_0 + p_s}{p_0^e} \right) \]
(25)
Under the assumption of no suction change, this gives
\[ \Delta \varepsilon_{qe} = \frac{1}{3} G \left( \frac{p_0 + p_s}{p_0^e} \right) \]
(26)
The elastic strain increment due to changes in net stresses, given by Equations (22) and (25), are inconvenient to use in the form of equivalent stress, unless some special loading case is considered. This is to be expected, as they were created to be simple and intuitive to use in the net stress space.
For implementation of the model, the elastoplastic tangent matrix is also required. Here, the matrix will be similar to that for Barcelona Basic Model (see e.g. Solowski & Gallipoli 2010), though the differentials of the yield locus and potential surface need to be computed with use of equivalent stresses.

4 \hspace{1cm} \text{COMPARISON OF NET STRESS AND EQUIVALENT STRESS FORMULATION OF BARCELONA BASIC MODEL}

The use of equivalent stress and the net stress for the Barcelona Basic Model is illustrated on a simple volumetric stress path ABCDEFG (see Solowski & Sloan 2011 for more comprehensive tests). Calculations were made with model parameters given in Table 1, whereas the initial conditions and conditions at the end of each loading stage are shown in Table 2.

The test (see Figure 1) consisted of loading (initially elastic, then elastoplastic, drying, loading (initially...
Table 1 Barcelona basic model parameters

<table>
<thead>
<tr>
<th>$N(0)$</th>
<th>$p_c$ [kPa]</th>
<th>$\kappa$</th>
<th>$\lambda(0)$</th>
<th>$p_m$ [kPa]</th>
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<td></td>
<td>10</td>
<td>0.02</td>
<td>0.2</td>
<td>100</td>
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</table>

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\kappa_i$</th>
<th>$\beta$ [kPa$^{-1}$]</th>
<th>$r$</th>
</tr>
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<tr>
<td>0.6</td>
<td>0.012</td>
<td>0.01</td>
<td>0.75</td>
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Table 2 Stress and hardening parameter at specified points

<table>
<thead>
<tr>
<th>Point</th>
<th>$p$ [kPa]</th>
<th>$s$ [kPa]</th>
<th>$p'$ [kPa]</th>
<th>$p''$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>0</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>200</td>
<td>27.7</td>
<td>30</td>
</tr>
<tr>
<td>D</td>
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<tr>
<td>E</td>
<td>65</td>
<td>200</td>
<td>47.6</td>
<td>55.3</td>
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<td>F</td>
<td>65</td>
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<td>65</td>
<td>65</td>
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<tr>
<td>G</td>
<td>95</td>
<td>0</td>
<td>95</td>
<td>95</td>
</tr>
</tbody>
</table>

Figure 1 Stress path ABCDEFG.

Figure 2 Specific volume versus suction variation on stress path ABCDEFG.

Figure 3 Specific volume and mean net stress variation on path ABCDEFG.

Figure 4 Variation of specific volume and equivalent mean stress on path ABCDEFG.

Figure 5 Specific volume and equivalent stress variation on path ABCDEFG.

elastic, then elasto-plastic, unloading and wetting (initially causing elastic swelling followed by collapse). Changes of specific volume with respect to suction, mean net stress and equivalent mean stress are given in Figures 2–5.

Figure 4 shows that the soil yields only when the maximum equivalent stress is exceeded. It confirms that the yield locus of the model is independent from suction. Figure 5 gives the same relationship between equivalent stress and specific volume, but now stress is in logarithmic scale. One can easily see that the yielding due to mean net stress change creates lines parallel to the normal consolidation line. The normal
consolidation line is also recovered once suction is reduced to zero.

5 CONCLUSIONS

The proposed transformation into the equivalent stress space may at first look like a purely mathematical operation—which to a large degree is certainly true. However, running calculations with the equivalent stress variables and doing the transition to total stresses only at the final stage of calculations may be beneficial in certain situations and may lead to less complex implementations of some models. Furthermore, in the case of models where no deformations depend on suction only, the equivalent stress is the only variable which is required to predict the behaviour of unsaturated soil. In this respect it is similar to the effective stress for saturated soil.

However, the equivalent stress does disappoint in the role of effective stress. First of all, it lacks any physical interpretation. Moreover, it is contrary to soil physics, as with the suction increase, the equivalent stress decreases, whereas the microstructural binding between soil grains increases. Finally, equivalent stress is constitutive model dependent, and as such it contains range of model parameters, unlike the formulation of effective stress for saturated soils. Nevertheless, introduction of equivalent stress may further the goal of finding the effective stress for unsaturated soils.

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