Upper Bound Rigid Block Analysis with rigid block subdivision

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Abstract

Rigid block limit analysis provides a simple method for computing rigorous upper bound solutions on the stability of geotechnical problems. The method requires the topology of a collapse mechanism to be defined a priori, from which an expression for stability is derived. For complex mechanisms optimizing these expressions using multivariate search algorithms requires enormous computational effort, and the initial feasible solution needed for local optimization algorithms is difficult to obtain. This paper describes a method for successively subdividing blocks within a collapse mechanism, increasing its complexity gradually and allowing simple univariate optimization algorithms to be used. The performance of this technique is demonstrated by computing the bearing capacity factor for a strip footing on weightless fissured clay and the undrained stability of a circular tunnel. The techniques developed in this paper are shown to provide a robust method for solving rigid block problems with demanding material behaviour and complex geometry.

Rigid block limit analysis (RBLA) provides a simple method for directly computing rigorous upper-bound solutions on the stability of geotechnical problems. The method requires the topology of a collapse mechanism to be defined a priori, which is then used in conjunction with the upper bound theorem of plasticity to formulate an expression for the stability of the mechanism. This expression is optimized to determine the geometry of the mechanism with the smallest collapse load. As the method is based upon the upper bound theorem of plasticity, the resulting solution provides a rigorous upper bound to the true collapse load. A comprehensive summary of solutions to familiar problems is provided by Chen (1975).

For simple collapse mechanisms, the stability expressions have only a few variables and global optimization methods such as grid searches can be used efficiently to find the smallest collapse load without any prior knowledge of a feasible solution. However, when mechanisms become larger, the stability expressions involve many unknown variables and global search algorithms require enormous computational effort. In such cases local search methods such as the Hooke-Jeeves method, can be used to efficiently obtain the minimum collapse load but these methods require a feasible solution within the vicinity of the global solution as a starting point.

This paper describes a method for successively subdividing blocks within a collapse mechanism to gradually increase the complexity of the mechanism. Starting with a relatively simple mechanism allows global search methods to find the optimal solution efficiently. The rigid block splitting technique then forms a more complex mechanism by subdividing one or more of the rigid blocks in the mechanism. An initial feasible solution for the new mechanism is obtained via interpolation of the optimised geometry of the previous mechanism. This feasible solution permits the new mechanism to be optimized using local search algorithms. The subdivision of rigid blocks and optimisation of the resulting mechanism can then be repeated to obtain progressively more complex collapse mechanisms.

The performance of this technique is demonstrated by computing the bearing capacity factor of a rigid strip footing sitting on a fissured soil and the undrained stability of a circular tunnel. The techniques developed in this paper are shown to provide a robust method for accurately and efficiently using rigid block methods to predict the stability of geotechnical structures.

1 RIGID BLOCK LIMIT ANALYSIS

Stability analysis of a geotechnical problem using RBLA requires a collapse mechanism composed of sliding rigid blocks separated by velocity discontinuities to be defined in advance. The motion of blocks within the mechanism must be kinematically admissible, satisfying the velocity boundary conditions, compatibility, and an associated flow rule. The collapse load is then calculated by forming an expression equating the rate of energy dissipation between rigid blocks to the rate of work done by external loads and body forces as some virtual displacement occurs. The upper bound theorem of plasticity (Drucker, 1953) guarantees a solution computed by the rigid block method is an upper bound to the true solution.

By defining the geometry of a mechanism in terms of an appropriate set of variables and optimizing the resulting energy expression for that specific mechanism, these methods are optimized efficiently. The Hooke-Jeeves optimization methods seek the minimum by evaluating it at regular domain. The grid point value is chosen as the carried out using either a multiple dimensions simultaneous minimum. Alternatively, and can be employed in which the function is performed on an individual minimum. To locate the process is repeated for a different solution converges. How search requires an initial grid point, making it less practical. Computational effort is required.

The Hooke-Jeeves method uses a grid search. The technique is demonstrated by computing the bearing capacity factor of a rigid strip footing sitting on a fissured soil and the undrained stability of a circular tunnel. The techniques developed in this paper are shown to provide a robust method for accurately and efficiently using rigid block methods to predict the stability of geotechnical structures.

2 RIGID BLOCK SPLITTIN

The basis of the rigid block analysis is the successively rigid block analysis using increasingly complex rigid block mechanisms. The key to applying the formulation of an expression for the mechanism so as to be an iterative definition of the process.
The splitting is achieved by adding variables for Melbourne, Australia, and the temporary displacement process in terms of plasticity between the virtual displacement and the flow of the material. The velocity discontinuities which divide an existing block into two or more new blocks. The key to applying this method is the use of simple grid search and Hooke-Jeeves algorithms to optimize the stability function precisely and shift the geometry of the rigid block failure mechanism closer to the exact solution. The accuracy of the solution is improved by successively splitting blocks and re-optimizing the geometry.

The splitting is achieved by adding variables for velocity discontinuities which divide an existing block into two or more new blocks. The purposes of this paper are to show that the block with the largest area was chosen for splitting, however other criteria, such as highest energy dissipation or rate of gradient change on the slip surface may also be suitable. By performing the split so that the new blocks fit exactly within the old one, a layout is formed which will have the same geometry as the pre-split mechanism. This new point is used to initialize the optimization for the higher dimension mechanism. A first approximation for the upper bound of the new mechanism can be found using a univariate grid search, and refined using the Hooke & Jeeves algorithm.

The rigid block splitting process continues until the desired dimensionality has been reached. As a layout for any split mechanism which has equal stability to the preceding mechanism is known, the process is guaranteed to be convergent or at worst, static.

3 RIGID FOOTING ON FISSURED CLAY

The stability of a smooth rigid strip footing on weightless fissured clay is a geotechnical problem that combines a relatively simple collapse mechanism with complex material behavior. Davis (1985) computed the bearing capacity factors for this problem which accounted for the reduced cohesion on fissure planes and the orientation of fissure planes within the soil. The analytical solution has been shown to satisfy both the upper and lower bound limit theorems and is therefore theoretically exact.
3.1 Rigid Block Mechanisms
The initial rigid block mechanism used to model the collapse of the footing is a simple fan failing in one direction. The mechanism, as shown in Figure 1, comprises of three triangular wedges radiating from the edge of the footing. The geometry of the mechanism is defined by the discontinuities that separate the wedges which in turn are described by unknown lengths and orientations of the discontinuities. Uniformly distributed angles, and radii equal to the footing width are suitable initial values.

Subsequent mechanisms are formed by subdivision of the largest rigid block into two new blocks. Each block added during the splitting process adds one additional discontinuity and requires two additional variables in order to describe the mechanism. Such a mechanism, with a total of B rigid blocks, requires 2B + 2 variables.

3.2 Cohesion in a Fissured Clay
Davis (1985) assumed that the cohesion varied discretely with orientation throughout the material; cohesion is given by c(y) on fissure planes and c(\text{elsewhere}) where, (c(y) \leq c(\text{elsewhere})). Modeling material strength in this fashion is problematic for RBLA. Unless an optimization algorithm evaluates the stability function at a point on which velocity discontinuities are aligned precisely with a fissure plane (an unlikely coincidence) it remains oblivious to the planes presence, and can miss the global minimum.

To overcome this problem in the implementation of the rigid block splitting method, discontinuities in the polar function for cohesion were replaced by high frequency sinusoidal curves, leading the Hooke & Jeeves algorithm to shift adjacent velocity discontinuities onto fissure planes. Figure 2 shows in polar co-ordinates an example of the numerical approximation for cohesion used to demonstrate the block splitting method.

Using the numerical approximation there is a potential for discontinuities to have an orientation which gives cohesion between the fissured and unfissured values, destroying the rigorous upper bound nature of the solution. For the purposes of this paper the upper bound nature was preserved by decreasing the sine curve period until the cohesion on all discontinuities lay very close to either c(y) or c(\text{elsewhere}).

3.3 Results
Convergence of the block splitting method was demonstrated by computing the bearing capacity factor Nc for two cases. The first was a soil with \(c(y) = 1\), and the second was a soil with \(c(y) = 0\) and orthogonal fissuring inclined at \(\xi = \pm 45^\circ\) to the vertical axis.

Optimizing the three block mechanism for the first set of soil parameters \((c(y) = 1)\) gave a solution accurate to 2.93%, this error was reduced to 0.05% using forty blocks after which no significant improvement was seen. Similarly for the second set of soil parameters \((c(y) = 0, \xi = \pm 45^\circ)\) the three block mechanism computed a solution accurate to 10.76%, which was improved to 3.94% using fifty blocks.

Figure 3 shows the numerical results approaching the exact values of Davis as the number of blocks increases. From these results it is evident that convergence is rapid in the early stages of the process, but slows as the number of iterations becomes large. The evolution of the rigid block mesh with \(c(y) = 1\) as the splitting process converges is shown in Figure 4.

To illustrate the practical value of the splitting method the bearing capacity factor for a footing on fissured material was computed over a range of \(c(y)\) values, and for sets of orthogonal fissures oriented at \(\xi = (0^\circ, 90^\circ), (20^\circ, 110^\circ),\) and \(\xi = \pm 45^\circ\). A comparison between results computed using the splitting process with fifty blocks and the solution by Davis (Fig. 5) shows a close correlation with a mean error of 0.82% and maximum of 3.94%.

4 UNDRAINED ANALYSIS TUNNEL
The undrained stability of a tunnel has recently been considered using modern Finite Element techniques to improve the probability numbers for the tunnel by Sivan and Assadi (1992). In this study, a rigid block analysis in conjunction with a block collapse mechanism to finite element results.

Wilson et al. (2011) con in which the undrained soil early with depth (see Fig. 6) below the ground surface expression:

\[
c(y) = c_0 + p
\]

in which \(c_0\) is the cohesion rate at which cohesion increases.

Stability of the tunnel is dimensionless parameter

\[
N = \frac{\sigma_r - \sigma_l}{c_0} = \left(\frac{H}{D}, \frac{D}{V}\right)
\]

where \(\sigma_r\) is the surcharge, \(\sigma_l\) is the radial stress within the tunnel, \(N\) is a function of the tunnel crown below, \(c_0\) is the cohesion of the tunnel crown below, \(D\) is the tunnel diameter, and \(V\) is the tunnel crown below.

In this paper, the soil will be considered.

4.1 Rigid Block Method
The upper bound analysis of the tunnel crown below the ground surface expression:
undrained capacity factor $N_c$, of soil with $q/c_s = 1$, and $q_c = 0$ and orthogonal $\theta$ to the vertical axis. (mechanism for the first 1) gave a solution reduced to 0.05% using significant improvement for a footing on all discontinui-

ties. The undrained stability of a plane strain circular tunnel has recently been considered by Wilson et al. (2011) who used modern Finite Element Limit Analysis (FELA) techniques to improve the published upper bound stability numbers for the tunnel as originally considered by Sloan and Assadi (1992) Wilson et al. (2011) used rigid block analysis in conjunction with a simple three block collapse mechanism to provide a validation of the finite element results.

Wilson et al. (2011) considered a tunnel in a soil in which the undrained shear strength increased linearly with depth (see Fig. 6). Cohesion at any distance below the ground surface was given by the expression:

$$c(z) = c_o + \rho z$$

where $c_o$ is the cohesion at the soil surface, $\rho$ is the rate at which cohesion increases with depth (m).

Stability of the tunnel is described in terms of the dimensionless parameter

$$N = \frac{\sigma - \sigma_r}{\sigma_0} = \frac{H \rho D c_o}{D' c_{so}}$$

where $\sigma$ is the surcharge at ground level, and $\sigma_r$ is the radial stress within the tunnel. The stability parameter $N$ is a function of the dimensionless variables: $H/D, \rho D c_o$, and $D'/c_{so}$, where $H$ is the depth of the tunnel crown below the soil surface, $D$ is the tunnel diameter, and $c_o$ is the soil unit weight (Sloan & Assadi, 1992). In this paper a tunnel in a weightless soil will be considered and hence $\rho D = 0$.

4.1 Rigid Block Mechanisms

The upper bound analysis was started using a symmetric three block mesh (See Fig. 7) as used in the previous work of Sloan and Assadi (1992) and Wilson et al. (2011). The application of rigid block splitting considered only subdivision of the middle block of the original mechanism. The complexity of the mechanism was increased by dividing every rigid block in the region at each splitting step. After applying rigid block splitting, the $N$th mechanism to be analyzed contained $B = 2 + 2^N$ blocks and required $4 + 2^N$ variables to be optimized. It was expected that refining the middle section of the initial mesh first would give the most rapid improvement in accuracy but further precision may be gained by splitting other regions of the mesh.

4.2 Results

To show the convergent behavior of the rigid block splitting method applied to a problem with complex collapse geometry, all previous splitting results computed with an increasing number of iterations were plotted against the FELA results of Wilson et al. (2011). Parameters corresponding to the greatest $H/D = 3$, $\rho D/c_o = 0.5$ and least $H/D = 10$, $\rho D/c_o = 0$ improvement in the rigid block solution were chosen for illustration. For the three block mechanism with $H/D = 3$ and $\rho D/c_o = 0.5$ the rigid block solution was 5.67% greater than the upper-bound FELA results, while the sixty six block mechanism was 4.43% greater than the FELA. For the $H/D = 10$, $\rho D/c_o = 0$ case the three block mechanism was 21.95% greater than the FELA and the sixty six block mechanism was 14.98% greater, giving an improvement of 6.97%. It is again seen that the convergence rate for the tunnel analysis is rapid at the outset of the splitting process, and slows as the number of block increases. The results of the convergence study are shown in Figure 8 and the development of the rigid block mesh can be seen in Figure 9.

A full comparison between the results of Wilson et al. (2011) and the block splitting method for a circular
Figure 5 Bearing capacity factor on fissured clay.

Figure 6 The tunnel stability problem.

Figure 7 Rigid block splitting for undrained circular tunnel.

Figure 8 Stability of circular tunnel.

Figure 9 Splitting of the tunnel mechanism.

Figure 10 Stability of a circular tunnel in undrained weightless soil ($K = pD/C_{u}$).

A new technique for performing rigid block analysis that enables large mechanisms to be used efficiently was presented. The technique involves the gradual development of complex mechanisms from an initial simple mechanism in order to improve the accuracy of the analysis. The most significant advantage of the method is that the initial mechanism can be solved using a global optimisation method that does not require an initial feasible solution. The solution of the subsequent mechanisms is then performed using optimisation methods that start from a kinematically feasible solution. The feasible solutions are obtained through interpolation of the optimal solution from simpler collapse mechanisms. As expected, it was observed that the accuracy of collapse mechanisms improved as the mechanisms became more complex.

On average, the RBLA results were 9.08% greater than the FELA values with a maximum of 15.42%.

5 CONCLUSION

The results computed using FELA for all values of $H/D$ and $pD/C_{u}$ examined. On average, the RBLA results were 9.08% greater than the FELA values with a maximum of 15.42%.

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REFERENCES

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