EM-Based Channel Estimation for Multicarrier Communication Systems

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DECLARATION

The thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying subject to the provisions of the Copyright Act 1968.

I hereby certify that the work embodied in this thesis contains published paper/s/scholarly work of which I am a joint author. I have included as part of the thesis a written statement, endorsed by my supervisor, attesting to my contribution to the joint publication/s/scholarly work.

______________________________
Rodrigo Carvajal
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**GLOSSARY**

**CIR**: Channel Impulse Response. Impulse response of the channel, understood as a finite impulse response digital filter.

**CFO**: Carrier Frequency Offset.

**CP**: Cyclic Prefix. Artificial extension of the transmitted signal in multicarrier systems.

**CRLB**: Cramér-Rao lower bound.

**DCT**: Discrete Cosine Transform.

**DFT**: Discrete Fourier Transform.

**EM**: Expectation-Maximization.


**IDFT**: Inverse Discrete Fourier Transform.

**IOT**: Inverse Orthogonal Transform.

**LLA**: Local Linear Approximation.

**LoS**: Line of sight.

**KF**: Kalman Filter.

**LO**: Local Oscillator.

**LQA**: Local Quadratic Approximation.

**LTE**: Long Term Evolution. Digital radio access technology for next-generation mobile networks.

**MAP**: Maximum a Posteriori.
MC: Multicarrier.

ML: Maximum Likelihood.

MLE: Maximum Likelihood Estimation.

OFDM: Orthogonal Frequency Division Multiplexing.

OT: Orthogonal Transform.

pdf: Probability density function.

PF: Particle Filter.

PHD: Phase distortion. The combined effect of phase noise and carrier frequency offset.

PHN: Phase Noise.

PS: Particle Smoother.

QAM: Quadrature Amplitude Modulation. Digital modulation technique.

QPSK: Quadrature Phase Shift Keying. Digital modulation technique.

VMGM: Variance Mean Gaussian Mixture.

\( \chi^2_l(\lambda) \): Chi-squared distribution with \( l \) degrees of freedom, where \( \lambda \) is the random variable.

Im \{ \cdot \}: The imaginary part of. Mathematical operator that extracts the imaginary part of a complex scalar/vector.

\( \mathcal{N}_\theta(\mu, \Sigma) \): Normal (Gaussian) distribution, where the random variable is \( \theta \), the mean is given by \( \mu \) and covariance matrix is given by \( \Sigma \).

Re \{ \cdot \}: The real part of. Mathematical operator that extracts the real part of a complex scalar/vector.
This thesis addresses the general problem of channel estimation in Multicarrier communication systems. This estimation problem, inter-alia, includes the joint estimation of channel noise variance, carrier frequency offset and phase noise bandwidth. A general state-space model is developed for multicarrier systems that represents any modulation scheme, by separating the signals into their real and imaginary parts. The approach presented in this thesis relies on the statistical representation of the signals of interest. The approach is valid for any statistical representation. In particular, we present a linear and Gaussian structure associated with the transmitted signal, which is exploited by utilizing the Kalman filter. For nonlinear signals, nonlinear filtering is carried out by utilizing sequential Monte Carlo techniques. The estimation problem is solved by using Maximum Likelihood (ML) and Maximum a Posteriori (MAP) estimation, for which the Expectation-Maximization (EM) algorithm is considered. For ML estimation, a novel selection of hidden variables and parameters is proposed, whilst the maximization step is carried out by concentrating the cost in one variable (carrier frequency offset). For MAP estimation, the prior terms are expressed as variance-mean Gaussian mixtures. In this case, the channel estimate can be obtained in closed form within the EM framework. In the maximization step of the EM algorithm, the cost function is also concentrated in one variable (carrier frequency offset). For sparse channel estimation, an $\ell_1$-norm regularization is considered. An Elastic Net penalty is also considered, which accounts for the different nature that communication channels can exhibit in a variety of environments. It is also shown that the utilization of variance-mean Gaussian mixtures present a general method for MAP estimation, which encompasses different penalizations and optimization methods, such as the Lasso, Group-Lasso, and local-linear/local-quadratic approximation for the Lasso, among others. The MAP estimation approach proposed in this thesis is illustrated with not only examples in MC communication systems, but also for sparse estimation with quantized data. Finally, it is also shown that the estimation of the channel noise variance is not straightforward, and that some modifications to the standard methods should be considered. It is shown that, in the
proposed MAP estimation approach, those modifications can be included in a simple manner.

The thesis also considers the impact of different levels of training on the overall parameter estimation problem. In particular, it is shown that the estimates of phase noise bandwidth are generally poor, and, hence, that high levels of training are required to obtain accurate channel estimates.