Nonhomogeneity in eastern Australian flood frequency data: Identification and regionalisation

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Tom Micevski
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Associated publications

The research work embodied in this thesis led to the following series of research papers:


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Abstract

Flood frequency data from the eastern Australian states of New South Wales (NSW) and Queensland (Qld) were investigated to determine the magnitude and extent of multidecadal variability (nonhomogeneity) in flood risk. Some flood data from NSW were found to be systematically in error because daily-read discharges were used instead of instantaneous peak discharges. A new approach, based on the method of maximum likelihood, was developed to overcome the potential artefacts introduced by the use of daily-read data in flood frequency analysis. However, it was shown for flood data typical of NSW, the treatment of daily-read data as instantaneous peaks did not introduce sufficiently large quantile bias and loss of mean-squared-error performance to warrant use of the new estimation method.

The flood data were stratified by Interdecadal Pacific Oscillation (IPO) value and flood frequency analyses performed on the IPO-stratified flood data — the IPO is a climate index of Pacific Ocean sea surface temperature anomalies, which displays variability on a long-term (multidecadal) time scale. The IPO was found to modulate the flood risk in NSW and southern Qld, with flood quantiles being increased, on average, by approximately 1.7 times during IPO-negative epochs, whereas little effect was detected for sites in northeast Qld located approximately north of the Tropic of Capricorn. The IPO modulation (nonhomogeneity) of flood risk has great practical significance — the use of at-site flood data with inadequate coverage of both IPO epochs may result in biased estimates of long-run flood risk.

A Bayesian regional flood model framework, based on hierarchical modelling concepts, was developed to overcome the possible bias in long-run flood risk associated with a nonhomogeneous flood record. Importantly, the model allows for the consideration of intersite correlation. Bayesian methods were used to enable a rigorous treatment of uncertainty in the flood regionalisation problem. The Gibbs sampler was used to infer uncertainty in the regional model parameters, while importance-sampling-based procedures were developed to compute the Bayesian predictive distribution and the posterior distribution of quantiles at a new site, which may be ungauged or gauged. This represents the first truly-general Bayesian solution for combining regional and gauged information in flood frequency analysis.

The flood data from eastern Australia were partitioned into four regions and analysed using the Bayesian hierarchical regional model. The (correlated-site) regional model found significant differences (at the 10% level) in the regional means for the two regions in NSW. The regional standard deviations showed significant differences for the two regions in Qld,
but these differences were opposite in sign, with IPO-positive standard deviations being greater than IPO-negative values. The equivalent gauged length provided by the regional model had a maximum of 4 to 8 years.

There is a large overlap in the probability limits between the IPO-positive and IPO-negative regional distributions for a flood quantile. However, because the regional model errors for the IPO phases are highly correlated, the difference in IPO-positive and IPO-negative quantiles is likely to be significant. For larger return periods, the opposite-in-sign differences in the regional mean and standard deviation may reduce the resultant IPO-related differences (in discharge).

The value of the regional model was demonstrated by pooling the regional information with the information in short gauged records at selected sites in each region. The pooled at-site flood frequency distribution provided substantial improvements over the gauged record alone (in terms of prediction limits and bias). Indeed, this was especially evident in a situation where a shortened (10-year) gauged record was found inconsistent with the true (long-run) gauged record — the shortened gauged record consisted mainly of years from the IPO-positive epoch. These results suggest that the use of the regional model may protect against bias in long-run flood risk at sites with short records sampled largely in one IPO epoch.
Chapter 1

Introduction

1.1 Overview

There are numerous influences, be they natural or man induced, that may affect climate variability over a range of time scales. For example, the short-term influences on climate variability can include the diurnal variations experienced as the Earth rotates about its own axis, and the seasonal effects associated with the tilt of the Earth and its rotation around the sun. Much attention has recently been focussed on longer-term influences, such as the El Niño/Southern Oscillation-associated impacts experienced in many parts of the world, and the possibility of the increasing levels of atmospheric carbon dioxide leading to “global warming”.

With regard to flood risk, recent studies have shown that flood risk in New South Wales (Australia) can vary on multidecadal time scales. For example, Erskine and Warner (1988) found that coastal rivers of New South Wales exhibited alternating multidecadal periods of high and low flood activity, which they termed flood- and drought-dominated regimes. Franks and Kuczera (2002) stratified the New South Wales flood record into pre- and post-1945 data sets and showed that the post-1945 data set had an elevated flood risk over the pre-1945 data set for vast majority of the sites analysed.

Often in flood studies, this long-term variability is ignored (or assumed to have a minor impact) and the flood records in question are treated as being homogeneous (“identically distributed”). Ignoring this variability may significantly impact (bias) the resultant long-term flood risk, especially if the data has insufficient samples to capture this variability. To illustrate this, assume that the pre-1945 record is the “drought” epoch, while the post-1945 record is the “flood” epoch. First consider Figure 1.1(a) where the flood record is relatively homogeneous with only a small difference between the flood and drought epochs. The long-term flood risks derived from data contained within either the drought epoch, the flood epoch, or a mixture of both epochs would be quite similar and the effects of multidecadal variability are minor, and probably may be safely ignored. However, consider Figure 1.1(b) where the effects of multidecadal variability are large and induce a significant difference in flood risk between the drought and flood epochs. Flood frequency analyses performed using a relatively even mixture of the pre- and post-1945 record should produce a reasonable estimate of the true long-term flood risk. In contrast, analyses using
mainly the pre-1945 record would significantly underestimate the true long-term flood risk, while analyses mostly using the post-1945 record would overestimate the true long-term flood risk.

In circumstances where site data inadequately samples the flood and drought epochs, it may be necessary to augment the limited site data using a regional flood analysis which adequately samples from both the drought and flood epochs. This highlights the need for a regional flood model which recognises interdecadal variability. A regional flood model utilises the flood data from (many) gauged sites and provides a mechanism to transfer the information contained in the gauged site records to another site of interest — this new site may have a short gauged record or may even be ungauged. It is expected that the additional information contained within the regional flood model/distribution will allow improved flood estimates at the site of interest.

1.2 Objectives

This study is motivated by the findings of Erskine and Warner (1988), Franks (2002b), and Franks and Kuczera (2002). It investigates the flood frequency records from the eastern Australian states of New South Wales and Queensland to determine the magnitude and spatial extent of decadal-scale variability in flood risk that exists in these records. A regional flood model is then developed to overcome the possible bias in long-term flood risk associated with this nonhomogeneous flood record.

These two main goals can be broken down into a set of more specific objectives:

1. Investigation of nonhomogeneity:

   - Analysis of flood records and the data errors present therein: During the data verification process, it was found that some flood data were systematically in error — some recorded flood peaks in the NSW flood record did not necessarily correspond to the peak instantaneous discharges, but instead consisted of “daily-read” flows. A daily-read flow is a discharge associated with a specific
reading time — river levels and, thus, discharges were manually recorded at, typically, 9 AM — and this results in an underestimate of the true peak instantaneous discharge because it is unlikely that the true peak occurred at 9 AM. The impacts of these “censored” data are investigated within the context of flood frequency analysis.

- Determining the magnitude and spatial extent of the multidecadal variability present within these flood records: The observed climate shift that was observed around 1945 provides a simple descriptor of multidecadal variability; however, there are other, larger scale, mechanisms at work. Climate indices describing sea surface temperature variations have been found to vary on multidecadal time scales, with shifts in these indices (e.g. from positive to negative values) being associated with changes in rainfall and streamflow in many parts of the world. One such climate index is used to stratify the flood data to allow for the investigation of the multidecadal variability in flood risk present within the flood records.

2. Development of a regionalisation methodology:

- Ability to deal with a nonhomogeneous flood record: The nonhomogeneity in the observed flood record is characterised by multidecadal epochs of reduced or elevated (at-site) flood risk. This nonhomogeneity should be incorporated into the regional model and the importance of these multidecadal variations should be investigated at a regional scale (i.e. does the regional model display a similar multidecadal variability?).

- Ability to deal with missing data in flood records: Flood frequency data sets often suffer from missing data and from unequal record lengths, possibly resulting in each year having a variable number of sites with data.

- Ability to deal with correlated sites: Flood frequency data is often spatially correlated; that is, there is intersite correlation between concurrent flows. This correlation has the effect of reducing the information content of the data. If it is ignored, and the sites assumed independent, the predictive power of the regional model may be overestimated. Thus, correlation should be incorporated into the model to allow for the investigation of its impact.

- Incorporate a statistically rigorous parameter uncertainty framework allowing for the quantification of predictive uncertainty: The overall aim of the regional flood model is to provide a prediction of the flood distribution at a site, whether that site be gauged or ungauged. A rigorous statistical framework should be used to explicitly account for uncertainty in the regional flood model parameters and in the quantification of uncertainty of the flood frequency distribution at a selected site using the available at-site and regional information.
1.3 Outline

Chapter 2 reviews the flood data from New South Wales and Queensland to be used in this study. The flood data is shown to consist of a mixture of instantaneous flood peaks and flood peaks derived from a single daily observation (say, at 9 AM). These “daily-read” observations may significantly underestimate the true flood peak because the instantaneous flood peak is unlikely to occur at the time that the daily observation was made. New estimation techniques are developed to deal with this problem. A Monte Carlo study is performed to investigate the effects that these daily-read flood peaks have with respect to flood frequency analysis.

The nonhomogeneity of the flood frequency record is investigated in Chapter 3, with the magnitude and spatial extent of this nonhomogeneity detailed. The relationship between flood risk and climate indices based on large-scale climate-related variables, such as sea surface temperature, is investigated. In particular, a promising association involving the Interdecadal Pacific Oscillation (IPO) is pursued — the IPO is a climate index based on sea surface temperature variations over the Pacific Ocean. The flood frequency data is stratified according to IPO value and then flood frequency analyses are performed on the IPO-stratified data.

Chapter 4 develops a new flood regionalisation methodology, based on a hierarchical model, that is able to deal with a nonhomogeneous flood frequency record. Model calibration is performed using Bayesian methods, which enables the quantification of parameter uncertainty. The hierarchical model and the Bayesian techniques for inferring its parameters are first described. An investigation is then performed using synthetic data to demonstrate the robustness of the Bayesian methods. Methods are also derived to evaluate and quantify the predictive uncertainty at both ungauged and gauged sites.

The eastern Australian flood data is analysed using the Bayesian hierarchical model in Chapter 5. The performance of the regional flood model is evaluated for both independent- and correlated-site implementations to determine whether the additional complexities associated with the correlated-site implementation improves the predictive performance over the independent-site implementation. The predictive performance of the regional flood model is then examined at both ungauged and gauged sites.

A summary of conclusions is provided in Chapter 6, along with possible future research directions.
Chapter 2

Flood frequency data affected by censoring errors

2.1 Introduction

Before the nature of multidecadal variability in the flood record can be investigated, it is necessary to carefully review the available flood record and deal with issues relating to the integrity of the record. This is the primary focus of this chapter. It assembles the flood frequency data from eastern Australia that will be used to investigate nonhomogeneity in the flood record and develops strategies to deal with shortcomings in the data. Investigation of these flood records shows that the flood data at some sites may be systematically in error due to the data collection procedures that have been used. New techniques to deal with this systematic error are developed, with their performance being evaluated through a Monte Carlo study.

2.2 Flood frequency data from eastern Australia

Annual maximum flood data from the eastern Australian states of New South Wales (NSW) and Queensland (Qld) were used in this study. The NSW data were obtained from the NSW Department of Land and Water Conservation’s Pinneena database (Department of Land and Water Conservation, 2000), while the Qld data were provided directly by the Qld Department of Natural Resources and Mines (Hans Mulder, 2004, personal communication). Flood data were obtained for a total of 127 sites, with Qld having 85 sites and NSW 42 sites. The locations of the sites are shown in Figure 2.1, along with the approximate location of the Great Dividing Range, while a summary of the site information is provided in Appendix A. The yearly peak flows were extracted using a water year from April to March, which corresponds to the typical ENSO cycle (see Chapter 3). Note that few sites were located to the west of the Great Dividing Range due to the lower rainfall regime and consequent lower stream gauging density. The sites west of the Divide were selected such that they were not influenced by extensive floodplains — floodplain storage can attenuate flood peaks resulting in a decrease of discharge with increasing catchment area. The sites within Qld were specifically selected such that they were minimally affected
Analyses performed regional only ffa and regional

Figure 2.1: Location of the 127 sites within New South Wales and Queensland, along with the approximate location of the Great Dividing Range (shown as the dashed line). The open circles represent sites which are analysed in Chapter 5 only, while the closed dots are analysed in both Chapters 3 and 5.

by regulation, while some of the NSW sites — mostly in western NSW — are situated on regulated rivers. A comparison (see Section 3.5) of analyses performed using both the regulated and unregulated sites and only the unregulated sites yielded similar results, so the regulated sites were retained for this study, providing additional coverage within NSW. The spatial coverage of sites is considerable. Indeed, the latitudinal extent ranges from about 11° to 38° south, with the climate varying from tropical in northern Qld to alpine in the Snowy Mountains of southern NSW.

Error checking was performed on the flood data, including checks for gross errors (“blunders”) and independence (between consecutive annual maxima at a site). The error-checking process revealed that the NSW flood data (especially pre-1970s) did not necessarily consist of the peak instantaneous discharge (“true peak”), but instead consisted of a mixture “daily-read” and true peak discharges. A daily-read discharge is a discharge associated with a specific reading time — river levels, and thus discharges, were manually
recorded at, typically, 9 AM (Department of Land and Water Conservation, 2000). This results in an underestimate of the true peak discharge because it is most unlikely that the true peak occurred at exactly 9 AM. The true peak discharges were derived either from multiple manual observations during flood events or from continuous (fixed interval) readings from automatic data loggers. Note that most sites have had continuous readings since the 1970s, with this continuous data being stored in the Pinneena database. In contrast, the Qld data consists almost entirely of true peak discharges and, so, contains few daily-read discharges.

This motivates an investigation into the impact of daily-read data on the integrity of the flood frequency analysis. Daily-read flows can be classified as censored-from-below observations or, alternatively, as random censoring thresholds, where the censoring is on the right. The data errors that result from this censoring will first be examined and then the consequences of these errors will be explored in the context of flood frequency analysis. This leads to a new technique, based on the method of maximum likelihood, to overcome, in a general way, the artefacts introduced by daily-read data.

### 2.3 Relationship between true peak and daily-read flows

An understanding of the relationship between true peak and daily-read flows is developed empirically through examination of the continuous portion of several flood gauging records obtained from the NSW Pinneena database. The data extracted consisted of river levels and rating curves.

For each day with recorded continuous river levels, the peak and “daily-reading time” water levels were extracted. Selection of the daily reading time is somewhat arbitrary, with 9 AM being used because it corresponds to the typical reading time used in the earlier periods of the NSW data set. The appropriate rating curve was then used to convert these water levels into discharges. The maximum values of the peak flow and daily-read flow, for each year, were then recorded and the ratio $R$ of these two flows calculated, where

$$R = \frac{\text{true annual maximum peak flow}}{\text{annual maximum daily-read flow}}$$  \hspace{1cm} (2.1)

These two maximum flows do not necessarily correspond to the same flood event; however, in practice, it was found that they were in correspondence about 85\% of the time. This analysis was performed for several catchments of varying size and the results are summarised in Table 2.1, with the results for the sites displayed in Figure 2.2. Note that the dashed line in the figure is the “perfect correlation” line; the closer that the points are plotted to this line, the closer that the daily-read flows are to the true peak flows.

Table 2.1 reveals that the statistics of the ratio $R$ are quite variable across the sites. No general rule, such as the ratio ($R$) reducing as catchment area increases, is evident, suggesting that catchment-specific (local) conditions exert a strong influence. It is shown that daily-read flows can significantly underestimate the true peak flow, with two sites, 203002 and 210006, displaying some very large differences, some greater than a factor of ten; e.g. site 210006 has a true peak of 154000 when the daily-read flow is 14500 ML/day.
Figure 2.2: Comparison of the annual maximum true peak and daily-read (9 AM) discharges for five NSW sites. Note that sixth site (422001) has not been shown because there is minimal difference between the true peak and daily-read discharges.
Table 2.1: Summary of statistics for several NSW sites

<table>
<thead>
<tr>
<th>Site</th>
<th>Area (km²)</th>
<th>N</th>
<th>mean</th>
<th>sd</th>
<th>med</th>
<th>min</th>
<th>max</th>
<th>CV[r]b</th>
</tr>
</thead>
<tbody>
<tr>
<td>204008</td>
<td>31</td>
<td>26</td>
<td>1.53</td>
<td>0.56</td>
<td>1.31</td>
<td>1.03</td>
<td>3.28</td>
<td>1.04</td>
</tr>
<tr>
<td>203002</td>
<td>62</td>
<td>25</td>
<td>2.67</td>
<td>2.14</td>
<td>2.02</td>
<td>1.00</td>
<td>11.12</td>
<td>1.28</td>
</tr>
<tr>
<td>401005</td>
<td>471</td>
<td>13</td>
<td>1.28</td>
<td>0.29</td>
<td>1.16</td>
<td>0.53</td>
<td>1.99</td>
<td>1.04</td>
</tr>
<tr>
<td>210006</td>
<td>3340</td>
<td>28</td>
<td>2.18</td>
<td>1.98</td>
<td>2.00</td>
<td>1.00</td>
<td>11.12</td>
<td>1.04</td>
</tr>
<tr>
<td>210001</td>
<td>16400</td>
<td>29</td>
<td>1.19</td>
<td>0.20</td>
<td>1.12</td>
<td>1.00</td>
<td>1.71</td>
<td>1.06</td>
</tr>
<tr>
<td>422001</td>
<td>132200</td>
<td>29</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.02</td>
<td>0.97</td>
</tr>
</tbody>
</table>

a Abbreviations N, sd, med, min, max, and CV respectively denote number of data, standard deviation, median, minimum, maximum, and coefficient of variation.

b The coefficient of variation is calculated for the reduced ratio \( r = R - 1 \).

These two sites have vastly different areas, with 203002 being quite small (62 km²) while 210006 is relatively large (3340 km²), further indicating the influence of catchment-specific conditions. Also, site 422001 has a ratio of one indicating that there is no difference between daily-read and true peak flows. This occurs because the catchment is very large and responds slowly. In such cases, it is appropriate to treat the daily-read flows as true peak flows. Note that selecting a different reading time, other than 9 AM, would cause the points on the graphs in Figure 2.2 to be shifted horizontally — the daily-read flows would either increase or decrease, while the peak flows remain the same. This should not significantly alter the analysis presented here.

The stream water level and discharge time series for site 210006 is presented in Figure 2.3 for the event where the peak flow was over ten times greater than the daily-read (9 AM) flow. It shows that the reason for the large anomaly between the censored and peak flows is that daily (9 AM) readings were far too infrequent to accurately capture the dynamic response of this catchment. Indeed, this peak would require readings spaced at intervals of around 2 hours to provide an adequate approximation.

Additional examination of the ratios — using the reduced ratio \( r \) — suggests that there is a structure within these ratios, where the reduced ratio is defined:

\[
 r = R - 1 
\]

The coefficients of variation \([CV = (standard deviation)/mean]\) of \( r \) are listed in Table 2.1 and show that this statistic is quite stable (approximately one) for most sites, except for the two sites that experience extremely large differences between true peak and daily-read flows. These two sites obviously have greater variability; thus, the increased CV is not unexpected. Furthermore, quantile-quantile (Q-Q) plots, shown in Figure 2.4, suggest that the reduced ratio follows a lognormal distribution — the Q-Q plots were constructed assuming that the true distribution of \( r \) was lognormal; i.e. \( \log r \sim N(m, s^2) \), where \( N(m, s^2) \) denotes a normal distribution with mean \( m \) and variance \( s^2 \). Thus, the relationship between the true peak flow and the daily-read flow can be transformed to approximate
Figure 2.3: Stage and hydrograph of an event showing a factor of ten difference between the daily-read (9 AM) and the true peak discharges for site 210006.

A common statistical distribution — this is referred to as the censoring distribution. Note that the Q-Q probability limits were derived using a parametric bootstrap procedure (for example Davison and Hinkley, 1998):

1. Calculate $R$ and $r$ for $n$ observed daily-read and true peak flows.

2. Calculate associated statistics: $E(R)$, $\text{Var}(R)$, $\text{CV}(r)$, $\text{Var}(r)$, and $E(r)$, where $E(\cdot)$, $\text{Var}(\cdot)$, and $\text{CV}(\cdot)$ denote the expectation, variance, and coefficient of variation, respectively.

3. Monte Carlo loop: randomly sample $r = R - 1$ from the lognormal distribution $\log r \leftarrow N(E(r), \text{Var}(r))$, for each of the $n$ observations and then order (rank) these $n$ terms. Note that $\leftarrow$ denotes “is sampled from”.

4. Extract the 90% probability limits for each of the $n$ (ordered) observations.

It is interesting to note that a similar empirical examination was performed by Langbein (1944) who created a figure — reproduced in Linsley et al. (1958, p. 75) — relating the true peak discharge to the daily mean discharge occurring on and either side of the day of the true peak. For example, if the true peak occurred on the 3rd day of the month, then the true peak discharge would be estimated using the daily mean discharges from the 2nd, 3rd, and 4th of the month.
Figure 2.4: Quantile-quantile plots, with 90% probability limits, of the censoring ratio $R = r + 1$ assuming a lognormal censoring distribution, $\log r \sim N(m, s^2)$. 
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2.4 Evaluation of methods

It was shown that estimating peak flows using daily-read flows can result in large errors which can affect the estimation of flood risk. Four methods that can be used to deal with these errors are now examined. Evaluation of these methods is performed using a Monte Carlo experiment.

These methods, all based on the maximum likelihood approach, are summarised below and are explained in greater detail in the following sections:

1. Exclude erroneous data: Identify and exclude the daily-read flows, analysing only the true peak flows.

2. Ignore data errors: Treat the daily-read flows as if they were true peak flows, ignoring the associated systematic underestimation of the true peak.


4. Random dependent censoring: Use a new procedure incorporating random censoring thresholds and the statistical dependence between the daily-read and true peak flows.

Note that Langbein’s approach is not considered because it does not relate the true peak to the daily-read discharge — it relates the true peak to the daily mean discharge instead. Also, it would require the development of a model relating the true peak flow to the flows that occur on and either side of the day of the true peak.

2.4.1 Monte Carlo Experiment

The flood distribution selected for the Monte Carlo experiment was the lognormal distribution. This distribution was selected because it is shown in Chapter 3 that the lognormal distribution provides an adequate fit to the (IPO-stratified) flood data used in this study. The IPO (Interdecadal Pacific Oscillation) is a climate index of long-term Pacific Ocean sea surface temperature anomalies (Folland et al., 1999) that has been shown to modulate the effects of El Niño/Southern Oscillation within Australia (Power et al., 1999; Kiem et al., 2003) — the IPO is discussed in more detail in Chapter 3.

The experiment describes the following scenario involving a total record length of \( n \) flow observations consisting of \( g \) true peak flows \( Q = \{Q_1, \ldots, Q_g\} \) and \( c \) daily-read flows \( q = \{q_{g+1}, \ldots, q_n\} \), where \( n = g + c \) — it is stressed that \( g \) and \( c \) are known constants. The true peak flows are assumed to follow a lognormal distribution, \( \log Q \sim N(\mu, \sigma^2) \), and the true peak and daily-read flows are related by the censoring distribution, \( \log r \sim N(m, s^2) \), where \( r = R - 1 = Q/q - 1 \) is the reduced ratio. Note that the terms gauged and censored flows are used interchangeably for true peak and daily-read flows respectively. The steps used within the Monte Carlo experiment are:

1. Randomly generate the true peak flow \( Q \) over the entire record length \( n \), \( \log Q_i \leftarrow N(\mu, \sigma^2) \), for \( i = 1, \ldots, n \).
2. Randomly censor \( c \) flows to obtain the daily-read flow \( q_j \), for \( j = g+1, \ldots, n \):

   (a) Randomly generate the censoring ratio \( R = Q/q = r + 1, \log r \leftarrow \mathcal{N}(m, s^2) \), where \( r \) is the reduced ratio from Section 2.3.

   (b) Compute the daily-read flow, \( q_j = Q_j/R_j \).

3. Estimate the parameters of the lognormal flood distribution \((\hat{\mu}, \hat{\sigma})\) using the four methods.

4. Repeat the above steps \( N = 1000 \) times.

5. Calculate the relative bias and rmse (root mean squared error) of selected flow quantiles, expressed as a percentage of the true quantiles (\( \theta \) and \( \hat{\theta} \) denote the true value and estimate of a quantile).

   \[
   \text{%bias} = \frac{100}{N\theta} \sum_{i=1}^{N} (\hat{\theta}_i - \theta) \quad (2.3)
   \]

   \[
   \text{%rmse} = \frac{100}{\theta} \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{\theta}_i - \theta)^2 / N} \quad (2.4)
   \]

A total record length of 40 years was used in the analysis because it approximately corresponds to the average length of the IPO-stratified flood record for NSW and to the average length of the (total) flood record for Qld (see Appendix A and Chapters 3 and 5). Additionally, these data sets have a mean, \( \mu \), and standard deviation, \( \sigma \), of approximately 10 and 1.2 respectively, where flow units are expressed in ML/day.

### 2.4.2 Method 0: Gauged Only (GO)

This method does not make any attempt to handle censored data; instead, the censored flows are excluded, with only the gauged flows being analysed. This method is used to provide a basis on which to judge the benefits provided by including the censored flows. The likelihood function is

\[
p(Q, q \mid \mu, \sigma) = \prod_{i=1}^{g} p(Q_i \mid \mu, \sigma) \quad (2.5)
\]

where \( p(x \mid \mu, \sigma) \) denotes a probability density function (pdf) evaluated at \( x \) with known parameters \( \mu \) and \( \sigma \). In this case, using the lognormal distribution, the likelihood is

\[
p(Q, q \mid \mu, \sigma) = \prod_{i=1}^{g} \text{LN} (Q_i \mid \mu, \sigma^2) \quad (2.6)
\]

where \( \text{LN} (x \mid \mu, \sigma^2) \) denotes a lognormal pdf evaluated at \( x \) with the underlying normal distribution having mean \( \mu \) and standard deviation \( \sigma \).
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2.4.3 Method 1: Censored As Gauged (CAG)

This method ignores the error associated with the censoring and treats the daily-read flow as if it were the true peak flow. The likelihood function simply becomes

\[ p(Q, q | \mu, \sigma) = \prod_{i=1}^{g} \ln (Q_i | \mu, \sigma^2) \prod_{j=1}^{c} \ln (q_j | \mu, \sigma^2) \] (2.7)

2.4.4 Method 2: Binomial Censoring (BC)

A daily-read flow underestimates the true peak flow because it is unlikely that the true peak flow occurred at the daily-reading time, say 9 AM. This situation is analogous to that of censored data where the daily-read flow is the censoring threshold, with the true peak flow being unobserved. Indeed, each daily-read flow is independent of another, resulting in what is known as random censoring (for example Schneider, 1986, pp. 2–3). This suggests that the binomial censoring procedure of Stedinger and Cohn (1986) may be applicable — each daily-read flow becomes the censoring threshold with a single exceedance above this threshold level (that exceedance being the unobserved true peak flow) and zero nonexceedances.

The likelihood function of Stedinger and Cohn (1986, Equation 10) is slightly modified to include multiple thresholds, each of which are exceeded once in one year

\[ p(Q, q | \mu, \sigma) = \prod_{i=1}^{g} p(Q_i | \mu, \sigma) \prod_{j=1}^{c} \left\{ \left( \frac{1}{1} \right) F(q_j | \mu, \sigma)^0 \left[ 1 - F(q_j | \mu, \sigma) \right]^1 \right\} \] (2.8)

where \( F(x | \mu, \sigma) \) denotes the cumulative distribution function (cdf) of \( x \), with known parameters \( \mu \) and \( \sigma \). In this case using the lognormal distribution, the likelihood becomes

\[ p(Q, q | \mu, \sigma) = \prod_{i=1}^{g} \ln (Q_i | \mu, \sigma^2) \prod_{j=1}^{c} \left[ 1 - \Phi(\log q_j | \mu, \sigma^2) \right] \] (2.9)

where \( \Phi(x | \mu, \sigma^2) \) denotes the normal cdf of \( x \) with mean \( \mu \) and standard deviation \( \sigma \).

Note that binomial censoring assumes that the censoring threshold is independent of the true peak flow. This assumption is valid for “historical data” cases where the threshold may be defined by a “water level on a wall” or as the magnitude of a “historic” flood. However, the threshold of a daily-read flow is not independent of the true peak flow — if the true peak flow is small then the daily-read flow will also be small, or if the true peak flow is large then the daily-read flow should also be large. The appropriateness of independent censoring thresholds for daily-read flows will be assessed. This motivates the development of the next method.

2.4.5 Method 3: Random Dependent Censoring (RDC)

The final method involves a new approach for censored flows. It incorporates the multiple random thresholds present in the data, as in the BC method. Significantly, however, it accounts for the dependence between the threshold (daily-read flow) and true peak flow.
The added complexities of this approach result in the likelihood function having to be evaluated using numerical integration.

The likelihood for a single daily-read flow can be expressed, using the total probability theorem, as

$$p(q | \mu, \sigma) = \int_{-\infty}^{\infty} p(q | Q, \mu, \sigma) p(Q | \mu, \sigma) dQ$$

where $p(q | Q)$ is the conditional pdf of the daily-read flow given the true peak flow, and $p(Q | \mu, \sigma)$ is the flood pdf. Note that the formulation of this likelihood is totally general — the relation used for $p(q | Q)$ is arbitrary and the assumed flood distribution $p(Q | \mu, \sigma)$ can also be freely chosen (with an associated change in the model parameters).

The likelihood function is developed for this particular case using the censoring distribution $\log r \sim N(m, s^2)$ and the assumed lognormal flood distribution $\log Q \sim N(\mu, \sigma^2)$ for $p(q | Q)$ and $p(Q | \mu, \sigma)$ respectively. Using derived distributions

$$p(q | Q) = \left| \frac{d \log r}{dq} \right| p(\log r | Q)$$

$$= \frac{-Q}{(Q - q)q} p(\log r | Q)$$

$$= \left[ \frac{Q}{(Q - q)q} \right] N \left( \log \left[ \frac{Q}{q} \right] - 1 \right) \left| m, s^2 \right)$$

with the simplification of the absolute value term arising since $Q > q > 0$ and the notation $N(x|a,b)$ denotes a normal pdf evaluated at $x$ with mean $a$ and variance $b$. The likelihood for a single daily-read flow can thus be written as

$$p(q | \mu, \sigma) = \int_{q}^{\infty} \left[ \frac{1}{(Q - q)q} \right] N \left( \log \left[ \frac{Q}{q} \right] - 1 \right) \left| m, s^2 \right) N(\log Q | \mu, \sigma^2) dQ$$

The full likelihood function — consisting of both true peak and daily-read flows — becomes

$$p(Q, q | \mu, \sigma) = \prod_{i=1}^{g} \prod_{j=1}^{c} \left\{ \int_{q_{ij}}^{\infty} \left[ \frac{1}{(Q - q_{ij})q_{ij}} \right] \right\} \times N \left( \log \left[ \frac{Q}{q_{ij}} \right] - 1 \right) \left| m, s^2 \right) N(\log Q | \mu, \sigma^2) dQ$$

This integral has no analytical solution and so must be integrated numerically. The use of an adaptive integration routine is recommended because the “region of interest” of the integral can be quite small and may be easily missed by fixed-point schemes, such as Gaussian quadrature. An adaptive Clenshaw-Curtis quadrature scheme (see Press et al., 1992, p. 190) was used, which varied the step size to ensure that the integral would reach the prescribed tolerance. Also, it is usually computationally advantageous to subdivide
Figure 2.5: Quantile rmse and bias for the CAG and GO methods for a record length of $n = 40$ years [$\log Q \sim N(8.0, 1.2^2)$, $E(r) = 1.0$, $CV(r) = 1.0$].

the integral into smaller panels.

2.4.6 Results and discussion

In the following sections, the GO method is used as the reference to evaluate the performance of the other methods. The Monte Carlo experiment was performed for the following scenarios: typical lognormal flood distribution parameter $(\mu, \sigma^2)$ values of $(8.0, 1.2^2)$ and typical censoring distribution $(E(r), CV(r))$ values of $(0.2, 1.0)$ and $(1.0, 1.0)$. Note that the choice of $\mu$ is arbitrary and does not influence the results, while censoring distribution values were derived from Table 2.1.

Method 1: CAG

The CAG method was analysed for typical flood and censoring distributions. The results of one of these analyses, for a total record length $n = g + c = 40$ years, are shown in Figure 2.5. The solid symbols, interconnected by lines, present the quantile rmse and bias for the CAG method, while the open symbols present the results for the GO method for the stated number of true peak flows. For example when $g = 10$, the CAG method uses of both true peak and daily-read flows ($n = g + c = 10 + 30$), while the GO method only uses the true peak flows ($n = g = 10$). The GO method provides a benchmark to evaluate performance gains from utilising daily-read flows. The results are shown for three recurrence intervals: 10, 50, and 100 years. Note that this analysis shows similar rmse behaviour to that shown for other combinations of $E(r)$, $CV(r)$, $\mu$, and $\sigma^2$. 
The CAG method clearly outperforms the GO method. Indeed, and somewhat surprisingly, the rmse for the CAG method is insensitive to the number of daily-read flows. While the bias in CAG exceeds that in GO, for \( g \geq 30 \) the differences are small. Moreover, though the CAG bias grows as \( g \) decreases, it is insufficient to make any noticeable impact on the rmse. This behaviour arises because as \( g \) is decreased, the increase in bias is compensated by a decrease in sampling error. To understand this, consider the first order variance approximation for the log-quantile from the lognormal distribution (Stedinger et al., 1993, Equation 18.4.4)

\[
\text{Var} (\log Q_T) = \frac{\sigma^2}{n} \left( 1 + \frac{z_T^2}{2} \right)
\]

where \( Q_T \) is the \( T \)-year quantile and \( z_T \) is the standard normal deviate with exceedance probability \( 1/T \). Note that \( n \) and \( z_T \) are fixed — the variance of \( \log Q_T \) only depends on \( \sigma \). In the Monte Carlo experiments, there was minimal bias in \( \sigma \) over all values of \( g \).

The variance of the quantile \( Q_T \) can be approximated using first-order analysis to give

\[
\text{Var} (Q_T) = [\text{E} (Q_T)]^2 \text{Var} (\log Q_T)
\]

It follows that the relative variance of \( Q_T \) is

\[
\frac{\text{Var} (Q_T)}{Q_T^2} = \left[ \frac{\text{E} (Q_T)}{Q_T} \right]^2 \text{Var} (\log Q_T)
\]

As \( g \) decreases, the bias in \( Q_T \) becomes more negative causing \( \text{E} (Q_T) / Q_T \) to fall well below one. Since \( \text{Var} (\log Q_T) \) remains virtually constant, the contribution of the sampling error \( \left( \text{Var} (Q_T) / Q_T^2 \right) \) to the relative rmse decreases and, in this case, offsets the contribution of the bias to the relative rmse.

Although the rmse is almost invariant, the bias is not. This is important because the flood frequency analyses performed in Chapter 3 rely on minimal bias in the resultant flood quantiles — Figure 2.5 shows that as \( g \) decreases, the bias in the flood quantiles grows appreciably. However, Figure 2.5 represents an extreme case for NSW flood data, with Figure 2.6 showing a plot for a more realistic value of \( \text{E} (r) = 0.2 \) (see Table 2.1). Figure 2.6 shows that the bias is quite small, even when dominated by daily-read data (i.e. bias is about -10% for \( g = 10 \)). Review of the NSW flood data revealed that the maximum observed proportion of daily-read flows \( c/(g+c) \) is about 0.5, with a proportion of about 0.15 being typical. In view of these considerations, the use of daily-read data in lieu of instantaneous peak discharges should not significantly compromise flood quantile rmse and bias.

**Method 2: BC**

The BC method was analysed for typical flood and censoring distributions. Figure 2.7 compares the BC and GO methods for a record length \( n \) of 40 years. It is clear the the BC method performs very poorly — indeed disastrously — with the bias exploding as...
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Figure 2.6: Quantile bias for the CAG and GO methods for a record length of $n = 40$ years [$\log Q \sim N (8.0, 1.2^2), E(r) = 0.2, CV(r) = 1.0$].

Figure 2.7: Quantile rmse and bias for the BC and GO methods for a record length of $n = 40$ years [$\log Q \sim N (8.0, 1.2^2), E(r) = 1.0, CV(r) = 1.0$].
$g$ decreases. The results shown in Figure 2.7 were typical of the other analyses, with the “errors” becoming more pronounced as the ratio $E(r)$ was decreased. This result becomes intuitive if the reverse case is considered: the “errors” become smaller as the ratio is increased. The increased ratio implies that there is greater variability between the daily-read and true peak flows, resulting in a “flatter” distribution between $Q$ and $q$. In the limiting case, this distribution becomes uniform and the variables $Q$ and $q$ lose their interdependence, which best corresponds to the binomial case. This situation was alluded to earlier in Section 2.4.4.

Method 3: RDC

The random censoring method was assessed using the typical scenarios, as well as additional cases where the parameters of the flood distribution $\mu$ and $\sigma$ and censoring distribution parameter $E(r)$ were varied to extreme observed levels, or beyond. The results are shown for a record length of $n = 40$ years in Figure 2.8, and are representative of the other analyses performed.

As for the CAG method, the rmse showed little change with $g$. However, the RDC method exhibited less bias than the CAG, particularly for low $g$ (compare Figure 2.5). This arises because $p(q \mid Q)$ is compensating for the underestimation of the true peak $Q$ by the daily-read flow $q$.

Figure 2.8 shows that the rmse remains quite constant for all values of $g$, and remains under the GO case; whilst the bias is quite similar to the GO case, except for small values
of $g$ where it is significantly less. This effect is not intuitive because one would expect that the rmse would decrease by a more significant amount as $g$ is increased; however, the explanation for this observed effect is that the variability present within the flood distribution ($\sigma$) overwhelms the data errors associated with the censored data.

This effect is demonstrated in Figure 2.9 where an unusually “tight” flood distribution is considered, with $\sigma$ lowered to an atypical level of 0.2 (versus the typical observed value of about 1.2). In this case, the rmse decreases as $g$ increases, and largely follows the GO curve. This indicates that the censored data has little useful information in this case — the censoring error has overwhelmed the flood distribution’s variability, negating any possible benefits. Note that despite the loss of information, the RDC method manages to control the bias.

In comparison, Figure 2.10 shows that the censored data errors overwhelm the CAG method causing both the rmse and bias to significantly increase, resulting in much poorer performance than both the RDC and GO methods.

**Misspecification errors**

The effect of censoring distribution misspecification errors is examined in this section. This is achieved by generating the censored data using an assumed, true, $E(r)$ value and then using another, erroneous, $E(r)$ value for the likelihood calculations. Obviously, these misspecification errors only affect the RDC method. The RDC method was analysed for the typical scenarios, as well as extra cases where the flood distribution parameter $E(r)$
Figure 2.10: Quantile rmse and bias for the CAG and GO methods for a record length of \( n = 40 \) years \([\log Q \sim N (8.0, 0.2^2), E(r) = 1.0, CV(r) = 1.0]\).

was increased to larger than typical values. The results of two RDC analyses are presented in Figures 2.11 and 2.12.

When the erroneous \( E(r) \) value is less than the true value, shown in Figure 2.11, the RDC method exhibits little change in rmse, but an increasing bias as \( g \) decreases. Though the bias deteriorates, it is insufficient to have much impact on the rmse.

When the erroneous \( E(r) \) value is greater than the true value, as in Figure 2.12, both the rmse and bias deteriorate for the RDC method as \( g \) decreases. Indeed, the rmse performance of the RDC method is little different from the GO method.

Thus, if the true censoring distribution cannot be determined, it is preferable to use a censoring distribution with a small \( E(r) \). This is because, with small \( E(r) \), the RDC method behaves like the CAG method, which has robust rmse performance for typical flood data.

2.5 Conclusion

It has been demonstrated that daily-read flows — flows derived from a single daily (say, 9 AM) observation — can underestimate the true peak flows by significant amounts, with underestimation factors of ten being possible. Several methods to deal with these daily-read, or censored, flows were evaluated using a Monte Carlo study.

The BC method was found to perform very poorly because the dependence between the true peak and daily-read flows is contrary to the assumption of independence made in method’s derivation.
Figure 2.11: Quantile rmse and bias for the RDC and GO methods with misspecification errors for a record length of $n = 40$ years [$\log Q \sim N (8.0, 1.2^2)$, true $E(r) = 1.0$, erroneous $E(r) = 0.2$, CV $(r) = 1.0$].

Figure 2.12: Quantile rmse and bias for the RDC and GO methods with misspecification errors for a record length of $n = 40$ years [$\log Q \sim N (8.0, 1.2^2)$, true $E(r) = 0.2$, erroneous $E(r) = 1.0$, CV $(r) = 1.0$].
Under typically encountered flood and censoring distributions, both the CAG and RDC methods were found to improve flood quantile estimates over the GO case. The RDC method is preferable over the CAG method because it is theoretically sound, provides a better performance when the censoring distribution is known, and is quite robust when the censoring distribution is misspecified. However, due to the computational burden of the RDC method, the CAG method may be used in these situations without a significant loss in performance (as measured by bias and rmse) over the RDC method.

Under atypical flood and censoring distributions, the CAG method’s performance may degrade considerably below that of the GO method, while the RDC method, at worst, performs similarly to the GO method.

The results presented here are of particular importance given the use of established databases in large scale flood studies. In particular, this investigation demonstrates the clear need to rigorously evaluate the meaning of data, especially where changes in monitoring practices have occurred.

Of particular importance to this study is the conclusion that treating daily-read flows as instantaneous peak flows should not impact significantly on the analyses performed in subsequent chapters.
Chapter 3

Multidecadal variability in flood risk: An investigation for eastern Australia

3.1 Introduction

This chapter investigates eastern Australian flood frequency records to determine whether these records are indeed homogeneous (i.e., independent and identically distributed). Some recent studies have suggested that eastern Australian flood data may be affected by multidecadal variability whereby decadal-long epochs experience varying periods of elevated and reduced flood risk. The Interdecadal Pacific Oscillation (IPO) is a climate index which has been identified by several researchers as being significantly associated with multidecadal variability in rainfall and streamflow. Here, the relationship between the IPO index and flood risk is investigated. Flood frequency analyses are performed on the IPO-stratified flood data and the effect and extent of this multidecadal variability in eastern Australia flood data is examined.

3.2 Climatic variability

A basic assumption of flood frequency analysis (FFA) is that flood peaks are independent and identically distributed (iid). However, recent studies have questioned the validity of this assumption with evidence showing the existence of more than one distribution within eastern Australian flood data, especially for New South Wales (NSW). Franks (2002b) analysed NSW flood data and identified a change in flood risk occurring around 1944, in both individual gauged sites and using a regional index. Franks and Kuczera (2002) stratified the NSW flood record into pre- and post-1945 data sets and showed that the post-1945 data set had an elevated 20-year flood risk over the pre-1945 data set for 37 of the 41 sites analysed. Also, Erskine and Warner (1988) showed that coastal rivers of NSW exhibit alternating periods of high and low flood activity, which they denoted flood- and drought-dominated regimes respectively. The duration of these regimes varied between 30
and 50 years, with the flood regime being characterised by significantly more, and usually larger, floods than the intervening drought regime. Indeed, the iid assumption has also been challenged for North American flood data with Olsen et al. (1999) reporting positive trends for flood risk over time for gauged sites within the Mississippi, Missouri, and Illinois River basins.

Many studies have demonstrated the relationship between El Niño/Southern Oscillation (ENSO) and observed climate variability (for quantities such as rainfall, temperature, and streamflow) in many parts of the world; for example, in Australia (Ropelewski and Halpert, 1987; Allan, 1988; Stone and Auliciems, 1992; Simpson et al., 1993; Chiew et al., 1998; Power et al., 1998; Piechota et al., 1998; Kiem and Franks, 2001; Westra and Sharma, 2006), in New Zealand (McKerchar and Pearson, 1994; McKerchar et al., 1998; Mosley, 2000), and throughout the rest of the world (Ropelewski and Halpert, 1989, 1996; Kiladis and Diaz, 1989; Redmond and Koch, 1991; Glantz et al., 1991; Kahya and Dracup, 1993; Moss et al., 1994; Piechota and Dracup, 1996; McCabe and Dettinger, 1999; Chiew and McMahon, 2002). ENSO occurs as a result of large-scale interactions between ocean and atmospheric circulation processes in the equatorial Pacific Ocean. These interactions result in fluctuations in both sea surface temperatures (SSTs) and sea-level pressures across the tropical Pacific Ocean (Kiem and Franks, 2001), whose effects can propagate far beyond the tropical Pacific Ocean. The ENSO phenomena consist of warm events (El Niño), cold events (La Niña), as well as the normal or neutral conditions. Many classification schemes have been developed to identify El Niño and La Niña events — a summary of various classification schemes is provided by Liu et al. (1998) — with many being based on climate indices, such as the Southern Oscillation index (Troup, 1965), the NINO3 index (Kaplan et al., 1998), or the multivariate ENSO index (MEI; Wolter and Timlin, 1998). The typical effects of ENSO within eastern Australia are drier conditions during El Niño and wetter conditions during La Niña, with a typical ENSO cycle usually initiating in April–May and concluding in March–April of the following year (for example, see Verdon et al., 2004).

Power et al. (1999) showed that the association between ENSO and Australian climate is modulated by the Interdecadal Pacific Oscillation (IPO), a climate index which uses a low-pass filter to highlight multidecadal sea surface temperature (SST) anomalies. A strong association was found between the magnitude of ENSO impacts during negative IPO phases, whilst positive IPO phases showed a weaker relationship. Kiem et al. (2003) analysed NSW flood data, using a regional flood index, stratifying the data based on IPO value. They found that the IPO modulated both the magnitude and frequency of ENSO events (El Niño and La Niña) resulting in multidecadal periods of elevated and reduced flood risk. In particular, they showed that La Niña events were the primary drivers of flood risk and that this was further enhanced under negative IPO phases. Kiem and Franks (2004) then analysed the performance of Grahamstown Reservoir in NSW and found that the drought risk, in terms of the probability of falling below a critical reservoir storage level, was significantly increased during the positive IPO phase for the various reservoir management and operational strategies investigated. Furthermore, Verdon et al. (2004)
examined the influence of ENSO and IPO on mean rainfall and streamflow in eastern Australia, using seasonal (September to January) totals. Both rainfall and streamflow were found to be significantly enhanced during the La Niña phase of ENSO. On multidecadal scales, the negative IPO phase was more associated with “wetter” conditions than the positive phase. Importantly, the magnitude of La Niña events was found to be further enhanced during the negative phase of the IPO.

Similar multidecadal effects have been displayed elsewhere around the world. For example, ENSO correlations to rainfall have been shown to vary on multidecadal time scales in America (Gershunov and Barnett, 1998; McCabe and Dettinger, 1999) and in the southwest Pacific (Salinger et al., 2001). McKerchar and Henderson (2003) showed that a decrease in flood size that occurred in parts of New Zealand was consistent with a shift in the phase of the IPO. Monthly and seasonal discharges in southeast Alaska were found (Neal et al., 2002) to vary on multidecadal scales, while the average annual discharge did not (possibly due to changes in evapotranspiration). Indeed, multidecadal variations have been noted in processes as varied as salmon production in the North Pacific Ocean (Mantua et al., 1997) to bushfire risk in eastern Australia (Verdon et al., 2004).

Proxy information, in the form of palaeoclimatic records, have been recently used to extend the IPO\(^1\) record by several centuries. Verdon and Franks (2006) used these proxy IPO records, along with reconstructed NINO3 (i.e. ENSO) records, to investigate the long-term behaviour of the IPO and ENSO. They found that over the past 400 years, climate shifts associated with changes in the IPO have occurred with a similar frequency to those observed in the 20th century and that changes in the IPO have a propensity to coincide with changes in the relative frequency of ENSO events — the positive phase of IPO is associated with more El Niño events, while the negative phase is more favourable for La Niña events. These results have obvious consequences for Australian flood risk.

This study will further examine the applicability of the iid assumption in FFA by investigating the effect and extent of multidecadal variability — that is, the IPO modulation of flood risk — for eastern Australian flood data. Specifically, the evidence of IPO dependence at individual, gauged sites will be investigated as well as the spatial extent of this IPO modulation being explored.

### 3.3 Interdecadal Pacific Oscillation (IPO)

The IPO is the coherent pattern of SST variability occurring on interdecadal time scales over the (entire) Pacific Ocean (Folland et al., 1999; Power et al., 1999; Allan, 2000). It is characterised by the third empirical orthogonal function of 13-year low-pass filtered global SSTs, projected onto annual data (Folland et al., 2002). The spatial structure of the Pacific Ocean SST patterns is similar to that associated with ENSO (Zhang et al., 1997).

The North Pacific Ocean manifestation of the IPO is similar to that of the Pacific Decadal Oscillation (PDO; Mantua et al., 1997; Zhang et al., 1997; Mantua and Hare, \(^1\)Or highly similar SST-based climate indices.
2002), which is defined as the leading principal component of North Pacific Ocean SST anomalies, poleward of \(20^\circ\)N — with both the IPO and PDO having very similar time series. In fact, both indices display marked multidecadal variability associated with the warming and cooling epochs exhibited in both Pacific and global temperature records (Franks, 2002a, 2004). This study prefers the use of the IPO over the PDO because the IPO is defined for the whole of the Pacific Basin (Folland et al., 2002).

The IPO annual time series is presented in Figure 3.1 and represents an average of four seasonal values — the IPO data were provided directly by the United Kingdom Met Office (Andrew Colman, 2004, personal communication). Note that the time series reveals extended epochs above and below the long-term average, with a few instances of abrupt sign changes (Mantua and Hare, 2002). The sign changes occurring around 1925, 1947, and 1977 have been labelled by several studies as “regime shifts” (Zhang et al., 1997; Mantua et al., 1997; Minobe, 1997). Also, there is some slight ambiguity about the final few years of the IPO data set because these values must be smoothed over a reduced number of years, so these values may change slightly as the full 13 years of data becomes available in the future.

3.4 Data and methodology

Annual maximum flood data from the eastern Australian states of NSW and Queensland (Qld) were used in the study. A total of 94 sites were analysed, with Qld having 52 sites and NSW 42 sites, with the locations of the sites used being shown by the closed dots in Figure 2.1. Note that all 127 possible sites were not used because of minimum data requirements.

The flood frequency analysis was performed using the FLIKE software (Kuczera, 1999).
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FLIKE performs a Bayesian flood frequency analysis and provides accurate estimates of expected probability design floods and confidence limits. It supports many commonly used flood distributions — lognormal, log-Pearson 3, Gumbel, GEV, and generalised Pareto.

It is noted that the NSW flood data consists of a mixture of daily-read and true peak discharges, with the daily-read discharges mainly occurring before the 1970s. A daily-read discharge is a discharge associated with a specific reading time — for example, river levels, and thus discharges, were manually recorded at 9 AM — resulting in an underestimate of the true peak discharge because it is most unlikely that the true peak occurred exactly at 9 AM. The true peak discharges were derived either from multiple observations during flood events or from continuous (fixed interval) readings. In contrast, the Qld data consists almost entirely of true peak discharges and, so, contains few daily-read discharges. Chapter 2 showed that the magnitude of the errors associated with daily-read flows can be quite large (up to a factor of ten); however, under normal circumstances, their impacts on flood quantile bias and root mean squared error were shown to be minimal. Thus, the daily-read discharges were treated as true peak discharges for this study.

The peak flows were stratified according to IPO value. The threshold used was an annual average IPO value of zero. Years that have an IPO > 0 belong to the IPO-positive (IPO+) epoch. Likewise, years with IPO < 0 belong to the IPO-negative (IPO-) epoch. Note that the ambiguity in IPO values would only result in the last few years possibly being misclassified into the wrong IPO epoch. This is considered to have minimal effect on the results. Figure 3.2 presents a histogram of record length for each IPO epoch, while Table 3.1 presents statistics on the record lengths. The records lengths up to 25 years are dominated by Qld data, indicating that the Qld record length is, on average, shorter than that of NSW. Also, the IPO+ epoch has a longer record length than the IPO- epoch. Finally, note that a minimum record length of 10 years has been used in the analyses.

Figure 3.2: Histograms of record length for both IPO epochs.
Table 3.1: Summary of record lengths.

<table>
<thead>
<tr>
<th>Epoch Number of flows (years)</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPO-</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td>IPO+</td>
<td>30</td>
<td>11</td>
</tr>
</tbody>
</table>

Lognormal probability plot: 136206A (IPO > 0)

Figure 3.3: A typical good lognormal fit to the IPO-stratified data. Mean quantile and 90% probability limits are also shown.

3.5 Results

The data sets of peak flows, stratified according to IPO value, were fitted using the lognormal distribution. The majority of analyses demonstrated a good lognormal fit to the data — about 10% to 20% of analyses showed a poor fit with one or more observations lying outside the 90% limits and/or a flattening in the right hand tail of the distribution. Examples of good and poor fits are presented in Figures 3.3 and 3.4 respectively. Overall, the lognormal distribution was deemed adequate. Note that when flood data are not stratified, the typical distribution used in Australia is the (three-parameter) log-Pearson 3 (Pilgrim and Doran, 1987, p. 206).

A more rigorous test for the lognormality of the IPO-stratified flood data was also performed using the Kolmogorov–Smirnov (K-S) test, which compares the maximum deviation between the observed empirical cumulative distribution functions and the assumed (lognormal) cumulative distribution function to determine whether the assumed distribution is appropriate (e.g. Ang and Tang, 2007, pp. 293–6). Figure 3.5 provides a histogram of the resultant $p$ values obtained from the K-S tests, which were performed using the R software package (R Development Core Team, 2005). The histogram shows that only 3 data sets (out of 188) have $p$ values < 0.10 (i.e. only 2% of the data sets reject the null hypothesis — that the data are lognormally distributed — at the 10% level). Thus, we can conclude that the lognormal distribution assumption is not inconsistent with the
CHAPTER 3. MULTIDECADAL VARIABILITY IN FLOOD RISK

Figure 3.4: A poor lognormal fit to the IPO-stratified data, with flattening in the right-hand tail.

Figure 3.5: Histogram of the $p$ values for the Kolmogorov–Smirnov test for an assumed lognormal distribution.

IPO-stratified flood data, so the lognormal distribution is acceptable.

The ratio of the fitted flood quantiles during IPO- periods to the fitted flood quantiles during IPO+ periods ("flood ratio") was calculated for several average recurrence intervals (ARIs), where the ARI is considered the reciprocal of the annual exceedance probability — the flood ratio, for an ARI of $Y$ years, is defined:

$$Y\text{-year flood ratio} = \frac{Y\text{-year IPO- flood discharge}}{Y\text{-year IPO+ flood discharge}} \quad (3.1)$$
Table 3.2: Summary of flood ratios.

<table>
<thead>
<tr>
<th>ARI</th>
<th>Flood ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>2</td>
<td>1.49</td>
</tr>
<tr>
<td>5</td>
<td>1.46</td>
</tr>
<tr>
<td>10</td>
<td>1.47</td>
</tr>
<tr>
<td>20</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Figure 3.6: Histograms of flood ratios for several ARIs.

The flood ratios are summarised in Table 3.2 and histograms of the flood ratios are presented in Figure 3.6. Figure 3.6 shows that the flood ratio is below one for between 11% and 27% of sites, depending upon the ARI selected; that is, between 73% and 89% of sites have a flood ratio > 1. Also, approximately 40% of sites have a flood ratio > 1.5. This indicates a significant difference between the IPO+ and IPO- epochs because these differences would not likely come about by chance alone. Indeed, Monte Carlo studies have shown that if the two IPO epochs were identically distributed, then about 50% of sites would have a ratio < 1. Moreover, the probability of a ratio > 1.5 at any site is approximately 0.1 (Franks and Kuczera, 2002, Figure 7). Using the binomial distribution, the probability of 38 or more sites out of 94 having a ratio > 1.5 is well below the 0.1% level (about 10^{-14}), under the iid assumption. This result assumes that the sites are not spatially correlated, which is not the case. Nonetheless, it still provides strong evidence that the flood distributions are not identically distributed between the two IPO epochs, especially after consideration of Table 3.2 which shows that the mean ratio is about 1.5.

The analysis was repeated with the minimum record lengths ranging from 15 to 25 years, with the results summarised in Table 3.3. Similar results to Table 3.2 and Figure 3.6 were found, with the varied minimum record lengths yielding binomial probabilities of
Table 3.3: Summary of results for varied minimum record lengths (averaged over ARIs 2, 5, 10, and 20 years).

<table>
<thead>
<tr>
<th>Min. record length</th>
<th>No. of sites</th>
<th>Proportion of sites ratio &lt; 1</th>
<th>No. of sites with ratio &gt; 1.5</th>
<th>Proportion of sites ratio &gt; 1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>64</td>
<td>0.15</td>
<td>0.4</td>
<td>26</td>
</tr>
<tr>
<td>20</td>
<td>39</td>
<td>0.10</td>
<td>0.5</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>31</td>
<td>0.05</td>
<td>0.5</td>
<td>16</td>
</tr>
</tbody>
</table>

Figure 3.7: Relationship between 10-year flood ratio and catchment area.

about $10^{-10}$ for encountering the number of observed sites having a ratio > 1.5. This suggests it is highly unlikely that sampling error is responsible for the observed IPO effect.

The graph of the 10-year flood ratio against catchment area is presented in Figure 3.7. There appears to be no relationship present, with quite a large amount of scatter being displayed. This absence of scale dependence suggests that the IPO dependence of flood risk is a regional phenomenon, independent of catchment area.

In contrast, a strong relationship is uncovered upon examination of the spatial distribution of the flood ratios. Figure 3.8 presents a plot of the 10-year flood ratio as a function of latitude, while Figure 3.9 shows the spatial distribution of the flood ratios. These figures show that the flood ratio increases when moving southward through Queensland, with the largest flood ratios occurring in southeast Queensland and northeastern New South Wales. The flood ratio is relatively constant throughout New South Wales. An interesting feature is the apparent clustering of ratios in Figure 3.8. The majority of the flood ratios < 1 occur north of latitude 22° south, while all but two flood ratios > 1.5 occur south of this latitude. Additionally, as shown in Figure 3.8, the mean flood ratios for sites north and south of this latitude are 1.0 and 1.7 respectively. Coincidently, this latitude is similar
to that of the Tropic of Capricorn (23.5° south).

It was noted in Chapter 2 that the NSW data consists of 15 sites which are influenced by regulation — the flood ratio statistics using both regulated and unregulated sites and the unregulated sites only are compared (in Table 3.4) to determine the effect of this regulation on the analysis. The comparison is only made for sites south of -22° latitude since all of the regulated sites are located south of this latitude. The various statistics for the flood ratio $R$ are very similar between the two sets of analyses, with the regulated analysis yielding slightly smaller flood ratios, possibly resulting from the attenuation of flood peaks due to the introduction of the reservoirs. Hence, it is appropriate to retain the regulated sites, which provide additional coverage within NSW.

### 3.6 Discussion

The mechanisms responsible for the association between the IPO and eastern Australian rainfall and climate are poorly understood. Arblaster et al. (2002) have offered some physical reasons for the influence of the IPO on Australian rainfall, based on the results of a single climate model. The interdecadal SST variations may affect convection in the equa-
Figure 3.9: Spatial distribution of the 10-year flood ratio.

Table 3.4: Comparison of the statistics of ratio $R$ using unregulated sites only and using both unregulated and regulated sites (south of -22° latitude).

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Unregulated only</th>
<th>Unregulated and regulated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARI=5</td>
<td>ARI=10</td>
</tr>
<tr>
<td>Mean</td>
<td>1.68</td>
<td>1.72</td>
</tr>
<tr>
<td>SD</td>
<td>0.41</td>
<td>0.51</td>
</tr>
<tr>
<td>Min</td>
<td>0.88</td>
<td>0.85</td>
</tr>
<tr>
<td>Quartile 1</td>
<td>1.41</td>
<td>1.34</td>
</tr>
<tr>
<td>Median</td>
<td>1.63</td>
<td>1.66</td>
</tr>
<tr>
<td>Quartile 3</td>
<td>1.96</td>
<td>2.11</td>
</tr>
<tr>
<td>Max</td>
<td>2.77</td>
<td>3.01</td>
</tr>
</tbody>
</table>
CHAPTER 3. MULTIDECADAL VARIABILITY IN FLOOD RISK

Tourial Pacific Ocean leading to shifts in the rising branch of the Walker circulation, which may then influence ENSO’s effect on Australian rainfall. Also, there may be increased (atmospheric) subsidence over Australia, i.e. lower rainfall, resulting from an anomalous regional Hadley circulation that may form above the warmer, western-Pacific SSTs.

An interpretation of the results may also be offered in terms of the movement of the regional convergence zones. Salinger et al. (1995) demonstrated that changes in the location of the South Pacific Convergence Zone (SPCZ) are responsible for relatively large changes in rainfall in the Southwest Pacific, about its mean position. The SPCZ is disrupted by El Niño (warm) events that prevent it from reaching its normal southerly position. La Niña (cool) events enhance the SPCZ, allowing it to move further south than normal (Salinger et al., 1995), delivering more rain-bearing cloud bands across eastern Australia. The IPO has been shown (Folland et al., 2002) to be associated with the location of the SPCZ in a similar manner to ENSO, except on multidecadal time scales. Whilst the SPCZ does not directly affect rainfalls in NSW, the SPCZ is related to the Intertropical Convergence Zone (ITCZ), which influences eastern Australian rainfalls. It is therefore expected that the IPO will have a marked effect on the location of the ITCZ, similar to that on the SPCZ, and thus explaining how New South Wales and the southern parts of Queensland generally experience greater rainfall (and discharge) during IPO- periods. Note, however, that the northern part of Queensland is not affected by this multidecadal variability because, despite changes in the location of the ITCZ, it nonetheless remains under the ITCZ’s influence.

The results show that the flood risk can be increased quite significantly during IPO-periods, except for sites in northern Queensland. Figure 3.10 shows two flood series stratified into IPO+ and IPO- epochs, along with the average annual peak discharge during each IPO epoch. The top panel shows a site in northern Queensland with a flood ratio of about 1.1. The IPO epochs have mean flows of 10.9, 11.7, and 9.3 GL/day, which indicates a fairly homogeneous time series. In contrast, the bottom panel presents a site in northeastern NSW that has a flood ratio of about 1.6. The IPO epochs have mean flows of 73.0, 143.7, and 80.2 GL/day, which illustrates a strong influence on flood risk related to the IPO phase.

A similar pattern of modulation of flood risk would be expected to continue further south into the state of Victoria. However, the effect of this modulation (i.e. the flood ratio) would be smaller than that observed in NSW due to the reduced ENSO impacts within Victoria. In general, the IPO modulation of flood risk should be expected in ENSO-affected areas because the IPO modulates both the magnitude and the frequency of ENSO events (Kiem et al., 2003; Kiem and Franks, 2004).

It is important to recognise the practical significance of IPO modulation of flood risk. The use of at-site flood data with an inadequate coverage of both IPO epochs may result in biased estimates of long-run flood risk. For example, Table 3.5 shows that using flood data from an IPO+ period would likely lead to a large underestimate of the long-run (marginal) flood risk for the bottom site in Figure 3.10; whereas a relatively unbiased estimate should be obtainable for the top site in Figure 3.10 using just IPO+ data. In
Figure 3.10: Flood series showing IPO stratifications and average discharges. Top panel is site 110003A (flood ratio \(\approx 1.1\)) and bottom panel is site 204001 (flood ratio \(\approx 1.6\)).

cases with inadequate coverage of one of the IPO epochs, the prospect of significant bias in long-run flood risk is high.

### 3.7 Conclusion

It has been shown that the IPO modulates the flood risk within parts of eastern Australia. Sites in eastern New South Wales and southeastern Queensland have flood quantiles about 1.7 times greater during IPO-negative periods than during IPO-positive periods (for ARIs ranging from 2 to 20 years). Northeast Queensland sites show little IPO modulation of flood risk. Decadal-scale movements of regional convergence zones may explain this IPO modulation of flood risk.

Although this study is somewhat limited by the geographic coverage of the data, its findings are, nonetheless, of considerable practical importance because the study area encompasses a significant proportion of the Australian population and associated infras-
Table 3.5: Discharges corresponding to various ARIs for both IPO epochs.

<table>
<thead>
<tr>
<th>Site</th>
<th>ARI (years)</th>
<th>Discharge (GL/day)</th>
<th>IPO-</th>
<th>IPO+</th>
</tr>
</thead>
<tbody>
<tr>
<td>110003A</td>
<td>2</td>
<td>6.4</td>
<td>6.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>16</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>26</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>40</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>204001</td>
<td>2</td>
<td>77</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>176</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>271</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>387</td>
<td>234</td>
<td></td>
</tr>
</tbody>
</table>

structure. Firstly, when considering flood risk, a decision must be made as to the time frame required — do we want to estimate the short-term or long-run flood risk? Interest will lie in the short-term risk if the desired time horizon is short. However, most infrastructure will require the long-run risk because their design lives will span multiple IPO epochs. If long-run flood risk is estimated using at-site data with poor coverage of one of the IPO epochs, then it is likely that long-run flood risk will be significantly biased. Therefore, it may be necessary to use a regional flood frequency distribution that has been obtained from a data set containing sufficient samples from both IPO-positive and -negative periods to augment the limited at-site data. This provides the motivation for the development of a regional flood model described in the following chapters.
Chapter 4

A Bayesian hierarchical regional flood frequency model

4.1 Overview

The previous chapter has demonstrated that eastern Australian flood time series are affected by multidecadal variability, indicating that the annual maximum flood peaks are not homogeneous — not identically distributed — with significantly increased at-site flood risk being experienced at many sites during the IPO-negative epochs as compared with IPO-positive epochs. Often in Australia, the flood record is relatively limited and may not adequately sample each IPO epoch. This is likely to produce significant bias in long-run flood risk estimates derived from these data.

This consideration motivates the development of a regional flood frequency distribution. A regional distribution, derived from a data set with sufficient samples from both IPO-positive and -negative epochs, may then be used to augment the limited at-site data and largely avoid the prospect of significant bias in long-run flood risk. For conceptual soundness, the proposed regional flood model should have various desirable properties. The IPO modulation of flood risk that is present within eastern Australia should be easily incorporated within the model. Flood frequency data sets often suffer from missing data and from unequal record lengths, and these data issues should be easily handled. Also, flood frequency data is often spatially correlated — that is, there is intersite correlation between concurrent flows — which has the effect of reducing the information content of the data. Typically, this correlation is ignored and the sites are assumed independent; however, this correlation should be incorporated into the model to allow for the investigation of its effects. Finally, the overall aim of a regional flood model is to provide a prediction of the flood distribution at a site, whether that site be gauged or ungauged, and the precision of these estimates should be quantified robustly (i.e. to provide estimates of the predictive uncertainty of the regional flood model).

These considerations lead to the following two objectives for the proposed regional flood model:

1. Develop a rigorous Bayesian regional flood model framework, exploiting recent de-
velopments in Bayesian inference, to fulfil the desirable requirements of a regional flood model. This is the main objective of this chapter.

2. Investigate if the regional flood model can avoid bias if the sample (flood data) is unrepresentative, in catchments affected by IPO modulation of flood risk. This is the main objective of the next chapter.

4.2 Background

Flood regionalisation aims to provide improved estimates of flood risk at either ungauged or poorly-gauged sites by utilising the information contained within the flood frequency records of neighbouring sites and transferring this information to the site in question. Flood regionalisation consists of two parts: delineation of regions and development of a regional flood distribution (to allow estimation of flood risk at the chosen site); although these two parts are often inter-related. Cunnane (1988) and GREHYS (1996a,b) provide reviews of the methods of flood regionalisation, while Smakhtin (2001) provides an extensive review focused in the area of low-flow hydrology. The following sections provide a summary of the major methods used in flood regionalisation.

4.2.1 Index flood regionalisation methods

An approach that has been in long use is the index-flood method (Dalrymple, 1960). The basic assumption of the index-flood method is that a group of $N$ sites form a homogeneous region and that the frequency distributions of these $N$ sites are identical, except for a site-specific scaling factor — the index flood — which reflects the size, rainfall, and runoff characteristics of each site (Hosking and Wallis, 1997; Stedinger et al., 1993). This can be written

$$Q_i(F) = \mu_i q(F)$$

(4.1)

where $Q_i(F)$ is the estimated flood at site $i$ for the quantile $F$ (e.g. $Q_i(0.99)$ denotes the 100-year flood), $\mu_i$ is the index flood (often the mean of the at-site flood frequency distribution is used), and $q(F)$ is the regional frequency curve of the dimensionless ratio $Q_i(F)/\mu_i$ that is common to all sites — these are also referred to as “growth curves” (National Environmental Resources Council, 1975). Another major assumption of the index-flood method is that the sites within the homogeneous region are independent (Hosking and Wallis, 1997, p. 8); that is, the effects of intersite correlation are ignored in the calculation of the growth curves.

The index-flood method has received much attention recently through the methods based on the use of probability-weighted moments (PWMs; Greenwood et al., 1979)/L moments (e.g. Wallis, 1981; Hosking et al., 1985a,b; Hosking and Wallis, 1997) — this index-flood approach is described here. L moments of a random variable were first introduced by Hosking (1986) and are similar to conventional moments, but are based based on linear combinations of order statistics (ranked observations). L-moment-based estimators are more robust than conventional moment-based estimators because they do not involve
the squaring or cubing of the observations, which can have major impacts with “outliers” or low values of log-transformed data (Stedinger et al., 1993; Hosking and Wallis, 1997). L moments can be expressed using PWM estimators, which can be defined as (Hosking, 1990):

$$\beta_r = E(X[F[X]]^r)$$ (4.2)

where $\beta_r$ is the $r$th order PWM for the random variable $X$ with cumulative distribution function $F(X)$. PWMs have been used as the basis of methods for estimating the parameters of probability distributions (e.g. Landwehr et al., 1979a,b; Hosking et al., 1985b; Hosking and Wallis, 1987); however, they are difficult to interpret directly as measures of the scale and shape of a probability distribution (Hosking and Wallis, 1997, p. 19). This information is contained in certain linear combinations of PWMs, with Hosking (1986, 1990) defining the first four L moments

$$\lambda_1 = \beta_0$$ (4.3)
$$\lambda_2 = 2\beta_1 - \beta_2$$ (4.4)
$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$$ (4.5)
$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0$$ (4.6)

where the first L moment $\lambda_1$ is the arithmetic mean and $\lambda_2$ is a measure of dispersion, analogous to the standard deviation. Dimensionless versions of L moments are often more convenient, with the L-moment ratios defined

$$\tau_r = \lambda_r / \lambda_2 \text{ for } r = 3, 4$$ (4.7)

where these L-moment ratios measure the shape of a distribution independently of its scale of measurement. $\tau_3$ is the L skewness and reflects the degree of symmetry, while $\tau_4$ is the L kurtosis and measures the peakedness of the distribution. The L CV is defined

$$\tau = \lambda_2 / \lambda_1$$ (4.8)

and is analogous to the ordinary coefficient of variation.

L moments are defined for a probability distribution, but must, in practice, be estimated from finite samples using order statistics. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the $n$ ordered samples. The following provides unbiased estimators of $\beta_r$ (Landwehr et al., 1979b)

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)\ldots(j-r)}{(n-1)(n-2)\ldots(n-r)} X_{(j)}$$ (4.9)
Thus, the sample L moments are defined

\[ \ell_1 = b_0 \] (4.10)
\[ \ell_2 = 2b_1 - b_2 \] (4.11)
\[ \ell_3 = 6b_2 - 6b_1 + b_0 \] (4.12)
\[ \ell_4 = 20b_3 - 30b_2 + 12b_1 - b_0 \] (4.13)

with the sample estimates for the L-moment ratios being

\[ t_r = \frac{\ell_r}{\ell_2} \quad \text{for } r = 3, 4 \] (4.14)

and

\[ t = \frac{\ell_2}{\ell_1} \] (4.15)

For the index-flood procedure, assume that the proposed region consists of \( N \) sites, with site \( i \) having record length \( n_i \) and sample L-moments ratios \( t^{(i)}_r, \tau^{(i)}_3, \tau^{(i)}_4 \) obtained using the unbiased PWM estimators \( b_r \). The regional-average L-moment ratios \( \ell^{(R)}_1, \tau^{(R)}_3, \tau^{(R)}_4 \) are then estimated by weighting the contribution of each site by its record length, that is

\[ t^{(R)}_r = \frac{\sum_{i=1}^{N} n_it^{(i)}_r}{\sum_{i=1}^{N} n_i} \] (4.16)

and

\[ t^{(R)}_r = \frac{\sum_{i=1}^{N} n_it^{(i)}_r}{\sum_{i=1}^{N} n_i} \] (4.17)

The regional-average mean is set to 1 (i.e. \( \ell^{(R)}_1 = 1 \)). Note that the regional averages may be calculated using L moments, instead of the L-moment ratios, with such a procedure described by Stedinger et al. (1993) and discussed in Hosking and Wallis (1997).

The regional distribution is then fitted by equating its L-moment ratios \( \lambda_1, \tau, \tau_3, \tau_4 \) to the regional-average L-moment ratios \( \ell^{(R)}_1, \tau^{(R)}_3, \tau^{(R)}_4 \) — see Hosking and Wallis (1997) or Stedinger et al. (1993) for expressions relating L moments to the parameters of various distributions. L-moment-ratio diagrams (L kurtosis/L skewness) may be useful in choosing the most appropriate distribution for the region, with the GEV often being used (Hosking et al., 1985a; Lettenmaier et al., 1987; Hosking and Wallis, 1988; Stedinger and Lu, 1995).

For a nonexceedance probability \( F \), the flood quantile is then estimated by

\[ \hat{Q}_i(F) = \hat{\mu}_i \hat{q}(F) \] (4.18)

where \( \hat{q}(\cdot) \) is the quantile function of the fitted regional distribution. If the site \( i \) is gauged, then the first sample L moment \( \ell^{(i)}_1 \) may be used for the index flood \( \hat{\mu}_i \). However, for ungauged sites, the index flood must be estimated using other means, such as regression (e.g. National Environmental Resources Council, 1975) or rainfall-runoff modelling.

The hierarchical regions approach of Gabriele and Arnell (1991) is similar in principle to the L-moment approach above, using weighted averages to obtain regional distribution.
parameters. Their method recognises that more sites are required to improve the precision of the higher-order regional moments as compared to the lower-order moments. They propose using a large region when estimating shape (skewness) parameters, with smaller subregions being used to estimate the dispersion (coefficient of variation) parameters. Thus, the method requires the specification of two sets of regions; however, the use of L-moment-based approaches helps to overcome some of the difficulties associated with the higher-order moments, so this approach loses much of its intuitive appeal.

L-moment-based methods may also be used to delineate (homogeneous) regions for the index-flood method, with Hosking and Wallis (1997) proposing such a regional heterogeneity/homogeneity measure for this purpose. L moments (or L-moment ratios) are used to test if the between-site variations observed at different sites can be expected to arise for a homogeneous region — the heterogeneity measure is used to test whether certain distribution parameters related to the kurtosis, skewness, and coefficient of variation are constant within the region. The procedure involves calculating the sample L moments at each site and then weighting these sample values to obtain regional-average L-moment values. A kappa distribution is fitted to the regional values and then a large number of simulations of an identical region are made using the regional kappa distribution. A heterogeneity measure, based on the observed and simulated regional values of L-moment statistics, is used to determine whether the region is “acceptably homogeneous”, “possibly homogeneous”, or “definitely heterogeneous”. The initial regional grouping of sites may be obtained through a multivariate technique such as cluster analysis (see Section 4.2.4), or through inspection of L-moment-ratio diagrams, with sites being added or removed based on the results of the homogeneity test, leading to the possibility of spatially-discontinuous regions. Note, however, that the homogeneity test does not account for intersite dependence and may lead to erroneous conclusions of homogeneity in cases with significant intersite correlation (Madsen and Rosbjerg, 1997).

A further development on the use of spatially discontinuous regions is the notion of “one-site regions” or “regionalisation without boundaries”, where each site has its own “region” and there are no fixed region boundaries. Acreman and Wiltshire (1989) were probably the first to formalise the idea, with their argument being that the homogeneity of the regions may be improved by increasing the number of regions, thus decreasing the membership of each region — in the limiting case, each site is its own region. Burn (1990) then developed this into the “region of influence” (ROI) approach — each site is considered to be at the centre of a collection of further sites, which comprise its region of influence. ROI uses a weighted Euclidean distance as a metric for the closeness between sites

\[
d_{jk} = \left[ \sum_{i=1}^{M} w_i (x_{ji} - x_{ki}) \right]^{1/2}
\]

(4.19)

where \(d_{jk}\) is the weighted Euclidean distance between sites \(j\) and \(k\), \(M\) is the number of catchment descriptors included in the distance measure, \(w_i\) is the weight associated with descriptor \(i\), and \(x_{ji}\) is the (standardised) value of the descriptor \(i\) for site \(j\), with the weights reflecting the relative importance of the descriptor for defining hydrologic
similarity. Cunderlik and Burn (2006) proposed replacing this weighted Euclidean-distance measure with a Mahalanobis (1936)-distance-based measure to account for the sampling variability and covariation of the descriptors:

\[ d_{jk} = \left[ (x_j - x_k)^T \Sigma^{-1} (x_j - x_k) \right]^{1/2} \]  

(4.20)

where \( x_j \) is the vector of (standardised) descriptors for site \( j \), \( \Sigma \) is the covariance matrix between \( x_j \) and \( x_k \), and \( x^T \) denotes the transpose of \( x \). Note that this Mahalanobis-distance-based approach was previously used by Latraverse et al. (2002). These distances are then used to construct a set of weights \( \eta_{jk} \) reflecting the weight given to site \( k \) in the ROI for site \( j \), with \( \eta_{jk} > 0 \) indicating that site \( k \) is in site \( j \)'s ROI. Zrinji and Burn (1994) replaced this somewhat subjective distance-based weight \( \eta_{jk} \) with a statistical test where sites are successively added to the ROI, starting with the most similar, until the additional site causes the ROI to be no longer homogeneous (according to their statistical test). The ROI approach is quite commonly used in association with the index-flood method, but may also be used with other regionalisation methods (e.g. Tasker et al., 1996).

### 4.2.2 Map-based regionalisation methods

Map-based methods are in use in both the United States and Australia. In the United States, *Bulletins 17A and 17B* (Interagency Advisory Committee on Water Data, 1977, 1982) use a map-based approach for skewness estimation. This method assumes that flood data \( Q \) have a log-Pearson 3 distribution (i.e. \( Z = \log_{10} Q \) has a Pearson 3 distribution) and that the first three sample moments of \( Z \) (i.e. \( \mu, \sigma^2, g \)) are estimated from the at-site data, using the method of moments. The site skewness \( g \) is plotted on a map and contours of skewness drawn, taking into account geographical features — ideally, this regional-map procedure should use at least 40 sites having record lengths of 25 years or more. The “refined” skewness \( g_w \) is estimated through a weighted average of the sample skewness \( g_s \) and the regional (map based, in this case) skewness \( g_r \). *Bulletin 17A* (Interagency Advisory Committee on Water Data, 1977) has the weights based on the at-site record lengths \( N \):

\[
g_w = \begin{cases} 
  g_r & N < 25 \\
  g_r + (g_s - g_r)(N - 25)/75 & 25 \leq N \leq 100 \\
  g_s & N > 100 
\end{cases} 
\]  

(4.21)

The weighting procedure was later adjusted in *Bulletin 17B* (Interagency Advisory Committee on Water Data, 1982) with the weights being dependent on the estimated sampling variance of the at-site and regional estimates of skewness:

\[
g_w = \frac{g_s/\text{MSE}(g_s) + g_r/\text{MSE}(g_r)}{1/\text{MSE}(g_s) + 1/\text{MSE}(g_r)} 
\]  

(4.22)

where \( \text{MSE}(\cdot) \) denotes the mean squared error.

The probabilistic rational method (McDermott and Pilgrim, 1982; Pilgrim, 1987; Pilgrim and Cordery, 1993) is used in (southeastern) Australia for rural catchments with
areas up to 250 km$^2$. The traditional rational formula (e.g. Pilgrim and Cordery, 1993, Equation 9.4.1)

$$Q = F CIA$$

(4.23)

where $Q$ is the peak discharge, $F$ is a units conversion factor, $C$ is a dimensionless runoff coefficient, $I$ is the rainfall intensity, and $A$ is the catchment area, can be written in a nondimensional form

$$C(Y) = \frac{Q(Y)}{I(t_c,Y)FA}$$

(4.24)

where the indices $Y$ and $t_c$ denote an average recurrence interval (ARI) of $Y$ years and the time of concentration respectively. This form provides the probabilistic interpretation of the rational method, with the (dimensionless) runoff coefficient representing the ratio of peak flow and rainfall rate (of selected duration) for the same ARI, which are derived from (probabilistic) frequency analyses of flood peaks and rainfall (see Pilgrim, 1987; Canterford et al., 1987, respectively). Runoff coefficients are calculated at each site in the region and plotted on a map and contours drawn. Typically, the map is created for a single ARI, with $C(10)$ being used in Australia (e.g. Pilgrim and Cordery, 1993, Figure 9.4.3), and runoff coefficients for other ARIs are estimated using

$$FF(Y) = \frac{C(Y)}{C(10)}$$

(4.25)

where $FF(Y)$ is a flood frequency conversion factor, which may be based on a regional-average value or on a regression equation. Flood quantiles are then estimated by

$$Q(Y) = FC(Y)I(t_c,Y)A$$

(4.26)

with *Australian Rainfall and Runoff* (Canterford, 1987) providing maps used in the estimation of the $C(Y)$ and $I(t_c,Y)$ terms.

### 4.2.3 Regression-based regionalisation methods

Another widely-used approach is based on regression methods, which use a set of catchment descriptor variables (e.g. area, slope) and then develop a relationship to map these descriptors into a set of hydrologic statistics (e.g. flood quantiles). Regression methods have been used to estimate hydrologic statistics at ungauged sites (e.g. Thomas and Benson, 1970; National Environmental Resources Council, 1975; Vogel and Kroll, 1990; Jennings et al., 1994; Meigh et al., 1997) or to augment limited at-site data with regional information to improve the precision of the estimated statistics (e.g. Interagency Advisory Committee on Water Data, 1982; Kuczera, 1982; Madsen and Rosbjerg, 1997).

The regression equation for site $i$ is typically assumed to be a linear function

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \varepsilon_i$$

(4.27)

where $y_i$ is the statistic of interest, \{x_{i1}, x_{i2}, \ldots\} are the catchment descriptor variables for site $i$, \{\beta_0, \beta_1, \beta_2, \ldots\} are the parameters of the model to be estimated, and $\varepsilon_i$ are the
regression model errors. This can be restated in vector form

\[ \mathbf{y} = \mathbf{X}\beta + \mathbf{\varepsilon} \quad (4.28) \]

The traditional approach used in hydrology is to estimate \( \beta \) using the ordinary least squares (OLS) method. OLS assumes that \( \mathbb{E}(\varepsilon) = \mathbf{0} \) and \( \text{Cov}(\varepsilon) = \sigma^2_\varepsilon \mathbf{I} \), where \( \mathbb{E}(\cdot) \) and \( \text{Cov}(\cdot) \) denote the expectation and covariance operators and \( \mathbf{I} \) is the identity matrix. This approach is appropriate when at-site flow estimates are equally reliable, the natural variability is the same at each site, and the observed concurrent flows at every pair of sites are independent. In practice, hydrologic data sets are not so homogeneous (Tasker and Stedinger, 1989).

A weighted least squares (WLS) approach (Tasker, 1980; Stedinger and Tasker, 1985) may be used to allow for variable precision at each site (e.g. from unequal record lengths), but still assuming independence between sites. Thus, \( \mathbb{E}(\varepsilon) = \mathbf{0} \) and \( \text{Cov}(\varepsilon) = \sigma^2_\varepsilon \mathbf{W} \), where \( \mathbf{W} \) is a diagonal matrix of weights (e.g. inversely proportional to record length at each site), with zeros on the off-diagonal terms.

The generalised least squares (GLS) method (Stedinger and Tasker, 1985, 1986a,b; Tasker and Stedinger, 1989; Reis et al., 2005) overcomes the conceptual difficulties of the OLS and WLS methods. It assumes that the total model error consists of two components: model errors \( \varepsilon_i \) that are assumed independently distributed with zero mean and covariance \( \sigma^2_\varepsilon \mathbf{I} \), and sampling errors that arise due to the fact that the actual values of \( y_i \) are unknown and estimates of the quantities are only available. The regression equation can be restated

\[ \mathbf{y} = \mathbf{X}\beta + \varepsilon + \omega = \mathbf{X}\beta + \mathbf{\eta} \quad (4.29) \]

where \( \omega \) is the sampling error in the sample estimators. The regression model errors \( \eta_i \) consist of both the time-sampling error in the sample estimates \( \hat{y}_i \) of \( y_i \) and the underlying model error \( \varepsilon_i \). The total error has the properties \( \mathbb{E}(\mathbf{\eta}) = \mathbf{0} \) and

\[ \text{Cov}(\mathbf{\eta}) = \mathbf{\Lambda}(\sigma^2_\varepsilon) = \sigma^2_\varepsilon \mathbf{I} + \mathbf{\Sigma} \quad (4.30) \]

where \( \mathbf{\Sigma} \) is the covariance matrix of the time-sampling errors. The time-sampling errors in \( \hat{y}_i \) are usually correlated due to the intersite correlation between concurrent flows that is typically present in most hydrologic data sets. Estimation of the sampling covariance matrix \( \mathbf{\Sigma} \) is an important task and should be independent of the estimators \( \hat{y}_i \) to ensure that an unbiased estimate of model parameters is obtained — guidance on the structure and estimation of \( \mathbf{\Sigma} \) is provided in Stedinger and Tasker (1985, 1986b), Tasker and Stedinger (1989), and Reis et al. (2005). Note that the Stedinger and Tasker (1985) GLS formulation is an operational implementation of the GLS earlier envisioned by Kuczera (1983b).

The GLS estimator of \( \beta \) is

\[ \hat{\beta} = [\mathbf{X}^T \mathbf{\Lambda}(\sigma^2_\varepsilon)^{-1} \mathbf{X}]^{-1} \mathbf{X}^T \mathbf{\Lambda}(\sigma^2_\varepsilon)^{-1} \mathbf{\hat{y}} \quad (4.31) \]
which has a sampling covariance matrix, for a given \( \sigma^2 \)

\[
\text{Cov} \left( \hat{\beta} \right) = [X^T \Lambda (\sigma^2)^{-1} X]^{-1}
\]  
(4.32)

The method of moments or the maximum likelihood method may then be used to estimate the model error variance \( \sigma^2 \).

The method-of-moments GLS estimator for \( \sigma^2 \) involves iteration to solve the generalised residual mean squared error equation

\[
(\hat{y} - X\hat{\beta})^T [\sigma^2 I + \Sigma]^{-1} (\hat{y} - X\hat{\beta}) = n - (k + 1)
\]  
(4.33)

Sometimes, all of the variability observed in the data is explained by the sampling covariance matrix, which leads to Equation (4.33) having a left-hand side less than \( n - (k + 1) \) for all \( \sigma^2 \), even when \( \sigma^2 = 0 \). Stedinger and Tasker (1985) suggest that \( \sigma^2 \) be set to zero in these cases.

The maximum likelihood estimator for \( \sigma^2 \) is obtained by maximising the log-likelihood function

\[
\log f \left( \hat{y} \mid \beta, \sigma^2 \right) = -\frac{1}{2} \log[\det(\sigma^2 I + \Sigma)] - \frac{1}{2} (\hat{y} - X\beta)^T (\sigma^2 I + \Sigma)^{-1} (\hat{y} - X\beta)
\]  
(4.34)

The log-likelihood function is derived assuming that the residuals are normally distributed with zero mean and covariance matrix as described in Equation (4.30), with \( \hat{\beta} \) and \( \sigma^2 \) being estimated jointly, subject to the constraint \( \sigma^2 \geq 0 \) (Reis et al., 2005).

A Bayesian treatment of the GLS model has recently been developed by Reis et al. (2005), using a “quasi-analytic” approach. The Bayesian approach allows for the full posterior distribution of the regression model parameters and model error variance. Importantly, it provides a sensible resolution in cases where \( \sigma^2 \) is estimated by Equation (4.33) to be zero.

### 4.2.4 Methods for the delineation of regions

A summary of some common methods for delineating regions is now provided. These delineation methods may be used with any of the flood regionalisation methods previously described.

A relatively simplistic method, which has been in relatively common use for many years, is geographic delineation of regions. Regions are selected to consist of sets of geographically-contiguous sites, often based on administrative boundaries or physiographic characteristics (e.g. National Environmental Resources Council, 1975; Slack et al., 1993). This method assumes that geographic proximity is a proxy for hydrologic similarity.

Over the last few decades, methods have been developed which measure similarity based on multivariate catchment-descriptor space (e.g. Bobée and Rasmussen, 1995; Bates et al., 1998), with these new methods often drawing from the field of multivariate analysis (e.g. Chatfield and Collins, 1980; Jobson, 1992). Note that the ROI method uses this multivariate approach, as does the L-moment-based homogeneity measure.
Cluster analysis attempts to allocate sites into a set of regions such that the region members are similar to each other and dissimilar to members of other regions. For example, cluster analysis has been used by Mosley (1981) to delimit hydrologically-similar regions in the South Island of New Zealand, while Nathan and McMahon (1990) used the method to identify homogeneous subregions in southeastern Australia and also provide a thorough discussion on the selection and weighting of descriptor variables. Also, Hughes (1987) used cluster analysis in combination with principal coordinates analysis (a generalisation of principal component analysis) to group hydrologic regions in Tasmania (Australia).

Canonical correlation analysis (CCA) has also been used as a method of delineating regions (Cavadias, 1990; Cavadias et al., 2001; Ouarda et al., 2000, 2001). CCA aims to identify the correlation structure between the sets of explanatory variables (catchment descriptors) and the response variables (flood quantiles). CCA identifies the dominant linear combinations of covariability between the two sets, attempting to reduce the high-dimensional relationship between the two sets of variables into a few pairs of canonical variables. Sites are then represented as points in the spaces of the canonical variables and the point patterns in theses spaces are examined for similarity and, if similar, then regions can be selected. If there are well-defined clusters, then fixed regions may be delineated and a new site is classified into one of these regions according to its coordinates in the response-related canonical variable space, which are themselves computed from the corresponding coordinates in the descriptor-related canonical variable space. Otherwise, the computed point may be used as the centre of a “hydrological neighbourhood” (i.e. a “one-site” region) in the response-related canonical variable space, in a somewhat similar manner to the ROI approach. Thus, sites can be classified into regions according to catchment characteristics alone, and the CCA approach can lead to spatially discontinuous regions. Note that while it is also possible to use the CCA method to estimate the flood risk at a new site, its use is usually restricted to delineating of regions (Cavadias et al., 2001, p. 501).

4.2.5 Discussion

The process of delineating regions for a flood regionalisation study is a major undertaking in its own right. In this thesis, the geographical method of delineating regions is used. This method is selected for multiple reasons: firstly, the primary focus of this thesis is the development of the concepts and techniques for the proposed regional flood model, rather than delineating regions, so the simpler, geographical method is considered suitable. Secondly, the flood frequency data that is used in the case studies in this thesis covers a large spatial area (eastern Australia), with a relatively sparse coverage of sites — this suggests that the cluster-analysis- and L-moment-based methods may result in a large number of regions, which will unduly confound this study. The one-site-region methods (ROI and CCA) are likely to produce small “regions of influence”/“neighbourhoods”, which will lead to greater variability and uncertainty in the model parameters. Finally, Merz and Blöschl (2005) found that using spatial proximity provided significantly better estimates of regional flood quantiles than those derived using (multivariate) catchment descriptors, with a mixture of spatial proximity and catchment descriptors yielding even
better predictive performance. If it is assumed that geographical regions can be taken as a proxy for spatial proximity, this suggests that the performance of geographical regions may not be too inferior to those derived using distance measures based on catchment descriptors.

The map-based methods are not deemed an appropriate method for regional flood estimation. These methods lack statistical rigour (e.g. Tasker and Stedinger, 1986) and can be considered subjective when there is a relatively poor coverage of sites. The predictive errors associated with the map-based method can be quite large; indeed, the PRM has a typical error of less than 30%, while errors of 200% are possible in extreme cases (McDermott and Pilgrim, 1982, Figure 8.1), with many users being unaware of the presence and extent of these errors.

The estimation of the growth curve in the index-flood method makes the assumption that sites are independent, which makes it very difficult to incorporate the effects of intersite correlation into the proposed regional flood model. The predictive uncertainty of the index-flood regional estimates is difficult to calculate; although the use of simulation provides a means of quantifying this uncertainty.

Thus, the regression-based approaches are favoured for use in the proposed regional flood model — a Bayesian hierarchical model. The regression approaches are quite versatile through the inclusion of various catchment descriptors (see Equation (4.27)). Also, the GLS method allows for the explicit inclusion of correlation into the regression if correlation can’t easily be incorporated into the regional flood model itself, with Reis et al. (2005) showing that a Bayesian treatment of the GLS model (and the OLS and WLS models) is possible. Additionally, the index-flood method may be considered a special case of regression when estimating flood quantiles (see Equation (4.18)), with the index flood \( \hat{\mu}_i \), itself, often being estimated by regression methods. Berliner (2003) has argued for the use of Bayesian hierarchical models in the geophysical sciences from a conceptual viewpoint, providing three examples from within this area. Note that the use of a Bayesian hierarchical regional flood model has also been suggested by Parent et al. (2004), but they provide little implementation detail, nor any results; while, recently, Seidou et al. (2006) used a spatial hierarchical model for regional estimation (interpolation) of the snow water equivalent within a Canadian catchment. Further examples of hierarchical models in a hydrological context (rainfall-runoff modelling) include Campbell and Bates (2001) and Marshall et al. (2006).

### 4.3 Flood regionalisation: A hierarchical model perspective

The flood regionalisation model can be posed as a hierarchical model in that observable outcomes are modelled conditionally on certain parameters, which themselves are given a probabilistic specification in terms of further parameters (Gelman et al., 1995, p. 119; Berliner, 2003). That is, the observable outcomes (e.g. annual maximum floods) are sampled from a site’s own flood model whose parameters (e.g. mean and standard deviation) are themselves sampled from a regional flood model. Thus, the hierarchical model consists of a two-level structure (see Figure 4.1), the two levels being the regional and site
models. The hierarchical model is, thus, parameterised as $\Theta = \{\theta_R, \theta_S\}$; where $\Theta$ is the vector of all model parameters, and $\theta_R$ and $\theta_S$ are the vectors of regional and site model parameters, respectively.

The site model assumes that for a homogeneous epoch, where this homogeneity may be achieved after IPO stratification (i.e. a IPO- or IPO+ epoch), the annual maximum flood is lognormally distributed $y_{it} \sim N(\mu_i, \sigma^2_i)$ for $i = 1, \ldots, n$ and $t = 1, \ldots, T_i$; where $y_{it}$ is the (natural) logarithm of the flood for site $i$ at year $t$, $n$ and $T_i$ are the number of gauged sites and years of data at each site respectively\(^1\), and $\mu_i$ and $\sigma_i$ are respectively the mean and standard deviation of $y_i = \{y_{i1}, \ldots, y_{iT_i}\}$. Note that use of the lognormal distribution was shown to be appropriate when the flood data were stratified according to the IPO (see Chapter 3) and that $\theta_S = \{\mu, \log \sigma\}$, where $\mu$ and $\log \sigma$ are vectors. Also, it is assumed that the (annual maximum) floods at each site are independent in time, but are spatially correlated. This is important because the information content of the flood data is reduced when floods are spatially correlated. Most regional models ignore this and, thus, may overestimate the predictive power of the regional model — the generalised least squares approach of Stedinger and Tasker (1985) is an exception. The intersite correlation may be described an exponential-type decay function:

$$\rho_{ij} = \exp \left( -\frac{[d_{ij}/A]^B}{B} \right)$$  \hspace{1cm} (4.35)

where $\rho_{ij}$ is the correlation between sites $i$ and $j$, $d_{ij}$ is the distance between the two sites, and $A > 0$ and $B > 0$ are variables to be estimated. The use of this smoothed, monotonically-decreasing\(^2\) correlation model helps ensure that the resultant covariance matrix is invertible/positive definite (Tasker and Stedinger, 1989, p. 366); although, many other correlation functions may be used (for example Cressie, 1991, Section 2; Gneiting et al., 2006).

\(^1\)The approach currently assumes a continuous flood record, subsequently it is generalised to handle a noncontinuous flood record, i.e. missing data at one or more sites and unequal record lengths between the sites.

\(^2\)Monotonically decreasing as a function of $d_{ij}$. 

Figure 4.1: Hierarchical model structure.
The regional model is assumed to take the form of a normal linear model with

\[ \mu = X\beta + \varepsilon \quad (4.36) \]

and

\[ \log \sigma = Z\gamma + \delta \quad (4.37) \]

where \( X \) and \( Z \) are matrices of known catchment characteristics (descriptors), \( \beta \) and \( \gamma \) are unknown regional-parameter vectors, and \( \varepsilon \) and \( \delta \) are random normally-distributed error \( n \)-vectors with properties \( E(\varepsilon) = E(\delta) = 0 \), \( \text{Cov}(\varepsilon) = \sigma_\varepsilon^2 I \), and \( \text{Cov}(\delta) = \sigma_\delta^2 I \) where \( \sigma_\varepsilon^2 \) and \( \sigma_\delta^2 \) are unknown regional parameters, \( I \) is the identity matrix, and \( E(\cdot) \) and \( \text{Cov}(\cdot) \) denote the expectation and covariance operators — these are ordinary least squares models (see Section 4.2.3). Note the use of \( \log \sigma \) in Equation (4.37) ensures that the site standard deviation \( \sigma \) remains positive.

It is assumed that the site standard deviation has no dependent variables, which results in \( Z \) taking a unit vector form \( (Z = [1, 1, \ldots, 1]^T) \) and \( \gamma \) simplifying to a scalar form \( (\gamma = \gamma) \), where \( Z^T \) indicates the transpose of \( Z \). The variables used for the site mean \( (X) \) are defined subsequently. Thus, the regional model parameter vector is \( \theta_R = \{\sigma_\varepsilon^2, \beta, \sigma_\delta^2, \gamma\} \).

The hierarchical model now consists of the regional model

\[ \mu_i \sim \mathcal{N}(x_i^T \beta, \sigma_\varepsilon^2) \quad (4.38) \]

\[ \log \sigma_i \sim \mathcal{N}(\gamma, \sigma_\delta^2) \quad (4.39) \]

and the site model, for independent sites

\[ y_{it} \sim \mathcal{N}(\mu_i, \sigma_i^2) \quad (4.40) \]

or, for correlated sites

\[ y_{it} \sim \mathcal{N}(\mu_t, \Sigma_t) \quad (4.41) \]

where \( \mathcal{N}(a, b^2) \) denotes the normal (Gaussian) distribution with mean \( a \) and variance \( b^2 \) and \( x_i \) is the vector of catchment descriptors of site \( i \). For the correlated case, \( \mathcal{N}(\mu_t, \Sigma_t) \) denotes the multivariate normal distribution with mean vector \( \mu_t \) and covariance matrix \( \Sigma_t \), where \( y_{it}, \mu_t, \) and \( \Sigma_t \) are the vector of flood data, vector of site means, and covariance matrix for year \( t \), respectively. Note that \( y_{it}, \mu_t \) and \( \Sigma_t \) can vary because different site data may be available in each year, since it is permissible to have nonoverlapping records and missing data. For example, \( y_{t=1} \) may consist of data from sites 1 and 3, while data from sites 2 and 3 might be present for \( y_{t=2} \) (i.e. \( y_{t=2} = \{y_{2,t=2}, y_{3,t=2}\}^T, \mu_{t=2} = \{\mu_2, \mu_3\}^T, \) and \( \log \sigma_{t=2} = \{\log \sigma_2, \log \sigma_3\}^T \)). Also, the \( ij \)th element of the covariance matrix \( \Sigma_t \) is \( \rho_{ij} \sigma_i \sigma_j \), where \( \rho_{ij} \) is defined by Equation (4.35).

The hierarchical model is illustrated in Figure 4.1. Observe that the regional flood model “sits above” the site model and furnishes different means and standard deviations, \( \mu_i \) and \( \sigma_i \), to each site; thus allowing for variation between sites.
4.4 Bayesian inference for the regional flood model

This section describes the methods used for the calibration (parameter estimation) of the regional model, with the objective being to develop a Bayesian framework to infer the regional flood (hierarchical) model parameters \( \Theta = \{ \theta_R, \theta_S \} \).

### 4.4.1 Bayesian inference framework

A Bayesian approach is used in the calibration of the hierarchical model. Bayesian methods are advantageous because they explicitly account for parameter uncertainty by making the parameters the object of inference; rather than simply relying on the optimal parameter values for which parameter uncertainty has not been taken into account.

The general Bayesian framework is now described — see, for example, Gelman et al. (1995, 2004) for more details on Bayesian analysis. The set of (annual maximum flood) observations \( Y = \{ y_1, \ldots, y_n \} \) are hypothesised to be a random realisation from a probability model \( M \) with probability density function \( f(Y \mid \theta, M, X, Z) \), where \( \theta \) is the unknown model parameter vector, \( X \) and \( Z \) are the matrices of explanatory variables (covariates), and \( Y \) is a matrix. Henceforth, the implicit conditioning on the probability model \( (M) \) and covariates \( (X \text{ and } Z) \) is dropped for notational convenience. The function \( f(Y \mid \theta) \) is known by two names depending upon the context in which it is used — either the sampling distribution or the likelihood function. When used to describe the probability model generating the sample data \( Y \) for a given \( \theta \), it is referred to as the sampling distribution; here, however, the data \( Y \) is known, and inference is sought on the parameter \( \theta \), so the term likelihood function is used.

The parameter vector \( \theta \) is estimated using Bayesian inference. Bayesian inference considers the parameter vector \( \theta \) to be a random vector whose probability distribution describes what is known about the true value of \( \theta \). Prior to the analysis of the data \( Y \), knowledge about \( \theta \) is summarised by the probability density function \( p(\theta) \). This is known as the prior density and can incorporate subjective belief about \( \theta \). Bayes Theorem is then used to revise, using the information contained in \( Y \), what is known about the true value of \( \theta \). Thus,

\[
p(\theta \mid Y) = \frac{f(Y \mid \theta)p(\theta)}{p(Y)} \propto f(Y \mid \theta)p(\theta)
\]

where \( p(\theta \mid Y) \) is the posterior density summarising the current knowledge of the true value of \( \theta \), given the observed data \( Y \), and \( p(Y) \) is the marginal likelihood and may be considered to be a normalising constant since, with fixed \( Y \), \( p(Y) \) is independent of \( \theta \) and is defined

\[
p(Y) = \int f(Y \mid \theta)p(\theta)d\theta
\]

Thus, the posterior density is proportional to the likelihood function multiplied by the prior density.
4.4.2 Overview of MCMC sampling techniques

A major objective of Bayesian inference is deriving and understanding the posterior distribution; however, it is not possible to derive an analytic expression for the posterior of most hierarchical models, so other techniques must be used to explore the posterior distribution. The techniques used in this study are Markov chain Monte Carlo methods, of which the Metropolis–Hastings algorithm and the Gibbs sampler are the most well known, with the latter being a special case of the former. MCMC methods provide a general means to randomly sample from from virtually any multivariate distribution. This discussion of MCMC methods is based on the lucid exposition given in Chib and Greenberg (1995) and also draws on Gelman et al. (1995) and Thyer (2000). Of special note is the Gibbs sampler, which is particularly well suited to hierarchical models (for example Gelman et al., 1995, p. 327; Arnold, 1993, p. 613). Additionally, a comparison of the performance of several MCMC methods in a hydrological modelling context is provided by Marshall et al. (2004).

The aim of MCMC simulation is to draw samples\(^a(1), a(2), \ldots, a(N)\) from a target distribution with pdf \(\pi(a) = kf(a)\), where \(f(a)\) is the unnormalised density and \(k\) is the (possibly unknown) normalising constant; thus, the target pdf need only be calculable up to its proportionality constant. MCMC methods generate a sequence of samples from a Markov chain whose stationary distribution converges to the target distribution — the posterior distribution in this case.

To prove that a Markov chain sequence converges to the target distribution involves two steps: first, show that the simulated sequence is a Markov chain with a unique stationary distribution, and second, show that the stationary distribution equals the target distribution (Gelman et al., 1995, p. 325). The first step requires that the Markov chain be irreducible, aperiodic, and not transient. Except for trivial exceptions, the last two conditions hold for a random walk on any proper distribution; with the term “random walk” describing a sequence of samples where the next value is equal to the current value plus some random increment. Irreducibility holds as long as the random walk has a positive probability of eventually reaching any part of the target distribution from any other part of the target distribution. The second step — to show that the stationary distribution equals the target distribution — requires more effort.

Continuous Markov chain theory specifies a transition kernel \(K(b \mid a)\), which is the conditional probability of moving from a point \(a\) to a point \(b\), as shown in Figure 4.2. It is important to recognise that the chain may make a transition from \(a\) to \(a\); i.e. there is a finite probability that the chain remains stationary and does not move from \(a\). An important consideration within Markov chain theory is to determine the conditions under which a stationary pdf \(\pi(\cdot)\) exists and under what conditions the iterations from the transition kernel \(K(\cdot \mid \cdot)\) converge to the stationary distribution. The stationary pdf must satisfy

\[
\pi(b) = \int K(b \mid a)\pi(a)da \tag{4.44}
\]

which has the intuitive interpretation that if the chain is stationary, then the total prob-

---

\(^a\)The samples \(a^{(1)}\) are assumed to be vectors.
Figure 4.2: Schematic representation of the Markov-chain transition kernel $K(b \mid a)$.

MCMC methods use a different approach to continuous Markov chain theory: the stationary density is already known (up to a proportionality constant) — it is $\pi(\cdot)$, the target density from which samples are desired — but the transition kernel is unknown. Thus, to generate samples from $\pi(\cdot)$, MCMC methods need to specify a transition kernel $K(\cdot \mid \cdot)$ that produces a sequence of samples which converge to $\pi(\cdot)$. This process is started at an arbitrary $a$ and iterated a large number of times. After this large number, the sequence of samples generated from the transition kernel approximates the target distribution.

The problem is now how to find an appropriate transition kernel. MCMC theory proceeds by constructing the following transition kernel, which describes the probability density of either moving from $a$ to a new point $b$ or remaining at $a$:

$$K(b \mid a) = p(b \mid a) + r(a)\delta(b - a)$$

$$= \begin{cases} 
  p(b \mid a) & \text{if } b \neq a \\
  r(a)\delta(0) & \text{if } b = a 
\end{cases}$$ (4.45)

where $p(b \mid a)$ is a nonnegative function with the property $p(a \mid a) = 0$, $\delta(b - a)$ is the (Dirac) delta function centred about $a$, and

$$r(a) = 1 - \int p(b \mid a)db$$ (4.46)
is the probability that the chain remains at \( a \) (noting that \( \int p(b \mid a) \, db \) is the probability of the chain moving away from \( a \)). The delta function (e.g. Spanier and Oldham, 1987, Chap. 10) is not a “proper” function; however, it has some properties which make it very useful. The delta function has zero probability mass at all values, except about the point \( x_0 \) where it has a spike of probability mass. The properties of the delta function include:

\[
\delta(x - x_0) = \begin{cases} 
\infty & \text{if } x = x_0 \\
0 & \text{if } x \neq x_0 
\end{cases} 
\]  

(4.47)

\[
\int_{-\infty}^{\infty} \delta(x) \, dx = 1 
\]  

(4.48)

\[
\int_{-\infty}^{\infty} f(x) \delta(x) \, dx = f(x_0) 
\]  

(4.49)

Now, for a Markov chain to be stationary, the reversibility condition (also known as “detailed balance”) must be satisfied:

\[
\pi(a) p(b \mid a) = \pi(b) p(a \mid b) 
\]  

(4.50)

Intuitively, the reversibility condition requires that the unconditional probability of moving from \( a \) to \( b \), where \( a \) is sampled from \( \pi(\cdot) \) — the left-hand side — is equal to the unconditional probability of moving from \( b \) to \( a \), where \( b \) is also sampled from \( \pi(\cdot) \) — the right-hand side.

If the reversibility condition is satisfied, then \( \pi(\cdot) \) is the stationary density of the transition kernel \( K(\cdot \mid \cdot) \). This can be verified by evaluating the right-hand side of Equation (4.44), first using a substitution from Equation (4.45) and then applying reversibility and the properties of the delta function:

\[
\int K(b \mid a) \pi(a) \, da = \int [p(b \mid a) + r(a)\delta(b - a)] \pi(a) \, da \\
= \int p(a \mid b) \pi(b) \, da + \int r(a)\delta(b - a) \pi(a) \, da \\
= \pi(b) \int p(a \mid b) \, da + r(b) \pi(b) \times 1 \\
= \pi(b)[1 - r(b)] + r(b)\pi(b) \\
= \pi(b) 
\]  

(4.51)

Thus, the reversibility condition is satisfied.

**Metropolis–Hastings algorithm**

The most general implementation of MCMC methods is the Metropolis–Hastings (M-H) algorithm. The Metropolis–Hastings algorithm (Hastings, 1970) is a generalisation of the Metropolis algorithm (Metropolis et al., 1953).

\footnote{In fact \( \delta(0) \) is undefined; however, its integral properties are well defined.}
A density \( q(b \mid a) \) is selected from which a sample \( b \) can be generated — this sample is known as a proposal. Since \( b \) is to form a Markov chain, the proposal density must be dependent on the current state of the chain \( a \). Now, it is also required that the pdf \( q(b \mid a) \) satisfies the reversibility condition (Equation (4.50)) for all \( a, b \). Most likely this will not occur for some \( a, b \), with

\[
\pi(a)q(b \mid a) > \pi(b)q(a \mid b)
\]

(4.52)

which states that the chain is more likely to move from \( a \) to \( b \) than to move from \( b \) to \( a \). A convenient way to correct this situation — and satisfy the reversibility condition — is to introduce a “probability of move” \( \alpha(b \mid a) < 1 \) that reduces the likelihood of the chain moving from \( a \) to \( b \). If the move is not made, the chain remains at \( a \). Thus, transitions from \( a \) to \( b \) (\( b \neq a \)) are made according to

\[
p_{\text{MH}}(b \mid a) \equiv q(b \mid a)\alpha(b \mid a)
\]

(4.53)

where \( \alpha(b \mid a) \) is yet to be determined.

To define the move probability \( \alpha(b \mid a) \) the inequality (4.52) is reconsidered. The likelihood of movement from \( b \) to \( a \) is too low and needs to be increased. This is achieved by setting \( \alpha(a \mid b) \) to be as large as possible, i.e. 1, since it is a probability. The reversibility condition is then applied to determine the probability of move \( \alpha(b \mid a) \)

\[
\pi(a)q(b \mid a)\alpha(b \mid a) = \pi(b)q(a \mid b)\alpha(a \mid b) = \pi(b)q(a \mid b)
\]

(4.54)

Hence, \( \alpha(b \mid a) = \pi(b)q(a \mid b)/[\pi(a)q(b \mid a)] \). If the inequality (4.52) is reversed, then \( \alpha(b \mid a) \) is set to 1 and \( \alpha(a \mid b) \) is derived in a similar manner to Equation (4.54). The move probabilities may be interpreted as being introduced to ensure that the two sides of the inequality (4.52) are in “balance”, allowing \( p_{\text{MH}}(b \mid a) \) to satisfy the reversibility condition. The probability of move is defined

\[
\alpha(b \mid a) = \begin{cases} 
\min \left[ \frac{\pi(b)q(a \mid b)}{\pi(a)q(b \mid a)}, 1 \right] & \text{if } \pi(a)q(b \mid a) > 0 \\
1 & \text{otherwise}
\end{cases}
\]

(4.55)

Several general issues related to the M-H algorithm must be highlighted. Firstly, a proposal density \( q(b \mid a) \) must be specified and the selection of an appropriate density is crucial to the efficiency of the algorithm. Secondly, if a proposal \( b \) is rejected, then the current value \( a \) is taken as the next step in the sequence — so the chain generates correlated samples. Thirdly, the calculation of \( \alpha(b \mid a) \) does not require the knowledge of the normalising constant of the target pdf \( \pi(\cdot) \) because the constant occurs in both the numerator and denominator of Equation (4.55). This is important because there may be considerable difficulty or computational effort required to calculate this constant.

The implementation of the M-H algorithm is summarised:

1. Initialise \( a \) with an arbitrary starting value \( a^{(0)} \).
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2. Repeat for $j = 1, 2, \ldots, N$:
   
   (a) Sample $b$ from $q(\cdot | a^{(j-1)})$ and $u$ from the uniform distribution $U(0,1)$.
   
   (b) If $u < \alpha(b | a^{(j-1)})$
       
       Set $a^{(j)} = b$ 
       
    Else
       
       Set $a^{(j)} = a^{(j-1)}$

3. Return the values $\{a^{(1)}, a^{(2)}, \ldots, a^{(N)}\}$.

As with any MCMC method, the draws are regarded as samples from the target density $\pi(\cdot)$ only after the chain has burned in — sufficient samples have been taken so that the effect of the starting values is no longer significant. Although the conditions required for the Markov chain to converge are known, they do not determine the rate of convergence. Convergence issues are very important to the performance of MCMC methods and are discussed in Section 4.4.3. As previously mentioned, the selection of an appropriate proposal density has a significant influence on the performance of the M-H algorithm. Typically, this density is chosen from a family of distributions that require the specification of such tuning parameters as the location and scale. Chib and Greenberg (1995) illustrate this subject with several examples.

**Metropolis algorithm**

The Metropolis algorithm (Metropolis et al., 1953) — a special case of the Metropolis–Hastings algorithm (Hastings, 1970) — employs a symmetric proposal pdf which satisfies $q(b | a) = q(a | b)$. As a consequence, the probability of move reduces to $\alpha(b | a) = \min[\pi(b)/\pi(a), 1]$. Thus, if $\pi(b) \geq \pi(a)$ the chain always makes the move from $a$ to $b$; otherwise, it moves with probability $\pi(b)/\pi(a)$. This may be interpreted such that if a jump goes “uphill” (to a higher density), then it is always accepted; while if a jump goes “downhill”, then it is accepted with nonzero probability.

Typically, a (multivariate) normal (or a Student $t$) density is chosen as the symmetric proposal density $q(b | a) = N(b | a, \Sigma)$, with the normal distribution being centred about the current sample $a$. The notation $N(b | a, \Sigma)$ denotes a multivariate normal distribution evaluated at $b$, with mean $a$ and covariance $\Sigma$. The symmetry arises from the fact that $N(b | a, \Sigma) = N(a | b, \Sigma)$. Since the proposal distribution is centred about $a$, this sampler is called a random walk sampler and provides an advantage that only the scale parameter $\Sigma$ needs to be tuned — the location parameter is given by the current value of the chain $a$.

The scale, or spread, of the proposal density is an important issue because it has implications for the efficiency of the algorithm. The spread of the proposal affects the behaviour of the chain in at least two ways: first is the “acceptance rate” (the percentage of times a move to a new point is made), and the second is the region of the sample space that is covered by the chain. This may be understood by considering the situation where the chain has converged and the density is being sampled around the mode. If the spread
is too large, then the samples from the proposal density will be far from the current value and will have a low probability of being accepted (because the target density far from the mode will be much smaller compared to the density at the mode). Conversely, if the spread is too small, then the chain will take a long time to explore the entire target density and it is likely that the low probability regions will be undersampled. This issue is discussed in more detail in Chib and Greenberg (1995).

**Block Metropolis (Gibbs) sampling**

The M-H algorithm can be applied to subsets (or blocks) of the vector $a$, rather than simultaneously to all elements of the vector. This “block at a time” algorithm provides a primary benefit in that it often simplifies the search for a suitable proposal density. Furthermore, a special case — the Gibbs sampler — is particularly efficient.

This block-sampling approach may be illustrated using two blocks, $a = \{a_1, a_2\}$. Suppose there exists a conditional transition kernel $K_1(b_1 \mid a_1, a_2)$ which, for a fixed value of $a_2$, has $\pi_1(\cdot \mid a_2)$ as its stationary pdf. Applying Equation (4.44) for this block case gives

$$
\pi_1(b_1 \mid a_2) = \int K_1(b_1 \mid a_1, a_2)\pi_1(a_1 \mid a_2)da_1
$$

(4.56)

Also, assume the existence of a conditional transition kernel $K_2(b_2 \mid a_2, a_1)$ which, for a given $a_1$, has $\pi_2(\cdot \mid a_1)$ as its stationary pdf. For example, $K_1$ could be transition kernel generated by a M-H chain applied to the block $a_1$ with $a_2$ fixed for all iterations. Now, the “product of kernels” principle shows that the product of the two transition kernels $K_1(\cdot \mid a_1, a_2)$ and $K_2(\cdot \mid a_2, a_1)$ has $\pi(a_1, a_2)$ as its stationary density. This principle has a huge practical significance because it is now possible to take draws in succession from each of these kernels, instead of having to run each of these kernels to convergence for every value of the conditioning variable. An additional advantage, as alluded to above, is that it is often easier to find several conditional kernels that converge to their respective conditional densities, rather than finding a single kernel that converges to the joint density.

To demonstrate the product-of-kernels principle, the order that the elements of $a$ are sampled must first be specified. Assume that the transition kernel $K_1(\cdot \mid \cdot, a_2)$ samples $b_1$, given $a_1$ and $a_2$, and that the transition kernel $K_2(\cdot \mid \cdot, b_1)$ samples $b_2$, given $a_2$ and $b_1$. This means that the vector $a = \{a_1, a_2\}$ is sampled in two steps: the first step samples $b_1$ from $K_1(\cdot \mid \cdot, a_2)$ to replace $a_1$, and in the second step $b_2$ is sampled from $K_2(\cdot \mid \cdot, b_1)$ to replace $a_2$. The kernel formed by multiplying these two conditional kernels together has
consider the case where a parameter vector \( \Theta \) is partitioned into \( d \) blocks. The Gibbs sampler is the most efficient implementation of the M-H algorithm. This result also applies for any number of blocks. Therefore, it can be concluded that the \( \alpha \) from the two-block case, and so on. Similarly, it can also be shown that the probability of move for the second step \( \alpha_{2|1}(b_2 | a_2, b_1) \) is also equal to one. This means that the candidate sample is always accepted. This result also applies for any number of blocks. Therefore, it can be concluded that the Gibbs sampler is the most efficient implementation of the M-H algorithm.

For the \( i \)th block (subvector), out of a total of \( d \) blocks (subvectors), the candidate (proposal) density can be written

\[
q_i(b_i | b_1, \ldots, b_{i-1}, a_i, \ldots, a_d) = \pi_i(b_i | b_1, \ldots, b_{i-1}, a_{i+1}, \ldots, a_d) \tag{4.59}
\]

The Gibbs sampler is very useful for drawing samples from a Bayesian posterior; for example, consider the case where a parameter vector \( \Theta \), with posterior density \( p(\Theta | Y) \), is partitioned into \( d \) blocks (subvectors). Defining \( \theta_i^{(j)} \) as the \( i \)th parameter block for the
jth iteration of the Gibbs sampler, the sample is drawn from

$$\theta^{(j)}_i \leftarrow p(\theta_i | \theta^{(j)}_1, \ldots, \theta^{(j)}_i, \theta^{(j-1)}_{i+1}, \ldots, \theta^{(j-1)}_d, Y)$$

(4.60)

This can be advantageous in situations where drawing samples of the full parameter vector directly from the posterior distribution is not possible. Often, it is easier to draw samples from the posterior distribution of a block of the parameter vector, conditioned on the remaining, fixed, parameter values.

In some cases, it may not be possible to easily draw a block from the full conditional distribution. Under these circumstances, the M-H algorithm can be used for that particular block — this is often referred to as a “Metropolis-within-Gibbs” step; however, Chib and Greenberg (1995, p. 332) argue against the use of this term.

4.4.3 Gibbs sampler implementation for the hierarchical regional flood model

As noted earlier in this chapter, the Gibbs sampler is well suited to hierarchical models and, so, is used for parameter inference in this thesis. The hierarchical model parameter vector is partitioned into two subvectors, $\Theta = \{\theta_R, \theta_S\}$, where $\theta_R = \{\sigma^2, \beta, \sigma^2_\delta, \gamma\}$ and $\theta_S = \{\mu, \log \sigma\}$. Each iteration of the Gibbs sampler cycles through the subvectors of $\Theta$, randomly sampling each subvector from its conditional posterior distribution — conditional on the (current) value of all of the other subvectors. For this case, there are two steps required in each iteration $j$ — sampling from the conditional posterior distributions $p(\theta_R | \theta_S^{(j-1)}, Y)$ and $p(\theta_S | \theta_R^{(j)}, Y)$; where $Y$ denotes the observed flood data (at all sites within the region). The conditional posterior distributions can be simplified:

$$p(\theta_R | \theta_S, Y) = \frac{p(Y | \theta_R, \theta_S)p(\theta_R | \theta_S)}{p(Y | \theta_S)} = \frac{p(Y | \theta_S)p(\theta_R | \theta_S)}{p(Y | \theta_S)} = p(\theta_R | \theta_S)$$

(4.61)

The first line rearranges using Bayes Theorem and the simplification in the second line exploits the hierarchical structure whereby the regional parameters only affect the data indirectly through the site parameters (see Equations (4.40) and (4.41)). The second conditional posterior distribution is similarly manipulated:

$$p(\theta_S | \theta_R, Y) = \frac{p(Y | \theta_S, \theta_R)p(\theta_S | \theta_R)}{p(Y | \theta_R)} = \frac{p(Y | \theta_S)p(\theta_S | \theta_R)}{p(Y | \theta_R)}$$

(4.62)

but this only yields a minor simplification, namely that the likelihood does not depend on $\theta_R$.

Thus, the implementation of this “two-block” $(\theta_R$ and $\theta_S)$ Gibbs sampler is summarised:
0. Assign starting value \( \theta_S^{(0)} \).

1. Randomly sample: \( \theta_R^{(j)} \leftarrow p(\theta_R | \theta_S^{(j-1)}, Y) = p(\theta_R | \theta_S^{(j-1)}) \).

2. Randomly sample: \( \theta_S^{(j)} \leftarrow p(\theta_S | \theta_R^{(j)}, Y) = \frac{p(Y | \theta_S)p(\theta_S | \theta_R^{(j)})}{p(Y | \theta_R^{(j)})} \).

3. Repeat steps 1 and 2, \( j = 1, \ldots, N \) times.

The implementation of these Gibbs sampler steps are now described in detail.

**Step 1 (regional-parameter block)**

A Gibbs step is required such that samples are produced from the conditional posterior distribution \( \theta_R^{(j)} \leftarrow p(\theta_R | \theta_S^{(j-1)}) \). Since the components of the regional model consist of normal linear models (Equations (4.36) and (4.37)), standard regression results may be used (see for example Gelman et al., 1995, pp. 235–7). Also, the independence between the “\( \mu \) and log \( \sigma \)” components of \( \theta_R \) (see Equations (4.36) and (4.37)) allows them to be sampled independently.

For the “\( \mu \)” component of \( \theta_R \):

\[
p(\beta, \sigma^2 \varepsilon | \mu^{(j-1)}) = p(\beta | \sigma^2 \varepsilon, \mu^{(j-1)})p(\sigma^2 \varepsilon | \mu^{(j-1)}) \tag{4.63}
\]

Now, the marginal posterior distribution of \( \sigma^2 \varepsilon \) has a scaled inverse \( \chi^2 \) form, thus

\[
\sigma^2 \varepsilon^{(j)} \leftarrow p(\sigma^2 \varepsilon | \mu^{(j-1)}) \tag{4.64}
\]

\[
\sim \chi^{-2} (n - n_\beta, s^2) \tag{4.65}
\]

where \( \chi^{-2} (\nu, s^2) \) is a scaled inverse chi squared distribution with \( \nu \) degrees of freedom and scale \( s \) (using the notation of Gelman et al. (1995, Table A.1, p. 474)), \( n_\beta \) is the size of (vector) \( \beta \),

\[
s^2 = \frac{1}{n - n_\beta} (\mu^{(j-1)} - X\hat{\beta})^T (\mu^{(j-1)} - X\hat{\beta}) \tag{4.66}
\]

and

\[
\hat{\beta} = (X^TX)^{-1}X^T\mu^{(j-1)} \tag{4.67}
\]

The conditional posterior distribution of \( \beta \), given \( \sigma^2 \varepsilon \), is normal

\[
\beta^{(j)} \leftarrow p(\beta | \sigma^2 \varepsilon^{(j)}, \mu^{(j-1)}) \tag{4.68}
\]

\[
\sim N \left( \hat{\beta}, (X^TX)^{-1}\sigma^2 \varepsilon^{(j)} \right) \tag{4.69}
\]

The “log \( \sigma \)” component follows a similar procedure as that above; however, further simplifications may be made by exploiting the unit vector form of \( Z \) and resultant scalar
form of $\gamma$, i.e. $\gamma = \gamma$. Thus,

$$p(\gamma, \sigma_\delta^2 | \log \sigma^{(j-1)}) = p(\gamma | \sigma_\delta^2, \log \sigma^{(j-1)})p(\sigma_\delta^2 | \log \sigma^{(j-1)})$$ (4.70)

$$\sigma_\delta^{2(j)} \sim \chi^2(n-1, s^2)$$ (4.71)

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left( \log \sigma^{(j-1)}_i - \hat{\gamma} \right)^2$$ (4.72)

$$\hat{\gamma} = \left( Z^T Z \right)^{-1} Z^T \log \sigma^{(j-1)}$$ (4.73)

$$\gamma^{(j)} \sim N \left( \hat{\gamma}, (Z^T Z)^{-1} \sigma_\delta^{2(j)} \right)$$ (4.75)

Thus, the original Gibbs step for the entire regional-parameter vector ($\theta_R$) is broken up into four separate Gibbs steps (for $\sigma_\delta^2$, $\beta$, $\sigma_\epsilon^2$, and $\gamma$).

**Step 2 (site-parameter block)**

A Gibbs step is required such that samples are produced from the conditional distribution $\theta_S^{(j)} \sim p(\theta_S | \theta^{(j)}_R, Y)$. Unfortunately, there are no standard results that can be used to allow the sampling of the site model parameters simply, so another technique must be used instead — a “Metropolis within Gibbs” step.

Before proceeding with the Metropolis-within-Gibbs step, the conditional posterior distribution must be manipulated into a form more readily usable within the Metropolis algorithm. From Equation (4.62),

$$p(\theta_S | \theta_R, Y) = \frac{p(Y | \theta_S)p(\theta_S | \theta_R)}{p(Y | \theta_R)} \propto p(Y | \theta_S)p(\theta_S | \theta_R)$$ (4.77)

where these terms are evaluated using:

$$p(Y | \theta_S) = p(Y | \mu, \log \sigma) = \prod_{t=1}^{T} N(y_t | \mu_t, \Sigma_t)$$ (4.78)

$$p(\theta_S | \theta_R) = p(\mu, \log \sigma | \theta_R) = p(\mu | \theta_R)p(\log \sigma | \theta_R)$$

$$= \prod_{i=1}^{n} N \left( \mu_i, X_i^T \beta^{(j)}, \sigma_\epsilon^{2(j)} I \right) N \left( \log \sigma, \gamma^{(j)}, \sigma_\delta^{2(j)} I \right)$$ (4.79)

where $y_t$, $\mu_t$, and $\Sigma_t$ are, respectively, the vector of flood data, vector of site means, and covariance matrix for the current year $t$ (see also Section 4.3), and $T$ is the total number of years where data is available (at any site). Additionally, if the sites are assumed to be independent, i.e. correlation is ignored, then Equation (4.78) is greatly simplified from a product of multivariate normal pdfs and becomes the double product (overs years and sites) of univariate normal pdfs.

Using the Metropolis description provided in Section 4.4.2, the “single block” implementation of the Metropolis algorithm — single block in the sense that all site parameters
are sampled at one time — is summarised for a single iteration:

1. Use existing values: $\theta_S^{(j-1)}$ and $\theta_R^{(j)}$.
2. Sample a trial parameter $\theta_S^*$ from a symmetric proposal distribution $q(\theta_S^* | \theta_S^{(j-1)})$.
3. Calculate the ratio of densities:

$$r = \frac{p(\theta_S^* | \theta_R^{(j)}, Y)}{p(\theta_S^{(j-1)} | \theta_R^{(j)}, Y)}$$

(4.80)

using Equation (4.77). Note that the unnormalised conditional posterior can be used because the normalising constants would cancel.
4. Perform an acceptance/rejection step. Set:

$$\theta_S^{(j)} = \begin{cases} 
\theta_S^* & \text{with probability } \min(r, 1) \\
\theta_S^{(j-1)} & \text{otherwise}
\end{cases}$$

**Implementation issues**

There are several implementation issues which need to be considered to ensure that adequate performance of the Gibbs sampler (and the Metropolis-within-Gibbs algorithm) is achieved.

The initial starting point $\theta_S^{(0)}$ (on the first Gibbs iteration) is obtained via a mode searching algorithm, such as the SCE (Duan et al., 1994), quasi-Newton (Dennis and Schnabel, 1996; Nocedal and Wright, 1999), or LBFGS (Byrd et al., 1995; Nocedal and Wright, 1999) methods, on the likelihood component $p(Y | \theta_S)$ of the unnormalised conditional posterior (Equation (4.77)). The mode of the likelihood, and not of the full posterior, is used because the likelihood involves only the available data and does not apply any additional model structure, which might significantly alter the resultant mode. Also, the parameters of the correlation function (Equation (4.35)) are determined at this time, if required. Thus, the maximum likelihood estimates of the site model parameters $\theta_S$ and the correlation parameters $A$ and $B$ are determined. Note, it is recommended to run the mode searching algorithm many times, with different settings and starting points, to ensure that a local optimum has not been found. It was found that local optima have, generally, not been a problem for the data sets (both synthetic and real) that were analysed in this thesis.

The jumping (or proposal) distribution was obtained from the Hessian matrix, with the approximate Hessian provided by the quasi-Newton method usually being adequate. Using a second-order Taylor expansion about the mode, it can be shown that the Hessian matrix — the matrix of second partial derivatives — of the logarithm of the posterior (or likelihood) is related to the covariance matrix of the parameters, with the negative Hessian being equal to the inverse covariance matrix (for example Seber and Wild, 1989). A more accurate Hessian, such as one based on an enhanced Ridders procedure (see Press et al., 1992, Chapter 5), may be computed if desired, but this usually provides little advantage
over the approximate quasi-Newton Hessian. Additionally, the jumping distribution may be adaptively scaled to try to achieve the desired acceptance rates — if the acceptance rate is too low, then the proposal distribution is “shrunk”; if the acceptance rate is too large, then the proposal distribution is “enlarged” (see Section 4.4.2). Gelman et al. (1995, pp. 334–335) provide guidance on the selection of optimal acceptance rates for normal posteriors.

The Metropolis algorithm must be allowed to “warm up” sufficiently before there may be confidence that the samples being produced are no longer being affected by the starting distribution and have, indeed, converged to the target (posterior) distribution. This convergence is monitored through the “R statistic” (Gelman et al., 1995, pp. 331-3), which compares between- and within-sequence variance of multiple sequences (of samples). Thus, multiple, simultaneous Metropolis sequences were run so that the R statistic could be estimated — these multiple sequences have the additional benefit of allowing more of the parameter space to be explored. Convergence to the target distribution occurs when the R statistic approaches one, with a value below 1.2 usually being acceptable close.

The principal difficulty that was initially encountered was a poor acceptance rate for the $\theta_S$ vector, even after attempting adaptive scaling of the proposal distribution. This problem was overcome by performing the acceptance/rejection step one site at a time, rather than for the entire $\theta_S$ vector at once — in effect, introducing $n$ Metropolis within Gibbs steps. This eliminated the possibility of a single poorly-sampled component of $\theta_S$ leading to the entire trial $\theta_S^*$ being rejected. However, this requires the Metropolis algorithm to be slightly modified, and is described below.

For notational convenience, rewrite the entire site parameter vector in terms of the current site-parameter vector and all of the remaining site-parameter vectors: let $\theta_S = \{\vartheta_i, \vartheta_{-i}\}$ where $\vartheta_i = \{\mu_i, \log \sigma_i\}$ and $\vartheta_{-i} = \{\vartheta_1, \ldots, \vartheta_{i-1}, \vartheta_{i+1}, \ldots, \vartheta_n\}$. The conditional posterior distribution (Equation (4.62)) must be modified because the entire site-parameter vector $\theta_S$ is no longer obtained from a single Metropolis-within-Gibbs step; instead, $n$ Metropolis-within-Gibbs steps are performed to obtain each site-parameter-vector sample $\vartheta_i^*$ individually. Using Bayes Theorem, the conditional posterior is:

$$p(\vartheta_i \mid \vartheta_{-i}, \theta_R, Y) = \frac{p(Y \mid \vartheta_i, \vartheta_{-i}, \theta_R) p(\vartheta_i \mid \vartheta_{-i}, \theta_R)}{p(Y \mid \vartheta_{-i}, \theta_R)}$$ (4.81)

The ratio of conditional posterior distributions becomes, upon exploiting the hierarchical structure:

$$r = \frac{p(Y \mid \vartheta_i^*, \vartheta_{-i}, \theta_R) p(\vartheta_i^* \mid \vartheta_{-i}, \theta_R)}{p(Y \mid \vartheta_i, \vartheta_{-i}, \theta_R) p(\vartheta_i \mid \vartheta_{-i}, \theta_R)} = \frac{p(Y \mid \vartheta_i^*, \vartheta_{-i}, \vartheta_R)}{p(Y \mid \vartheta_i, \vartheta_{-i})} \frac{p(\vartheta_i \mid \theta_R)}{p(\vartheta_i^* \mid \theta_R)}$$ (4.82)
To evaluate the terms in this ratio, first consider the independent case, for simplicity:

\[ r_{\text{IND}} = \frac{p(y_1, \ldots, y_n | \vartheta_1, \ldots, \vartheta^*_i, \ldots, \vartheta_n)p(\vartheta^*_i | \vartheta_R)}{p(y_1, \ldots, y_n | \vartheta_1, \ldots, \vartheta_i, \ldots, \vartheta_n)p(\vartheta_i | \vartheta_R)} \]

\[ = \left( \prod_{j=1, j \neq i}^{n} p(y_j | \vartheta_j) \right) \frac{p(y_i | \vartheta^*_i) p(\vartheta^*_i | \vartheta_R)}{p(y_i | \vartheta_i) p(\vartheta_i | \vartheta_R)} = \frac{p(y_i | \vartheta^*_i) p(\vartheta^*_i | \vartheta_R)}{p(y_i | \vartheta_i) p(\vartheta_i | \vartheta_R)} \quad (4.83) \]

For the correlated case, the terms in Equation (4.82) are evaluated in a similar manner to Equations (4.78) and (4.79); however, the \( \mu_t \) and \( \Sigma_t \) terms in Equation (4.78) may now require a combination of the current trial site-parameter vector \( \vartheta^*_i \) and the other site-parameter vectors from the previous Metropolis iteration. For example, \( p(Y | \vartheta^*_2, \vartheta_{-2}) = p(Y | \vartheta_1, \vartheta^*_2, \vartheta_3, \ldots) \). Note that this increases the computational effort because the likelihood (Equation (4.78)) must now be recomputed for each of the \( n \) Metropolis-within-Gibbs steps.

The modified “site at a time” implementation of the Metropolis-within-Gibbs algorithm is summarised for a single iteration:

1. For \( i = 1, \ldots, n \) and existing \( \theta_S^{(j-1)} \) and \( \theta_R^{(j)} \):
   
   (a) Sample a trial parameter \( \vartheta^*_i \) from a symmetric jumping distribution \( q(\vartheta^*_i | \vartheta_i^{(t-1)}) \).
   
   (b) Calculate the ratio of densities \( r \) (using Equation (4.82) or (4.83), depending on whether the sites are assumed correlated or independent).
   
   (c) Perform an acceptance/rejection step. Set:

   \[ \vartheta_i^{(j)} = \begin{cases} \vartheta^*_i & \text{with probability } \min(r, 1) \\ \vartheta_i^{(j-1)} & \text{otherwise} \end{cases} \]

Summary of Gibbs sampler algorithm

For the sake of clarity, the full Gibbs sampler implementation is summarised:

1. Use a mode searching algorithm to determine the maximum likelihood estimates (using Equation (4.78)) of the site model parameters \( \theta_S^{(0)} \) and the correlation parameters \( A \) and \( B \), if required, as well as the Hessian matrix.

2. Obtain the initial (proposal) covariance matrix \( \Sigma^{(0)} \) from the Hessian matrix.

3. For \( j = 1, \ldots, N \):
   
   (a) Step 1: sample the regional-parameter vector \( \theta_R \) (using four Gibbs steps):
      
      i. \( \sigma_e^{2(j)} \leftarrow \chi^{-2} (n - n_\beta, s^2) \) — Equation (4.65), with \( s^2 \) defined in Equation (4.66).
ii. $\beta^{(j)} \sim N(\hat{\beta}, (X^T X)^{-1} \sigma^2_{\beta}^{(j)})$ — Equation (4.69), with $\hat{\beta}$ defined in Equation (4.67).

iii. $\sigma_{\beta}^{2(j)} \sim \chi^{-2}(n-1, s^2)$ — Equation (4.71), with $s^2$ defined in Equation (4.72).

iv. $\gamma^{(j)} \sim N(\hat{\gamma}, \sigma^2_{\beta}^{(j)}/n)$ — Equation (4.76), with $\hat{\gamma}$ defined in Equation (4.73).

(b) Step 2: sample the site-parameter vector $\theta_S$ (using $n$ Metropolis-within-Gibbs steps).

For $i = 1, \ldots, n$:

i. Sample a trial parameter $\vartheta^*_i$ from a symmetric jumping distribution $q(\vartheta^*_i | \vartheta_i^{(t-1)}) = N(\vartheta_i^{(t-1)}, c_i^2 \Sigma_i^{(0)})$, where $c_i$ is a scaling factor (initially set to 1) and $\Sigma_i^{(0)}$ is obtained from the appropriate elements of $\Sigma^{(0)}$.

ii. Calculate the ratio of densities $r$ (using Equation (4.82) or (4.83), depending on whether the sites are assumed correlated or independent).

iii. Perform an acceptance/rejection step. Set:

$$
\vartheta_i^{(j)} = \begin{cases} 
\vartheta^*_i & \text{with probability } \min(r, 1) \\
\vartheta_i^{(j-1)} & \text{otherwise}
\end{cases}
$$

iv. Update the covariance scaling factor $c_i$ (based on the acceptance rate), if desired. Note that this updating is only performed during the “warm up”/“burn in” phase, with the updates occurring after a large number (say, 1000) of iterations have elapsed since the last update.

The above Gibbs-sampler algorithm describes the model calibration (i.e. parameter inference) procedure for the hierarchical regional flood model using a rigorous Bayesian framework. This Gibbs sampler is implemented in a computer program using the Fortran 95 programming language.

4.4.4 Assessment of hierarchical regional flood model inference using synthetic data

Generation of synthetic data

The Gibbs sampler proposed in Section 4.4.3 is complex. To gain insights about its behaviour and to assure the integrity of the computer code implementing the sampler, it is necessary to conduct experiments involving synthetic data. For simplicity, the synthetic data is generated assuming all sites to be independent, i.e. spatial correlation is ignored; however, coordinates (latitude and longitude) were also assigned to all sites to facilitate a correlated analysis. The number of sites analysed were then significantly increased to ensure that the variability present in the fitted (regional) parameters was reduced.

The algorithm to generate the synthetic data is:

These coordinates were derived in a somewhat random manner whereby the coordinates of actual “towns” in eastern Australia were assigned to the generated sites — the coordinates were obtained from the Geoscience Australia web site (http://www.ga.gov.au/)
1. Assign $\theta_R = \{\sigma^2_\varepsilon, \beta, \sigma^2_\delta, \gamma\}$ and $X$.

2. Generate data for each site $i; i = 1, \ldots, n$:
   
   (a) Generate “true” site mean (Equations (4.36) and (4.38)): $\varepsilon_i \sim N\left(0, \sigma^2_\varepsilon\right)$ and $\mu_i = x_i^T\beta + \varepsilon_i$.

   (b) Generate “true” site standard deviation (Equations (4.37) and (4.39)): $\delta_i \sim N\left(0, \sigma^2_\delta\right)$ and $\log \sigma_i = \gamma_i + \delta_i$.

   (c) Generate $T = 100$ years of data (Equation (4.40)): $y_{it} \sim N\left(\mu_i, \sigma^2_i\right)$; $t = 1, \ldots, T$.

   (d) Assign “random” coordinates.

Typical values are used for the regional parameters $\theta_R$: $\beta = \{0.5, 5.0\}$, $\sigma^2_\varepsilon = 0.05$, $\gamma = 0.2$, and $\sigma^2_\delta = 0.1$, with these values being derived from ordinary least squares regressions using the eastern Australian flood data. The catchment characteristics $X$ represent the logarithm of the catchment area and an additive constant, with “logareas” ranging from 0 to 10. The use of a record length of $T = 100$ years is well in excess of most Australian record lengths.

Additionally, further analyses were performed where the regional variance terms were significantly reduced such that the “regional prior” dominates the “likelihood” (respectively, the $p(\theta_S | \theta_R)$ and $p(Y | \theta_S)$ terms in Equation (4.77)). This atypical case ($\sigma^2_\varepsilon = 0.0001$ and $\sigma^2_\delta = 0.0005$) permits a check on the robustness of the proposed Gibbs sampler.

**Analysis of “typical” synthetic data**

For the “typical” synthetic data experiment, the regional parameters are: $\beta = \{0.5, 5.0\}$, $\sigma^2_\varepsilon = 0.05$, $\gamma = 0.2$, and $\sigma^2_\delta = 0.1$.

Firstly, an independent-site run was performed (see Equation (4.83)) with the number of sites being increased from 30 to 100 to 500 sites. All runs proceeded satisfactorily with good acceptance rates (9–24%) and small R statistics, i.e. well below 1.2, using three sequences. Histograms of the variance terms are presented in Figures 4.3 and 4.4 and show that the true values of the variance terms are located within the region of high posterior density and also that the widths of the histograms are reduced as the number of sites are increased.

A correlated-site analysis (see Equation (4.82)) was repeated for 30 and 100 sites. In this experiment the sites are independent; however, the purpose of the experiment was to confirm the results are consistent with the uncorrelated case. These analyses also achieved very good acceptance rates (17–55%) and R statistics well below 1.2 for three sequences. The correlated-site analysis gave an almost identical result to that of the independent-site analysis, as shown in Table 4.1, which suggests that the Gibbs sampler has been implemented correctly.

---

*Natural logarithms are used.*
Figure 4.3: Histograms of $\sigma^2_\varepsilon$ (true value = 0.05; independent-site scenario).

Table 4.1: Comparison of the posterior regional parameters derived from independent- and correlated-site analyses for 30 sites using the “typical” synthetic data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>&quot;True&quot; value</th>
<th>Independent Mean</th>
<th>SD</th>
<th>Correlated Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.50</td>
<td>0.4952</td>
<td>0.0188</td>
<td>0.4957</td>
<td>0.0190</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>5.00</td>
<td>5.0000</td>
<td>0.1077</td>
<td>4.9982</td>
<td>0.1093</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.05</td>
<td>0.0720</td>
<td>0.0257</td>
<td>0.0727</td>
<td>0.0257</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.20</td>
<td>0.2645</td>
<td>0.0513</td>
<td>0.2633</td>
<td>0.0517</td>
</tr>
<tr>
<td>$\sigma^2_\delta$</td>
<td>0.10</td>
<td>0.0744</td>
<td>0.0224</td>
<td>0.0749</td>
<td>0.0226</td>
</tr>
</tbody>
</table>
CHAPTER 4. A BAYESIAN HIERARCHICAL REGIONAL FLOOD MODEL

Analysis of “atypical” synthetic data with small variance terms

For the “atypical” synthetic data experiment, the regional parameters are: \( \beta = \{0.5, 5.0\} \), \( \sigma^2_\varepsilon = 0.0001 \), \( \gamma = 0.2 \), and \( \sigma^2_\delta = 0.0005 \). While the use of near-zero values for the regional variance terms \((\sigma^2_\varepsilon, \sigma^2_\delta)\) is somewhat atypical, these near-zero values (or even zero values) sometimes occur when using GLS procedures, as noted in Section 4.2 — see Reis et al. (2005) for a recent application where the variance term (their model error variance \( \sigma^2_\delta \)) takes on values on or near zero.

An independent-site run was performed for 30 sites, but was found to be unsatisfactory. The R statistic was near one, but acceptance rates were too low to foster confidence in the results — the maximum acceptance rate was about 5%, with the sites in one sequence having rates around 0.2%. This is not unexpected because the “regional prior/variance” is so small that it is dominating the observed data (i.e. \( \sigma_\varepsilon \ll \sigma_i \)), meaning that the initial starting value \( \theta_S^{(0)} \) is quite poor because the influence of the regional prior has been ignored. This situation can be understood by consideration of Figure 4.5. Under normal circumstances, the regional prior is quite flat and has less information content than the
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Figure 4.5: Schematic representation of distributions for a site mean or standard deviation for “typical” and “atypical” cases.

likelihood (e.g. $\sigma_{\epsilon} > \sigma_i$), so the posterior is dominated by the likelihood — this is shown in (a) — and the high-posterior region is focused around the mode of the likelihood. Under atypical circumstances, the regional prior is “spiked” and very much dominates the posterior so that the likelihood has negligible influence (e.g. $\sigma_{\epsilon} \ll \sigma_i$) — as shown in (b) — and the high-posterior region is far removed from the mode of the likelihood. The limited width of the prior spike has a large influence on the convergence behaviour because it makes it very unlikely for the Gibbs sampler to actually sample within the spike, so few acceptances can occur. The problem is compounded because of the interdependence of the regional parameters and the site parameters in the Gibbs sampling procedure (see Equations (4.65) to (4.76)).

Several approaches to improve on this initial starting point (and proposal) were investigated. The first was a “pooled” or weighted estimate combining the initial site parameters, an initial estimate of the regional parameters\(^7\), and the initial proposal. The mean and variance of the proposal distribution, for each element of $\theta_S$, were calculated (for example Ang and Tang, 2007, pp. 361–7):

\[
\mu'' = \frac{\mu' (\sigma^2/n) + \bar{x} (\sigma')^2}{(\sigma^2/n) + (\sigma')^2} \tag{4.84}
\]

\[
(\sigma'')^2 = \frac{(\sigma^2/n)(\sigma')^2}{(\sigma^2/n) + (\sigma')^2} \tag{4.85}
\]

where $\mu''$ and $\sigma''$ denote the pooled proposal mean and variance estimate (for each element of $\theta_S$), $\mu'$ and $\sigma'$ are obtained using the initial estimate of the regional parameters, $\bar{x}$ is the initial site-parameter value, and $(\sigma^2/n)$ is the initial proposal variance (obtained from the Hessian). Next, a “greedy start” procedure, where the mean and covariance of the proposal were estimated using only the first (say) 50 accepted samples (for example Haario et al., 2001, p. 226), was tested. The greedy start procedure performed slightly better than the pooled estimate; however, it is not an ideal solution because problems with poor acceptance rates remained. Thus, a more robust approach is needed.

\(^7\)The initial estimate of the regional parameters is obtained using ordinary least squares regression on the initial site parameters (see Equations (4.36) and (4.37)).
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To ensure that appropriately low values of $\sigma^2_\varepsilon$ and $\sigma^2_\delta$ were able to be sampled, a uniform sampling procedure was implemented for $\sigma^2_\varepsilon$ and $\sigma^2_\delta$. This involves a slight modification of the existing procedure with a uniform proposal distribution being used for these (variance) terms via a Metropolis-within-Gibbs step, rather than the inverse chi squared distribution with the “standard” Gibbs step — this new procedure is described below.

An independent-site analysis using the “uniform sampling” procedure was performed with 30 sites. The analysis proceeded with good acceptance rates (31–46% for the site parameters and 11–13% for the two regional variance terms) and produced R statistics near 1.0, using three sequences. Histograms of the variances, $\sigma^2_\varepsilon$ and $\sigma^2_\delta$, and the $\gamma$ parameter are given in Figure 4.6. The posteriors of the variance parameters ((a) and (b)) are somewhat “J shaped”, having a mode near zero; however, the true value of these parameters has been located within the region of high posterior density indicating that the uniform sampling approach has worked satisfactorily in this difficult situation — the uniform sampling procedure should also be suitable for the scenario described in Reis et al. (2005).

Modified Metropolis-within-Gibbs procedure for regional variance terms

A Metropolis-within-Gibbs step, using a uniform proposal distribution, is described to ensure that appropriately low values of the regional variance terms $\sigma^2_\varepsilon$ and $\sigma^2_\delta$ are able to be sampled. The uniform distribution used has a lower bound of zero, while the upper bound is set empirically by scaling the modal ordinary least squares estimators for $\sigma^2_\varepsilon$ and $\sigma^2_\delta$ (using Equations (4.36) and (4.37)). This scaling allows the domain of the problem to be reduced considerably (from a theoretical maximum of $\infty$), with a scaling factor $k_{OLS} = 2$ found to be adequate — the adequacy of the scaling factor can easily be checked by analysing the histograms of the posterior samples and ensuring that the right-hand tail is not truncated. If the tail is truncated, then the scaling factor should be increased and the posterior rechecked. Note, assuming the upper limit is specified correctly, the uniform sampling procedure is guaranteed to work because it is a “brute force” approach and will eventually sample near the “true” value; however, this method may be quite inefficient. Also note that the uniform proposal distribution is symmetric, so the Metropolis algorithm can be used.

Using Bayes Theorem, the full conditional posterior is

$$p(\sigma^2_\varepsilon | \beta, \sigma^2_\delta, \gamma, \theta_S, Y) \propto p(Y | \sigma^2_\varepsilon, \beta, \sigma^2_\delta, \gamma, \theta_S)p(\sigma^2_\varepsilon | \beta, \sigma^2_\delta, \gamma, \theta_S) \propto p(Y | \theta_S)p(\sigma^2_\varepsilon | \beta, \sigma^2_\delta, \gamma, \theta_S)$$ (4.86)

where these terms are evaluated using Equations (4.78) and (4.79). Similarly, the full conditional posterior for the $\sigma^2_\delta$ term is

$$p(\sigma^2_\delta | \sigma^2_\varepsilon, \beta, \gamma, \theta_S, Y) \propto p(Y | \theta_S)p(\sigma^2_\delta | \sigma^2_\varepsilon, \beta, \gamma, \theta_S)$$ (4.87)

The Metropolis-within-Gibbs procedure is summarised for a single iteration for the $\sigma^2_\varepsilon$ term:
Figure 4.6: Histograms of regional parameters (a) $\sigma^2_{\varepsilon}$, (b) $\sigma^2_\delta$, and (c) $\gamma$ for 30 sites (true values: (a) 0.0001, (b) 0.0005, and (c) 0.2; independent-site scenario using the uniform sampling procedure). Note, for clarity, (a) zooms in on the region of highest posterior density for $\sigma^2_{\varepsilon}$ and the right-hand tail continues (and is not truncated).
1. Sample a trial parameter $\sigma_2^* \epsilon^{*}$ from a symmetric proposal distribution $q(\sigma_2^* \epsilon^{*} | \sigma_2^2(j-1)) = U(0, k_{OLS} \times \hat{\sigma}_2^2)$, where $k_{OLS}$ is defined above and $\hat{\sigma}_2^2$ indicates the ordinary least squares estimator of $\sigma_2^2$ (Equation (4.36)).

2. Calculate the ratio of densities:

$$\begin{align*}
r &= \frac{p(Y | \theta_S^{(j-1)})p(\sigma_2^2 \epsilon^{*} | \beta(j-1), \sigma_2^2(j-1), \gamma(j-1), \theta_S^{(j-1)})}{p(Y | \theta_S^{(j-1)})p(\sigma_2^2(j-1) | \beta(j-1), \sigma_2^2(j-1), \gamma(j-1), \theta_S^{(j-1)})}
\end{align*}$$

(4.88)

using Equations (4.78) and (4.79).

Similarly, the Metropolis-within-Gibbs procedure for a single iteration for the $\sigma_2^2$ term is:

1. Sample a trial parameter $\sigma_2^* \delta^{*}$ from a symmetric proposal distribution $q(\sigma_2^* \delta^{*} | \sigma_2^2(j-1)) = U(0, k_{OLS} \times \hat{\sigma}_2^2)$.

2. Calculate the ratio of densities:

$$\begin{align*}
r &= \frac{p(Y | \theta_S^{(j-1)})p(\sigma_2^2 \delta^{*} | \sigma_2^2(j-1), \beta(j), \gamma(j-1), \theta_S^{(j-1)})}{p(Y | \theta_S^{(j-1)})p(\sigma_2^2(j-1) | \sigma_2^2(j-1), \beta(j), \gamma(j-1), \theta_S^{(j-1)})}
\end{align*}$$

(4.89)

Summary

The synthetic data studies, which generated data assuming that there was no correlation between sites, have provided good evidence that the Gibbs sampler procedure has been implemented correctly. For the “typical” data, both the independent- and correlated-site implementations identified the “true” regional parameters; indeed, they produced almost identical results to each other. Using the uniform sampling procedure for the “atypical” data, the posterior distributions of the regional parameters were consistent with their “true” values. Note that the “normal” Gibbs sampler procedure (as described in Section 4.4.3) is the first choice method on account of its efficiency, and that the uniform sampling procedure need only be used in “atypical” situations where the normal Gibbs sampler procedure encounters convergence difficulties.

4.4.5 Predicting the flood frequency distribution at a new site

In this section, attention turns away from the regional model to the prediction of the flood frequency distribution at a new site, where this site was not used in the development of the regional model. This new site may have no gauged data (i.e. ungauged) or may have (possibly limited) gauged data. The objective is to develop a rigorous Bayesian solution to this problem.

Prediction at an ungauged site

Consider first an ungauged site. Out of necessity, the flood frequency distribution must be derived solely from the regional flood model.
**Bayesian predictive distribution** The predictive pdf at an ungauged site can be evaluated using total probability to yield:

\[
p(q \mid Y, x) = \sum_{i=1}^{2} \int p(q \mid \theta_S, IPO_i)p(\theta_S \mid Y, x, IPO_i)d\theta_SP(IPO_i)
\]  

(4.90)

where \(q\) is the (annual maximum) flood at the site in question; \(Y\) is the observed flood data (that have been used in the derivation of the regional model); \(x\) is the catchment characteristics vector; \(\theta_S = \{\mu, \log \sigma\}\); IPO\(_i\) denotes the IPO epoch (IPO- or IPO+); \(P(IPO_i)\) is the probability of IPO epoch \(i\) occurring in any year (see page 75); and \(p(\theta_S \mid Y, x, IPO_i)\) is the posterior predictive pdf of \(\theta_S\) (from the regional distribution). This integral can be evaluated using the Monte Carlo importance sampling methods (for example Arulampalam et al., 2002; Press et al., 1992).

**Monte Carlo importance sampling** To understand importance sampling, Monte Carlo integration is first reviewed through consideration of the multidimensional integral

\[
I = \int f(a)da
\]  

(4.91)

Define a pdf \(p(a)\) which spans the domain of integration, so the integral can be rearranged to yield:

\[
I = \int f(a)da = \int \frac{f(a)}{p(a)}p(a)da = E\left(\frac{f(a)}{p(a)}\right) \approx \frac{1}{N} \sum_{i=1}^{N} f(a_i) \frac{1}{p(a_i)}, \quad a_i \leftarrow p(a)
\]  

(4.92)

This shows that \(I\) is simply the expected value of the function \(f(a)/p(a)\).

To illustrate importance (or particle) sampling, consider the expected value of \(g(a)\)

\[
E(g(a)) = \int g(a)f(a)da
\]  

(4.93)

which may be computed using Monte Carlo integration to yield

\[
E(g(a)) \approx \frac{1}{N} \sum_{i=1}^{N} g(a_i), \quad a_i \leftarrow f(a)
\]  

(4.94)

In this case, each sample from \(f(a)\) is given an equal weight of \(1/N\) — this weight can be considered as the probability mass assigned to the sample \(\{a_i; i = 1, \ldots, N\}\). If it is difficult to sample from the density \(f(a)\), a more convenient sampling distribution \(p(a)\)
may be used:
\[
E(g(a)) = \int g(a)f(a)da = \int g(a)\frac{f(a)}{p(a)}p(a)da
\] (4.95)
which may be approximated:
\[
E(g(a)) \approx \frac{1}{N} \sum_{i=1}^{N} g(a_i)\frac{f(a_i)}{p(a_i)}, \quad a_i \sim p(a)
\] (4.96)
This leads to the following interpretation: the \(N\) samples \(\{a_i; i = 1, \ldots, N\}\) (often called “particles”) are samples from \(f(a)\) with probability mass (or weight)
\[
P(a_i) = w_i = \frac{1}{N} \frac{f(a_i)}{p(a_i)}, \quad i = 1, \ldots, N
\] (4.97)
Note that the pdfs \(f(a)\) and \(p(a)\) need only be computable up to a normalising constant \(K\). For example, suppose that \(f(a) = Kh(a)\) (4.98)
The sum of the particle probabilities must equal one because \(f(a)\) is a pdf, so the particle probability mass is:
\[
P(a_i) = \frac{Kh(a_i)}{\sum_{k=1}^{N} Kh(a_k)} = \frac{h(a_i)}{\sum_{k=1}^{N} h(a_k)}, \quad i = 1, \ldots, N
\] (4.99)
showing that the normalising constant cancels out.

**Evaluation of the integral**  The predictive pdf integral in Equation (4.90) can now be rewritten
\[
I = \int p(q | \theta_S, IPO)p(\theta_S | Y, x, IPO) \theta_S
\]
\[
\approx \frac{1}{N} \sum_{j=1}^{N} p(q | \theta_S^{(j)}, IPO), \quad \theta_S^{(j)} \sim p(\theta_S | Y, x, IPO)
\] (4.100)
where \(\theta_S^{(j)}\) is obtained in multiple steps. First, randomly sample the IPO epoch, then \(\theta_R^{(j)} \sim p(\theta_R | Y, x, IPO) = p(\theta_R | Y, IPO)\), which is a regional-parameter sample from the Gibbs sampler, and finally \(\theta_S^{(j)} \sim p(\theta_S | \theta_R, Y, x)\), using Equations (4.38) and (4.39). Note that each particle is given equal weight.

**Posterior of flood quantiles**  If one is interested in the posterior distribution of the selected flood quantiles, the above considerations lead to the following algorithm:

1. Randomly sample the IPO epoch: \(IPO^{(j)} \sim P(IPO)\).
2. Randomly sample \(\theta_S^{(j)} = \{\mu^{(j)}, \log \sigma^{(j)}\}\) from the regional distribution in two steps: \(\theta_R^{(j)} \sim p(\theta_R | Y, IPO^{(j)})\) and \(\theta_S^{(j)} \sim p(\theta_S | \theta_R^{(j)}, x)\).
CHAPTER 4. A BAYESIAN HIERARCHICAL REGIONAL FLOOD MODEL

3. Compute the $T$-year flood: $q_T^{(j)} = \mu^{(j)} + z_T\sigma^{(j)}$, where $z_T$ is the standard normal deviate corresponding to an annual exceedance probability of $1/T$ (i.e. an ARI of $T$ years).

4. Repeat steps 1 to 3 for $j = 1, \ldots, N$.

5. Sort $q_T$ and extract probability limits.

Probability of an IPO epoch The probability $P(IPO_i)$ is the probability of observing IPO epoch $i$ (i.e. $P(IPO-)\text{ and } P(IPO+)$), which is then used as a weighting when calculating the flood frequency distribution at a new site. Since the long-run flood risk is usually desired, $P(IPO-i)$ should be calculated using the longest record possible, so as to best represent the long-run conditions — either from the IPO time series in Figure 3.1 (i.e. $P(IPO-) \approx 0.45$), or, even better, from the reconstructed IPO record in Verdon and Franks (2006, Figure 3) (i.e. $P(IPO-) \approx 0.50$). Thus, $P(IPO-) = P(IPO+) = 0.50$ should be used. Note that the values used for $P(IPO-i)$ and $P(IPO+) should have a much-reduced impact on the resultant flood frequency distribution.

Prediction at a gauged site

Suppose at the site under consideration there exists some gauged data $y_g$, where $y_g$ is a vector of length $n_g$. This can be used to augment the information contained in the regional model. The predictive pdf becomes:

$$ p(q \mid y_g, Y, x) = \frac{1}{k} \sum_{i=1}^{N} p(q \mid \theta_S, IPO_i)p(y_g \mid \theta_S, Y, x, IPO_i) \left\{ p(\theta_S \mid Y, x, IPO_i) \right\} \text{d}\theta_S P(IPO_i) \tag{4.101} $$

where the integral can be rewritten, according to Equation (4.95), as:

$$ I = \int p(q \mid \theta_S, IPO)p(\theta_S \mid y_g, Y, x, IPO) \text{d}\theta_S $$

$$ = \int p(q \mid \theta_S, IPO) \left\{ \frac{p(y_g \mid \theta_S, Y, x, IPO)p(\theta_S \mid Y, x, IPO)}{p(y_g \mid Y, x, IPO)} \right\} \text{d}\theta_S $$

$$ = k \int p(q \mid \theta_S, IPO)p(y_g \mid \theta_S)p(\theta_S \mid Y, x, IPO) \text{d}\theta_S $$

$$ \approx \frac{k}{N} \sum_{j=1}^{N} p(q \mid \theta_S^{(j)}, IPO)p(y_g \mid \theta_S^{(j)}) \left\{ p(\theta_S^{(j)} \mid Y, x, IPO) \right\} \tag{4.102} $$

where $k = 1/p(y_g \mid Y, x, IPO)$ is the normalising constant from the use of Bayes Theorem.

Equation (4.102) shows how one can sample from the posterior $p(\theta_S \mid y_g, Y, x, IPO)$. The $p(y_g \mid \theta_S)$ term, the product of $n_g$ normal pdfs (i.e. similar to Equation (4.78)), can be interpreted as the “particle weight” for importance sampling from the posterior.
CHAPTER 4. A BAYESIAN HIERARCHICAL REGIONAL FLOOD MODEL

\( p(\theta_S \mid y_g, Y, x, IPO) \). The particle weights are normalised

\[
    w^{(j)} = \frac{p(y_g \mid \theta_S^{(j)})}{\sum_{k=1}^{N} p(y_g \mid \theta_S^{(k)})}
\]  

(4.103)

This leads to the following algorithm to obtain the posterior distribution of a flood quantile:

1. Randomly sample the IPO epoch: \( IPO^{(j)} \leftarrow P(\text{IPO}) \).
2. Randomly sample \( \theta_S^{(j)} = \{\mu^{(j)}, \log \sigma^{(j)}\} \) from the regional model.
3. Compute the \( T \)-year flood: \( q_T^{(j)} = \mu^{(j)} + z_T \sigma^{(j)} \).
4. Compute the (unnormalised) particle weight: \( p(y_g \mid \theta_S^{(j)}) \).
5. Repeat steps 1–4 for all \( j = 1, \ldots, N \) posterior samples.
6. Compute the normalised particle weights \( w^{(j)} \) for all \( j = 1, \ldots, N \) (using Equation (4.103)).
7. Sort \( q_T \) and compute the (sorted) cumulative particle weights \( W^{(j)} \) for all \( j = 1, \ldots, N \).
8. Extract probability limits using the cumulative particle weights.

Summary

This section has described a new and general approach to inferring the Bayesian predictive distribution and the posterior distribution of selected quantiles for a site which may be ungauged or may have some gauged data, whose information can be augmented by the regional model. The approach incorporates the \( \theta_R \) samples generated by the Gibbs sampler in the regional model with the aid of importance sampling.

4.5 Discussion

The hierarchical model implementation presented in this chapter has assumed that the data is lognormally distributed. Other distributions, such as the log-Pearson 3 (LP3) or generalised extreme value, may be more appropriate for other flood data; however, if the corresponding, explicit, multivariate distribution is not easily available, then the hierarchical model must then assume the sites to be independent and correlation not be considered. For example, assuming that the site data is LP3 distributed — i.e. the logarithm of the site data \( y \) have a Pearson 3 distribution with parameters mean \( \mu \), standard deviation \( \sigma \), and skewness \( g \) — the site model would be

\[
y_i \sim P3(\mu_i, \sigma_i^2, g_i)
\]  

(4.104)
while the regional model would become

\[
\mu_i \sim N(x_i^T \beta, \sigma^2)
\] (4.105)

\[
\log \sigma_i \sim N(\gamma, \sigma^2)
\] (4.106)

\[
g_i \sim X(a, b)
\] (4.107)

where \(X(a, b)\) denotes a yet-to-be-specified distribution with parameters \(a\) and \(b\). For example, \(X(a, b)\) may be chosen to be a normal distribution, where its parameters \(a\) and \(b\) would be estimated using a Metropolis-within-Gibbs step.

Alternatively, if the flood data are not lognormally distributed — and the corresponding, explicit, multivariate distribution is not easily available — a correlated-site analysis may still be performed by employing a more general transformation to render the transformed data normally distributed. For example, a Box-Cox transformation (Kuczera, 1983a) or the normal quantile transformation (Krzysztofowicz, 1997) may be used to normalise the flood data.

4.6 Conclusion

A regression-based hierarchical regional flood model has been developed. It has been implemented using Bayesian methods allowing for a rigorous treatment of the flood regionalisation problem. The model has many desirable properties including the ability to incorporate the IPO modulation of flood risk present within the eastern Australian flood record. It also can deal with spatial correlation between sites and missing data within the flood record. Moreover, it can rigorously allow for uncertainty in the regional model parameters.

The performance of this Bayesian hierarchical regional flood model was investigated using synthetic flood data and was found to be consistent with the synthetic data parameters. This provides confidence in the software implementation of the inference algorithms which will be used in the next chapter.

Algorithms have been developed to compute the Bayesian predictive distribution and the posterior distribution of quantiles for sites with and without gauged data. An approach based on importance sampling makes effective use of the regional parameters obtained by the Gibbs sampler when calibrating the regional flood model. This represents the first truly-general Bayesian solution for combining gauged and regional information in flood frequency analysis.

The performance of this Bayesian hierarchical regional flood model is investigated using IPO-influenced eastern Australian flood data in the next chapter.
Chapter 5

Case study investigations of eastern Australian flood data using the Bayesian hierarchical flood model

5.1 Overview

The previous chapter developed a Bayesian hierarchical regional flood model and investigated its performance using synthetic data sets. This chapter analyses the flood frequency data from eastern Australia, described in the earlier chapters, using the Bayesian hierarchical model for both IPO- and IPO+ epochs. Analyses are performed using both independent- and correlated-site models. The model from Chapter 4 is revised in light of evidence of strong correlation between model errors from both IPO epochs and the analyses are done using data from both IPO epochs jointly. Predictions of long-run (marginal) flood risk are made for both ungauged and gauged sites, with the uncertainty associated with predictions being quantified. Finally, an assessment of the value of the regional flood model, to avoid bias in marginal flood risk due to IPO modulation, is made.

The objectives of this chapter are to:

1. Implement the Bayesian hierarchical model using eastern Australian flood data.

2. Determine whether the regional model can detect the IPO modulation that has been identified in the at-site data (i.e. is there a difference in regional models between the IPO epochs?).

3. Demonstrate the pooling of regional and gauged information using the methods described in Chapter 4.
CHAPTER 5. CASE STUDY INVESTIGATIONS

5.2 Case studies

The case studies implement the Bayesian hierarchical model using flood data from the eastern Australian states of New South Wales and Queensland (as described in Chapters 2 and 3). Before the regional analysis can be performed, however, the variables to be used in the site-mean regression (see Equation (4.36)) must be defined and the regions delineated.

5.2.1 Selection of catchment variables for the site-mean regression equation

This study uses two variables (area and rainfall) for the site-mean regression. These have been found (or analogues thereof) to, generally, have a large influence on streamflow characteristics, such as the mean annual flood (for example Thomas and Benson, 1970; Acreman, 1985; Mimikou and Gordios, 1989; Nathan and McMahon, 1990; Meigh et al., 1997). Note that the selection of an optimal set of catchment descriptors for the site-mean regression is not an objective of this thesis; it is noted that Bayes factors (Kass and Raftery, 1995; DiCiccio et al., 1997; Marshall et al., 2005) may be a useful tool were this to be attempted.

It is assumed that the site mean is a function of the logarithm\(^1\) of the catchment area \(A\) and the 2-year, 12-hour rainfall intensity \(IFD\), while the (logarithm of the) site standard deviation has no dependent variables (as noted earlier in Section 4.3). Thus,

\[
X = \begin{bmatrix}
\log A_1 & IFD_1 & 1 \\
\vdots & \vdots & \vdots \\
\log A_n & IFD_n & 1
\end{bmatrix}
\quad \text{and} \quad
Z = \begin{bmatrix}
1 \\
\vdots \\
1
\end{bmatrix}
\]

(5.1)

Note that these rainfall intensity data \(IFD\) were obtained from the maps in *Australian Rainfall and Runoff* (Canterford, 1987). It is noted that these maps were derived disregarding the effects of IPO modulation. Therefore, the \(IFD\) variable refers to the long-run 2-year intensity.

5.2.2 Delineation of regions

The data from each state was split into two geographical regions, giving a total of four regions — these regions are shown in Figure 5.1. Table 5.1 summarises statistics for the variables used for each region. The rationale for the selection of the regions is the spatial distribution of the IPO modulation of flood risk, as shown in the map of flood ratios in Figure 3.9. The northern Queensland region (region 1) consists of sites above the 22°S line of latitude, which show little IPO modulation of at-site flood risk. Southern Queensland (region 2) experiences quite large IPO modulation of flood risk, especially in the southeast corner. At-site flood risk in northern New South Wales (region 3) is also significantly affected by IPO modulation, particularly in the northeast corner. The sites in region 4 in southern New South Wales are also affected by the IPO modulation of at-

\(^1\)Natural logarithms are used.
site flood risk; however, these sites are quite a distance away from the “hot spot” around southeastern Queensland and northeastern New South Wales and so are allocated to their own region. Note that there is only a total of 127 sites. This limits the opportunity to subdivide the overall region into further subregions — four regions with region 4 having only 16 sites is judged to be the limit.

5.2.3 Results of regional analysis for IPO-stratified flood data

The flood data for each of the four regions were stratified into data sets corresponding to years where IPO < 0 (IPO- or IPO negative) and years where IPO ≥ 0 (IPO+ or IPO positive) — Figure 5.2 presents histograms of record lengths for both IPO epochs. These IPO-stratified data sets were analysed using the hierarchical model with the aid of the Gibbs sampler described in Section 4.4.3. These analyses were performed for both the independent- and correlated-site hierarchical models, resulting in a grand total of sixteen model calibrations (eight for independent and eight for correlated). The Gibbs sampler runs used three sequences and were allowed to “burn in” for 50000 iterations, where the first 15000 were used to adaptively scale the (normal/Gaussian) proposal distribution for the site parameters so as to improve the acceptance rate (see Section 4.4.3), with a further 5000 iterations being stored (giving 15000 samples). Very good acceptance rates (around 30–40%) and small R statistics (i.e. below 1.2, and quite near to 1.0; see Section 4.4.3) were achieved, providing strong evidence that convergence to the posterior distribution.
CHAPTER 5. CASE STUDY INVESTIGATIONS

Table 5.1: Summary of basin variables\textsuperscript{a,b}.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>All four regions (127 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log A$</td>
<td>2.3</td>
<td>10.9</td>
<td>6.7</td>
<td>2.0</td>
</tr>
<tr>
<td>$IFD$</td>
<td>3.55</td>
<td>22.5</td>
<td>8.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Region 1 (North Qld, 49 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log A$</td>
<td>2.7</td>
<td>10.8</td>
<td>6.5</td>
<td>1.7</td>
</tr>
<tr>
<td>$IFD$</td>
<td>6.5</td>
<td>22.5</td>
<td>11.5</td>
<td>3.9</td>
</tr>
<tr>
<td>Region 2 (South Qld, 36 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log A$</td>
<td>2.3</td>
<td>10.9</td>
<td>6.6</td>
<td>2.2</td>
</tr>
<tr>
<td>$IFD$</td>
<td>5.3</td>
<td>12.8</td>
<td>8.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Region 3 (North NSW, 26 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log A$</td>
<td>3.4</td>
<td>10.2</td>
<td>7.4</td>
<td>2.0</td>
</tr>
<tr>
<td>$IFD$</td>
<td>4.4</td>
<td>11.5</td>
<td>6.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Region 4 (South NSW, 16 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log A$</td>
<td>3.3</td>
<td>10.2</td>
<td>6.7</td>
<td>2.4</td>
</tr>
<tr>
<td>$IFD$</td>
<td>3.55</td>
<td>13.0</td>
<td>5.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Units of $A$ and $IFD$ are km\textsuperscript{2} and mm/h, respectively.
\textsuperscript{b} Natural logarithms are used.

Table 5.2: Summary of the modal correlation parameters\textsuperscript{a} (and associated variables\textsuperscript{b}).

<table>
<thead>
<tr>
<th>Region</th>
<th>$\log A$ IPO-</th>
<th>$\log A$ IPO+</th>
<th>$\log B$ IPO-</th>
<th>$\log B$ IPO+</th>
<th>CLS (km) IPO-</th>
<th>CLS (km) IPO+</th>
<th>$d_{av}$ (km) IPO-</th>
<th>$d_{av}$ (km) IPO+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.3754</td>
<td>5.3410</td>
<td>-0.7143</td>
<td>-0.4259</td>
<td>587</td>
<td>209</td>
<td>385</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.8982</td>
<td>5.4506</td>
<td>-0.2383</td>
<td>-0.4850</td>
<td>364</td>
<td>233</td>
<td>306</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.9542</td>
<td>5.4450</td>
<td>-0.3810</td>
<td>-0.2966</td>
<td>385</td>
<td>232</td>
<td>236</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5.6032</td>
<td>5.7442</td>
<td>-0.7637</td>
<td>-0.5356</td>
<td>271</td>
<td>312</td>
<td>217</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a} The correlation parameters $A$ and $B$ are defined in Equation (4.35).
\textsuperscript{b} CLS is the correlation length scale and $d_{av}$ is the average distance between sites.

had occurred. The modal correlation parameters and the posterior distributions of the regional parameters are summarised in Tables 5.2 and 5.3, respectively.

Several questions now arise. Is there a difference in the regional models between the IPO epochs (i.e. is the IPO stratification warranted for the regional model)? Does the use of a correlated-site model improve the estimation of regional distribution over the use of the simpler independent-site model? These issues will be investigated in the remainder of this chapter.

Initial examination of the regional parameters in Table 5.3 indicates some trends. For the independent-site analyses, the $\beta_3$ and $\gamma$ terms are larger during the IPO- epochs for regions 2–4, while the $\beta_1$ and $\beta_2$ terms are quite similar, leading to generally larger means $\mu$ and standard deviations $\sigma$ during the IPO- epoch; while region 1 does not show the same pattern. For the correlated-site analyses, no trends are apparent. However, further examination of the regional parameters reveals that there is no apparent significant difference between the IPO- and IPO+ epochs, for both the independent- and correlated-
site analyses, after consideration of the associated standard deviations. This appears to contradict the earlier results in Chapter 3. The regional model appears to be affected by considerable noise, due to the large areal extent of the regions (see Figure 5.1), the wide range of areas, and the possible nonhomogeneity of the regions due to the geographical nature of the regional delineation. The noise in the regional model appears to swamp any differences due to IPO. This can easily be demonstrated by estimating the information content of the regional model for the mean, expressed as an equivalent record length. For a sample of size \( n \), the variance of the arithmetic mean is \( \text{Var}(\bar{x}) = \frac{\sigma^2}{n} \); thus, the variance of the site mean can be stated as \( \text{Var}(\mu) = \frac{\sigma^2}{n_e} \), where \( \sigma^2 \) is the site variance and \( n_e \) is the equivalent record length. If the variability in \( \beta \) is ignored (i.e. \( \text{Var}(\beta) = 0 \), see Equation (4.38)), then \( \text{Var}(\mu) = \sigma^2_e \), while \( \sigma^2 \) is taken as the expected site variance (see Equation (4.39)), so the equivalent record length is:

\[
n_e = \frac{[\exp(\gamma)]^2}{\sigma^2_e} \tag{5.2}
\]

Table 5.4 summarises the equivalent record lengths provided by the regional models. The record lengths range from 2–12 years. However, these are overestimates of the equivalent record lengths (since \( \text{Var}(\beta) \) is assumed to be zero, which Table 5.3 shows is not the case), so it is not surprising that the regional model cannot detect IPO-related differences in the regional parameters. Indeed, with the short equivalent record lengths, the sampling variability swamps any IPO-related differences. Noting that \( \exp(\mu) \) is the 2-year flood
### Table 5.3: Summary of the posterior regional parameters derived from independent- and correlated-site analyses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Independent</th>
<th></th>
<th></th>
<th></th>
<th>Correlated</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IPO- Mean</td>
<td>SD</td>
<td>IPO+ Mean</td>
<td>SD</td>
<td>IPO- Mean</td>
<td>SD</td>
<td>IPO+ Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Region 1 (North Queensland, 49 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.6708</td>
<td>0.0625</td>
<td>0.6273</td>
<td>0.0566</td>
<td>0.6350</td>
<td>0.0559</td>
<td>0.6073</td>
<td>0.0589</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.1270</td>
<td>0.0270</td>
<td>0.1256</td>
<td>0.0250</td>
<td>0.1139</td>
<td>0.0246</td>
<td>0.1185</td>
<td>0.0258</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.7150</td>
<td>0.6508</td>
<td>4.8471</td>
<td>0.5943</td>
<td>4.8140</td>
<td>0.5957</td>
<td>5.0893</td>
<td>0.6338</td>
</tr>
<tr>
<td>$\sigma^2_2$</td>
<td>0.2874</td>
<td>0.0774</td>
<td>0.2523</td>
<td>0.0674</td>
<td>0.2421</td>
<td>0.0641</td>
<td>0.2694</td>
<td>0.0660</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.2065</td>
<td>0.0728</td>
<td>0.0272</td>
<td>0.0523</td>
<td>-0.0272</td>
<td>0.0574</td>
<td>0.0454</td>
<td>0.0456</td>
</tr>
<tr>
<td>$\sigma^2_3$</td>
<td>0.1914</td>
<td>0.0541</td>
<td>0.1076</td>
<td>0.0482</td>
<td>0.0814</td>
<td>0.0389</td>
<td>0.0556</td>
<td>0.0182</td>
</tr>
<tr>
<td>Region 2 (South Queensland, 36 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.5171</td>
<td>0.0557</td>
<td>0.5150</td>
<td>0.0580</td>
<td>0.6118</td>
<td>0.0577</td>
<td>0.5256</td>
<td>0.0618</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.2716</td>
<td>0.0561</td>
<td>0.2866</td>
<td>0.0618</td>
<td>0.3000</td>
<td>0.0561</td>
<td>0.2885</td>
<td>0.0633</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.2411</td>
<td>0.7417</td>
<td>3.7420</td>
<td>0.7872</td>
<td>3.1757</td>
<td>0.7954</td>
<td>3.7049</td>
<td>0.8509</td>
</tr>
<tr>
<td>$\sigma^2_2$</td>
<td>0.1924</td>
<td>0.0882</td>
<td>0.3314</td>
<td>0.1015</td>
<td>0.1799</td>
<td>0.0663</td>
<td>0.3282</td>
<td>0.0968</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3083</td>
<td>0.0414</td>
<td>0.2541</td>
<td>0.0437</td>
<td>0.3685</td>
<td>0.0481</td>
<td>0.2407</td>
<td>0.0460</td>
</tr>
<tr>
<td>$\sigma^2_3$</td>
<td>0.0168</td>
<td>0.0183</td>
<td>0.0482</td>
<td>0.0182</td>
<td>0.0389</td>
<td>0.0181</td>
<td>0.0556</td>
<td>0.0182</td>
</tr>
<tr>
<td>Region 3 (North New South Wales, 26 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.6498</td>
<td>0.0646</td>
<td>0.6672</td>
<td>0.0570</td>
<td>0.6669</td>
<td>0.0624</td>
<td>0.6631</td>
<td>0.0584</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.4601</td>
<td>0.0773</td>
<td>0.4322</td>
<td>0.0676</td>
<td>0.4168</td>
<td>0.0765</td>
<td>0.4407</td>
<td>0.0724</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>2.5648</td>
<td>0.8409</td>
<td>2.1232</td>
<td>0.7472</td>
<td>2.6235</td>
<td>0.8654</td>
<td>2.0939</td>
<td>0.8046</td>
</tr>
<tr>
<td>$\sigma^2_2$</td>
<td>0.2720</td>
<td>0.1033</td>
<td>0.2033</td>
<td>0.0771</td>
<td>0.2489</td>
<td>0.0883</td>
<td>0.2207</td>
<td>0.0815</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1042</td>
<td>0.0473</td>
<td>0.0593</td>
<td>0.0467</td>
<td>0.1885</td>
<td>0.0590</td>
<td>0.1447</td>
<td>0.0594</td>
</tr>
<tr>
<td>$\sigma^2_3$</td>
<td>0.0394</td>
<td>0.0182</td>
<td>0.0418</td>
<td>0.0179</td>
<td>0.0716</td>
<td>0.0258</td>
<td>0.0763</td>
<td>0.0267</td>
</tr>
<tr>
<td>Region 4 (South New South Wales, 16 sites)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.5601</td>
<td>0.0963</td>
<td>0.5798</td>
<td>0.0999</td>
<td>0.5482</td>
<td>0.0980</td>
<td>0.5191</td>
<td>0.0893</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.1971</td>
<td>0.0972</td>
<td>0.2196</td>
<td>0.1006</td>
<td>0.1815</td>
<td>0.0979</td>
<td>0.1583</td>
<td>0.0886</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.6225</td>
<td>1.1008</td>
<td>4.0087</td>
<td>1.1504</td>
<td>4.8184</td>
<td>1.1245</td>
<td>4.8639</td>
<td>1.0465</td>
</tr>
<tr>
<td>$\sigma^2_2$</td>
<td>0.3410</td>
<td>0.1863</td>
<td>0.3666</td>
<td>0.2114</td>
<td>0.3661</td>
<td>0.1930</td>
<td>0.2856</td>
<td>0.1617</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1143</td>
<td>0.0781</td>
<td>0.0900</td>
<td>0.0896</td>
<td>0.1364</td>
<td>0.0849</td>
<td>0.2009</td>
<td>0.1133</td>
</tr>
<tr>
<td>$\sigma^2_3$</td>
<td>0.0804</td>
<td>0.0433</td>
<td>0.1092</td>
<td>0.0544</td>
<td>0.0985</td>
<td>0.0500</td>
<td>0.1909</td>
<td>0.0882</td>
</tr>
</tbody>
</table>

quantile, the ratio

$$\frac{\exp(\mu + 2\sigma^2_\mu)}{\exp(\mu)}$$

is approximately the ratio of the upper 95th probability limit (the 97.5th percentile) to the expected value of the 2-year flood quantile and provides a measure of the sampling variability. Typical values are: $\sigma = \exp(0.2) \approx 1.2$ and $n \approx 5$ (from Tables 5.3 and 5.4 respectively); so the (sampling variability-related) ratio is $\exp(0.6) \approx 1.8$, which overwhels the typical IPO-related ratio of about 1.7 for the shorter equivalent record lengths.

Table 5.2 also presents the average distances between sites $d_{av}$ and the correlation length scales (CLSs) within each region — the CLS is the distance at which the intersite correlation drops to $\exp(-1) \approx 0.37$ and provides a distance-based measure of the strength of the correlation that is present in the data set. The fact that $d_{av}$ tends to be less than CLS would suggest that intersite correlation may have an influence for all four IPO-
Table 5.4: Summary of the equivalent record lengths.

<table>
<thead>
<tr>
<th>Region</th>
<th>nₑ (years)</th>
<th>Independent</th>
<th>Correlated</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPO-</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>IPO+</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>IPO-</td>
<td>10</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>IPO+</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>IPO-</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>IPO+</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>IPO+</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

analyses and also for the region 4 IPO+ analysis. However, comparison of the regional parameters from the independent- and correlated-site analyses (Table 5.3) shows little discernible difference between the two sets of analyses.

5.2.4 Analysis of residuals for IPO-stratified flood data

The conclusion that the regional model cannot detect any IPO effect implicitly assumes that the regional analyses for IPO- and IPO+ are statistically independent. It is essential to scrutinise this assumption. Here, posterior diagnostics based on residuals are used to check the assumption of statistical independence.

The residual is defined here as the “estimated value” minus the “true value” so that a positive (negative) residual indicates an over- (under-)estimation of the true value; thus, the residual $r$ for the site mean is

$$ r = x^T \beta - \hat{\mu} $$

(5.4)

where $r$ is the residual and $\hat{\mu}$ is obtained from the maximum likelihood estimate of $\theta_S$.

The residual for the (logarithm of the) site standard deviation is similarly defined.

Is there a difference in regional models between IPO epochs?

The IPO- and IPO+ residuals for the site mean and (logarithm of the) standard deviation are plotted in Figures 5.3 to 5.4 respectively for the independent-site case, and Figures 5.5 to 5.6 respectively for the correlated-site case. The figures show that there is limited scatter and a high degree of correlation between the IPO- and IPO+ residuals. This strong dependence structure between the residuals means that the IPO- and IPO+ regional models are not independent. As a result, inferences based on the assumptions of independence (between IPO epochs) are invalid. To overcome this problem, the regional models need to be analysed jointly (that is, using all the data).

This insight can be further developed by considering the regional parameters in Table 5.3 and the residual plots. The results suggest it is reasonable to assume that $\beta^-_1 \approx \beta^+_1$, $\beta^-_2 \approx \beta^+_2$, and $\epsilon^- \approx \epsilon^+$. Noting the existing regional model for the mean for the two IPO

---

2The residual terms defined here are opposite in sign to to regression error terms (i.e. $r_\mu = -\epsilon$ and $r_{log \sigma} = -\delta$).
CHAPTER 5. CASE STUDY INVESTIGATIONS

Figure 5.3: Residual plots for the site mean comparing IPO- and IPO+ analyses for all four regions (independent-site analysis).

Figure 5.4: Residual plots for the (logarithm of the) site standard deviation comparing IPO- and IPO+ analyses for all four regions (independent-site analysis).
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Figure 5.5: Residual plots for the site mean comparing IPO- and IPO+ analyses for all four regions (correlated-site analysis).

Figure 5.6: Residual plots for the (logarithm of the) site standard deviation comparing IPO- and IPO+ analyses for all four regions (correlated-site analysis).
epochs is:

\[ \mu^- = \beta_1^- \log A + \beta_2^- IFD + \beta_3^- + \epsilon^- \]  
\[ \mu^+ = \beta_1^+ \log A + \beta_2^+ IFD + \beta_3^+ + \epsilon^+ \]  

(5.5)

(5.6)

it follows that the difference in regional IPO-dependent means is largely accounted by the difference in regional intercepts:

\[ \mu^- - \mu^+ = \beta_3^- - \beta_3^+ \]  

(5.7)

5.2.5 Joint procedure for regional model with correlated errors

The regional model must be reformulated to properly represent the strong dependence in its errors. One approach is to jointly model the mean using a dummy indicator variable so that the mean depends on the state of the IPO (e.g. Coles, 2001, pp. 107–108; Theil, 1971, pp. 155–6):

\[ \mu^- = \beta_1 \log A + \beta_2 IFD + \beta_3(IPO^-) + \beta_4(IPO^+) + \epsilon^- \]  
\[ \mu^+ = \beta_1 \log A + \beta_2 IFD + \beta_3(IPO^-) + \beta_4(IPO^+) + \epsilon^+ \]  

(5.8)

(5.9)

where \( I(IPO^-) \) is an indicator function with a value of 1 in IPO- years and 0 otherwise, and \( I(IPO^+) \) is an indicator function with a value of 1 in IPO+ years and 0 otherwise. In effect, each IPO epoch uses a separate intercept and set of model errors. It should also be noted that the \( \beta_1 \) and \( \beta_2 \) parameters (the nonintercept terms) are now forced to be common across both IPO epochs, a reasonable assumption given the results presented in Table 5.3. Similarly, the site standard deviation is modelled:

\[ \log \sigma^- = \gamma_1(IPO^-) + \gamma_2(IPO^+) + \delta^- \]  
\[ \log \sigma^+ = \gamma_1(IPO^-) + \gamma_2(IPO^+) + \delta^+ \]  

(5.10)

(5.11)

Thus, the catchment descriptor matrices become:

\[ X = \begin{bmatrix}
\log A_1 & IFD_1 & 1 & 0 \\
\log A_2 & IFD_1 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\log A_n & IFD_n & 1 & 0 \\
\log A_n & IFD_n & 0 & 1 
\end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\vdots & \vdots \\
0 & 1 
\end{bmatrix} \]  

(5.12)

with each matrix having \( 2n \) rows. Additionally, the site-parameter vector \( \theta_S = \{ \mu, \log \sigma \} \) is now of length \( 4n \), where:

\[ \mu = \{ \mu_1^-, \mu_1^+, \ldots, \mu_n^-, \mu_n^+ \} \]  

(5.13)

\[ \log \sigma = \{ \log \sigma_1^-, \log \sigma_1^+, \ldots, \log \sigma_n^-, \log \sigma_n^+ \} \]  

(5.14)

---

3This discussion focuses on the regional mean; however, a similar procedure is also used for the regional (logarithm of the) standard deviation.
However, this formulation still does not account for the dependence structure in the model errors. The correlation in the model errors must be explicitly accounted for:

$$\text{Cov} (\mu^-, \mu^+) = V = \sigma^2_R R$$  \hfill (5.15)

where $V$ and $R$ are the residual covariance and correlation matrices, respectively (with both matrices having $2n$ rows and columns). Noting the structure of the $\theta_S$ vector and the previous residual plots, the residual correlation matrix is constructed with a banded, tri-diagonal structure:

$$R = \begin{bmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{bmatrix} \ldots$$  \hfill (5.16)

where $\rho$ is the correlation between the IPO- and IPO+ model errors. Determining the correct value for $\rho$ will require iteration — a value is first assumed, the regional analysis performed, and the resultant correlation between residuals checked (and the procedure repeated until the correlation no longer changes appreciably).

**Testing for a difference between IPO epochs**

Referring to Equation (5.7), it is noted that the difference in regional means is largely due to the difference in regional intercept terms:

$$\mu^- - \mu^+ = \beta_3 - \beta_4$$  \hfill (5.17)

From the at-site analyses of Chapter 3, which generally showed increased flood risk in IPO-epochs, it is to be expected that $\beta_3 > \beta_4$ for regions 2 to 4. Thus, to test for a difference in regional mean models between IPO epochs, we must test whether $\beta_3$ is significantly larger than $\beta_4$. Now, the posterior difference in regional mean intercepts is approximately normally distributed:

$$\beta_3 - \beta_4 \sim N \left( \hat{\beta}_3 - \hat{\beta}_4, \text{Var} \left( \hat{\beta}_3 - \hat{\beta}_4 \right) \right)$$  \hfill (5.18)

where

$$\text{Var} \left( \hat{\beta}_3 - \hat{\beta}_4 \right) = \text{Var} \left( \hat{\beta}_3 \right) + \text{Var} \left( \hat{\beta}_4 \right) - 2 \text{Cov} \left( \hat{\beta}_3, \hat{\beta}_4 \right)$$  \hfill (5.19)

with the (co)variances obtained from the Gibbs sampler output. To test for a difference in regional means between IPO epochs, the test statistic $z$ is computed

$$z = \frac{\hat{\beta}_3 - \hat{\beta}_4}{\sqrt{\text{Var} \left( \hat{\beta}_3 - \hat{\beta}_4 \right)}}$$  \hfill (5.20)
which corresponds to the standard normal deviate (i.e. $z$ score). Hence, the corresponding normal cdf can be estimated ($\Phi(z)$) and the nonexceedance probability $1 - \Phi(z)$ can be considered the significance probability (i.e. the probability that $\beta_3 > \beta_4$ by random chance alone)\(^4\).

**Modifications to the Gibbs sampler procedure**

Since the model error correlation matrix $R$ (and the $V$ covariance matrix) is no longer strictly diagonal, the existing ordinary least squares-based procedures used in the Gibbs sampler are no longer applicable. Instead, generalised least squares (GLS)-based procedures (as first discussed in Section 4.2) must be implemented. Additionally, the correlation structure between model errors will require a slight modification of the Metropolis-within-Gibbs step (for the site-parameter subblocks).

The required modifications to the Gibbs sampler procedures described in Section 4.4.3 are now outlined.

**Modification 1: Generalised least squares procedure**

Step 1 of the Gibbs sampler (regional-parameter block) must be modified to use a GLS-based procedure. Since the model error variance matrix $V$ is known up to an unknown scalar factor ($V = \sigma^2_{\epsilon}R$), generalisations of the results used for the original OLS-based procedure (where $V = \sigma^2_{\epsilon}I$; see Section 4.4.3) may be used (see for example Gelman et al., 1995, p. 257).

For the “$\mu$” component of $\theta_R$. The marginal posterior distribution of $\sigma^2_{\epsilon}$ has a scaled inverse $\chi^2$ form, so

$$\sigma^{2(j)}_{\epsilon} \sim p(\sigma^2_{\epsilon} | \mu^{(j-1)}) \sim \chi^{-2} (2n - n_\beta, s^2)$$

(5.21)

where

$$s^2 = \frac{1}{2n - n_\beta} (\mu^{(j-1)} - X\hat{\beta})^T R^{-1} (\mu^{(j-1)} - X\hat{\beta})$$

(5.22)

and

$$\hat{\beta} = (X^T R^{-1} X)^{-1} X^T R^{-1} \mu^{(j-1)}$$

(5.23)

The conditional posterior distribution of $\beta$, given $\sigma^2_{\epsilon}$, is normal

$$\beta^{(j)} \sim p(\beta | \sigma^{2(j)}_{\epsilon}, \mu^{(j-1)}) \sim \mathcal{N} (\hat{\beta}, (X^T R^{-1} X)^{-1} \sigma^{2(j)}_{\epsilon})$$

(5.24)

Note, the above four equations respectively replace existing Equations (4.65), (4.66), (4.67), and (4.69).

The “log $\sigma$” component follows a similar procedure as that above, thus

$$\sigma^{2(j)}_{\delta} \sim p(\sigma^2_{\delta} | \log(\sigma^{(j-1)}) \sim \chi^{-2} (2n - n_\gamma, s^2)$$

(5.25)

\(^4\)Note that an alternative method is to obtain the cumulative distribution of $\beta_3 - \beta_4$ directly from the Gibbs sampler output.
CHAPTER 5. CASE STUDY INVESTIGATIONS

where \( n_\gamma \) is the size of (vector) \( \gamma \),

\[
s^2 = \frac{1}{2n - n_\gamma} (\log \sigma^{(j-1)} - Z\hat{\gamma})^T R^{-1} (\log \sigma^{(j-1)} - Z\hat{\gamma})
\]

(5.26)

and

\[
\hat{\gamma} = (Z^T R^{-1} Z)^{-1} Z^T R^{-1} \log \sigma^{(j-1)}
\]

(5.27)

While

\[
\gamma^{(j)} \leftarrow p(\gamma | \sigma^2_\delta^{(j)}, \log \sigma^{(j-1)}) \sim N \left( \hat{\gamma}, (Z^T R^{-1} Z)^{-1} \sigma^2_\delta^{(j)} \right)
\]

(5.28)

Note, the above four equations respectively replace existing Equations (4.71), (4.72), (4.73), and (4.75). Also, the \( R \) matrices for these two components are different (the \( \rho \) “diagonal” elements differ for the \( \mu \) and \( \log \sigma \) components).

**Modification 2: Correlated model errors procedure** Step 2 of the Gibbs sampler (regional-parameter block) — the Metropolis-within-Gibbs steps — must be modified to account for the correlation structure within the mean and (log) standard deviation model errors. Therefore, the existing series of univariate normal densities (from Equation (4.79)) are rewritten as a series of bivariate normal densities:

\[
p(\theta_S | \theta_R) = p(\mu | \theta_R)p(\log \sigma | \theta_R)
\]

\[
= \prod_{i=1}^{n} \left\{ N \left( \begin{bmatrix} \mu_i^- \\ \mu_i^+ \end{bmatrix}, \begin{bmatrix} 1 & \rho_\mu \\ \rho_\mu & 1 \end{bmatrix} \right), \sigma^2_\varepsilon \right\}
\]

\[
= N \left( \begin{bmatrix} \log \sigma_i^- \\ \log \sigma_i^+ \end{bmatrix}, \begin{bmatrix} 1 & \rho_{\log \sigma} \\ \rho_{\log \sigma} & 1 \end{bmatrix} \right) \}
\]

(5.29)

This equation is then used when evaluating the ratio of conditional posterior distributions in Equations (4.82) or (4.83).

**Summary of modified Gibbs sampler algorithm** Referring to the summary of the original Gibbs sampler provided in Section 4.4.3, the modified Gibbs sampler for the joint analysis procedure is summarised:

1. Use a mode searching algorithm to determine the maximum likelihood estimates (using Equation (4.78)) of the site model parameters \( \theta_S^{(0)} \) and the correlation parameters \( A \) and \( B \), if required, as well as the Hessian matrix.

2. Obtain the initial (proposal) covariance matrix \( \Sigma^{(0)} \) from the Hessian matrix.

3. For \( j = 1, \ldots, N \):

   a. Step 1: sample the regional-parameter vector \( \theta_R \) (using four Gibbs steps):
      i. \( \sigma^2_\varepsilon^{(j)} \leftarrow \chi^2 (2n - n_\beta, s^2) \) — Equation (5.21), with \( s^2 \) defined in Equation (5.22).
      ii. \( \beta^{(j)} \leftarrow N \left( \hat{\beta}, (X^T R^{-1} X)^{-1} \sigma^2_\varepsilon^{(j)} \right) \) — Equation (5.24), with \( \hat{\beta} \) defined in Equation (5.23).
iii. $\sigma^2_{\delta(j)} \leftarrow \chi^{-2}(2n - n_\gamma, s^2)$ — Equation (5.25), with $s^2$ defined in Equation (5.26).

iv. $\gamma(j) \leftarrow N(\hat{\gamma}, (Z^T R^{-1} Z)^{-1} \sigma^2_{\delta(j)})$ — Equation (5.28), with $\hat{\gamma}$ defined in Equation (5.27).

(b) Step 2: Sample the site-parameter vector $\theta_S$ (using $n$ Metropolis-within-Gibbs steps).

For $i = 1, \ldots, n$:

i. Sample a trial parameter $\vartheta^*_i$ from a symmetric jumping distribution $q(\vartheta^*_i | \vartheta^{(t-1)}_i) = N(\vartheta^{(t-1)}_i, c_i^2 \Sigma^{(0)}_i)$, where $c_i$ is a scaling factor (initially set to 1) and $\Sigma^{(0)}_i$ is obtained from the appropriate elements of $\Sigma^{(0)}$.

ii. Calculate the ratio of densities $r$ (with Equation (5.29) being used in conjunction with either Equation (4.82) or (4.83), depending on whether the sites are assumed correlated or independent).

iii. Perform an acceptance/rejection step. Set:

$$\vartheta^{(j)}_i = \begin{cases} 
\vartheta^*_i & \text{with probability } \min(r, 1) \\
\vartheta^{(j-1)}_i & \text{otherwise}
\end{cases}$$

iv. Update the covariance scaling factor $c_i$ (based on the acceptance rate) during the “warm up”/“burn in” phase, if desired.

5.2.6 Results of regional analysis using joint procedure

The flood data — jointly using data from both IPO epochs — for each of the four regions were reanalysed using the hierarchical model with the aid of the modified Gibbs sampler described in Section 5.2.5. These analyses were performed for both the independent- and correlated-site hierarchical models, resulting in a grand total of eight model calibrations (four for independent and four for correlated). The Gibbs sampler runs used three sequences and were allowed to “burn in” for 10000 iterations — the first 5000 of these were used to adaptively scale the (normal/Gaussian) proposal distribution for the site parameters — with an additional 5000 iterations being stored (giving 15000 samples). Very good acceptance rates (around 25–35%) and small R statistics (i.e. below 1.2, and quite near to 1.0; see Section 4.4.3) were achieved, providing strong evidence that convergence to the posterior distribution had occurred. The modal correlation parameters and the posterior distributions of the regional parameters are summarised in Tables 5.5 and 5.6, respectively. Additionally, Table 5.7 gives the approximate equivalent records lengths provided by the regional models (see Equation (5.2)), while Table 5.8 provides the model error correlations (see Equation (5.16)) — convergence of these model error correlations occurred within three iterations, with the initial correlations obtained by examining the residuals from the previous IPO-stratified analyses (Section 5.2.4).

Initial examination of the regional parameters in Table 5.6 indicates no obvious trends, except that the differences between intercept terms ($\beta_3 - \beta_4$) are larger for the independent-
Table 5.5: Summary of the modal correlation parameters\(^a\) (and associated variables\(^b\)) for the joint analysis procedure.

<table>
<thead>
<tr>
<th>Region</th>
<th>log (A)</th>
<th>log (B)</th>
<th>CLS (km)</th>
<th>(d_{av}) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.6474</td>
<td>-0.5227</td>
<td>284</td>
<td>385</td>
</tr>
<tr>
<td>2</td>
<td>5.7629</td>
<td>-0.4340</td>
<td>318</td>
<td>306</td>
</tr>
<tr>
<td>3</td>
<td>5.7148</td>
<td>-0.3425</td>
<td>303</td>
<td>236</td>
</tr>
<tr>
<td>4</td>
<td>5.6778</td>
<td>-0.6506</td>
<td>292</td>
<td>217</td>
</tr>
</tbody>
</table>

\(^a\) The correlation parameters \(A\) and \(B\) are defined in Equation (4.35).
\(^b\) CLS is the correlation length scale and \(d_{av}\) is the average distance between sites.

Site analyses. However, surprisingly, the equivalent record lengths associated with the independent-site analyses are smaller than those for the correlated-site analyses.

Table 5.9 summarises the significance tests for differences in the regional mean (and standard deviation) between IPO epochs (see Equation (5.20)). Focusing on the regional mean, the tests indicate that the independent-site analyses all show highly significant differences, with the least significant difference being for region 1 (at the \(< 0.01\) level). This result for region 1 is quite unexpected — the at-site analysis from Chapter 3 showed an average flood ratio of about 1 for this region. This suggests that the independent-site analysis cannot be relied upon to detect IPO-related effects. Indeed, Table 5.5 shows that there is considerable correlation present between the sites. For example, at a distance of \(d_{av}\) (the average distance between sites in each region), the correlation is about 0.30 in region 1 and about 0.42 in region 4. Thus, it appears that the independent analysis may be making erroneous inferences because it overestimates the information content of the data, since it ignores the considerable correlation structure that is present in the flood data.

The significance tests for the correlated-site analyses appear more reasonable. Region 1 does not show a significant difference between \(\beta_3\) and \(\beta_4\), which is consistent with the results of Chapter 3. Region 2 shows a possible difference, but it cannot be considered conclusive (significant at the \(0.33\) level). Both regions 3 and 4 show moderately strong differences, which are significant at the \(< 0.10\) level. Hence, it is appropriate to use correlated-site analyses only for the remainder of this study.

For the regional standard deviation, there exists significant differences in regions 1 and 2 (using correlated-site analyses). Since these significance probabilities are near one, this is interpreted as \(\gamma_2\) being significantly larger than \(\gamma_1\) (at the \(< 0.01\) level). This suggests that when extrapolating to larger (say, 100 year) events, the resultant IPO-related differences may diminish because the difference in the regional mean may be cancelled out by the difference in the regional standard deviation, which is opposite in sign. This does not contradict the results of Chapter 3 because those results focused on the smaller ARIs, where the differences due to \(\sigma\) are largely suppressed (e.g., for an ARI of 10 years, the frequency factor \(k\) is 1.28; while \(k\) is 2.33 for an ARI of 100 years). Thus, the opposite-in-sign behaviour of the regional mean and standard deviation may result in a lesser-than-
Table 5.6: Summary of the posterior regional parameters derived from independent- and correlated-site analyses (using the joint analysis procedure).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Independent</th>
<th>Correlated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean  SD</td>
<td>Mean  SD</td>
</tr>
<tr>
<td>Region 1</td>
<td>(North Queensland, 49 sites)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.6432 0.0507</td>
<td>0.6287 0.0485</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.1257 0.0224</td>
<td>0.1180 0.0215</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.8919 0.5306</td>
<td>4.8364 0.5253</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>4.7437 0.5305</td>
<td>4.9023 0.5297</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.2333 0.0465</td>
<td>0.2085 0.0384</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.1992 0.0618</td>
<td>-0.1060 0.0516</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.0170 0.0550</td>
<td>0.0683 0.0462</td>
</tr>
<tr>
<td>$\sigma^2_\delta$</td>
<td>0.1260 0.0246</td>
<td>0.0804 0.0161</td>
</tr>
<tr>
<td>Region 2</td>
<td>(South Queensland, 36 sites)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.5141 0.0506</td>
<td>0.5484 0.0521</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.2795 0.0533</td>
<td>0.2885 0.0530</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.1921 0.6839</td>
<td>3.7252 0.7332</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>3.8071 0.6801</td>
<td>3.6441 0.7198</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.2694 0.0694</td>
<td>0.2511 0.0572</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.2570 0.0472</td>
<td>0.2161 0.0473</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.2512 0.0425</td>
<td>0.3345 0.0416</td>
</tr>
<tr>
<td>$\sigma^2_\delta$</td>
<td>0.0420 0.0136</td>
<td>0.0432 0.0107</td>
</tr>
<tr>
<td>Region 3</td>
<td>(North New South Wales, 26 sites)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.6599 0.0525</td>
<td>0.6648 0.0507</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.4421 0.0623</td>
<td>0.4271 0.0620</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>2.6055 0.6858</td>
<td>2.5453 0.6960</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>2.1120 0.6850</td>
<td>2.1958 0.7009</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.1979 0.0542</td>
<td>0.1817 0.0475</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.1023 0.0442</td>
<td>0.1431 0.0543</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.0540 0.0432</td>
<td>0.1891 0.0543</td>
</tr>
<tr>
<td>$\sigma^2_\delta$</td>
<td>0.0331 0.0100</td>
<td>0.0590 0.0156</td>
</tr>
<tr>
<td>Region 4</td>
<td>(South New South Wales, 16 sites)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.5708 0.0816</td>
<td>0.5307 0.0759</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.2085 0.0818</td>
<td>0.1700 0.0762</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.4753 0.9343</td>
<td>4.9855 0.8762</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>4.1444 0.9342</td>
<td>4.7713 0.8815</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.2735 0.1060</td>
<td>0.2478 0.0926</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.1129 0.0746</td>
<td>0.1711 0.0911</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.0904 0.0762</td>
<td>0.1630 0.0903</td>
</tr>
<tr>
<td>$\sigma^2_\delta$</td>
<td>0.0756 0.0279</td>
<td>0.1126 0.0387</td>
</tr>
</tbody>
</table>

Table 5.7: Summary of the equivalent record lengths for the joint analysis procedure.

<table>
<thead>
<tr>
<th>Region</th>
<th>$n_e$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>IPO- IPO+ IPO- IPO+</td>
</tr>
<tr>
<td>1</td>
<td>3 4 4 5</td>
</tr>
<tr>
<td>2</td>
<td>6 6 6 8</td>
</tr>
<tr>
<td>3</td>
<td>6 6 7 8</td>
</tr>
<tr>
<td>4</td>
<td>5 4 6 6</td>
</tr>
</tbody>
</table>
**Table 5.8:** Summary of the model error correlations for the mean and (log) standard deviation for the independent- and correlated-site analyses.

<table>
<thead>
<tr>
<th>Region</th>
<th>Correlation</th>
<th>Independent</th>
<th>Correlated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$\log \sigma$</td>
</tr>
<tr>
<td>1</td>
<td>0.82</td>
<td>0.69</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>0.86</td>
<td>0.67</td>
<td>0.86</td>
</tr>
<tr>
<td>3</td>
<td>0.86</td>
<td>0.67</td>
<td>0.90</td>
</tr>
<tr>
<td>4</td>
<td>0.86</td>
<td>0.80</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**Table 5.9:** Summary of the $z$ statistics and associated significance probabilities $(1 - \Phi(z))$ for testing for a difference in regional means and standard deviations between IPO epochs (i.e. $P(\beta_3 > \beta_4)$ and $P(\gamma_1 > \gamma_2)$ respectively).

<table>
<thead>
<tr>
<th>Region</th>
<th>$\mu$</th>
<th>$\log \sigma$</th>
<th>$1 - \Phi(z)$</th>
<th>$1 - \Phi(z)$</th>
<th>$1 - \Phi(z)$</th>
<th>$1 - \Phi(z)$</th>
<th>$1 - \Phi(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.40</td>
<td>0.008</td>
<td>-0.50</td>
<td>0.690</td>
<td>-3.79</td>
<td>1.000</td>
<td>-3.56</td>
</tr>
<tr>
<td>2</td>
<td>4.73</td>
<td>0.000</td>
<td>0.44</td>
<td>0.331</td>
<td>0.12</td>
<td>0.453</td>
<td>-2.40</td>
</tr>
<tr>
<td>3</td>
<td>6.99</td>
<td>0.000</td>
<td>2.35</td>
<td>0.009</td>
<td>1.05</td>
<td>0.146</td>
<td>-1.08</td>
</tr>
<tr>
<td>4</td>
<td>3.34</td>
<td>0.000</td>
<td>1.38</td>
<td>0.084</td>
<td>0.36</td>
<td>0.359</td>
<td>0.12</td>
</tr>
</tbody>
</table>

expected IPO-related difference for larger ARIs.

### 5.2.7 Analysis of residuals for joint procedure

**Independence of regional model errors**

The residuals for the site mean and (logarithm of the) standard deviation are plotted in Figures 5.7 to 5.8 respectively for the correlated-site case. The figures show that there is limited scatter and a high degree of correlation between the IPO- and IPO+ residuals, and justifies the joint regional analysis.

Figure 5.9 provides scatter plots of the mean and (logarithm of the) standard deviation residuals (for the correlated-site case) to check for independence between these two variables, with Table 5.10 reporting the corresponding correlation coefficients. The “+” and “−” signs correspond to the residuals in the IPO+ and IPO- epochs, respectively. The plots show no apparent linear relationship, both overall and when considering each IPO epoch separately. It is noted that there is a suggestion of clustering in region 4. Table 5.10 shows that the correlations for regions 2, 3, and 4 are not significant; while for region 1, the correlations for IPO- and overall (combining IPO- and IPO+) are significant. This is somewhat unexpected and considered an anomaly. Indeed, Figure 5.10 (to be discussed) highlights further anomalous behaviour for region 1, suggesting the need to subdivide the region or add extra explanatory variables. Overall, it is concluded that the mean and (log) standard deviation regional model errors are statistically independent.
Figure 5.7: Residual plots for the site mean comparing IPO- and IPO+ analyses for all four regions (correlated-site analysis).

Figure 5.8: Residual plots for the (logarithm of the) site standard deviation comparing IPO- and IPO+ analyses for all four regions (correlated-site analysis).
Figure 5.9: Residual plots comparing the site mean and (logarithm of the) standard deviation for both IPO epochs for all four regions (correlated-site analysis).

Table 5.10: Summary of the correlations between the site mean and (logarithm of the) standard deviation model errors for various IPO epochs (correlated-site analysis)\textsuperscript{a}.

<table>
<thead>
<tr>
<th>Region</th>
<th>n</th>
<th>Correlation \textsuperscript{IPO-}</th>
<th>Correlation \textsuperscript{IPO+}</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
<td>-0.45</td>
<td>-0.22</td>
<td>-0.34</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>0.18</td>
<td>-0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>-0.20</td>
<td>-0.23</td>
<td>-0.22</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>0.03</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\textsuperscript{a} n is the number of sites and “overall” combines both IPO epochs.
Spatial independence of regional model errors

The signs (positive “+” or negative “−”) of the residuals are examined in Figure 5.10 to determine whether there are any obvious spatial patterns present that the regional regression has not accounted for. The site mean does not show any obvious pattern or clustering, with there being a relatively even mix of “+” and “−” signs throughout eastern Australia. The site standard deviation shows a similar mix of “+” and “−” signs, except for the most-northern part of Queensland where there is a cluster of “+” signs and no “−” signs for both IPO epochs (north of approximately -16° latitude). So, if desired, slightly improved results could be achieved for region 1 by splitting it into two regions, or by adding some catchment descriptors to the regression equation for log $\sigma$. Overall, the regional regression performs adequately for the selected regions. Adding more structure to the regional model for log $\sigma$ is not warranted in this study as this is not central to the stated objectives of the chapter.

Normality of regional model errors

The regional regression equations (Equations (5.8) to (5.11)) make the assumption that the model error terms (i.e. $\varepsilon$ and $\delta$) are normally distributed. This normality assumption can be tested through the use of normal probability plots. However, since the residuals have two components of variance — sampling error and model error — the residuals must first be standardised before the plots are made. Considering the residual equation as defined in Equation (5.4), the residual variance (for the site mean) becomes:

$$\text{Var}(r) = \text{Var}(x^T \beta) + \text{Var}(\hat{\mu}) = \sigma^2_e + \text{Var}(\hat{\mu})$$

(5.30)

where $\sigma^2_e$ is the model error (for the site mean, in this case), and $\text{Var}(\hat{\mu})$ is the sampling error (which can be obtained from the Hessian matrix). These residuals are standardised:

$$r' = \frac{r}{\sqrt{\text{Var}(r)}} = \frac{r}{\sqrt{\sigma^2_e + \text{Var}(\hat{\mu})}}$$

(5.31)

and Figures 5.11 to 5.12 present normal probability plots for the (standardised) site mean and standard deviation residuals for all four regions.

For the mean residual, regions 3 and 4 show some variability from the straight line, while also showing single high or low outliers. The standard deviation residuals generally follow the straight line quite well, while all regions, except region 4, having at least one high or low outlier. Overall, the normal probability plots suggest that the normal assumption has been satisfied by the residuals.

A more precise test for normality uses the Kolmogorov–Smirnov (K-S) test statistic, which compares the maximum deviation $D_{\text{max}}$ between the observed empirical cumulative distribution functions and the assumed (normal) cumulative distribution function (see also Section 3.5). Table 5.11 summarises the results of the K-S tests on the standardised residuals for all four regions. The resultant $p$ values show that none of the tests were significant at the 10% level. Thus, the regional regression model error normality assumptions
Figure 5.10: Spatial patterns of the residuals of the site mean and (logarithm of the) standard deviation showing all four regions (correlated-site analysis).
Figure 5.11: Normal probability plots for the standardised site mean residuals (correlated-site analysis).

Figure 5.12: Normal probability plots for the standardised (logarithm of the) site standard deviation residuals (correlated-site analysis).
Table 5.11: Results of the Kolmogorov–Smirnov tests on the standardised residuals.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\mu$ residual</th>
<th>$\log \sigma$ residual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_{\text{max}}$</td>
<td>$p$ value</td>
</tr>
<tr>
<td>1</td>
<td>0.1323</td>
<td>0.5844</td>
</tr>
<tr>
<td>2</td>
<td>0.0974</td>
<td>0.6708</td>
</tr>
<tr>
<td>3</td>
<td>0.1141</td>
<td>0.2835</td>
</tr>
<tr>
<td>4</td>
<td>0.0457</td>
<td>0.9810</td>
</tr>
</tbody>
</table>

$a$ $D_{\text{max}}$ is the maximum deviation between the empirical and assumed cumulative distribution functions.

Figure 5.13: Posterior distribution of selected flood quantiles, derived using the regional model, for IPO- and IPO+ epochs for site 210001 in region 3. Circle represents the 50th percentile, while the bars represent 90% probability limits.

are deemed to be not inconsistent with the data.

5.2.8 Prediction at an ungauged site

The previous results, using significance tests on the regional mean parameters, have shown that there are significant differences in the regional mean and standard deviation between the IPO- and IPO+ epochs for two regions (and a possible difference for a third region), results which are consistent with those of Chapter 3. In this section, we focus on differences in the derived at-site flood frequency curves (i.e. at-site frequency distributions).

Are IPO- and IPO+ distributions different?

A set of at-site flood frequency curves for both the IPO- and IPO+ epochs are shown in Figure 5.13. These curves were obtained using the procedures outlined in Section 4.4.5. The site (210001 in region 3) was selected because the flood risk in the IPO- epoch is approximately 1.7 times larger than the IPO+ epoch (see Chapter 3) and also because there was a significant difference in regional means for this region, so it should make it easier to detect any IPO-related difference. The figure shows that the median discharge for a particular ARI is larger during the IPO- epoch, with the probability limits for both
IPO epochs largely overlapping each other. This large overlap seemingly suggests that there is no distinguishable difference between the two IPO epochs. However, this assumes regional-model-error independence between the IPO epochs, which is not the case here with the high correlation in the regional errors for both $\mu$ and $\log \sigma$.

This is best explained by considering the distributions of the regional means in Figure 5.14. The solid lines give the (marginal) distributions for the IPO+ and IPO- epochs (top and bottom panels respectively), which have a high degree of overlap. To account for the correlation present between the two IPO epochs, the conditional distribution must be calculated. The (multivariate) conditional normal distribution of $X_1$ given $X_2$ is (e.g. Judge et al., 1988, p. 50, Equation 2.5.13):

$$X_1 | X_2 \sim N \left( \mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (X_2 - \mu_2), \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} \right)$$

(5.32)

where $X_1$ and $X_2$ correspond to the (regional mean in the) IPO- and IPO+ epochs respectively. This can be simplified for the bivariate case:

$$X_1 | X_2 \sim N \left( \mu_1 + \frac{\rho \sigma_1}{\sigma_2} (X_2 - \mu_2), \sigma_2^2 (1 - \rho) \right)$$

(5.33)

The bottom panel also shows the conditional distributions ($X_1 | X_2 = a$) for correlations of 0.8 and 0.9 (typical correlation values for the regional mean errors, see Table 5.8). It is clear that the conditional distributions $X_1 | X_2$ show a high degree of separation from $a$. In other words, it is highly probable that the IPO- mean is greater than the IPO+ mean.

Note that inspection of Equation (5.33) shows that when $\rho = 0$, the conditional distribution will be equal to the marginal distribution and that as $\rho$ is increased to 1, the conditional distribution becomes a “spike” (the variance becomes zero). In this second case, by knowing $X_2$ the value of $X_1$ is also known exactly because of the “perfect” correlation — this value of $X_1$ corresponds to point $b$.

We can conclude that the IPO- and IPO+ distributions derived using the regional model produce significantly different quantiles.

**Long-run flood risk**

The remainder of this chapter considers the long-run — or marginal — distribution because this is the usual concern in flood frequency analysis.

To gain an understanding of the variability associated with the regional parameters and its influence on the resultant (marginal) at-site flood frequency curves, Figure 5.15 compares the regional at-site probability limits (labelled as “true”) to the at-site limits obtained assuming that there is no regional-parameter uncertainty, but with regional noise (i.e. $\text{Var} (\beta) = \text{Var} (\gamma) = 0$, labelled as “$\text{Var} = 0$” — using the posterior mean values of $\beta$ and $\gamma$) and to the at-site limits obtained assuming that there is no regional noise, but with regional-parameter uncertainty (i.e. $\sigma^2_\beta = \sigma^2_\delta = 0$, labelled as “$\sigma^2 = 0$” — assuming that the regional regression is perfect).

The figure shows that the uncertainty is reduced for both the “$\text{Var} = 0$” and “$\sigma^2 = 0$” cases; however, considerable uncertainty still remains. The biggest reduction in
CHAPTER 5. CASE STUDY INVESTIGATIONS

Figure 5.14: Posterior distribution of the regional mean for site 210001 in region 3 \((\log A = 9.7, IFD = 5.0)\). Top panel is IPO+ and bottom panel is IPO-. Solid curves are the marginal distributions, while the broken curves give the conditional distributions for IPO- (given the IPO+ value is equal to \(a\)).

Figure 5.15: Posterior distribution of selected flood quantiles, derived using the regional model, for the “true”, “perfect parameter” \((\text{Var} = 0)\) and “perfect regression” \((\sigma^2 = 0)\) cases for site 210001 in region 3. Circle represents the 50th percentile, while the bars represent 90% probability limits.
was uncertainty was obtained for the “$\sigma^2 = 0$” case, suggesting that the regional noise has a bigger impact on the overall uncertainty than does the variability in the regional parameters. Nonetheless, the possibility of combining at-site data — no matter how limited — with the regional distribution, may have considerable utility because this would reduce the variability present within the regional mean and, thus, reduce overall variability.

### 5.2.9 Prediction at a gauged site

The study now examines the improvements that may be achieved, if any, by augmenting the regional flood distributions with some at-site gauged data, using the procedures described in Section 4.4.5. This additional gauged data (5 or 10 years’ worth) is randomly sampled from the existing full gauged record; that is, 5 or 10 years of data are randomly selected from a site’s full gauged record, which is then used to augment the regional flood distribution. Figure 5.16 compares, for four sites, the flood frequency distributions for the full at-site gauged record [G(full)], the shortened 10-year gauged record [G(10)], the “ungauged” record (i.e. regional distribution only) [UG], the ungauged record + 5 years of gauged data [UG(5)], and the ungauged record + 10 years of gauged data [UG(10)] — these results are also tabulated in Table 5.12. Note that the same 10 years of data are used for the shortened 10-year gauged record and for augmenting the ungauged record, with UG(5) using the first 5 of 10 years.

The results show that augmenting the regional flood distribution with the additional gauged data reduces the variability in the resultant flood frequency distribution (i.e. reduces the probability limits) and these results are consistent with the full at-site gauged record. Indeed, except for site 136206A, augmenting the regional distribution with only 5 years of gauged data produces tighter limits than those achieved using at-site analysis with the longer 10-year gauged record. This is a reassuring result because it shows that even with a noisy regional flood model, augmenting the regional distribution with a short gauged record can provide substantial improvements over the gauged record alone. Note that these three sites (112003A, 210001, and 222007) have shortened gauged records with a relatively even mix from both IPO epochs (5, 5, and 6 of the 10 years are sampled from the IPO- epoch, respectively) and also have a wide range of flood ratios (ratio of flood risk in IPO- epoch to flood risk in IPO+ epochs — 0.9, 1.7, and 2.3, respectively).

Site 136206A behaves somewhat anomalously because the shortened 10-year gauged record is quite different to the full 35-year gauged record — the probability limits for these two gauged records do not even overlap! This provides an excellent illustration of the situation where the (10-year) gauged record is unrepresentative of the long-run (35-year) flood record; indeed, 7 of the 10 years are sampled from the IPO+ epoch, so these 10 years of gauged data consist mainly of lower-flow years. The ungauged record performs well, with the ungauged limits reflecting the full (35-year) at-site limits. Augmenting the regional distribution with the (unrepresentative) gauged data shows that the resultant frequency distributions are still quite good, even though the regional distribution and the 10-year gauged record are contradicting each other — the probability limits of the augmented analyses are now inflated above those of the 10-year at-site analysis because
Figure 5.16: At-site and regionalised flood frequency distributions for sites located in all four regions, with the full (at-site) gauged record [G(full)], 10 years of gauged data [G(10)], ungauged (i.e. regional model only) [UG], ungauged + 5 years of gauged data [UG(5)], and ungauged + 10 years of gauged data [UG(10)], respectively used. Circle represents the 50th percentile, while the bars represent 90% probability limits.

The “ungauged” and “gauged” data are opposing each other. The average/median flood levels are decreased as more gauged data is used, with the gauged data exerting more influence over the resultant distribution; however, the probability limits are still consistent with the full (35-year) probability limits and the resultant bias is less than that obtained using the 10-year record alone. This illustrates that the use of the regional model can help reduce the bias in the resultant flood frequency distribution if the gauged record is unrepresentative of the “true” (long-run) flood record.
Table 5.12: Summary of at-site and regionalised flood distributions (units: \(\times 1000\) ML/day)\(^a,b,c\).

<table>
<thead>
<tr>
<th>Analysis</th>
<th>10-year flood quantile</th>
<th>100-year flood quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
<td>50%</td>
</tr>
<tr>
<td>Site 112003A (region 1, 44 years)</td>
<td>G(44)</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>G(10)</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>UG</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>UG(5)</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>UG(10)</td>
<td>26</td>
</tr>
<tr>
<td>Site 136206A (region 2, 35 years)</td>
<td>G(35)</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>G(10)</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>UG</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>UG(5)</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>UG(10)</td>
<td>8.0</td>
</tr>
<tr>
<td>Site 210001 (region 3, 97 years)</td>
<td>G(97)</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td>G(10)</td>
<td>218</td>
</tr>
<tr>
<td></td>
<td>UG</td>
<td>101</td>
</tr>
<tr>
<td></td>
<td>UG(5)</td>
<td>167</td>
</tr>
<tr>
<td></td>
<td>UG(10)</td>
<td>209</td>
</tr>
<tr>
<td>Site 222007 (region 4, 51 years)</td>
<td>G(51)</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>G(10)</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>UG</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>UG(5)</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>UG(10)</td>
<td>14</td>
</tr>
</tbody>
</table>

\(^a\) \(G(n)\) represents the gauged record (i.e. at-site flood distribution) with \(n\) years of gauged data.

\(^b\) UG represents the regional flood distribution.

\(^c\) UG(\(n\)) represents combining the information in the regional flood distribution with \(n\) years of gauged data.

5.3 Discussion

5.3.1 Regional-parameter variability

The regional-parameter estimates show reasonable amounts of variability, which most likely is the result of too few sites being used over the large regions. Some possibilities to reduce the variability include increasing the number of sites as well as reducing the size of the regions in an attempt to reduce the intersite variability within the regions. Another possible approach would be to include more explanatory variables in the regression equations (i.e. the \(X\) and \(Z\) terms in Equation (5.12)) which may produce a more accurate regional estimator; however, this is a major data-mining exercise which is beyond the scope of this thesis. Nonetheless, even with this variability, a significant difference in the regional mean was detected for two regions and a large difference for another region — results consistent with the findings of Chapter 3.
5.3.2 Intersite correlation

The use of a correlated-site model decreases the information content of the data. However, due to the structure and formulation of the hierarchical model, it may be possible for some of this loss of information, due to intersite correlation, to be counterbalanced by a gain in information achieved through implicit infilling — the correlated-site model effectively fills in the missing years of data using nearby sites which contain these years of data, if the degree of correlation is sufficiently high. This can be demonstrated visually using Figure 5.17 — the bottom site has missing data from about 1960 onwards and this gap may be filled in using the corresponding data contained in nearby sites. The infilling achieved by the correlated model is, in effect, performing the same task as that described by Wang (2001) with his Bayesian joint probability approach. Wang’s likelihood function (Equation (12)) is

\[
p(x, y \mid \theta) = \prod_{k=1}^{K} f_{X,Y}(x_k, y_k \mid \theta) \prod_{i=1}^{I} f_X(x_i \mid \theta) \prod_{j=1}^{J} f_Y(y_j \mid \theta)
\]  

(5.34)

where \(x\) and \(y\) are two flood records which partially overlap. The first product represents the period where the two records overlap, while the second and third products denote the periods where the flood data exists only in records \(x\) and \(y\), respectively. It is clear that the hierarchical model’s (correlated-site) likelihood function (Equation (4.78)) is formulated, conceptually, in the same way as Wang’s, but the hierarchical model provides a more general implementation — Wang’s approach uses a bivariate joint density, while the hierarchical model provides a multivariate joint density, allowing for more than two sites to be used. Even though Wang (2001) found his procedure improved the precision of flood quantile estimates, the infilling hypothesis associated with the correlated model needs to be confirmed through further analysis, such as a realistic synthetic data study where the data records contain missing values and the records are of variable length with different starting and ending years.

An important point follows from the consideration of a realistic data set: the availability of several sites, or even a single site, with long data records may be very advantageous to an analysis because the infilling achieved could be significant, something that an independent-site analysis could not exploit — the advantage presented by long records in correlated analysis has also been recognised (Stedinger and Tasker, 1985, 1986a; Hosking and Wallis, 1988, p. 594). However, some studies that claim that the effects of correlation are can be ignored in practice, including Hosking and Wallis (1988), preclude the possibility of infilling in their experimental design by using a contiguous block of data (e.g. 20 sites with the same 30 years of data).

5.4 Conclusion

The Bayesian hierarchical regional model was first used to analyse IPO-stratified flood data from eastern Australia using both independent- and correlated-site formulations. This separate, IPO-stratified analysis was found to be inappropriate because the regional
Figure 5.17: Years of available data for some sites in region 3 (to illustrate possible infilling of missing years). The bottom site is 210005 with the remaining sites in the region being sorted by distance, up to the correlation length scale (about 300 km).
model errors are highly correlated. Thus, the analysis was appropriately modified and performed using data from both IPO epochs jointly. The independent-site analysis was found to be unsatisfactory because it contradicted the findings of Chapter 3. In contrast, the correlated-site analysis could detect significant differences in the regional means for regions 3 and 4 (and a large difference for region 2), which is consistent with the findings of Chapter 3; while the regional standard deviation showed significant differences for regions 1 and 2, but with these differences opposite in sign (i.e. larger in the IPO+ epoch). The resultant IPO- and IPO+ distributions largely overlap; however, by considering the high correlation in the regional model errors and the conditional distribution (of IPO- given IPO+), it is highly probable that the IPO- mean is greater than the IPO+ mean at a particular site. For larger ARIs, the opposite-in-sign differences in the regional mean and standard deviation may reduce the resultant IPO-related differences (in discharge). Additionally, if a more extensive spatial coverage of gauged sites and more catchment descriptors were available, a more informative regional model may be obtained.

The formulation and structure of the correlated-site model indicates the potential possibility that the regional model may be able to implicitly “infill” missing data using nearby sites. This suggests that further studies should be conducted to determine the importance of this infilling.

It was found that augmenting the regional distribution with a short gauged record improved the resultant at-site flood frequency distribution over that achieved using the gauged record alone. Importantly, an example with a short gauged record that was unrepresentative of the true flood record showed that the resultant at-site flood frequency distribution remained consistent with the true flood frequency distribution, and the bias associated with the augmented record was less than the bias associated with using the short record alone.
Chapter 6

Conclusions

6.1 Introduction

This thesis had two objectives. The first involved the investigation of flood frequency records from eastern Australia to determine the magnitude and spatial extent of decadal-scale variability (nonhomogeneity) in flood risk that exists in these records. The second was to develop a regional flood model to overcome the possible bias in long-term flood risk associated with this nonhomogeneous flood record. This chapter summarises the progress towards fulfilling these objectives, presents the principal findings of this thesis, and examines future research directions.

6.2 Summary

6.2.1 Development of a new procedure for censored data in flood frequency analysis

Chapter 2 showed that some flood data from New South Wales (NSW) were systematically in error due to the data collection techniques that were used. These erroneous discharge observations were not instantaneous peak discharges — as are typically used in flood frequency analysis — but were, instead, daily-read (censored) discharges. The daily-read discharges may underestimate the true (instantaneous) peak discharge by significant amounts, with factors of ten being observed.

A new approach (RDC method), based on the method of maximum likelihood, was developed to overcome, in a general way, the artefacts introduced by the daily-read data in flood frequency analysis. This new method incorporated the dependence that exists between the daily-read and instantaneous peak discharges, which is reflected through the censoring distribution. Importantly, even though the method was developed using log-normal distributions for both the flood-frequency and censoring distributions, the form of these two distributions is arbitrary, so the RDC method may be used with virtually any distribution. The RDC method improved the flood quantile estimates over the GO method, which was used as a reference case — the GO method excludes the daily-read data. Another technique (CAG method), where the daily-read data were treated as instan-
CHAPTER 6. CONCLUSIONS

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Taneous peak data, also provided improved flood quantile estimates over the GO method for parameters typical of eastern Australia, but did not perform as well as the RDC method. Nonetheless, the use of this daily-read data should not significantly impact upon the reliability of subsequent analyses of eastern Australian flood data. The BC method, which was based on the binomial censoring approach of Stedinger and Cohn (1986), performed poorly because the BC method assumes the censoring threshold is independent of the peak discharge while, in fact, there is strong dependence between the daily-read and instantaneous peak discharges.

These results demonstrate the clear need to rigorously evaluate the meaning of data, especially where changes in monitoring practices have occurred.

6.2.2 Investigation of the nonhomogeneity of eastern Australian flood data

Chapter 3 investigated the nonhomogeneity of eastern Australian flood data from the states of NSW and Queensland (Qld). Previously-published studies have questioned the homogeneity of flood risk in NSW, with decadal-long periods of elevated and reduced flood risk being observed. Many studies have also shown that a relationship between observed climate variability and large-scale ocean-atmosphere processes, with El Niño/Southern Oscillation (ENSO) being known to have an influence on rainfall and streamflow in eastern Australia. Additionally, the Interdecadal Pacific Oscillation (IPO), a climate index of multidecadal Pacific Ocean sea surface temperature anomalies, has been found to modulate the magnitude and frequency of ENSO events, leading to multidecadal periods of elevated or reduced flood (and drought) risk. Hence, the eastern Australian flood data were stratified according to IPO value (IPO positive or IPO negative) to permit the investigation of the (non)homogeneity of the flood record on multidecadal time scales.

Flood frequency analyses were performed on the IPO-stratified flood data, with the lognormal distribution found to be appropriate. Flood ratios — the ratios of IPO-flood discharge to the IPO+ flood discharge — were calculated for recurrence intervals between 2 and 20 years. About 80% of sites had flood ratios above 1, while approximately 40% of sites had flood ratios greater than 1.5, with this difference between IPO epochs shown to be very unlikely to occur by chance alone.

The flood ratio did not display any clear relation with catchment area, suggesting that the IPO dependence of flood risk is a regional phenomenon, independent of catchment area. However, the flood ratios exhibited a strong spatial relationship, with a cluster of sites in northern Qld (north of latitude 22°S) showing little IPO dependence, while the sites below this latitude displayed significant IPO dependence, with an average flood ratio of about 1.7. Possible mechanisms for the observed IPO dependence were postulated; however, it is recognised that the mechanisms responsible for the association between the IPO and eastern Australian rainfall and climate are poorly understood.

This study is somewhat limited by the geographic coverage of the data, with sites being mainly restricted to the coastal regions of NSW and Qld; however, its findings are, nonetheless, of considerable importance because the study area encompasses a significant
It is important to recognise the practical significance of the IPO modulation of flood risk. Firstly, when considering flood risk, a decision must be made as to the time frame required — do we want to estimate the short-term or long-run flood risk? Most infrastructure will require the long-run risk because their design lives will span multiple IPO epochs. The use of at-site flood data with an inadequate coverage of both IPO epochs may result in biased estimates of long-run flood risk. Therefore, it may be necessary to use a regional flood frequency distribution that has been obtained from a data set containing sufficient samples from both IPO-positive and -negative epochs to augment the limited at-site data. This is the motivation for the development of the regional flood model.

6.2.3 Development of a hierarchical regional flood model using a Bayesian framework

Chapter 4 developed a Bayesian regional flood model framework based on hierarchical model concepts. The flood regionalisation procedure consists of two, often inter-related, components: delineation of regions and flood estimation at the site of interest. The major methods used in flood regionalisation were examined. From this review, a regression-based approach was selected for the regional flood model.

The Bayesian hierarchical model has several desirable qualities which improve its conceptual soundness. Firstly, the IPO modulation of flood risk is easily incorporated using the model. Common data-related issues of missing data and unequal record lengths pose no difficulty. Intersite correlation, which has the effect of reducing the information content of the flood data, can be considered, if desired. Finally, parameter inference is performed using Bayesian methods, which explicitly account for parameter uncertainty, allowing for a rigorous treatment of the flood regionalisation problem. Specifically, the Gibbs sampler was used, which is a Markov chain Monte Carlo method particularly well suited to hierarchical models. A detailed algorithmic implementation of the Gibbs sampler was provided, along with implementation guidelines to help improve the performance of the Gibbs sampler and ensure the integrity of the results.

A synthetic data study was conducted to ensure that the implementation of the Gibbs sampler could reproduce parameters consistent with the synthetic data parameters, with the performance being satisfactory for “typical” cases. Under “atypical” cases, where the regional variances were near zero, the performance of the Gibbs sampler was unsatisfactory. To overcome this problem, the Gibbs sampler procedure was modified, using a uniform-sampling approach for the regional variance. Satisfactory performance and consistent results were then achieved. Importantly, the development of the uniform-sampling approach makes the Gibbs-sampler implementation of the hierarchical model robust, albeit with a minor disadvantage of extra computational expense.
6.2.4 Techniques for prediction at a new (ungauged or gauged) site using the regional posterior distribution

Chapter 4 also developed procedures to the compute the Bayesian predictive distribution and the posterior distribution of quantiles at a new site, which may be ungauged or gauged. An approach based on importance sampling makes effective use of the regional (posterior) parameters obtained during the Gibbs sampler calibration and allows for a robust quantification of the prediction limits. This represents the first truly-general Bayesian solution for combining regional and gauged information in flood frequency analysis.

6.2.5 Application of the regional flood model to flood data from eastern Australia

Chapter 5 analysed the flood data from eastern Australia using the Bayesian hierarchical model. The data from each state (NSW and Qld) were split into two geographical regions, giving a total of four regions, with the region boundaries being guided by the observed spatial distribution of the IPO dependence of flood risk reported in Chapter 3. The limited number of available sites (127) over a large spatial area restricted the opportunity to further delineate regions.

The catchment characteristics used for the site-mean regression equation were catchment area and $IFD$, the 2-year 12-hour rainfall intensity. They were selected because these variables (or analogues thereof) have been found by many studies to have a large influence on streamflow. The $IFD$ variable was obtained from a (map-based) regionalisation procedure. It therefore represents a long-run value and disregards the effects of IPO modulation. However, to stratify and recalculate $IFD$ would have been a major data-mining exercise, well beyond the scope of the thesis.

Initially, the hierarchical model from Chapter 4 was used to analyse the IPO-stratified flood data. This separate, IPO-stratified analysis was deemed inappropriate because the two epochs were found to have highly correlated model errors and, so, should not be analysed separately. The analysis procedure was appropriately modified and performed using data from both IPO epochs jointly.

Analysis of residuals indicated that the regression assumption of normality was satisfied and that spatial clustering of residuals was not a major concern. Some clustering did occur in northern Queensland for the (logarithm of the) standard deviation residual suggesting that slightly improved results could be achieved by splitting that region into two separate regions or by adding some catchment descriptors to the regression equation; however, splitting the northern Queensland region would have resulted in too few sites for a credible analysis.

The independent-site implementation was found to be unsatisfactory because it contradicted the findings of Chapter 3, with all regions showing very strong differences in the regional mean. In contrast, the correlated-site implementation could detect significant differences in the regional means for regions 3 and 4 (both in NSW). However, no significant differences were found for regions 1 and 2 (both in Qld). The regional standard deviations showed significant differences for regions 1 and 2, but these differences were
opposite in sign, with IPO+ standard deviations greater than IPO- values. It is noted that the findings of Chapter 3 were for ARIs up to 10 years, which are less affected by differences in standard deviation.

The IPO- and IPO+ distributions for a quantile largely overlap. However, this does not mean that there is no difference between the IPO+ and IPO- frequency distributions. By accounting for the high correlation in the regional model errors, it was shown that it is highly probable that the IPO- mean is greater than the IPO+ mean at a particular site. For larger ARIs, the opposite-in-sign differences in the regional mean and standard deviation may reduce the resultant IPO-related differences (in discharge).

Additionally, if a more extensive spatial coverage of gauged sites and more catchment descriptors were available, a more informative regional model may be obtained. Indeed, the equivalent gauged length provided by the correlated-site regional model had a maximum of 4–8 years.

The regional posterior distribution — expressed in the form of samples from the posterior distribution obtained using the Gibbs sampler — was combined with a short gauged record to predict the flood frequency distribution for selected sites in each region. The resultant (combined) at-site flood frequency distribution provided substantial improvements over the gauged record alone, a reassuring result given the considerable variability present within the regional model. Indeed, even where a shortened (10-year) gauged record was found inconsistent with the true (long-run) gauged record (site 136206A in region 2) — the shortened record consisted mainly of years from the IPO+ epoch — the resultant at-site flood frequency distribution remained consistent with the full-record flood frequency distribution, and the bias associated with the combined record was less than the bias associated with the short gauged record alone. These results suggest that combining regional and gauged information may provide significant benefits when used to predict at-site flood frequency distributions. In particular, the use of the regional model may help protect against bias in long-run flood risk at sites with short records sampled largely in one IPO epoch.

6.3 Future work

The main contributions of this thesis include demonstrating that regions of eastern Australian flood data are affected by (multidecadal) nonhomogeneity and the development of a Bayesian hierarchical regional model that can help to overcome the possible bias associated with this nonhomogeneous flood record and that rigorously allows combining at-site and regional flood information. However, opportunities remain for extending this work.

6.3.1 Further investigations relating to censored data

There exists some possibilities to extend the investigations relating to censored data. The analysis could be repeated using daily average flow data — which are more common outside of Australia — to determine the extent of the data errors associated with these type of observations. Also, the computation of the RDC method might be implemented using
Markov chain Monte Carlo simulation. This would replace the numerical quadrature used to evaluate the convolution integral in Equation (2.13) by a Monte Carlo evaluation.

### 6.3.2 Areas affected by the nonhomogeneity of flood risk

The current investigation of the nonhomogeneity of (at-site) flood risk was restricted to coastal areas of NSW and Qld. The coverage of sites was sparse in some areas, so additional sites could be analysed to improve the reliability of these results. Further sites may also be analysed to determine whether IPO-related nonhomogeneity exists in other Australian flood records. For example, sites in the tropical areas of Western Australia and the Northern Territory and additional Queensland sites around the Gulf of Carpentaria could be analysed to determine whether there is a consistent lack of IPO modulation of flood risk observed in the northern (tropical) latitudes of Australia.

### 6.3.3 Reducing the variability in the regional analysis

There exists considerable variability in the regional models. Fortunately, even with the presence of this variability, it is still possible to detect significant IPO-related differences in the regional mean and standard deviation. Nonetheless, various approaches may be taken in an attempt to reduce this variability. Additional sites might be analysed to provide better spatial coverage throughout each region — if sufficiently large numbers of sites are added, then further regions may be also delineated. More advanced methods, such as the region of influence or canonical correlation approaches, may provide a better regional delineation. Finally, a more extensive assessment of different catchment descriptors could produce regional models with stronger predictive capability. However, it is recognised that this represents a major data-mining exercise.

Bayes factors could be used to provide a rigorous assessment of optimal set of catchment descriptors. They may also be used to evaluate the relative performance of various model structures; e.g. independent- versus correlated-site analysis, lognormal versus log-Pearson 3 distribution, and hierarchical versus GLS model.

### 6.3.4 Testing the correlated-site infilling hypothesis

It was hypothesised that the correlated-site regional model implicitly “infills” missing data using the information contained at nearby sites. While the formulation of the likelihood function shows that the correlated-site model is essentially a generalisation of the Bayesian joint probability approach of Wang (2001), the infilling hypothesis should be confirmed through a realistic synthetic data study that characterises typical features of real data sets such as missing data and sites with varying record lengths and start/end years.

### 6.3.5 Comparison between joint and two-stage models

The traditional approach to regionalisation involves a two-stage process. In the first stage, at-site estimates are obtained. In the second stage, the regional model is identified and calibrated using the at-site data. In this study, the Bayesian hierarchical model jointly
performed at-site and regional model calibration. The joint approach overcomes some of the difficulties with the two-stage approach, such as evaluating sampling uncertainty. It would be useful to assess the relative performance of the two approaches.

6.4 Final remarks

A new general approach for dealing with daily-read (censored) data in flood frequency analysis has been developed in this thesis. Fortunately, this work showed that treating the daily-read data as instantaneous peaks does not significantly degrade the performance of the flood frequency analysis for parameters typical of eastern Australia.

It is important to recognise the (IPO-related) multidecadal nonhomogeneity of flood risk, with much of coastal eastern Australian flood data being affected. Flood frequency analysis based on data with inadequate coverage of both IPO epochs has a high prospect of significant bias in long-run flood risk. The proposed Bayesian hierarchical regional model provides a method to overcome this possible bias by augmenting the limited at-site data with a regional flood distribution that has been obtained from a data set containing sufficient samples from both IPO epochs. Importantly, the work presented in this thesis represents the first truly-general Bayesian solution for combing at-site and regional information in flood frequency analysis.
Appendix A

Summary of site data

Overview

This appendix summarises the site data from the eastern Australian states of Queensland and New South Wales that have been used in this study.

<table>
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<tr>
<th>Site</th>
<th>Location</th>
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<th>Long (deg)</th>
<th>Length (year)</th>
<th>Area (km²)</th>
<th>IFD (mm/h)</th>
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*IFD is the 2-year, 12-hour rainfall intensity (see Section 5.2.1).*
Table A.2: Summary of site data for New South Wales.

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*IFD is the 2-year, 12-hour rainfall intensity (see Section 5.2.1).
Bibliography


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BIBLIOGRAPHY


