The case for a mathematics of structural transformation and genesis: Whitehead against Badiou?

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Abstract:
From a mathematical perspective, this paper will compare the 'Speculative' Realist Philosophy of Alain Badiou with the Process Philosophy of Alfred North Whitehead. To this end, each philosophy will be examined in terms of how it departs from the strictures of Kantian Critical Philosophy, specifically, in terms of how it articulates the structural relationship between subjects, objects, and relata. Moreover, each philosophy will also be judged both in terms of how it addresses the mereological distinction between internal and external relations, and how it conceives of the relationship holding between philosophy and its others: the four Althusserian 'generics' of science, politics, art, and love.

1. Introduction: orthodox and heterodox economic approaches to the mathematical

It is common for heterodox critics of the highly mathematical 'upper reaches' of the realm of neoclassical economics to emphasize the fact that economists of this tradition are at a loss to explain structural transformations with the meager quantitative resources placed at their disposal. Personally, I would clearly separate the mathematics away from the economics and attribute the blame solely to the coupling of methodological individualism with rational choice theory along with the presumption that agents form their expectations in an anticipative (and risk-sensitive) manner.

Certainly, the generation that included Arrow, Debreu and Hahn, introduced rigorous new, and to a large extent, Bourbakian, mathematical techniques and conceptions of structure to mainstream economics. These new structures and techniques, however, were largely applied to existence proofs of equilibrium (and more rarely, stability) based on the analysis of Walrasian excess demand functions constructed over multi-dimensional Euclidean spaces (albeit, including equilibria defined over intertemporally diverse vectors of contingent claims). Mathematical foundations were largely stolen from mathematical physics, for which the (homotopic) analysis of functions defined over order-dense subsets, which could be characterized by continuity properties defined with respect to the real number line, was a reasonable approximation. This, however, is much less the case for economics, which actually deals, in ontological terms, with finite sets of indivisible entities such as commodities. Modern topological notions of linear operators and fixed point theorems defined over vector spaces, were effectively deployed, in modern versions of General Equilibrium theory, to generalize microeconomic behavior beyond the constraints of classical calculus. Nevertheless, this deployment was both narrowly based, and inadequate, as shown, on one hand, by the manifold pitfalls of game theory, the implications of the Mantel-Debreu-Sonnenschein theorem for aggregation, and the reliance on imbecilic notions of tâtonnement and, on the

1 A recent example of this is Bouleau's (2011: 90) endorsement in Real World Economics of Hudson's contribution to the same journal, which Bouleau describes as focusing "...an important issues that escape mathematical models, such as the structural and historical evolution of societies, prevention of crises, psychological phenomena, long-term thinking." In contrast, Hudson's contribution apparently "...emphasizes the normative nature of marginal analysis and equilibrium models, and denounces rough quantifications such as GDP and the staggering increase in debt."
other hand, by the vast terrain of contemporary mathematical analysis that, as it were, has remained largely unmolested by mainstream mathematical economics. This paper will examine aspects of this terrain. To this end it will compare the 'Speculative' Realist Philosophy of Alain Badiou with the Process Philosophy of Alfred North Whitehead. Each philosophy will be examined in terms of how it departs from the strictures of Kantian Critical Philosophy; more pecifically, in regard to how it articulates the structural relationship between subjects, objects, and relata. Moreover, each philosophy will also be judged both in terms of how it addresses the mereological distinction between internal and external relations. For both Whitehead and Badiou, the mathematical sciences have clearly played a central role in the formation of their ontological conceptions. In the two volumes of his magnum opus, Badiou has drawn on the formal resources of both set theory and category theory. Accordingly, the next section of the paper provides a brief overview of category theory. This is followed by an investigation of Badiou and Whitehead's approach to ontology. For each thinker, this investigation is framed by how each has responded to the Kantian "circle of objectivity": that is, in each case it addresses the way that their ontological positions have subverted the experiential (and epistemological) relationship between the knowing subject and the known object. More specific aspects of the relationship between mathematics and the social sciences are considered in the final section of the paper, with conclusions following.

2. Modeling structural transformation

Steven Awodey (2006: 1) observes that category theory, as a branch of abstract algebra, was invented in the tradition of Felix Klein's Erlanger Programme, as a way of studying and characterizing different kinds of mathematical structures in terms of their "admissible transformations." The general notion of a category "provides a characterization of the notion of a 'structure-preserving transformation,' and thereby of a species of structures admitting such transformations." While Category Theory (henceforth, CT) is often portrayed as an alternative foundation to that afforded by Set Theory for the "working mathematician", the Elementary Theory of the Category of Sets, which has displaced Set Theory on the basis of an intuitionist rather than a classical logic, is only one small part of CT. Of no less value are the revolutions in algebra accomplished by Eilenberg and Mac Lane on the basis of a "general theory of natural equivalences", Grothendieck's subsequent reworking of algebraic geometry, the transformations of meta-mathematics and model theory wrought by Lawvere's research into the Category of Categories and, more recently, the revolutions in theoretical physics and algebra that have been inspired by Grothendieck's dream of a new form of Higher Dimensional Algebra. The obvious question here must be what relevance these untouched resources might have for thinking about processes of production within the social sciences!

The richness of Category Theoretic formulations has yet to be appreciated within the social sciences. Along Hilbertian lines, Landry (2011: 448-9), for example, argues that while the Eilenberg-Mac Lane (EM) axioms suffice for the conceptual and transitional analysis of abstract mathematical structures in any given branch of mathematics, the Elementary Theory of the Category of Sets (ETCS) axioms suffice for the conceptual analysis of branches that are specifically organized in set-theoretic terms, and the Category of Categories as Foundation (CCAF) axioms suffice for conceptual analysis of the category itself, each in turn can serve as the vehicle for the logical analysis of axiomatic systems (at either the abstract, set-structured, or cat-structured level, respectively). That is, the resources of the various categorical logics can
be deployed to analyze concepts within each of the systems organized by the EM, ETCS or CCAF axioms, irrespective of whether the theorist is engaged in semantic analysis of the model-theoretic concepts of satisfiability, relative consistency, interpretation and truth (via multi-sorted languages); the syntactic analysis of deductive systems (in terms of the category of deductive systems, which takes formulas as objects and deductions as morphisms), or the finitistic analysis of the content of constructive mathematical systems (via topos theory) (Landry, 2011: 450-51).

3. Philosophical realism in the post-Kantian milieu

It is well known that Marx never fully set out his own philosophy, which is one reason why he can be interpreted in so many different ways. Both Badiou and Deleuze would call themselves Marxists. For his part, Whitehead was a Social Liberal like Keynes. Despite their differences, Whitehead and Deleuze, on one side, and Badiou on the other, share a passion for mathematical philosophy. In this regard, both Deleuze and Badiou have been greatly influenced by the work of Albert Lautman. Nevertheless, Lautman’s work was terminated by his execution at the hands of the French Police in 1944. Accordingly, he never had the chance to participate in, or discuss the philosophical implications of the development of Category Theory from the 1940s on. Despite his post-Principia fascination with the mathematics of pattern, Whitehead is unfortunately seated in the same boat with Lautman. Nevertheless, Badiou’s *Logics of Worlds* is category-theoretic from beginning to end.

Many interpreters of category theory are Structural Realists. Nevertheless, Structural Realism is constrained in what it can say about mathematics by its role of being a philosophy of science (and unlike Critical Realism, it does not go so far as advocate an ethical philosophy). As such, questions of metaphysics, ethics, or aesthetics are displaced by its epistemological efforts to explain the success of scientific praxis in structural terms. In contrast, as will be explained below, the process philosophy of Whitehead subsumes epistemological considerations within an ethical and aesthetic frame, to avoid what Whitehead describes as the bifurcation of nature. Badiou, too, privileges ethics over questions of epistemology.

In *Logics of Worlds*, Badiou (2009: 9) introduces a distinction between *democratic materialism* (for which there are only bodies and languages) and the *materialist dialectic* (for which there are only bodies and languages, except that there are truths). For Badiou, the cause of the subject is neither truth (its “stuff”), nor the infinity whose finitude it is, but rather

1 Landry (2011: fn. 35, p. 451) observes that although statements of consistency and coherence might be assertory within a stronger meta-mathematical theory than the theory under examination, the categorical logic of this stronger theory does not, in itself, need to be conceived as truthful (in the externally assertory sense), because strength can be defined strictly in computational or information-theoretic terms.

2 Structural Realism recognises the importance of two fundamental problems which must be overcome for an evolving scientific practice. First, there is the ‘No Miracles Argument’ (NMA) as set out by John Worrall (1989), which stipulates that realism is only philosophy that does not make the success of science a miracle. Second there is the ‘Pessimistic Meta-induction Argument’ (PMIA), which accepts inductive evidence that current theories are likely to be discarded despite their current success (the dual to this principle is the ‘Optimistic Meta-induction Argument’ (OMIA), which accepts evidence that if current theories are false they too will be discarded despite their current success).

3 Badiou introduces a further distinction between what he sees as two French traditions, namely: that of Brunschvicg (a mathematising idealism) and that of Bergson (a vitalist mysticism), suggesting that the first passes through Cavailles, Lautman, Desanti, Althusser, Lacan and himself, while the second passes through Canguilhem, Foucault, Simondon and Deleuze. Needless to say, this convenient distinction pits him in opposition to Deleuze despite the latter’s obvious affinity with the work of Albert Lautman. Notably, Badiou and Deleuze follow Lautman closely in their respective interpretations of the important contributions made by Galois and Abel to contemporary mathematics.

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the "truth event"! As such, the void (conceived in Lacanian and Cantorian terms as both the empty set and "trace" of the lost object of desire) is no longer the "eclipse of the subject". Instead, it is on the side of being, while truth, he suggests, is on the side of the indiscernible. The basis for his critique of democratic materialism (which he interprets as a form of social constructivism) is that,

A truth, if it is thought as being solely a generic part of the situation, is a source of veracity once the subject forces an undecidable in the future anterior. But if veracity touches on language (in the most general sense of the term), truth only exists insofar as it is indifferent to the latter, since its procedure is generic inasmuch as it avoids the entire encyclopaedic grasp of judgements. (Badiou, 2009: 433)

For his part, Whitehead follows Leibniz in asserting the monadic uniqueness of relata (his term for relata is "actual occasions"). Actual occasions are the only ontological realities in his philosophy of process. That does not mean that relata are not determined in their uniqueness by structure. One way to appreciate this metaphysical stance is to consider a simple example. Dipert (1997) draws on the notion of an asymmetric graph to provide a formal (iconic) illustration of how structure can determine both invariances and the uniqueness of relata. In an asymmetric graph, each vertex (i.e. each of the relata) can be uniquely identified by the overarching structure (that of an undirected graph of edges and vertices), while objects can be conceived as subgraphs enclosed within the asymmetric graph. While some of these subgraphs may be isomorphic to one another, their vertices will all be uniquely determined.

One of the crucial differences between Whitehead and Badiou concerns their approach to mereology, or the relationship between parts and wholes. This is the theme of the following section of the paper.

4. Mereological considerations in the work of Badiou and Whitehead

In his Prologomena, Kant set out arguments to justify the importance of strong mereological conceptions of unity insofar as they apply to the various categories of understanding. In his table of categories, for example, each third category is a combination of the previous two. So limitation brings together reality and negation, totality brings together unity and plurality, community both subsistence and causality, and necessity/contingency both possibility/impossibility and existence/non-existence. In each case the third category is organic, in the strong mereological sense, insofar as it relies on internal relations between its component parts. For example, community entails mutual or reciprocal causality and interdependence rather than one-way causality or substantive independence.

Bell (2001: 1) observes, that the whole-part relation also plays a fundamental role in respect to Kant's notions of the manifold, number, synthesis, and synthetic unity; his doctrines of the unity of apperception; his distinctions between judgments of perception and experience; and, in his arguments about the composition of time and space. In defining the concept of analytic unity or juxtaposition, which arises when there are two of more things in a manifold, this manifold is nothing different from the many, implying zero mutual involvement. Similarly, in speaking of the union or formation of a synthetic whole Kant concedes that in its weakest form, this union is a mere aggregation constituted on the basis of common though external relation, such that parts are prior to whole (AK XXIV: 87 ). Nevertheless, in regard to the formation of a synthetic unity (B201a), he claims that the parts are, instead, internally related (belonging) to one another.
While mereological concerns can be traced back to Aristotle, they played an important role in Analytical Philosophy. G. E. Moore's attack on the notion of "internal relations" was clearly directed against the organicism of the Oxford Hegelians. The refusal of Whitehead and Husserl to abandon the notion of internal relations however, is entirely understandable. For Husserl this ontological notion grounded his theories of quality in perception and unity in Arithmetic, whereas Whitehead drew upon the distinction between internal and external relations to characterize both abstractive hierarchy and complexity.

In his seminal work, The Concept of Model, Badiou (2007) argues that logic is the very thought of what is structural, whereas mathematics is the thought of the system so structured. Crucially, this model-theoretic conception is related to the properties of the universal quantifier: "for all x something" is logical if it is valid for every possible model, whereas it is mathematical if it is only available for determinate models (Badiou, 2007: 86).

In Logics of Worlds, this conception of the mathematical merely serves as a point of departure. Now mathematics is conceived as the thought of the pure multiple, while logic is considered to be the formal structure of possible worlds (Badiou, 2009: 85). Although mathematics "touches being", logic "touches the localization of being", but that does not mean that mathematics, in any sense, is narrower. As Tzuchien Tho argues, with the move from set theory to the theory of the topos in Logics of worlds, every localization finally includes the presence of pure multiplicity as a matter of localization. In other words, it is no longer a question of the extension of the universal quantifier's value but a question of a localization of the global itself (i.e. the possibility of thinking being as such in a determined world). This process of localization is formally and logically conceived in terms of the \( \Omega \)-complete Heyting algebra, which grounds Badiou's conception of the topos.

Badiou now conceives of mathematics as the science of being qua being, as the disclosure of the structure of the 'there is', the latter regarded in turn as a presentation, or a situation. The 'there is' can only be grasped initially as a multiplicity due to the non-inherence of unity. For Badiou, this multiplicity is rigorously conceptualised as a set. Thus, for Badiou no less than for Kant, objects as such are only unitary due to a subsequent operation of unification extrinsic to their being. Even in Logics of Worlds, the multiple must still be counted-for-one before being submitted to transcendental laws. Set theory is still the formal structure of any presentation. It is not so much that being is mathematical, but that mathematics pronounces what is expressible of being qua being (Fraser, "Introduction" to Badiou, 2009: xxxix; xli). It is not so much a question of an identity holding between sets and presentations; rather, all being is thought in and by the set. As a mode of thought, therefore, a set is that aspect that is produced of each thing when that thing is thought in its being.

Badiou observes that in Kant's analysis of experience, the process of 'gathering in' is accomplished by the synthetic power of transcendental apperception, which is, in turn, conceived as the originary structure of all presentations. He notes the mereological distinction supporting Kant's analysis. On the other hand there is the binding of representations. In this case, it is the faculty of binding rather than the bind itself, which Badiou views as characterizing all representable structures (Badiou, 2004: 137).

What makes boundedness possible is not the bind as such, which, from this point of view, in-exists, but then pure faculty of binding, which is not reducible to effective relations since only the one can account for it; it is the originary law for the consistency of the multiple, the capacity for 'bringing the manifold of given representations under the unity of apperception'.

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Nevertheless, Badiou observes that Kant, in response to his famous epistemological problem—how are a priori judgements possible?—conceives of each form of aggregation in terms of the object as a horizon of knowledge. Through the introduction of objectivity and object-hood Kant can now assign both unity and binding to the faculty of understanding under the logical form of the judgement. Badiou, however, complains that, as a result, the very being of the object is left undetermined by the process of binding.

Badiou’s conception of truth conforms neither to that of adequation nor correspondence. Rather, he reflects on the capacity of a truth to “subtract itself from” the existing regime of knowledge so as to achieve a re-ordering of all the basic possibilities of presentation and representation within existing order (Badiou, 2009: 9-10). Any resulting transformation will always and of necessity be partial and capable of infinite renewal through the fidelity of the “subject” to its consequences (Badiou, 2009: 33-5).

Badiou’s notion of a subtractive ontology is closely related to his championing of an ethical fidelity to the truth event. For him, fidelity provides the force necessary for unifying the two voids—one of which is associated with the transcendental subject (for Kant, this is the subject conceived as constituted by the transcendental apperception) while the other is associated with the transcendental object (or object considered solely as a representative of the pure form of objectivity). A similar unifying force is described in the context of Lacan’s discussions of the ethics of psychoanalysis, which stipulates that we as ‘analysands’ should not give up on our desire. On the basis of his declaration that “there are truths” it would seem that Badiou has effectuated a methodological surpassing of the Circle of Objectivity. In the process, however, he abandons any further discussion discussion of mereological considerations. He also goes on to distance himself from Lacan—the Master—in arguing.

What Lacan lacked—despite this lack being legible for us solely after having read what, in his texts, far from lacking, founded the very possibility of a modern regime of the true—is the radical suspension of truth from the supplementation of a being-in-situation by an event which is a separator of the void. (Badiou, 2009: 443)

Nevertheless, seen in the light of recent developments in set theory and forcing, Badiou’s interpretation of Cohen’s work is probably one of the weaker aspects of his analysis of the forcing phenomenon in Being and Event. Many set theorists seem to be following Hamkins’s (2011) argument that there are many distinct concepts of set, each mediated by a corresponding set-theoretic universe. This contrasts markedly with the absolute universe view, which stipulates that there is an absolute background set concept, with a corresponding absolute set-theoretic universe. In holding to the former position, Hamkins contends that

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**Footnote:** For his part, Badiou entitled part VIII of Being and Event “Forcing: Truth and the Subject: Beyond Lacan” and explicitly engaged with Lacan in Book VII, section 2 of Logics of Worlds. And, in his essay “On Subtraction”, published in his Theoretical Writings, Badiou describes each of the stages of forcing in Lacanian psychoanalytic terms. Lacan (1961-2) draws on the concept of the unary trait (the einziger zug initially described by Freud in Group Psychology) to explain the nature of symbolic identification, which arises for Freud when one arbitrary aspect of someone’s mannerisms or personality comes to assume the full burden of representation. For Lacan, the signifier, as the “simplest form of mark” (like a notch scratched on the handle of a gun) also marks the point of conjunction with jouissance or enjoyment, insofar as the paired terms serve as the motive for repetition. Far from representing a surpassing of the signifier or a transgression of the law, jouissance functions more as a form of ‘waste’ or entropy: as what serves no purpose. Through this coupling the signifier operates as something “intersignifying”, as a gap internal to the linguistic field. Here, the gap reflects the inadequacy of the signifier to itself insofar as it is unable to function purely, nevertheless, for Lacan the gap has two names: as a negative magnitude (say, √-1) it relates to the subject as: and, as a positive magnitude (i.e. waste) it relates to the object cause of desire (the objet a). Crucially, in this role the signifier does not represent an object for subject; rather, it represents the subject for another signifier.
interesting set-theoretic questions (such as the continuum hypothesis) have definitive, final answers; confirmed by the increasingly stable consequences of the large cardinal hierarchy. For adherents of this view the pervasive independence phenomenon is a distraction or side discussion about provability rather than truth. Hamkins suggests that his favoured multiverse view, is one of higher-order realism insofar as it asserts the actual (though Platonic) existence of alternative set-theoretic universes. Whitehead seems to concur with this viewpoint.

While mereological concerns can be traced back to Aristotle, they played an important role in Analytical Philosophy. G. E. Moore’s (1919-20) attack on the notion of “internal relations" was clearly directed against the organicism of the Oxford Hegelians. The refusal of Whitehead and Husserl to abandon the notion of internal relations however, is entirely understandable. For Husserl this ontological notion grounded his theories of quality in perception and unity in Arithmetic, whereas Whitehead drew upon the distinction between internal and external relations to characterize both abstractive hierarchy and complexity.

For his part, Whitehead escapes the Kantian Circle of Objectivity holding between the knowing subject and known object, by expanding it to encompass all entities (irrespective of whether they are conscious or unconscious, and organic or inorganic) and associated processes of prehension (a correlated generalisation of cognition as comprehension). As he puts it,

The philosophy of organism is the inversion of Kant’s philosophy. The Critique of Pure Reason describes the process by which subjective data pass into the appearance of an objective world. The philosophy of organism seeks to describe how objective data pass into subjective satisfaction, and how order in the objective data provides intensity in the subjective satisfaction. For Kant, the world emerges from the subject; for the philosophy of organism, the subject emerges from the world. (Whitehead, 1928: 135)

Nevertheless, for our purposes, Whitehead’s mereological conceptions have the greatest import for economics. Initially, Whitehead developed the logical foundations for a mereological conception of space-time (the extensive continuum) in the years after the publication of Principia Mathematica. He has always intended to complete a fourth volume of the Principia on geometry, but never wrote it. His 1914 correspondence with Bertrand Russell reveals that his intended approach to geometry can be viewed, with the benefit of hindsight, as mereological in essence. This work culminated in Whitehead (1916) and the mereological systems of Whitehead (1919, 1920). Stanislaw Leśniewski was the logician who actually coined the term “mereology" in 1927, based on the Greek word µέρος (mêros, "part"), which he adopted to refer to a formal theory of part-whole in papers that he published between 1916 and 1931 (translated in Leśniewski, 1992). Subsequently, Leśniewski’s student Alfred Tarski (1984) simplified Leśniewski’s original formalism. Point-free geometry was first formulated in Whitehead (1919, 1920), initially as a theory of “events" given an “extension relation" defined as holding between events rather than as a theory of geometry or of spacetime. As such, Whitehead’s intentions were both philosophical and mathematical. By showing how Whitehead’s theories could be fully formalized and repaired, Clarke (1981, 1985) founded contemporary mereo-topology. Pointless topology is an approach to topology that avoids mentioning points. The name ‘pointless topology’ was originally due to John von Neumann and described a topological approach, in which regions (opens) are treated as foundational without explicit reference to underlying point sets.

More general versions of pointless topology, deploy spectral rather than Hausdorff spaces. Johnstone (1983: 47) cites Wallman’s 1938 demonstration that any Hausdorff space may be embedded as a retract of spectral space. While classical topologists may complain that
this amounts to representing something well known (Hausdorff spaces) in terms of what is less
known (spectral spaces), he observes that from a localic perspective, the latter are much
better behaved. Moreover, the ‘pointless topologist’ can take regularity as his or her
separation axiom rather than the Hausdorff property based on points. Furthermore, in
response to the criticism that “pointless topology” is topology for logicians rather than
topologists, Johnstone (1983: 49) observes that there are many contexts in which a
mathematician would like to do topology but the axiom of choice or law of excluded middle
is simply not available. He instances the example of toposes, which are a category sufficiently
like the category of sets in that one can carry out set-theoretic constructions in it, but only if
the law of excluded middle is refused. Other examples that he cites include: sheaves as a
topology over a fixed base, proper maps, zero-dimensional locales (or spreads), open maps of
spaces, and equivariant topologies of groups or monoids.

In his 2008 critique of Logics of Worlds, Peter Hallward observes that, despite the
dazzling ambition and breadth of the Badiou’s analysis of “the being-there of the being of a
being”, he is obliged to develop such an elaborate and laborious theory of logical worlds” to
compensate “for the consequences of his enthusiastically simple if not simplistic conception of
being (without beings), of appearing (without perception), of relation (without relation), of
change (without history), of decision (without alternatives), of exception (without mediation)”.
Instead, Hallward (2008: 121) pleads for the constitution of a thoroughly relational ontology—
one that “privileges history rather than logic as the most fundamental dimension of the
world”.

The reason for Hallward’s dissatisfaction is obvious: Badiou conceives of existence in
terms of “the ongoing process of its relation to itself” (i.e. Badiou’s identity function is
constructed as a self-reflexive ‘morphism’ varying between minimal and maximal limits)
(Hallward, 2008: 115). For Badiou this is necessitated by the fact that set theory excludes
relation from being (i.e. a function is defined solely through membership in terms of the set of
elements that it generates). Hallward (2008: 115-116) instances class conflict as an example
where a mere partial ordering of relative differences in intrinsic strength, or even a revaluation
of the intensity of a singular object fails to capture the richness of history or to explain
processes of genuine change. Although Badiou’s analysis of appearance represents an
improvement over his previous notion of the ‘evental site’, which conceived of singularity “in
terms of exclusion pure and simple”, Hallward (2008: 118) cautions that the knowledge of
being derived from such an analysis is unable to account for Foucauldian, if not Gramscian
forms of power that “do not merely exclude or prohibit but rather modulate, guide or enhance
behaviour and norms conducive to the status quo”.

Hallward (2008: 118) attributes this restricted palette to the absence of any middle or
mediating terms in Badiou’s analysis:

Between the being of a pure simple multiplicity and an appearing as docile or insurgent
lies an abyss without mediation. The space that in other philosophies might be filled by an
account of material actualisation or emergent self-realization (or any number of alternatives) is
one that Badiou, so far, prefers to consign to contingency.

One crucial question is whether the non-organic Vitalism of Deleuze and Whitehead
might afford the antidote to such a restricted palette.

While Hallward is ‘heading in the right direction’ with this critique, from a category-
theoretic perspective his arguments are somewhat misplaced. For one thing, Badiou has a
remarkably diverse range of colours available to him on his “palette”. Free functors defined
over an indexed base category Set, and another category, can expose the points located within
such a set to an remarkable diversity of logics and topologies. Moreover, as Awodey argues, in
the category of Set, products (as adjoints) provide for relations as propositional functions on
the product, and exponentials (as adjoints) provide for propositional functions of propositional
functions. Clearly, this is not where Badiou’s weaknesses are to be found. Rather, they reside
in his desire to continually revert back to the category of Set as a base category for his chosen
system of Ω-complete Heyting Algebras. As argued above, this restriction—one that is
effectively imposed by Badiou’s atomistic “principles of materialism”—precludes a wide range
of on-going developments in pure mathematics and algebraic topology, especially those
motivated by mereological concerns about the relationship between parts and wholes.
Recently, mereologists and mereo-topologists have contributed to the task of formalizing both
weak and strong mereological notions of unity (Mormann, 1998; Petitot, 1993). From a
categorical perspective, Ellerman’s work on the disjunctive syllogism, however, may provide
some direction to those intending to apply category theory to political economy.

5. Category Theory and the Social Sciences

Underpinning the ability of category theory to identify and characterize structural
invariances are universal mapping properties. Ellerman (2006) points out that, while both set-
theory and category theory involve universals, in each respective case, however, the universals
differ significantly in nature. This difference directly involves the issue of a self-participating
universal. He explains (Ellerman, 2006: 133) that Frege’s hubris found expression in the
attempt to construct a general theory of universals that could be either self-participating or
non-self-participating. In an effort to avoid ensuing paradoxes of logic, Russell’s theory of types
led to the construction in set theory of non-self-participating universals. Although various
instances of a property can be collected together there is no explicit theory of determination
with respect to a given property. While determination by morphisms is analogous to the
notion of collection (membership) in set theory, the notion of an always “self-participating”
and concrete universal was realized in category theory by requiring that objects possess
universal mapping properties (UMPs) (Ellerman, 2006: 134). In effect, all instances of a
property are determined to have that property by virtue of a morphism “participating” in that
paradigmatic instance. Universality is thus achieved by reconceptualizing atoms (x, y) from
being determinees to being their own determiner (e.g. in case of product, the determiner of X
and Y is set of all ordered pairs (x, y) from X and Y denoted X ⊗ Y isomorphic to all other sets
with that UMP (Ellerman, 2006: 135). Moreover, the universals of category theory are self-
participating insofar as each “participates” in itself via the identity morphism.

Ellerman goes on to observe that universals always part of what is called an adjunction
in category theory, which raises the question, What are adjoint functors about? During the
early stages in the development of category theory, categories were used to define functors,
and functors were used to define natural transformations. Although adjoints came later, they
are now seen as foundational. Ellerman explains that adjoints arise from bi-representations of
what he calls chimera or hetero-morphisms (i.e. morphisms between objects of different
categories) and it is these representations that provide universal properties. Accordingly,
Ellerman introduces the notion of Het-bifunctors (represented by Het: X ⊗ Y ⊗ Set), to treat
object-to-object chimera morphisms between categories, along with the Adjunction
Representation Theorem, which states that every adjunction,

\[ F: X', A: G \]
can be represented (up to an isomorphism) as arising from the left and right representing universals of a het-bifunctor \( \text{Het}: X \circ \text{Set} \to \text{Set} \), giving the chimera morphisms from the objects in a category,

\[
X \cong \hat{X}
\]
to the objects in a category,

\[
A \cong \hat{A}
\]

Each determination achieved through a given adjunction factors uniquely through sending/receiving universals at each end of the determination (zig-zag factorization). Ellerman (2006: 173) notes that the universal constructs the object representing all possibilities that might be directly determined (via the UMP). Particularization only comes with the indirect factor map that selects certain possibilities. These two aspects of the adjunction give rise to a composite effect, reflecting the fact that the composition of the specific factor map then implements the possibilities to agree with the direct determination.

Having described the nature of universals in category theory Ellerman goes on to consider the applicability of this concept to certain developments in the Social Sciences. To this end, he distinguishes two mechanisms: instruction and selection.

Instruction is applicable to relations between the environment and the organism, teacher and pupil, and antigen and the immune system. In each of these cases the first member of the pair can be conceived as actively instructing the second member about adaptive features, capabilities, or the required anti-pathogenic response. A selectionist mechanism working within the population, amongst students, or internal to the immune system explores possibilities already generated by variation, already "imprinted" on the soul, or constructed by the immune system (Ellerman, 2006: 175-8). For example, in the process of biological evolution the DNA sequence takes on both roles, first as a determinee, then after the splitting of double-helix, as a determiner. In linguistics, the Chomskyian notion of a generative grammar conceives linguistic competence as an innate capacity to acquire language that unfolds according to experience of child. In Edelman's conception of neural Darwinism internal selection processes lead to the privileging of one neuronal group over another. In each case, a composite effect obtains when process of external instruction, potential self-reproduction, and internal selection lead to the differential reproduction of selected variants. Ellerman (2006: 176) summarises the implications of recent developments in immunology by suggesting that organisms function more like juke-boxes (characterised by an internally structured repertoire of capabilities) than phonographs (which can only play records that have been chosen by some external process). Finally, Ellerman (2006: 179) turns to a telling political example in the form of (heteronomous) governance relations between employer and employee. Under his category-theoretic conception of composition these relations can be restructured (i.e. factored via the indirect form) to form a political democracy characterised by self-management. In this way, once again under the sway of chimera adjunctions, the determinee becomes a self-governing determiner.

In his concluding comments Ellerman observes that his work considers identifies a series of "unreasonably effective analogies" between mathematical concepts on one hand, and some central philosophical themes, on the other hand. Specifically, his chimeric conception of adjunction as both organised and self-organizing, achieves a reconciliation between the two poles of the Kantian antinomy between external and efficient cause and internal and final cause. At one stage Ellerman also refers to the conceptual structure of determination through universals as a "normative model".
Significantly, Ellerman demonstrates that his chimeric formalization of adjunctive squares carries over to both limits and colimits. In the category-theoretic literature introduced in the next section of the paper, colimits are used to characterise the structural properties of complex, hierarchical systems. Ehresmann and Vanbremeersch (1987: 17-18) argue that the categorical notion of the colimit is a good model for hierarchical organization. Further refinements of the colimit concept—including the notion of collective links, sub-limits (i.e. colimits situated within a hierarchical ordering), and comparisons—enable them to formally represent the well-known phenomenon of hierarchy within complex biological systems.

Introducing a 'transition functor' whose morphisms (between sub-categories) can be temporally indexed, Ehresmann and Vanbremeersch construct what they choose to call a 'transitive system', which allows them to embed their hierarchical system of sub-limits within a dynamic frame. This formal machinery can, in turn, account for (phenomenological) processes of creation, destruction, gluing, and persistence. Accordingly, biological (and social) processes of maintenance and reproduction as well as the emergence of novelty can be accommodated within an integrative framework. Clearly, the same arguments would apply with equal force to social systems.

6. Concluding thoughts on the application of category theory to economics

The following comments about the applicability of category to economics must, of necessity, be brief. Firstly, commodities are neither fractal nor infinitely divisible, and even in obvious cases where individual units could be further divided, as with the grinding of wheat into flour, the process of division is effectively a process of production, which changes the very nature of the commodity. Neoclassical General Equilibrium theorists such as Gerard Debreu, attempted to bypass the reality of indivisibility at the level of the commodity by spuriously assuming that it was the attributes of the commodity rather than the commodity itself, that could be endlessly divided. This simple insight into indivisibility has profound implications: the economic analysis of processes of arbitrage and the determination of equilibrium must now proceed along combinatorial lines, forsaking the mathematical convenience of order-dense metric spaces.

By the same token, analysis can gainfully draw on the resources of category theory to the represent economic phenomena, in particular, through the use of adjunctions, along with hierarchies of limits and colimits, and where necessary, the application of transition functors. The universal mapping properties of limits and colimits can be deployed to represent bilateral social relationships, which are inscribed within more extensive hierarchical structures of authority. For example, while the King has authority over vassals (e.g. knights and sheriffs), these same vassals, through the authority of the King, exercise domination over villeins (peasants). The totality of social control that any individual is able to exercise over objects characterizing a process of production can now be represented by a family of morphisms over functor homomorphisms, representing the production process in turn, the temporal nature of production can be formally described through the use of transition functors. Finally, bifunctor relationships can be deployed to distinguish between relations of consent and coercion, on one hand, and the respective flows of rewards and obligations (the beneficium and servicium) that obtain through these respective relations, on the other hand. Transformations in social relation can then be represented by changes in the nature and admixture of the respective morphisms.

These observations draw on the unpublished work of Tony Atley (2011), who was influenced by the observations of the now-deceased Macquarie University mathematician, A. J. van der Poorten.

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References


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