LARGE DEFORMATION ANALYSIS IN GEOMECHANICS USING ADAPTIVE FINITE ELEMENT METHODS

By

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A Thesis submitted for the
Degree of Doctor of Philosophy at
The University of Newcastle

March 2012
DECLARATION

“This Thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying subject to the provisions of the Copyright Act 1968.”

(Signed)
ACKNOWLEDGEMENTS

This research study would not have been possible without the support of many people who have helped and inspired me during my study.

First and foremost I offer my sincerest thanks to my supervisor Prof. Daichao Sheng for his unending help and invaluable assistance, support and guidance from the initial to the final level of my research. His unlimited availability and encouragement made my research life smooth and rewarding. Also, I would like to appreciate my co-supervisors Prof. Scott Sloan and Dr. Andrew Abbo for their helpful advice.

I was delighted to have valuable discussion with Prof. Peter Wriggers in some critical stages of this research work. His guidance is really appreciated.

I offer my regards and blessings to the colleagues and postgraduates in the Geotechnical Group of the University of Newcastle who supported me in many respects and provided a convivial environment to work.

I acknowledge the University of Newcastle for giving me a postgraduate scholarship and providing the financial support for this research.

I owe my deepest gratitude to my husband Majid, for his unflagging love and support and encouragement. I am indebted to him for his abundant help, guidance and valuable assistance in the preparation and completion of this study. Last but not least, I am grateful to my gorgeous son who has constantly reminded me that life without a challenge is going to be really boring.
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ABSTRACT

The finite element method (FEM) is extensively used in analysis of a wide range of nonlinear geotechnical problems. The finite element method can handle simple and complex constitutive soil models, and solve problems with complicated geometries and boundary conditions with reasonably accurate results. On the other hand, mesh distortion and entanglement of elements, occurring inevitably in failure zones with high stress/strain concentration, are main drawbacks of the common finite element solutions such as the Updated Lagrangian method. In addition, efficacious application of the method requires experience and a certain amount of trial and error, particularly when choosing an optimal time and spatial discretisation.

Adaptive finite element methods provide a means for obtaining more reliable solutions by continuously adjusting the discretisation in time and space according to the current solution. These procedures automatically refine, coarsen, or relocate a mesh to achieve a solution with a specified accuracy in an optimal fashion. Although a significant amount of research has been devoted to adaptive finite element analysis in solid mechanics, the application of adaptive methods has been less considered in nonlinear geotechnical problems due to the complexity. Modelling of problems in geomechanics is typically sophisticated due to nonlinear constitutive laws, large deformations, changing boundary conditions and time-dependent behaviour. A variety of adaptive finite element techniques have been developed to tackle nonlinear problems in solid mechanics. However, the
application of these methods to geomechanics is still a challenge. Amongst the various adaptive techniques, the $r$-adaptive and $h$-adaptive finite element methods are probably the most favoured and most established. $r$-adaptive finite element method attempts to eliminate the mesh distortion by refining the mesh in the finite element domain. On the other hand, $h$-adaptive finite element method is based on the idea of generating a new mesh by dividing the area of original elements where the interpolation should be improved to achieve higher accuracy or to avoid mesh distortion.

In this Thesis, the $h$-adaptive finite element technique will be employed to solve some complex geotechnical problems involving material nonlinearity, large deformation, changing boundary conditions and time-dependent nonlinearity. To achieve this, the main features of the technique including advanced mesh generation algorithms, error estimation methods and a procedure for remapping of state variables will be discussed and developed in company of a robust analysis program. The performance of the $h$-adaptive finite element method is then represented by considering the accuracy and efficiency of the method in solving some classical geomechanics problems such as the bearing capacity of footings, expansion of cavities, and the stability of slopes.

In addition, this Thesis will address the performance and the efficiency of alternative error estimation techniques for particular geotechnical applications involved with changing boundary conditions and inertia forces, such as static and dynamic penetration of an object into soil. Such problems are categorised as one of the most sophisticated problems of computational geomechanics due to their extreme nonlinearity.
This Thesis will also present a new and innovative combined adaptive method for tackling geotechnical problems with relatively large deformations. This robust method is based upon an elegant combination of the Arbitrary Lagrangian-Eulerian (ALE) method and the $h$-adaptive finite element method developed as a part of the Thesis. The proposed method takes advantage of $r$-refinement as well as $h$-refinement finite element techniques, and yet eliminates the individual drawback of each method.
PREFACE

The research work presented in the Thesis has been performed in the Discipline of Civil, Surveying and Environmental engineering, school of Engineering, at the University of Newcastle, Australia under supervision of Prof. Daichao Sheng from September 2007 to September 2011. During the term of candidature, a number of papers were published which are listed below:


