# Reliability of Interconnection Networks

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I would like to dedicate this thesis to my loving parents Professor Shiying Wang and Mrs. Zhifen Mu.

# Declaration

I hereby certify that the work embodied in the thesis is my own work, conducted under normal supervision. The thesis contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made. I give consent to the final version of my thesis being made available worldwide when deposited in the University's Digital Repository, subject to the provisions of the Copyright Act 1968 and any approved embargo.

Mujiangshan Wang

June 2019

# Acknowledgement of Authorship

I hereby certify that the work embodied in this thesis contains published papers of which I am a joint author. I have included a written declaration below endorsed in writing by my supervisor, attesting to my contribution to the joint publications. By signing below I confirm that

I contributed the main proofs of lemmas, theorems and corollaries, the discussion of other proofs and the proofreading to the papers entitled as follows:

[1] Wang, M., Lin, Y., and Wang, S. (2016). The 2-good-neighbor diagnosability of Cayley graphs generated by transposition trees under the PMC model and MM* model. *Theoretical Computer Science*, 628:92–100.

[2] Wang, M., and Wang, S. (2016). Diagnosability of Cayley graph networks generated by transposition trees under the comparison diagnosis model. *Annals of Applied Mathematics*, 32(2):166–173.

[3] Wang, M., Guo, Y., and Wang, S. (2017). The 1-good-neighbour diagnosability of Cayley graphs generated by transposition trees under the PMC model and MM* model. *International Journal of Computer Mathematics*, 94(3), 620–631.

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[6] Wang, M., Lin, Y., and Wang, S. (2017). The nature diagnosability of Bubble-sort star graphs under the PMC Model and MM* Model. *International Journal of Engineering and*

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[7] Wang, M., Lin, Y., and Wang, S. (2018). The 1-good-neighbor connectivity and diagnosability of Cayley graphs generated by complete graphs. *Discrete Applied Mathematics*, 246:108–118.

[8] Wang, M., Lin, Y., Wang, S., and Wang, M. (2018). Sufficient conditions for graphs to be maximally 4-restricted edge connected. *The Australasian Journal of Combinatorics*, 70(1): 123—136.

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[3] Zhao, L., Wang, M., Zhang, X., Lin, Y., and Wang, S. (2017). An algorithm for the orientation of complete bipartite graphs. *International Conference on Applied Mathematics*.

[4] Wang, S., Wang, Z., Wang, M., and Han, W. (2017). $g$ -good-neighbor conditional diagnosability of star graph networks under PMC model and MM$^*$ model. *Frontiers of Mathematics in China*, 12(5):1221–1234.

[5] Lin, Y., Wang, M., Xu, L., and Zhang, F. (2017). The maximum forcing number of a polyomino. *The Australasian Journal of Combinatorics*, 69(3):306—314.

Name of Supervisor,
Yuqing Lin

Signature of Supervisor

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# Publication

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# Abstract

Graph is a type of mathematical model to study the relationships among entities. The theory on graphs is called Graph Theory. It started in 1736 and has 283 years of history since the paper was written by Leonhard Euler on the Seven Bridges of Königsberg.

In computer science, the term "Interconnection Networks" has been used to refer to a set of interconnected elements. For example, a computer network where computers was connected by wires or Internet of Things (IoT) is connected via wireless connection. There are two types of network: static and dynamic.

Static networks are hard-wired and their configurations do not change. The structure, which is also called topology signifies that the nodes are arranged in specific shape and the shape is maintained throughout the networks. In this thesis, we focus on the static networks.

In graph theory, graphs are used to model the topology of network, whether it is networks of communication, data organization, computational devices, the flow of computation. For instance, the link structure of a local area network can be represented by an undirected graph, in which the vertices represent computers and edges represent connections between two computers. A similar approach can be applied to problems in social media, travel, biology, computer design, mapping the progression of neuro-degenerative diseases, and many other fields. Graph models could be directed, undirected and weighted, depending on the properties of the network we are studying. Fault-tolerance of networks is an important property. Fault-tolerance is the property that enables a system to continue operating properly in the event of the failure of some (one or more faults) of its components. Fault-tolerance is particularly sought after in high-availability or life-critical systems.

We are interested in the fault-tolerance of networks. Considering the corresponding graph model of the networks, connectivity of the graphs measures how resistant a graph can be against the nodes (link) removal. In graph theory, there is a set of fault-tolerance related parameters, such as restricted-connectivity, extra-connectivity etc., which gave refined information about how robust is a network.

Performance of the distributed system is significantly determined by the choice of the network topology. Desirable properties of an interconnection network include low degree, low diameter, symmetry, low congestion, high connectivity, and high fault-tolerance. For the past several decades, there has been active research on a class of graphs called Cayley graphs because this type of graph possesses many of the above properties. Many Cayley graphs based on permutation groups has proven to be suitable for designing interconnection networks, such as Star graph [1, 2, 47], Hypercubes [8], Pancake graphs [2, 79], Shuffel-Exchange Permutation Network [50], the Rotation-Exchange Network [110]. These graphs are symmetric, regular, and share the desirable properties described above.

In this thesis, we studied the connectivity and diagnosability of some popular network structures. For instance, Cayley graphs generated by transpositions, expanded $k$-ary $n$-cube and locally twisted cube.

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