Aghbolagh, H. D., Schmid, R. & Zamani, M. et al. (2018) Nonovershooting bipartite output regulation of linear multi-agent systems. Australian and New Zealand Control Conference (ANZCC 2018): Proceedings (Melbourne, Vic. 07-08 December, 2018) p. 204-209

Available from: http://dx.doi.org/10.1109/ANZCC.2018.8606568

Nonovershooting Bipartite Output Regulation of Linear Multi-Agent Systems

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Abstract— **We consider the problem of output regulation for linear multi-agent systems over signed digraphs. A design method is presented that solves the bipartite output regulation problem and also ensures that all agents track the desired reference signal without overshoot in their transient response. Finally, the results are validated via a numerical example.**

I. Introduction

The classic output regulation problem is central to modern control theory. The aim of the problem is to design a feedback controller which internally stabilizes the system while rejecting the disturbances and ensuring the output converges asymptotically to the desired reference signal. The solvability conditions and extensive compilations of results are given in [1].

The problem of output regulation of multi-agent systems has been the subject of a number of papers recently, including [2], [3] and [4]. As some of the agents cannot access the exogenous signal, the problem cannot be solved by the methods of the classical output regulation. [4] proposed a so-called distributed control scheme and gave conditions under which their multi-agent cooperative regulation problem could be solved. They showed that their framework could accommodate the methods of [2] and [3] as special cases.

To date there has been little consideration given to the transient response of the regulated outputs of the agents under these cooperative control schemes. For many control systems, there is a need to avoid undesirable transient phenomena such as high-frequency oscillations and large magnitudes of the output [5]. Using a state feedback, [6] implemented a nonovershooting algorithm on a single LEGO® robot and showed that this algorithm is capable of driving the robot to the desired point without overshoot. This example can be generalized for the case of robotic groups. Consider a multi-robot system performing a task in a dangerous environment or carrying a highly sensitive load. In such situations, an overshoot in any of the agents may cause irreversible damage such as losing the robot itself or the load.

The design of control laws to achieve a nonovershooting step response for a single linear time invariant (LTI) plant (N = 1) was considered in [7]. Several methods were given for the design of a linear state feedback control law to deliver a nonovershooting step response for an LTI system, subject to a known non-zero initial condition. This requires the closed-loop system to be stable, and that the tracking error of the step response converges to zero without changing sign in any of its components. In [8], the methods were adapted to the problem of avoiding undershoot in the step response, and in [9] the methods were used to achieve nonovershooting output regulation.

In most works in the literature, cooperative interactions are considered. That is, it is assumed that the agents communicate over graphs with positive signs. However, using the notion of signed graphs, researchers have been able to consider negative interactions which have been interpreted as competition, antagonism, or disagreement [10]. Just to mention a few works available in the literature, one may refer to [11], which uses a distributed feedback law to achieve output bipartite consensus for heterogeneous multi-agent systems with a virtual exosystem. Furthermore, [12] employed output feedback laws to study bipartite output synchronization of heterogeneous agents. The work of this paper was further expanded in [13]. Bipartite output regulation of multi-agent systems with antagonistic interactions was discussed in a recent paper by [14].

In [15], the authors provide nonovershooting transient response for a group of agents using dynamic output feedback. However, this paper only considers cooperative interactions. As a result, in this paper, we address the nonovershooting bipartite output regulation problem of multi-agent systems in such a manner that all agents achieve exact output regulation with a nonovershooting transient response.

The paper is organized as follows. In Section II, we give some essential mathematical preliminaries. In Section III, we introduce the output feedback control architecture introduced by [14], and define our nonovershooting bipartite output regulation problem. In Section IV, we briefly discuss the nonovershooting controller design methods of [7]. The main result of the paper is presented in Section V, where we show how these nonovershooting methods can be employed within the controller architecture of [14] to solve our problem. Section VI applies the control method to the example system discussed in [14], and Section VII offers some concluding thoughts.

Notation. Throughout this paper, the symbol 0_n represents the zero vector of length n, and I_n is the n-dimensional identity matrix. For a square matrix A, we use $\sigma(A)$ to denote its spectrum. We say that a square matrix A is *Hurwitz-stable* if $\sigma(A)$ lies within the open left-hand complex plane. For any complex scalar λ, we define $\mathbf{Re}(\lambda)$ to be its real part. We

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use $\mathbf{1}_N$ to denote an $N \times 1$ matrix with unit elements, and $\otimes$ denotes the Kronecker product of matrices.

II. Preliminaries

A. Signed Graphs

A signed digraph can be represented by $G^s = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{v_1, \dots, v_N\}$ is the finite set of N nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges, and $\mathcal{A}$ is the interaction matrix. We assume that $a_{ij}a_{ji} \geq 0$, where a_{ij} and a_{ji} are the elements of the interaction matrix. It is worth mentioning that a signed digraph G^s is said to be structurally balanced if it admits a bipartition of the sets $\mathcal{V}_1, \mathcal{V}_2$ such that $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ also $a_{ij} \geq 0$ $\forall v_i, v_j \in \mathcal{V}_{\alpha(\alpha \in \{1,2\})}$, $a_{ij} \leq 0$ $\forall v_i \in \mathcal{V}_\alpha$ and $v_j \in \mathcal{V}_{\beta(\alpha \neq \beta \text{ and } \alpha, \beta \in \{1,2\})}$. It is structurally unbalanced otherwise [10].

For a signed digraph G^s, the Laplacian matrix is denoted as $\mathcal{L} = \mathcal{C} - \mathcal{A}$, where $\mathcal{C}$ is the diagonal degree matrix defined as $[\mathcal{C}]_i = \sum_{j \in adj(i)} |a_{ij}|$ [10], [16].

Consider a *structurally balanced signed digraph* $\bar{G}^s$ with nodes $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{0\}$ with 0 being the index of the leader, and weights a_{ij}. Its Laplacian, $\bar{\mathcal{L}}$, can be partitioned as

$$\bar{\mathcal{L}} = \left(\begin{array}{c|c} \sum_{j=1}^N |a_{0j}| & [a_{01} \dots a_{0N}] \\ \hline -\Delta \mathbf{1}_N & H \end{array} \right) \tag{1}$$

where Δ is an $N \times N$ diagonal matrix with diagonal entries of a_{i0} [14]. Moreover, $H = \mathcal{L} + \Delta$, where $\mathcal{L}$ is the Laplacian of the signed digraph after removing the leader. It is assumed that $a_{i0} \geq 0$.

B. Gauge Transformation

A gauge transformation is a change of orthant order in $\mathbb{R}^N$ performed by a matrix $\Phi = \text{diag}(\phi)$. Denote $\mathcal{M} = \{\Phi = \text{diag}(\phi), \phi = [\phi_1, \dots, \phi_N], \phi_i \in \{\pm 1\}\}$ the set of all gauge transformations in $\mathbb{R}^N$ [10]. A gauge transformation can render $\mathcal{A}$ to be a nonnegative matrix for a structurally balanced digraph.

C. Exponentially decaying sinusoids.

Our analysis will require some discussion of the properties of exponentially decaying sinusoids.

Definition 1 ([15]): For any positive integer n, let μ_i, ω_i, α_i and β_i with $i \in \{1, \dots, n\}$ be sets of real numbers such that for all $i \in \{1, \dots, n\}$ we have $\mu_i < 0$ and $\omega_i \geq 0$. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(t) = \sum_{i=1}^{n} e^{\mu_i t}[\alpha_i \sin(\omega_i t) + \beta_i \cos(\omega_i t)] \tag{2}$$

Also let $\mu < 0$ be given by

$$\mu = \max\{\mu_i : i \in \{1, \dots, n\}\} \tag{3}$$

We say that the scalar function f is the *sum of exponentially decaying sinusoidal* (SEDS) functions with rate μ. If $v : \mathbb{R} \to \mathbb{R}^m$ is a vector-valued function with $v(t) = [v_1(t) \dots v_m(t)]^T$, and each component v_j is a scalar SEDS function of rate $\mu_j < 0$, then we say that v is a *SEDS function with rate* $\mu = \max\{\mu_j : j \in \{1, \dots, m\}\}$. If f is such that $\omega_i = 0$ for all $i \in \{1, \dots, n\}$, then we say that f is the *sum of exponentially decaying (SED) functions.*

We note some straightforward properties of SEDS functions.

Lemma 1 ([15]): Let $f_1 : \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$ be SEDS functions with rates $\mu_1 < 0$ and $\mu_2 < 0$ respectively. Then $f_1 + f_2$ and $f_1 f_2$ are SEDS functions with rates $\mu = \max\{\mu_1, \mu_2\}$, and $\mu = \mu_1 + \mu_2$, respectively.

Lemma 2 ([15]): Consider the linear system

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t), \quad x(0) = x_0 \\ y(t) & = & Cx(t) + Du(t) \end{array} \tag{4}$$

where A is Hurwitz. Let $\lambda_0 = max\{\mathbf{Re}(\lambda) : \lambda \in \sigma(A)\}$.

(i) For any x_0, the zero-input solution x and zero-input response y arising from the input u with $u(t) = 0$ for all $t \geq 0$ are SEDS functions with rate λ_0. Moreover, for any x_0, there exists a constant $k_1 > 0$ such that $|y(t)| \leq k_1 |x_0|$ for all $t \geq 0$.

(ii) If the input u is a SEDS function with rate μ, then the zero-state response y arising from $x_0 = 0$ is a SEDS function with rate μ. If $|u(t)| \leq k_1$ for some $k_1 > 0$, then $|y(t)| \leq k_2 k_1$ for some $k_2 > 0$.

Lemma 3 ([15]): Let $f : \mathbb{R} \to \mathbb{R}$ be a SEDS function of the form (2) with rate μ, and for some positive integer m, let $g : \mathbb{R} \to \mathbb{R}$ be a SED function given by

$$g(t) = \sum_{i=1}^{m} \beta_i e^{\lambda_i t} \tag{5}$$

where $\{\beta_1, \dots, \beta_m\}$ are arbitrary real numbers, and $\{\lambda_1, \dots, \lambda_m\}$ are distinct negative real numbers satisfying $\mu_i < \lambda_j$ for all $i \in \{1, \dots, n\}$ and $j \in \{1, \dots, m\}$. Assume $g(t) \neq 0$ for all $t \geq 0$. Then there exists a positive real number δ such that $g(t) + \delta f(t) \neq 0$ for all $t \geq 0$.

III. Problem formulation

We assume that the digraph G^s is structurally balanced. As a result, the set of nodes $\mathcal{V}$ can be divided into two disjoint subsets $\mathcal{V}_1$ and $\mathcal{V}_2$. Moreover, without loss of generality, we consider that $\mathcal{V}_1 = \{1, \dots, k\}$ and $\mathcal{V}_2 = \{k+1, \dots, N\}$. As a result, we can define a diagonal matrix

$$\Phi = \text{diag}(\phi_i) \tag{6}$$

where $\phi_i = 1$ if $i \in \mathcal{V}_1$ and $\phi_i = -1$ if $i \in \mathcal{V}_2$.

In this paper, we consider the nonovershooting bipartite output regulation in a group of linear multi-agent systems in the form

$$\Sigma_i : \left\{ \begin{array}{rcl} \dot{x}_i(t) & = & A_i x_i(t) + B u_i(t) + E_i \bar{v}_i(t) \\ e_i(t) & = & C_i x_i(t) + D_i u_i(t) + F_i \bar{v}_i(t) \end{array} \right. \tag{7}$$

where, for all $t \geq 0$, the signal $x_i(t) \in \mathbb{R}^{n_i}$ represents the state, $u_i(t) \in \mathbb{R}^{m_i}$ represents the control input, and $e_i(t) \in \mathbb{R}^{p_i}$ represents the tracking error of the i-th agent, for $i \in \{1, \dots, N\}$. Note that the exogenous signal is produced by $\dot{v} = Sv$. Defining $\bar{v}_i = \phi_i v$, we get $\dot{\bar{v}}_i = \phi_i \dot{v} = \phi_i^2 S \bar{v}_i$ which yields

$$\dot{\bar{v}}_i = S \bar{v}_i \tag{8}$$

For more details on this model, one could refer to [14].

Assumption 1: Assume system Σ_i in (7) and (8) satisfy the following assumptions:

(A.1) S has no eigenvalues with negative real parts.

(A.2) The pairs (A_i, B_i) are stabilizable, for $i = 1, \ldots, N$.

(A.3) The linear matrix equations

$$X_i S = A_i X_i + B_i U_i + E_i \quad (9)$$

$$0 = C_i X_i + D_i U_i + F_i, \quad i = 1, \ldots, N \quad (10)$$

have solutions (X_i, U_i), respectively.

These conditions are standard for the solution of the classical linear regulator problem [1]. Condition (A.2) ensures the existence of a feedback matrix K_{1i} such that $A_i + B_i K_{1i}$ is Hurwitz.

In [14], the authors proposed the following distributed observer.

$$\dot{\eta}_i = S\eta_i + \mu_0 \sum_{j=1}^{N} (a_{ij}\eta_j - |a_{ij}|\eta_i) + a_{i0}(\bar{v}_i - \eta_i) \quad (11)$$

which was incorporated into a feedback law of the form

$$u_i = K_{1i} x_i + K_{2i} \eta_i \quad (12)$$

In this paper, we consider an extension of the multi-agent output regulation problem considered by [4] and [14]. We aim to show how the control law (12) can be used to achieve bipartite output regulation without overshoot in all components of the tracking error, for all agents.

We use $e_{ij}(t)$ to denote the tracking error in the jth component of the ith subsystem. Since overshoot occurs when the tracking error changes sign, we define our multi-agent nonovershooting tracking control problem as follows.

Problem 1: For the system (7)-(8) with signed digraph $\mathcal{G}^s$, and initial conditions $x_i(0)$, $\eta_i(0)$ with $i = 1, \ldots, N$ and $\bar{v}(0)$, find suitable gain matrices K_{1i}, K_{2i} and parameter μ_0 such that the control law (12) ensures that for each $i \in \{1, \ldots, N\}$, the tracking error arising from the initial conditions satisfies $e_i(t) \to 0$ without changing sign in any component, i.e., sgn$(e_{ij}(t))$ is constant for all $t \geq 0$.

Our method shows how to choose the gain matrices K_{1i}, K_{2i} and parameter μ_0 such that the control law (12) solves Problem 1.

IV. Nonovershooting Tracking Controller Design Methods

In this section, we revisit the state feedback control design methods of [7] to deliver a nonovershooting step response for a single LTI plant ($N = 1$). We consider the *nominal system* which arises when the exosystem (8) is excluded from consideration ($S = 0$ and $\bar{v}(0) = 0$). In this case each agent in (7), for $i \in \{1, \ldots, N\}$ simplifies to a system

$$\Sigma_{i,nom} : \begin{cases} \dot{\tilde{x}}_i(t) = A_i \tilde{x}_i(t) + B_i \tilde{u}_i(t), \ \tilde{x}_i(0) = \tilde{x}_{i0} \\ \tilde{e}_i(t) = C_i \tilde{x}_i(t) + D_i \tilde{u}_i(t), \end{cases} \quad (13)$$

The reference [7] gave several methods for the design of a linear state feedback control law $\tilde{u}_i = K_{1i}\tilde{x}_i$ to deliver a nonovershooting tracking response for a system in the form (13) subject to a non-zero initial condition $\tilde{x}_i(0) \neq 0$. This is equivalent to solving Problem 1 for the case $N = 1$ and $S = 0$ and involves finding a state feedback gain matrix K_{1i} to ensure that $A_i + B_i K_{1i}$ is Hurwitz and that the tracking error $\tilde{e}_i(t) \to 0$ without changing sign in any component.

The design method of [7] assumes the nominal system (13) is at a known initial equilibrium $(\tilde{u}_0, \tilde{x}_0)$, and that the closed loop poles are to be selected from within a user-specified interval $[a, b]$ of the negative real line. The algorithm selects candidate sets L of distinct closed-loop eigenvalues within the specified interval and then associates them with candidate sets of eigenvectors in such a way that only a small number (generally one or two, or at most three) of the closed-loop modes contribute to each output component. The error signal e is then formulated in terms of the candidate set of eigenvectors and a test is used to determine if the system response is nonovershooting in all components. If the test is not successful, then a new candidate set L is chosen, and the process is repeated. If it succeeds, then the desired matrix K can be obtained by applying Moore's algorithm [18]. The tests are analytic in nature, and do not require simulating the system response to test for overshoot. Recently the design method was incorporated into a public domain MATLAB® toolbox, known as **NOUS** [19].

The **NOUS** controller design method can be applied to multiple-input multiple-output systems, and these may be of non-minimum phase. The designer has considerable freedom to select the desired closed-loop eigenvalues, in order to accommodate requirements on the convergence rate, or to avoid actuator saturation. The algorithm involves a search for suitable feedback matrices to deliver a nonovershooting response, and a successful search cannot be guaranteed for any given system, for any given initial condition. The reference [7] gave some discussions about the circumstances in which a successful search is likely. The condition was that

$$n - 3p \geq z$$

where n is the number of states, p is the number of inputs/outputs, and z is the number of minimum-phase zeros. Some further discussion on this question was given in [9].

To address Problem 1, we shall assume the existence of feedback matrices that yield a nonovershooting response for the nominal system of each agent:

Assumption 2: Assume that for each system $\Sigma_{i,nom}$ in (13), a feedback gain matrix K_{1i} exists such that

(A.4) the eigenvalues of $A_i + B_i K_{1i}$ are real, distinct and negative, and

(A.5) applying the control law $\tilde{u}_i = K_{1i}\tilde{x}_i$ to $\Sigma_{i,nom}$ yields nonovershooting outputs $\tilde{e}_i$ from the initial condition $\tilde{x}_{i0} = x_{i0} - X_i \bar{v}_{i0}$.

V. Problem Solution

In this section, we present the main result of our paper, providing a solution for Problem 1. We adopt all of Assumptions (A.1)-(A.5). Thus for all $i \in \{1, \ldots, N\}$, we have, for any initial condition $x_i(0)$ and $\bar{v}_i(0)$ of (7)-(8), gain matrices K_{1i} such that applying the control law $\tilde{u}_i = K_{1i}\tilde{x}_i$ to the

i-th nominal system (13) yields a nonovershooting response, from the initial condition $\tilde{x}_{i,0} = x_{i,0} - X_i\bar{v}_{i,0}$. Our first task is to obtain suitable gain matrices K_{2i} and parameter μ_0 so that the control laws (12) will solve Problem 1. Thus we introduce

$$K_{2i} = U_i - K_{1i}X_i, \quad \text{for } i \in \{1,\dots,N\} \qquad (14)$$

We define

$$\lambda_0 = \min\{\lambda : \lambda \in \sigma(A_i + B_iK_{1i}),\ i \in \{1,\dots,N\}\} \quad (15)$$

According to [17], the eigenvalues of H in (1) are non-zero with positive real part, so we may choose $\mu_0 > 0$ such that

$$\mathbf{Re}(\lambda_i(S) - \mu_0\lambda_j(H)) < \lambda_0 \qquad (16)$$

for all $i \in \{1,\dots,q\}$ and $j \in \{1,\dots,N\}$, where $\lambda_i(S)$ and $\lambda_j(H)$ denote the eigenvalues of S and H, respectively. To compactly represent the closed-loop system of (7)-(8) under the control law (12), we use the coordinate change

$$\begin{aligned} \xi_i &= x_i - X_i\bar{v}_i \\ \varepsilon_i &= \eta_i - \bar{v}_i \end{aligned} \qquad (17)$$

and then define variables and matrices

$$\begin{aligned}
x &= \left[x_1^T,\dots,x_N^T\right]^T && (18)\\
\eta &= \left[\eta_1^T,\dots,\eta_N^T\right]^T && (19)\\
\bar{v} &= \left[\bar{v}_1^T,\dots,\bar{v}_N^T\right]^T && (20)\\
\xi &= [\xi_1^T,\dots,\xi_N^T]^T && (21)\\
\varepsilon &= [\varepsilon_1,\dots,\varepsilon_N^T]^T && (22)\\
e &= [e_1^T,\dots,e_N^T]^T && (23)\\
\hat{v} &= \mathbf{1}_N \otimes v && (24)\\
A &= \text{blkdiag}(A_1,\dots,A_N) && (25)\\
B &= \text{blkdiag}(B_1,\dots,B_N) && (26)\\
C &= \text{blkdiag}(C_1,\dots,C_N) && (27)\\
D &= \text{blkdiag}(D_1,\dots,D_N) && (28)\\
E &= \text{blkdiag}(E_1,\dots,E_N) && (29)\\
F &= \text{blkdiag}(F_1,\dots,F_N) && (30)\\
X &= \text{blkdiag}(X_1,\dots,X_N) && (31)\\
U &= \text{blkdiag}(U_1,\dots,U_N) && (32)\\
K_1 &= \text{blkdiag}(K_{11},\dots,K_{1N}) && (33)\\
K_2 &= \text{blkdiag}(K_{21},\dots,K_{2N}) && (34)
\end{aligned}$$

Then we have

$$\begin{aligned} \xi &= x - X\bar{v} && (35)\\ \varepsilon &= \eta - \bar{v} && (36) \end{aligned}$$

Before we state our theorem, we need the following lemma from [14].

Lemma 4: The following equality holds given the matrices H, Δ from (1), and Φ from (6).

$$(H \otimes I_q)\bar{v} = (\Delta\Phi \otimes I_q)\hat{v} \qquad (37)$$

where $\hat{v} = (\Phi \otimes I_q)\bar{v}$.

Theorem 1: Consider the multi-agent system (7) with exosystem (8). For $i = 1\dots,N$, let x_{i0} and $\bar{v}_i(0)$ be their initial conditions. Assume that (A.1)-(A.5) hold and further assume that the signed digraph $\mathcal{G}^s$ is structurally balanced and its Laplacian matrix can be partitioned as (1). For each $i = 1,\dots,N$, let K_{1i} be gain matrices such that $A_i + B_iK_{1i}$ is Hurwitz stable, and that when the control law $\tilde{u}_i = K_{1i}\tilde{x}_i$ is applied to the homogeneous system

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= A_i\tilde{x}_i(t) + B_i\tilde{u}_i(t), \\ \tilde{e}_i(t) &= C_i\tilde{x}_i(t) + D_i\tilde{u}_i(t), \end{aligned} \qquad (38)$$

yields $\tilde{e}_i \to 0$ without overshoot from initial condition $\tilde{x}_{i0} = x_{i0} - X_iv_{i,0}$. Let $K_{2i} = U_i - K_{1i}X_i$, and let μ_0 satisfy (16). Then the distributed dynamic state feedback control law (12), with these K_{1i}, K_{2i} and μ_0, solves Problem 1, provided the initial exosystem observer error $\varepsilon(0)$ is sufficiently small.

Proof: The closed-loop system of the whole network can be written as

$$\begin{aligned}
\dot{x} &= (A + BK_1)x + BK_2\eta + E\bar{v} \\
\dot{\eta} &= ((I_N \otimes S) - \mu_0(H \otimes I_q))\eta + \mu_0(\Delta\Phi \otimes I_q)\hat{v} \\
e_i &= (C_i + D_iK_{1i})x_i + D_iK_{2i}\eta_i + F_i\bar{v} && (39)
\end{aligned}$$

By (8) and (9),

$$\begin{aligned}
\dot{\xi} &= \dot{x} - X(I_N \otimes S)\bar{v} \\
&= (A + BK_1)x + BK_2\eta + (E - X(I_N \otimes S))\bar{v} \\
&= (A + BK_1)x + B(U - K_1X)(\varepsilon + \bar{v}) - (AX + BU)\bar{v} \\
&= (A + BK_1)(x - X\bar{v}) + B(U - K_1X)\varepsilon \\
&= (A + BK_1)\xi + BK_2\varepsilon && (40)
\end{aligned}$$

Noting Lemma 4, we can write

$$\begin{aligned}
\dot{\varepsilon} &= \dot{\eta} - \dot{\bar{v}} \\
&= ((I_N \otimes S) - \mu_0(H \otimes I_q))\eta + \mu_0(\Delta\Phi \otimes I_q)\hat{v} - (I_N \otimes S)\bar{v} \\
&= ((I_N \otimes S) - \mu_0(H \otimes I_q))\eta + (H \otimes I_q)\bar{v} - (I_N \otimes S)\bar{v} \\
&= ((I_N \otimes S) - \mu_0(H \otimes I_q))\varepsilon && (41)
\end{aligned}$$

Using (10),

$$\begin{aligned}
e &= (C + DK_1)x + DK_2\eta + F\bar{v} \\
&= (C + DK_1)x + D(U - K_1X)(\varepsilon + \bar{v}) \\
&\qquad -(CX + DU)\bar{v} \\
&= C(x - Xv) + DK_1x + DU\varepsilon - DK_1X\varepsilon \\
&\qquad -DK_1X\bar{v} \\
&= (C + DK_1)(x - Xv) + D(U - K_1X)\varepsilon \\
&= (C + DK_1)\xi + DK_2\varepsilon && (42)
\end{aligned}$$

The overall closed-loop system can be put into the form

$$\begin{aligned}
\dot{\xi} &= (A + BK_1)\xi + BK_2\varepsilon && (43)\\
\dot{\varepsilon} &= ((I_N \otimes S) - \mu_0(H \otimes I_q))\varepsilon && (44)\\
e &= (C + DK_1)\xi + DK_2\varepsilon && (45)
\end{aligned}$$

For brevity we define

$$\begin{aligned} A_{cc} &= (I_N \otimes S) - \mu_0(H \otimes I_q) \\ A_c &= \begin{bmatrix} A + BK_1 & BK_2 \\ 0 & A_{cc} \end{bmatrix} \\ C_c &= \begin{bmatrix} C + DK_1 & DK_2 \end{bmatrix} \end{aligned}$$

and introduce coordinates $x_c = [\xi^T, \varepsilon^T]^T$. Then we can write (43)-(45) as

$$\begin{aligned} \dot{x}_c &= A_c x_c \\ e &= C_c x_c \end{aligned} \tag{46}$$

The stability of $A + BK_1$ follows from Assumption (A.4), and the stability of A_{cc} follows from the choice of μ_0 in (16). Due to its diagonal structure, A_c is also Hurwitz stable, so $e(t) \to 0$ as $t \to \infty$.

To observe that the response is nonovershooting, we decompose the state vector ξ of (43) according to $\xi = \xi_A + \xi_B$ where ξ_A and ξ_B are the zero-input solution and zero-state solutions, respectively. Similarly we can decompose the output e into $e = e_A + e_B$, the zero-input response and zero-state responses, respectively. By Assumption (A.4)-(A.5), for each agent, $A_i + B_i K_{1i}$ is Hurwitz with real, negative and distinct eigenvalues, and the output e_i of the nominal system $\Sigma_{i,nom}$ in (13) from initial condition $x_{i,0} - X_i \bar{v}_{i,0}$ is nonovershooting. Since e_A is composed of the zero-input responses from all the informed agents, we conclude that ξ_A and e_A are SED functions and $e_A(t) \to 0$ as $t \to \infty$ without changing sign in any component.

Considering the dynamics for ε in (44), we know by (16) that A_{cc} is Hurwitz and satisfies $\max\{\mathbf{Re}(\lambda) : \lambda \in \sigma(A_{cc})\} \leq \lambda_0$. By Lemma 2(i), ε is a SEDS function with rate λ_0, and $|\varepsilon(t)| \leq k_1|\varepsilon_0|$, for some $k_1 > 0$. As ε is the input for (43), by Lemma 2(ii), we conclude that ξ_B and e_B are both SEDS functions with rate at most λ_0, and $|e_B(t)| \leq k_2 k_1 |\varepsilon_0|$, for some $k_2 > 0$. We may now apply Lemma 3 with $g = e_A$ and $f = e_B$. Provided $|\varepsilon_0|$ is sufficiently small, we have $e(t) \to 0$ as $t \to \infty$ without changing sign in any component, and hence Problem 1 is also solved. ■

VI. Example

In this section we compare the performance of our method with that of [14]. This example involves six agents whose dynamic connections are shown in the network of Fig. 1.

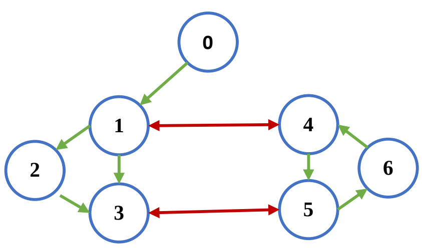

Fig. 1. Network of agents with agent 0 as the leader. Note that red links represent negative interactions.

For $i = 1, \ldots, 6$, consider the following parameters for the agents shown in Fig. 1,

$$A_i = \begin{pmatrix} 0 & 0.5 * i \\ 1 & 0 \end{pmatrix} \quad B_i = \begin{pmatrix} 0.5 * i \\ 0.25 \end{pmatrix} \quad C_i = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad D_i = 0$$

$$E_i = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.25 * i & 0 & 0 & 0 \end{pmatrix} \quad F_i = \begin{pmatrix} 0 & 0.25 * i & -1 & 0 \end{pmatrix}$$

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 \end{pmatrix}$$

where $i = 1, \ldots, 6$. We shall assume initial conditions of

$$x_{10} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \; x_{20} = \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} \; x_{30} = \begin{pmatrix} 0.3 \\ -1.5 \end{pmatrix} \; x_{40} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$

$$x_{50} = \begin{pmatrix} 2.5 \\ 5 \end{pmatrix} \; x_{60} = \begin{pmatrix} 6 \\ 7 \end{pmatrix} \; \bar{v}(0) = \begin{pmatrix} -1 \\ 5 \\ 0 \\ -7 \end{pmatrix}$$

for the six agent systems and the exosystem.

Fig. 2 shows the tracking error of the agents under the control scheme proposed by [14] using the "place" command of MATLAB. We observe that the system response achieves the desired reference tracking, but exhibits overshoot in all six outputs.

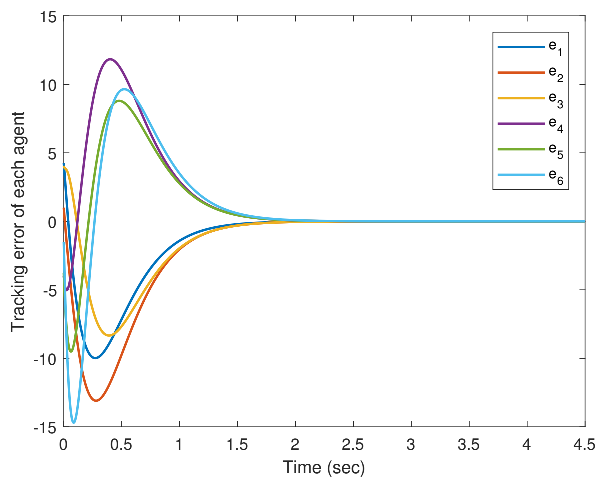

Fig. 2. Tracking error of agents using the scheme proposed in [14]

To address Problem 1 for this multi-agent system, we used the **NOUS** MATLAB® toolbox [19] to obtain gain matrices K_{1i} that achieved a nonovershooting response for each system (38), from the specified initial conditions $\tilde{x}_{i0} = x_{i0} - X_i \bar{v}_{i,0}$. We refer the reader to [19] for the details on how to use the toolbox, and to [7] for a detailed discussion of the algorithm itself. We obtained

$$K_{11} = \begin{pmatrix} -11.60 & -1.00 \end{pmatrix} \; K_{12} = \begin{pmatrix} -5.50 & -1.00 \end{pmatrix}$$

$$K_{13} = \begin{pmatrix} -3.35 & -1.00 \end{pmatrix} \; K_{14} = \begin{pmatrix} -3.23 & -1.00 \end{pmatrix}$$

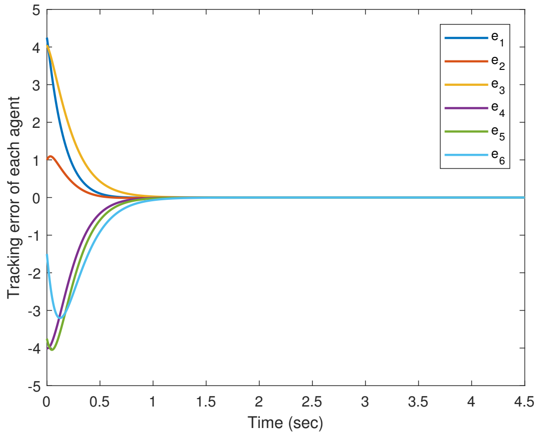

Fig. 3. Tracking error of agents using Theorem 1

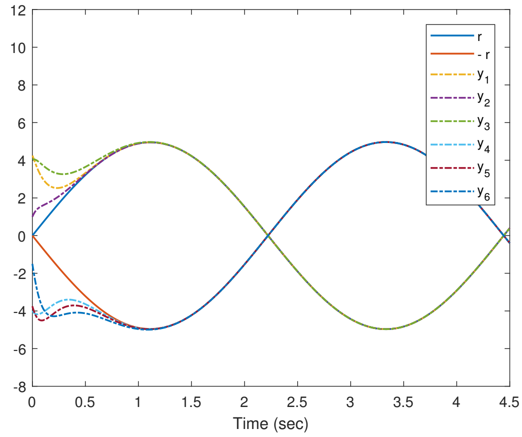

Fig. 4. The outputs of the agents. Note that the solid lines represent the reference signal with positive and negative sign to be tracked.

$$K_{15} = \begin{pmatrix} -2.52 & -1.00 \end{pmatrix} \; K_{16} = \begin{pmatrix} -1.76 & -1.00 \end{pmatrix}$$

Using X_i and U_i, we can obtain $K_{2i} = U_i - K_{1i}X_i$ for $i = 1, \ldots, 6$. Computing (15) gives $\lambda_0 = -6.53$; we found that choosing $\mu_0 = 35$ satisfies (16). The error from the six multi-agent systems are presented in Fig. 3. We observe that all the errors converge to zero without overshoot, and hence Problem 1 is solved. In addition, the outputs of the agents are depicted in Fig. 4, which clearly shows the bipartition.

It should be noted that the **NOUS** algorithm involves a search for suitable feedback matrices to deliver a nonovershooting response, and a successful search cannot be guaranteed for any given system, for any given initial condition. The reference [7] gives some discussion of the circumstances in which a successful search is likely.

The successful avoidance of overshoot indicates that initial value of exosystem observer error $\varepsilon(0)$ is sufficiently small, as required by Theorem 1. If this initial error is too large, then the nonovershooting performance may not be observed, for some or all of the agents. Thus, successful application of the design method requires that a reasonably accurate estimate of the initial state of the exosystem is available.

VII. Conclusion

We have combined the methods of [4], [9], and [14] to obtain a design methodology for the problem of nonovershooting bipartite output regulation of linear multi-agent systems. Using the proposed scheme, it was shown that proper gain matrices can be found to avoid overshoot while achieving bipartite output regulation. Finally, the results were validated by a numerical example.

References

[1] A. Saberi, A. Stoorvogel and P. Sannuti, Control of linear systems with regulation and input constraints, *ser. Communications and Control Engineering*, Springer, 2000.

[2] Y. Hong, L. Gao, D. Cheng, and J. Hu, Lyapunov-based approach to multiagent systems with switching jointly connected interconnection, *IEEE Transactions on Automatic Control*, 52 (2007) pp. 943-948.

[3] J. Xiang, W. Wei, and Y. Li, Synchronized output regulation of networked linear systems, *IEEE Transactions on Automatic Control*, 54, (2009) pp. 1336-1341.

[4] Y. Su and J. Huang, Cooperative Output Regulation of Linear Multi-Agent Systems, *IEEE Transactions on Automatic Control*, 57 (2012) pp. 1062–1066.

[5] Gajic, Z. and Lelic, M., Modern control systems engineering, (1996), Prentice-Hall, Inc.

[6] M. Hughes, R. Schmid and Y. Tan, Implementation of novel non-overshooting tracking control method on a LEGO® Robot, *Proceedings 2nd IEEE AUCC*, Sydney, 2012.

[7] R. Schmid, and L. Ntogramatzidis, A unified method for the design of nonovershooting linear multivariable state-feedback tracking controllers, *Automatica*, 46 (2010) pp. 312–321.

[8] R. Schmid, and L. Ntogramatzidis, The design of nonovershooting and nonundershooting multivariable state feedback tracking controllers, *Systems & Control Letters*, 61 (2012) pp. 714–722.

[9] R. Schmid, and L. Ntogramatzidis, Nonovershooting and nonundershooting exact output regulation, *Systems & Control Letters*, 70 (2014) pp. 30–37.

[10] C. Altafini, Consensus problems on networks with antagonistic interactions, *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 935946, 2013.

[11] M. Cheng, H. Zhang, and Y. Jiang, Output bipartite consensus of heterogeneous linear multi-agent systems, in Control Conference (CCC), 2016 35th Chinese. IEEE, 2016, pp. 82878291.

[12] F. A. Yaghmaie, R. Su, and F. L. Lewis, Bipartite output synchronization of linear heterogeneous multi-agent systems via output feedback, in American Control Conference (ACC), 2016. IEEE, 2016, pp. 1024 1029.

[13] F. A. Yaghmaie, R. Su, F. L. Lewis, and S. Olaru, Bipartite and cooperative output synchronizations of linear heterogeneous agents: A unified framework, Automatica, vol. 80, pp. 172176, 2017.

[14] H. Dehghani Aghbolagh, M. Zamani, and Z. Chen, Bipartite output regulation of multi-agent systems with antagonistic interactions, *11th Asian Control Conference (ASCC)*, IEEE, 2017.

[15] R. Schmid, and H. Dehghani Aghbolagh, Nonovershooting Cooperative Output Regulation for Linear Multi-Agent Systems, *IEEE Transactions on Control of Network Systems*, 2018 DOI:10.1109/TCNS.2018.2846180.

[16] L. Dou, L. Wang, and X. Wang, Leader-following bipartite consensus of high-order multi-agent systems on signed networks, in Control Conference (ASCC), 2015 10th Asian. IEEE, 2015, pp. 15.

[17] J. Hu, X. Hu, and Y. Zhang, Bipartite consensus strategies for coopetition networks, in Control Conference (CCC), 2014 33rd Chinese. IEEE, 2014, pp. 15051510.

[18] B.C. Moore, On the Flexibility Offered by State Feedback in Multivariable systems Beyond Closed Loop Eigenvalue Assignment, *IEEE Transactions on Automatic Control*, 21 (1976) pp. 689–692.

[19] A. Pandey and R. Schmid, **NOUS**: a MATLAB® toolbox for the design of nonovershooting and nonundershooting multivariable tracking controllers, *Proceedings of the Second IEEE Australian Control Conference* Sydney, 2012,