A Unified Framework for the Analysis and Design of Networked Control Systems

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Eduardo I. Silva, 18/02/2009

I hereby certify that parts of this thesis (in particular Chapter 3, and early versions of Chapters 4 and 7) are the result of collaboration with my supervisors Prof. G.C. Goodwin and Dr. D.E. Quevedo. This collaboration has been based on normal candidate-supervisors relations. Chapters 5 and 6 are the result of collaboration with Mr. M.S. Derpich and Dr. J. Østergaard (University of Aalborg, Denmark). Again, this collaboration has been based on a normal collaborative practice, where the bulk of work and intellectual effort has been undertaken by myself.

I hereby certify that parts of this thesis, and early versions of some of the results herein, have been previously published in peer reviewed conferences and journals. In such publications, my contributions have been either significant or fundamental.

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Contents

Abstract

This thesis studies control systems with communication constraints. Such constraints arise due to the fact that practical control systems often use non-transparent communication links, i.e., links subject to data-rate constraints, random data-dropouts or random delays. Traditional control theory cannot deal with such constraints and the need for new tools and insights arises.

We study two problems: control with average data-rate constraints and control over analog erasure channels with i.i.d. dropout profiles.

When focusing on average data-rate constraints, it is natural to ask whether information theoretic ideas may assist the study of networked control systems. In this thesis we show that it is possible to use fundamental information theoretic concepts to arrive at a framework that allows one to tackle performance related control problems. In doing so, we show that there exists an *exact* link between control systems subject to average data-rate limits, and control systems which are closed over *additive i.i.d. noise channels subject to a signal-to-noise ratio constraint.*

On the other hand, in the case of control systems subject to i.i.d. data-dropouts, we show that there exists a *second-order moments equivalence* between a linear feedback system which is interconnected over an analog erasure channel, and the same system when it is interconnected over an *additive i.i.d. noise channel subject to a signal-to-noise ratio constraint.*

From the results foreshadowed above, it follows that the study of control systems closed over signal-to-noise ratio constrained additive i.i.d. noise channels is a task of relevance to many networked control problems. Moreover, the interplay between signal-to-noise ratio constraints and control objectives is an interesting issue in its own right. This thesis starts with such a study. Then, we use the resultant insights to address performance issues in control systems subject to either average data-rate constraints or i.i.d. data-dropouts. Our approach shows that, once key equivalences are exposed, standard control intuition and synthesis machinery can be used to tackle networked control problems in an *exact* manner. It also sheds light into fundamental results in the literature and gives (partial) answers to several previously open questions. We believe that the insights in this thesis are of fundamental importance and, to the best of the author's knowledge, novel.

Symbols and Acronyms

The following is a list of symbols and acronyms commonly used throughout this thesis. In some cases, the definitions are vague; proper definitions will be given where appropriate.

EC	entropy coder
ECDQ	entropy coded dithered quantizer
ED	entropy decoder
i.i.d.	independent and identically distributed
LTI	linear time invariant
LMI	linear matrix inequality
MIMO	multiple-input multiple-output
MP	minimum phase
MSS	mean square stable or mean square stability
NMP	non-minimum phase
PDF	probability density function
PSD	power spectral density
SISO	single-input single-output
wss	wide sense stationary
$\triangleq$	equal by definition
$\{\cdot\}$	a set
$\{\cdot\}_m$	a multi-set (see [11])
$\#$	cardinality (number of elements)
$\in$	set membership

$\subseteq$	contained in or equal to
$\subset$	contained but not equal to
$\cap$	intersection
$\cup$	union
$\setminus$	set exclusion
$\mathbb{N}_0$	$\{0, 1, 2, 3, \cdots\}$
$\mathbb{Z}$	$\{\cdots, -2, -1, 0, 1, 2, \cdots\}$
$\mathbb{R}$	the real numbers
$\mathbb{R}_0^+$	$\{x \in \mathbb{R} : 0 \leq x < \infty\}$
$\mathbb{R}^+$	$\{x \in \mathbb{R} : 0 < x < \infty\}$
$\mathbb{C}$	the complex plane
$\mathbb{R}^n$	$\mathbb{R} \times \cdots \times \mathbb{R}$ (n times)
$\mathbb{C}^n$	$\mathbb{C} \times \cdots \times \mathbb{C}$ (n times)
∞	infinity (note that $\infty \notin \mathbb{N} \cup \mathbb{Z} \cup \mathbb{R} \cup \mathbb{C}$; see [141].)
(a, b)	$\{x \in \mathbb{R} : a < x < b\}$
$(a, b]$	$\{x \in \mathbb{R} : a < x \leq b\}$
$[a, b)$	$\{x \in \mathbb{R} : a \leq x < b\}$
$[a, b]$	$\{x \in \mathbb{R} : a \leq x \leq b\}$
$\|x\|$	magnitude (absolute value) of $x \in \mathbb{C}$
$\bar{x}$	conjugate of $x \in \mathbb{C}$
$\mathrm{Re}\{x\}$	real part of $x \in \mathbb{C}$
$\mathrm{Im}\{x\}$	imaginary part of $x \in \mathbb{C}$
A^T	transpose of the matrix A
A^H	conjugate transpose of the matrix A (Hermitian operator)
$A \geq 0$	the matrix A is positive semi-definite
$A > 0$	the matrix A is positive definite
$\mathrm{diag}\{a_1, \cdots, a_n\}$	diagonal matrix containing $a_1, \cdots, a_n$ in its diagonal
ε_i	i^{th} vector of the canonical basis in $\mathbb{R}^n$, where n is understood from the context

z	argument of the Z-transform or forward shift operator, where the meaning is clear from the context
$A(z)$	transfer function in discrete time (scalar or multidimensional)
$\{A(z)\}\|_{z=0}$	$A(0)$
$A(\infty)$	$\lim_{z\to\infty} A(z)$
$A(z)^{\sim}$	shorthand for $A(z^{-1})^T$
$[A(z)]_{\mathcal{H}_2^{\perp}}$	strictly unstable part of $A(z) \in \mathcal{L}_2$
$[A(z)]_{\mathcal{H}_2}$	stable and strictly proper part of $A(z) \in \mathcal{L}_2$
$\|\|A(z)\|\|_2^2$	squared 2-norm of $A(z) \in \mathcal{L}_2$
$\otimes$	Kronecker product (see, e.g., [11,19])
$\operatorname{vec}\{\cdot\}$	column stacking operator (see, e.g., [11,19])
$\operatorname{vec}^{-1}\{\cdot\}$	inverse of the column stacking operator
$\mathcal{E}\{\cdot\}$	expectation operator
$\mathcal{E}\{\cdot\|\cdot\}$	conditional expectation
$\mu_x(k)$	mean at time k of the process x
$\sigma_x^2(k)$	variance at time k of the process x
μ_x	stationary mean of x, i.e., $\lim_{k\to\infty} \mu_x(k)$
σ_x^2	stationary variance of x, i.e., $\lim_{k\to\infty} \sigma_x^2(k)$
$\Omega_x(z)$	spectral factor of the process x
$S_x(z)$	power spectral density of the process x
x	shorthand for $\{x(k)\}_{k\in\mathbb{F}}$, with $\mathbb{F} = \mathbb{N}_0$ or $\mathbb{F} = \mathbb{Z}$
x^k	shorthand for $x(0), x(1), \cdots, x(k)$
x_i^k	shorthand for $x(i), x(i+1), \cdots, x(k)$
ln	natural logarithm
$\log_a$	logarithm in base a
inf	infimum
min	minimum
arg inf	argument that “infimizes” (see Section 2.2)
arg min	argument that minimizes

$\mathcal{R}$	set of all real rational transfer functions
$\mathcal{R}_p$	subset of $\mathcal{R}$ containing all proper transfer functions
$\mathcal{R}_{sp}$	subset of $\mathcal{R}$ containing all strictly proper transfer functions
$\mathcal{RH}_\infty$	subset of $\mathcal{R}_p$ containing all stable transfer functions
$\overline{\mathcal{RH}}_\infty$	subset of $\mathcal{R}_p$ containing all marginally stable transfer functions
$\mathcal{RH}_2$	subset of $\mathcal{R}_{sp}$ containing all stable transfer functions
$\mathcal{RH}_2^\perp$	subset of $\mathcal{R}$ containing all transfer functions that have only unstable poles (or no poles at all)
$\mathcal{U}_\infty$	subset of $\mathcal{RH}_\infty$ containing all square transfer functions with inverses in $\mathcal{RH}_\infty$
$\overline{\mathcal{U}}_\infty$	subset of $\mathcal{RH}_\infty$ containing all square transfer functions with inverses in $\overline{\mathcal{RH}}_\infty$
$\mathcal{L}_2$	subset of $\mathcal{R}$ containing all transfer functions whose frequency response is square integrable over the unit circle (see, e.g., [113, 182])

(The previous definitions are valid irrespective of the dimension of the transfer functions. If necessary, we will add an $n \times m$ superscript to refer to transfer functions of a specific size.)

$H(x)$	discrete entropy of (the discrete random variable) x
$H(x\|y)$	conditional discrete entropy of x given y
$h(x)$	differential entropy of (the continuous random variable) x
$h(x\|y)$	conditional differential entropy of x given y
$I(x;y)$	mutual information between x and y
$I(x;y\|z)$	conditional mutual information between x and y, given z
$D(x\|\|y)$	divergence between the distributions of x and y

$\bar{h}(x)$	entropy rate of the process x
$I_\infty(x;y)$	average scalar mutual information between the processes x and y
$I_\infty(x \to y)$	average directed mutual information between the processes x and y

$x \leftrightarrow y \leftrightarrow z$	x, y and z form a Markov chain (in that order)