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Citation: Physics of Fluids **29**, 051702 (2017); doi: 10.1063/1.4983724
View online: http://dx.doi.org/10.1063/1.4983724
View Table of Contents: http://aip.scitation.org/toc/phf/29/5
Published by the American Institute of Physics

PHYSICS OF FLUIDS **29**, 051702 (2017)

A note on the velocity derivative flatness factor in decaying HIT

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(Received 6 April 2017; accepted 5 May 2017; published online 17 May 2017)

We develop an analytical expression for the velocity derivative flatness factor, F, in decaying homogenous and isotropic turbulence (HIT) starting with the transport equation of the third-order moment of the velocity increment and assuming self-preservation. This expression, fully consistent with the Navier-Stokes equations, relates F to the product between the second-order pressure derivative $(\overline{\partial^2 p/\partial x^2})$ and second-order moment of the longitudinal velocity derivative $(\overline{(\partial u/\partial x)^2})$, highlighting the role the pressure plays in the scaling of the fourth-order moment of the longitudinal velocity derivative. It is also shown that F has an upper bound which follows the integral of $k^{*4}E_p^*(k^*)$ where E_p and k are the pressure spectrum and the wavenumber, respectively (the symbol * represents the Kolmogorov normalization). Direct numerical simulations of forced HIT suggest that this integral converges toward a constant as the Reynolds number increases. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4983724]

Soon after Kolmogorov proposed his similarity hypotheses, hereafter denoted K41,[1] Landau drew attention to the effect of fluctuations of the turbulent kinetic energy dissipation rate, ϵ, on the small-scale properties of turbulence (see, for example, the footnote on page 126 of the book by Landau and Lifshitz[2]). This prompted Kolmogorov to modify his initial similarity hypotheses. The modified hypotheses widely known as K62[3] were first presented in 1961 at an international colloquium in Marseille partly to inaugurate the establishment of the Institut de Mécanique Statistique de la Turbulence (IMST). Manifestations of the salient differences between K41 and K62 can be observed in the skewness factor ($S \equiv \overline{(\partial u/\partial x)^3}/\overline{(\partial u/\partial x)^2}^{3/2}$) and flatness factor ($F \equiv \overline{(\partial u/\partial x)^4}/\overline{(\partial u/\partial x)^2}^2$) of the longitudinal velocity derivative. While K41 predicts that S and F are universal constants, K62 suggests $S \sim Re_\lambda^{\mu_1}$ and $F \sim Re_\lambda^{\mu_2}$, μ_1 and μ_2 are small positive numbers (Re_λ is the Taylor microscale Reynolds number). It should be stated that these power-laws are obtained empirically.

While the merits and demerits of K41 or K62 continue to be vigorously debated, there is a lack of exact analytical expressions for S and F rigorously deduced from the Navier-Stokes equations, and which can be used for testing K41 and K62. Further, a major hurdle for testing K41 and K62 is the lack of an "infinitely" large Reynolds number. Consequently, researchers examined the behaviours of S and F as the Reynolds number increases from low to moderately high values.

In this letter, we develop an analytical expression for F in homogeneous and isotropic turbulence (denoted hereafter HIT), based on the Navier-Stokes equations and assuming self-preservation (hereafter denoted SP). An expression of S which accounts for the dependence on Re_λ was reported elsewhere.[4–6]

Starting with the Navier-Stokes equations written at two separate and independent spatial points x and x', the transport equation for $\delta u_3 (\equiv \overline{(\delta u)^3} : \delta u = u(x',t) - u(x,t), x' = x + r)$ for decaying HIT can be expressed as[7]

$$\delta u_4 = \frac{6}{r^2}\int_0^r s(\overline{\delta u_2 \delta v_2})ds - \frac{1}{r^2}\int_0^r s^2 T_{111} ds - \frac{2\nu}{r^2}\int_0^r s^2 E_{111} ds + \frac{2\nu}{r^2}\int_0^r \left\{4 + 4s\frac{\partial}{\partial s} + s^2\frac{\partial^2}{\partial s^2}\right\}\delta u_3 ds - \frac{1}{r^2}\int_0^r s^2 \frac{\partial \delta u_3}{\partial t} ds, \tag{1}$$

where $\delta u_4 \equiv \overline{(\delta u)^4}$, $\overline{(\delta u_2 \delta v_2)} \equiv \overline{(\delta u)^2 (\delta v)^2}$, and

$$T_{111} = \frac{3}{\rho}\overline{(\delta u)^2\left(\frac{\partial \delta p}{\partial X}\right)}, \tag{2a}$$

$$E_{111} = 3\overline{(\delta u)\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u'}{\partial x'}\right)^2\right\}} = 3\overline{(\delta u)\left\{\left(\frac{\partial \delta u}{\partial x}\right)^2 + \left(\frac{\partial \delta u}{\partial x'}\right)^2\right\}}. \tag{2b}$$

In the above equation and expressions, we used the transformation $X = (x + x')/2$ and the fact that $\partial u/\partial x' = \partial u'/\partial x = 0$; $\delta p = p(x',t) - p(x,t)$ is the pressure increment (for the sake of simplicity, we incorporate $1/\rho$ into p). We now assume SP and write

$$\delta u_2 = u_2^2(t) f(r^*), \tag{3a}$$

$$\delta u_3 = u_3^3(t) g(r^*), \tag{3b}$$

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$$\delta u_4 = u_4^4(t)h(r^*), \tag{3c}$$

$$\delta v_2 = u_2^2(t)f_v(r^*), \tag{3d}$$

$$\overline{\delta u_2 \delta v_2} = u_2^4(t)f_{uv}(r^*), \tag{3e}$$

$$T_{111} = \frac{u_2^4(t)}{l(t)}\tilde{T}_{111}(r^*), \tag{3f}$$

$$E_{111} = \frac{u_2^3(t)}{l^2(t)}\tilde{E}_{111}(r^*), \tag{3g}$$

where $r^* = r/l(t)$, $l(t)$ is a length scale, $u_2(t)$, $u_3(t)$, and $u_4(t)$ are the velocity scales function of t while $f, g, h, f_v, f_{uv}, \tilde{T}_{111}$, and $\tilde{E}_{111}$ are the dimensionless functions of r^* only. For convenience, we hereafter drop the variable t. We introduce the skewness and flatness coefficients of the longitudinal velocity increment, viz., $S(\equiv \delta u_3/\delta u_2^{3/2})$ and $F(\equiv \delta u_4/\delta u_2^2)$ as well as the mixed flatness coefficient $F_{uv}(\equiv \overline{(\delta u)^2(\delta v)^2}/(\delta u_2)^2)$. This allows us to express δu_4, δu_3, and $\overline{(\delta u)^2(\delta v)^2}$ in terms of δu_2. Substituting (3) into (1), we obtain

$$\begin{aligned} Re_l F f^2 = &-\frac{Re_l}{r^{*2}}\int_0^{r^*} s^{*2}\tilde{T}_{111}ds^* - \frac{2}{r^{*2}}\int_0^{r^*} s^{*2}\tilde{E}_{111}ds^* \\ &+ \frac{6Re_l}{r^{*2}}\int_0^{r^*} s^* F_{uv} f^2 ds^* \\ &+ \frac{2}{r^{*2}}\int_0^{r^*}\left\{4 + 4s^*\frac{\partial}{\partial s^*} + s^{*2}\frac{\partial^2}{\partial s^{*2}}\right\} S f^{3/2} ds^* \\ &- \frac{l^2}{\nu u_2^3}\frac{\partial u_2^3}{\partial t}\frac{1}{r^{*2}}\int_0^{r^*} s^{*2} S f^{3/2} ds^* \\ &+ \frac{l}{\nu}\frac{\partial l}{\partial t}\frac{1}{r^{*2}}\int_0^{r^*} s^{*3}\frac{\partial (S f^{3/2})}{\partial s^*} ds^*, \end{aligned} \tag{4}$$

where s^* is a dummy variable of integration. It can be shown[8] that when $r^* \to \infty$, the integrals approach finite values. Since the coefficients of the second and fourth terms on the right side of (4) are constant, then the SP constraints immediately follow

$$Re_l = C_1, \tag{5a}$$

$$\frac{l^2}{\nu u_2^3}\frac{\partial u_2^3}{\partial t} = C_2, \tag{5b}$$

$$\frac{l}{\nu}\frac{\partial l}{\partial t} = C_3, \tag{5c}$$

where $Re_l = u_2 l/\nu$ is a scaling Reynolds number. The above constraints, i.e., C_1, C_2, and C_3 must be independent of t, apply for all separations r^*; the values of the constants depend on the choice of the scaling variables l and u_2.

We now shift our attention on the behaviour of (4) when $r^* \to 0$. We first rewrite (4) as

$$\begin{aligned} F_0 f^2 = &F_{uv,0}\frac{6}{r^{*2}}\int_0^{r^*} s^* f^2 ds^* - \frac{1}{r^{*2}}\int_0^{r^*} s^{*2}\tilde{T}_{111}ds^* \\ &- \frac{1}{Re_l}\frac{2}{r^{*2}}\int_0^{r^*} s^{*2}\tilde{E}_{111}ds^* \\ &+ \frac{S_0}{Re_l}\frac{1}{r*^2}\int_0^{r^*}\left[2\left(4 + 4s^*\frac{\partial}{\partial s^*} + s^{*2}\frac{\partial^2}{\partial s^{*2}}\right)(f^{3/2})\right. \\ &\left. - C_2 s^{*2} f^{3/2} + C_3 s^{*3}\frac{\partial (f^{3/2})}{\partial s^*}\right] ds^*, \end{aligned} \tag{6}$$

where we used $\frac{\partial S}{\partial r^*} \simeq 0$ as $r^* \to 0$; the subscript 0 refers to the values when $r^* \to 0$. When $r^* \to 0$, and using a Taylor series expansion up to r^{*4}, we have $f \sim (\alpha r^{*2} - \frac{1}{12}\beta r^{*4})$, $\tilde{T}_{111} \sim 3(\gamma_1 r^{*3} + \gamma_2 r^{*4})$, and $\tilde{E}_{111} \sim 6(\zeta_1 r^* + \zeta_2 r^{*2} + \zeta_3 r^{*3} + \zeta_4 r^{*4})$ where α, β, γ_i, and ζ_i are constants under SP and whose values depend on the choice of l and u_2, and, at this stage of the analysis, cannot be assumed to be Re_λ-independent. Substituting these expressions into (6) and equating terms of order 4 in r^* leads to

$$\begin{aligned} F_0 = &F_{uv,0} - \frac{1}{2}\gamma_1\alpha^{-2} \\ &+ \frac{1}{Re_l}\left\{-2\zeta_3\alpha^{-2} + \left[2 + \frac{1}{6}(3C_3 - C_2)\right] S_0\alpha^{-1/2}\right\}. \end{aligned} \tag{7}$$

Further, $\alpha = \frac{l^2}{u_2^2}\overline{(\frac{\partial u}{\partial x})^2}$, $\beta = \frac{l^4}{u_2^2}\overline{(\frac{\partial^2 u}{\partial x^2})^2}$, $\gamma_1 = \frac{l^4}{u_2^4}\overline{(\frac{\partial u}{\partial x})^2\frac{\partial^2 p}{\partial x^2}}$, and $\zeta_3 = \frac{l^5}{u_2^3}\overline{(\frac{\partial u}{\partial x})^2\frac{\partial^3 u}{\partial x^3}}$. If we use $l = \lambda$ ($\lambda \equiv u'/\overline{(\partial u/\partial x)^2}^{1/2}$ is the Taylor microscale) and $u_2 = u'$ (i.e., λ and u' are the scaling variables in conformity with SP) then $\alpha = 1$ and (7) become

$$F_0 = F_{uv,0} - \frac{1}{2}\gamma_1 + \frac{1}{Re_\lambda}\left\{-2\zeta_3 + \left(2 + \frac{1}{6}(3C_3 - C_2)\right) S_0\right\}. \tag{8}$$

As $Re_\lambda \to \infty$, $S_0/Re_\lambda \to 0$, regardless of whether S_0 is constant (K41) or varies as Re_λ^a ($0 < a < 1$) (K62). Thus, as $Re_\lambda \to \infty$, (8) reduces to

$$F_0 = F_{uv,0} - \frac{1}{2}\gamma_1 - \frac{2\zeta_3}{Re_\lambda}. \tag{9}$$

It was shown[9,10] that $\frac{6}{r^2}\int_0^r s(\overline{\delta u_2 \delta v_2})ds \simeq \delta u_4$ at all separations or equivalently $F \simeq F_{uv}$. This is consistent with the measured ratio $F_{uv,0}/F_0$ in several flows,[11] where it is shown that this ratio is practically independent of Re_λ, although its magnitude depends on the flow geometry. It was also shown[12] that for HIT, the fourth-order velocity derivative moments can be expressed in terms of four rotational invariants of the velocity-deformation tensor, I_α ($\alpha = 1, 2, 3$, and 4) such as $\overline{(\partial u/\partial x)^4} = 4I_1/105$, $\overline{(\partial u/\partial x)^2(\partial v/\partial x)^2} = I_1/105 + I_2/70 - I_3/105$, where $I_1 = \overline{(s_1^2 + s_2^2 + s_3^2)^2}$, $I_2 = \overline{(\omega_1^2 + \omega_2^2 + \omega_3^2)(s_1^2 + s_2^2 + s_3^2)}$, and $I_3 = \overline{(\omega_1^2 s_1^2 + \omega_2^2 s_2^2 + \omega_3^2 s_3^2)}$, and ω_i and s_i are the components of the vorticity and strain rate, respectively. Accordingly, we can write

$$\frac{F_{uv,0}}{F_0} = \frac{I_1/105 + I_2/70 - I_3/105}{4I_1/105} = \frac{1}{4} + \frac{105}{280}\frac{I_2}{I_1} - \frac{1}{4}\frac{I_3}{I_1}. \tag{10}$$

Thus, (9) becomes

$$F_0\left(\frac{3}{4} - C\right) = -\frac{1}{2}\gamma_1 - \frac{2\zeta_3}{Re_\lambda}, \tag{11}$$

with $C = \frac{105}{280}\frac{I_2}{I_1} - \frac{1}{4}\frac{I_3}{I_1}$. A study[15] reported measured I_i in grid turbulence and found no discernible Reynolds number dependence on the ratios I_2/I_1 and I_3/I_1 over a range of $Re_\lambda = 27$–100, despite scatter in the data. The results of this study are consistent with others.[12,13,16] We report in Fig. 1 the dependence of the ratios $\frac{I_i}{I_1}$ ($i = 2, 3$, and 4) on Re_λ.[13,14] There is a clear plateau in each ratio when $Re_\lambda \geq 200$, which implies

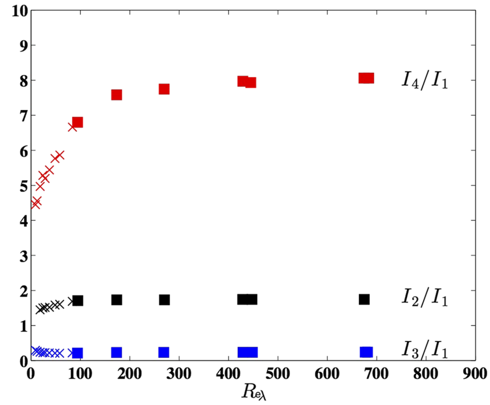

FIG. 1. Dependence of the ratios $\frac{I_i}{I_1}$ ($i = 2$, 3, and 4) on Re_λ. ×;[13] □.[14]

that C is a constant independent of Re_λ. Estimating $\frac{I_2}{I_1} \simeq 1.75$ and $\frac{I_3}{I_1} \simeq 0.25$ gives $C \simeq 3/5$, thus yielding

$$F_0 \simeq -\frac{10}{3}\gamma_1 - \frac{40}{3}\frac{\zeta_3}{Re_\lambda}. \quad (12)$$

Since experimental and numerical evidence shows that F_0, which is positive, does not approach zero as $Re_\lambda \to \infty$, the right side of (12) must also remain positive. It is straightforward to show that keeping the dominant terms in (6) when $r^* \to 0$ leads to

$$3\zeta_1 r^{*2} \simeq 11 r^{*2}, \quad (13)$$

yielding $\zeta_1 \simeq 11/3$, a constant independent of Re_λ. Recalling the expression $\tilde{E}_{111} \sim 6(\zeta_1 r^* + \zeta_2 r^{*2} + \zeta_3 r^{*3} + \zeta_4 r^{*4})$ as $r^* \to 0$, the coefficients ζ_2, ζ_3, and ζ_4 must be either independent of Re_λ or becoming Re_λ-independent as Re_λ increases if indeed $\tilde{E}_{111} \sim r^{*2}$ as $r^* \to 0$ at all Re_λ (ζ_2 is in fact zero since there is no r^{*3} term in (6) when $r^* \to 0$). This would lead to $\zeta_3/Re_\lambda \to 0$ as $Re_\lambda \to \infty$, yielding

$$F_0 \simeq -\frac{10}{3}\gamma_1 = -\frac{10}{3}\frac{\lambda^4}{u'^4}\overline{\left(\frac{\partial u}{\partial x}\right)^2\left(\frac{\partial^2 p}{\partial x^2}\right)}. \quad (14)$$

We note that γ_1 is a (negative) constant under the framework of SP used to derive (14) as the turbulence decays. However, it remains to determine whether or not it is Re_λ-dependent. Of course, from the definition of F_0, we also have

$$F_0 = 225\left(\frac{\nu}{\bar{\epsilon}}\right)^2\overline{\left(\frac{\partial u}{\partial x}\right)^4}, \quad (15)$$

which leads to

$$-\frac{10}{3}\overline{\left(\frac{\partial u}{\partial x}\right)^2\left(\frac{\partial^2 p}{\partial x^2}\right)} \simeq \overline{\left(\frac{\partial u}{\partial x}\right)^4}. \quad (16)$$

Using the Kolmogorov scales to normalize both sides of (16) leads to

$$-\frac{10}{3}\overline{\left(\frac{\partial u^*}{\partial x^*}\right)^2\left(\frac{\partial^2 p^*}{\partial x^{*2}}\right)} \simeq \overline{\left(\frac{\partial u^*}{\partial x^*}\right)^4} = F_0/15^2, \quad (17)$$

(the symbol * represents the Kolmogorov normalization; we used $(\lambda/u')^4 = 15^2(\eta/v_K)^4$ where η and v_K are the Kolmogorov length and velocity scales, respectively). Applying now the Schwarz inequality on the left side of (17), we obtain

$$F_0 \le \left(\frac{150}{3}\right)^2\overline{\left(\frac{\partial^2 p*}{\partial x^{*2}}\right)^2} \quad (18)$$

or

$$F_0 \le \left(\frac{150}{3}\right)^2\int_0^\infty k^{*4}E_p^*(k^*)dk^* \quad (19)$$

where E_p^* is the Kolmogorov normalized pressure spectrum. Expression (19) reveals that whether or not F_0 becomes a constant as $Re_\lambda \to \infty$ depends on the behaviour of the integral of $k^{*4}E_p^*(k^*)$. Since at very high Reynolds numbers, the small-scale turbulence in decaying HIT becomes stationary, (19) should also apply to a stationary (forced) 3D periodic box turbulence. We thus reproduce in Fig. 2 the distributions of $k^{*4}E_p^*(k^*)$ at several Re_λ in high Reynolds number direct numerical simulations (DNSs) of forced 3D periodic box turbulence.[17] The distributions appear to converge toward a universal distribution with increasing Re_λ, which would imply that the integral of $k^{*4}E_p^*(k^*)$ approaches a constant. DNS results at Re_λ higher than the maximum one reported in Fig. 2 should confirm whether or not the left side of (19) converges toward a true constant (i.e., independent of Re_λ).

In summary, an expression for the velocity derivative flatness factor, F_0 (see Eq. (14)), is derived from the transport equation for the third-order moment of the longitudinal velocity increment by assuming self-preservation for decaying homogeneous and isotropic turbulence. It is found that F_0 should remain constant during the decay under self-preservation conditions. Further, it is shown that F_0 has an upper bound which follows the integral of $k^{*4}E_p^*(k^*)$ (see (19)). Results of DNS of forced periodic box turbulence suggest that this integral approaches a universal constant when Re_λ becomes relatively large. If this result is confirmed at higher Reynolds numbers than those reported in Fig. 2, it would then imply that F_0 reaches a constant, likely to be smaller than the right side of (19), as $Re_\lambda \to \infty$, in agreement with K41 but in contradiction with K62.

An important observation that can be drawn from this work is that an accurate assessment of the behaviour of F_0

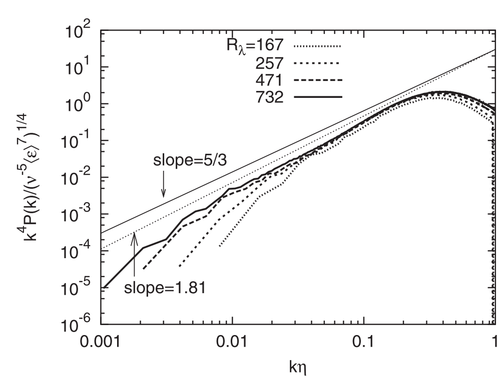

FIG. 2. Normalized spectra $k^{*4}E_p^*(k^*)$ at several Re_λ (denoted R_λ in the figure) in a forced 3D periodic box turbulence. This figure[17] is partly reproduced with permission from Ishihara *et al.*, "Spectra of energy dissipation, enstrophy and pressure by high-resolution direct numerical simulations of turbulence in a periodic box," J. Phys. Soc. Jpn. **72**, 983–986 (2003). Copyright 2003 Journal of the Physical Society of Japan.

in HIT as the Reynolds number increases requires results at Re_λ at least larger than the largest one reported in Fig. 2. It is likely that only DNS will provide those results. However, DNS faces a difficult challenge imposed by the resolution requirement associated with very high Reynolds number simulations. For example, it was estimated[18] that DNS with a mesh size resolution of about $\eta \simeq LRe^{-3/4}$ (L is an integral length scale and Re a large-scale Reynolds number) cannot accurately predict the properties of violent (small-scale) events.

The financial support of the Australian Research Council is acknowledged. L.D. thanks the Institut National des Sciences Appliquées (INSA, Rouen, France) for its financial support for this work during his stay at CORIA (University of Rouen Normandie). The Labex EMC3 is thanked for its financial support.

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