A validity framework theory and fatigue damage function for an S–N plane

Hoda Eskandari
BSc, Mechanical Engineering-Solid Design

Faculty of Engineering and Built Environment
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Statement of originality

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I hereby certify that the work embodied in this thesis contains a published paper/s/scholarly work of which I am a joint author. I have included as part of the thesis a written statement, endorsed by my supervisor, attesting to my contribution to the joint publication/s/scholarly work.

Signed ________________________

Hoda Eskandari
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Abstract

Fatigue life predictions associated with S–N curves have been largely based on empiricism due to the complexity involving multiple variables such as fatigue life, applied stress, number of loading cycles and stress ratio. A damage model proposed by Palmgren in 1924 and popularised by Miner in 1945 may be one of the most important milestones in the history of fatigue damage research. However, its validity and the principle it adopts have been questioned, and many researchers have attempted to refine the model without much success. The ultimate objectives of the current work were to provide a validity framework theory for fatigue damage associated with the S–N curve and to derive a damage function capable of predicting fatigue life. The validity framework is designed as the fundamental basis to ensure the validity of a damage function in the development process. In the current work, a validity framework theory consisting of axioms, relative conditions for compatibility and boundary conditions is developed for fatigue damage on an S–N plane at a stress ratio of zero. The compatibility in fatigue damage was conceptualised. Manifestation points for accumulated damage were defined and conceptualised for boundary conditions by differentiating between damage accumulated before failure, and failure caused by damage. A selected equation for damage at failure as the reference damage was theoretically validated for further validation of damage on the S–N plane. Also, a damage function capable of predicting fatigue damage is proposed following the validation process. Comparisons between experimental results from two stress level sequence loading and theoretical fatigue life predictions were made and a close agreement between them was found. An evaluative review on conventional fatigue damage models is presented along with the benefits of the new compatibility concept and criteria developed here.
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Constant</td>
</tr>
<tr>
<td>$B$</td>
<td>Constant or parameter</td>
</tr>
<tr>
<td>$b$</td>
<td>Parameter of S–N curve</td>
</tr>
<tr>
<td>$C$</td>
<td>Constant or parameter</td>
</tr>
<tr>
<td>$D$</td>
<td>Damage</td>
</tr>
<tr>
<td>$E$</td>
<td>Reduced elastic modulus due to damage</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Elastic modulus</td>
</tr>
<tr>
<td>$F$</td>
<td>Load</td>
</tr>
<tr>
<td>$K$</td>
<td>Parameter of S–N curve</td>
</tr>
<tr>
<td>$R$</td>
<td>Stress ratio</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of fatigue loading cycles</td>
</tr>
<tr>
<td>$\Delta N$</td>
<td>Additional number of fatigue loading cycles at a different stress level</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Number of fatigue loading cycles at failure</td>
</tr>
<tr>
<td>$N_{\infty}$</td>
<td>Number of fatigue loading cycles at the initial endurance limit</td>
</tr>
<tr>
<td>$N_n$</td>
<td>Number of fatigue loading cycles at $n$th stress level</td>
</tr>
<tr>
<td>$N_{fn}$</td>
<td>Number of fatigue loading cycles at failure at $n$th stress level</td>
</tr>
<tr>
<td>$W$</td>
<td>Network absorbed at failure</td>
</tr>
<tr>
<td>$\Delta W$</td>
<td>Total energy input</td>
</tr>
<tr>
<td>$w_{\Delta N_n}$</td>
<td>Work done due to $\Delta N_n$ cycles</td>
</tr>
<tr>
<td>$\sigma_{R(i)}$</td>
<td>Residual strength of the material after cyclic load at $i$th stress level</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>Residual strength</td>
</tr>
<tr>
<td>$\sigma_{UT}$</td>
<td>Ultimate tensile strength</td>
</tr>
<tr>
<td>$\sigma_{\max}$</td>
<td>Applied peak stress</td>
</tr>
<tr>
<td>$\sigma_{\min}$</td>
<td>Applied valley stress</td>
</tr>
<tr>
<td>$\sigma_{\infty}$</td>
<td>Fatigue limit</td>
</tr>
<tr>
<td>$\Delta \sigma$</td>
<td>Stress amplitude $= (\sigma_{\max} - \sigma_{\min})/2$</td>
</tr>
</tbody>
</table>