Reconsidering Bovill's method for determining the fractal geometry of architecture

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Abstract: Throughout the 1970s the mathematician Benoit Mandelbrot developed an argument which proposes that natural systems frequently possess characteristic geometric or visual complexity over multiple scales of observation. This argument lead to the formulation of fractal geometry and it was central to the rise of the sciences of non-linearity and complexity. During the 1990s, researchers Michael Batty and Paul Longley, Bill Hiller and Carl Bovill developed this concept in relation to, respectively, the city, urban neighborhoods and individual buildings. More recently, architectural scholars and building scientists have suggested that such models might be used to determine quantitative measures of visual complexity in architectural form. In parallel, a range of computational tools have also been developed to assist in the determination of the characteristic visual complexity of architecture. At the heart of such approaches is a set of rules developed by Bovill for analyzing buildings. However, despite its growing importance, the assumptions implicit in Bovill's method have never been adequately questioned. The present paper returns to the origins of Bovill's analytical method to reconsider his assumptions, arguments and the evidence he uses to support his case. A series of alternative variations on Bovill's method are then proposed and discussed.

Conference theme: Digital Architecture, Virtual Environments
Keywords: fractal architecture, computational tools, design assessment

INTRODUCTION

While a growing number of scientific or computational tools have been developed for the analysis of architectural plans, the production of similar tools for the investigation of the visual attributes of a building proved more problematic (Lynch, 1960; Hillier and Hanson, 1984) until fractal geometry began to be applied to architecture in the 1990s (Ostwald and Moore, 1993; 1996). However, despite these more recent developments, the few methods that do exist for the visual analysis of architecture have not been subjected to the same level of scholarly scrutiny as the previous generation of planning tools. As a result of this situation, there is a considerable gap in quality and consistency between the application of methods (like Space Syntax) for the analysis of architectural plans and those for the analysis of building elevations (encompassing visual qualities). This introduction provides a brief background to both fractal geometry and methods of visual analysis, to position the research, before the paper focuses on one key analytical method developed by Bovill for investigating the visual qualities of elevations.

In 1975 the mathematician Benoit Mandelbrot published *Les Objects Fractals: Form Hasard et Dimension.* At the core of Mandelbrot's research is an attempt to understand the geometric rules that underlie nature. In this work Euclidean geometry, the traditional tool used in science to describe natural objects, is viewed as fundamentally unable to fulfil this purpose. Clearly, to paraphrase Mandelbrot (1977), mountains are not conical in form, clouds are not spherical and rivers are not orthogonal. While historically, science considered roughness and irregularity an aberration disguising underlying ordered systems with a fixed-state or finite values, Mandelbrot argues that the fragmentation of all naturally occurring phenomena cannot so easily be disregarded (Ostwald 2003a). A coastline is not straight and no Euclidean Geometric construct can approximate the form of a coastline without serious abstraction or artificiality (Mandelbrot, 1977). As a result of this natural fragmentation, mathematicians have shown that the length of the coastline cannot be determined at all (Feder, 1988). Yet, the characteristic irregularity of a coastline may be measured by imagining that the increasingly complicated and detailed path of the coastline is actually somewhere between a one-dimensional line and a two-dimensional surface (Schroeder, 1991). The more complicated the line, the closer it becomes to being a two-dimensional surface. Therefore, coastlines and many similar natural lines can be viewed as being fractions of integers, or what Mandelbrot describes as *fractal* geometric forms. Thus, fractal geometry describes irregular or complex lines, planes and volumes that exist between whole number integer dimensions. This implies that instead of having a dimension, or D, of 1, 2, or 3, fractals might have a D of 1.51, 1.93 or 2.74.

Architects adopted fractal geometry as a design tool in the 1980s but, despite some interesting outcomes, it rarely produced an enduring architectural response (Jencks 1995; Ostwald 2001). In contrast, the history of applications of fractal geometry to the analysis of architectural and urban forms is still evolving and is displaying more promising results. For example, Oku (1990) and Cooper (2003; 2005) have separately attempted to use fractal geometry to provide a quantitative measure of the visual qualities of an urban skyline. Yamagishi, Uchida and Kuga (1988) have sought to determine geometric complexity in street vistas and Kakei and Mizuno (1990) have applied fractal geometry to the analysis of historic street plans; a project that has been extended by Rodin and Rodina (2000). At a much finer scale, Capo (2004) has provided an explanation of the complexity of the architectural orders (Doric, Ionic and Corinthian columns) using fractal geometry and Eaton (1998) has interpreted the layout and decoration of some of Frank Lloyd Wright's Houses as being fractalesque. At a larger scale Cartwright (1991) offered an overview of the importance of fractal geometry and complexity science in town planning and Batty and Longley (1994) and Hillier (1996) have each developed increasingly refined methods for using fractal geometry to understand the visual qualities of macro-scale urban environments. More recently, Batty (2005) has combined fractal geometry with computational automata to produce a further detailed explanation for the visual and formal complexity observed in cities. In particular, Batty has analyzed the fractal dimension of various urban plans allowing different cities to be ranked or described in terms of their growth patterns, geometric distribution and graphic density (see figure 1).

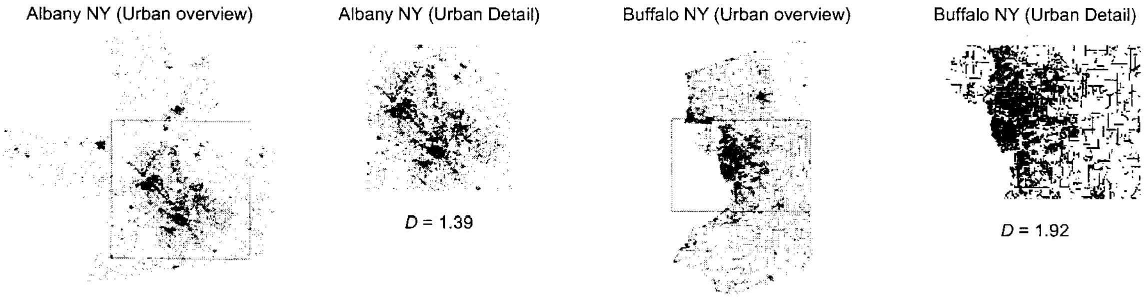


Figure 1: Comparative analysis of fractal dimension (*D*) of the cities of Albany and Buffalo in New York State, USA. The comparison of *D* values suggests that Buffalo has a consistently more dense, and complex urban pattern than Albany. (Source: Batty 2005: 510)

Despite these examples, one of the more commonly repeated methods for the analysis of visual character in architecture is Bovill's (1996) extrapolation of Mandelbrot's box-counting approach to determining fractal dimension (Mandelbrot, 1977; 1982). Bovill's original contribution to the box-counting method rests primarily in his explanation of its potential application in architecture, design and the arts. Bovill's interpretation of Mandelbrot's box counting method—henceforth "Bovill's method"—has been used to analyse historic and modern building facades along with streetscapes and skylines; all situations where visual complexity is important and quantitative methods have not previously been available. Since his original publication, Bovill has offered an extrapolation of its use (1997) and Bechhoefer and Appleby (1997) have used the method to consider the visual qualities of vernacular architecture. Bovill's method has also been repeated by a range of researchers including Makhzoumi and Pungetti (1999) and Burkle-Elizondo, Sala and Valdez-Cepeda (2004) and Sala (2006). Controversially, it has recently been suggested by Taylor (2006), that Bovill's method may be useful in helping provide some determination of the psychological impact of architectural form.

The following sections examine Bovill's method in detail. The purpose of this analysis is not to criticise Bovill's approach—fundamentally his method is accepted as a valuable procedure and his work on the topic was groundbreaking—but to subject it to a range of tests and begin the process of exploring and exposing its potential limits.

1. BOVILL'S METHOD

1.1 Founding Assumptions

Before considering Bovill's mathematical method, the assumptions implicit in his application of fractal geometry to architecture are worth examining. For example, Bovill commences his work with the argument that architecture is necessarily produced through the manipulation of rhythmic forms. He expands this to propose that fractal geometry will allow a "quantifiable measure of the mixture of order and surprise" (1996: 3) in such rhythmic forms to be determined and, moreover, that this will reveal the essence of the architectural composition.

> Architectural composition is concerned with the progression of interesting forms from the distant view of the facade to the intimate details. This progression is necessary to maintain interest. As one approaches and enters a building, there should always be another smaller-scale, interesting detail that expresses the overall intent of the composition. This is a fractal concept. Fractal geometry is the formal study of this progression of self-similar detail from large to small scales. (Bovill 1996: 3)

Contrary to this claim, the desire to "maintain interest" or produce a cascade of detail from different perspectives is not a primary formal motivation in any major architectural theory since Roman times (Kruft 1994). Indeed, the exact opposite is true for much Ancient Greek and Renaissance architecture. In the former case elaborate geometric strategies (including *entasis* in columns) were employed to artificially correct a range of changes that occur when a building is viewed from different ranges. In the latter case, Renaissance architecture was designed to be appreciated from a singular, almost Platonic, perspective viewpoint. European Baroque architecture is marginally closer to Bovill's argument but the underlying theory of its form is still essentially identical to Renaissance architecture regardless of its extravagant decoration (Portughesi 1970). While a person experiencing a building may appreciate detail over a range of scales it is not, and probably never has been, a primary theory for shaping architectural form.

In the second stage of his proposition Bovill maintains that the use of fractal analysis in architecture might explain why some modern buildings have never been fully appreciated by the general public, whereas some vernacular architecture is more widely liked (1996: 6). Here Bovill assumes that modern architecture (by which he means the international style architecture of mid-career Le Corbusier or Mies van der Rohe) will have a lower fractal dimension and, therefore, a lower correlation with natural geometry than, say, historic architecture. In this proposition Bovill repeats Mandelbrot's argument which has as its founding assumption the Kantian belief that nature is innately beautiful and that people are drawn to the appreciation of natural forms because of this. For Bovill, fractal images;

> are pleasant because they capture the character and depth of texture that nature displays. Our perceptual mechanisms evolved in nature and therefore respond to a similar textural quality. The study of fractal geometry should help the designer achieve a better understanding of the cascade of detail all around us in the natural world. (Bovill 1996: 70)

Yet, as philosophers have observed, the Kantian belief in the essential rightness, goodness or beauty of nature is not supported by strong evidence and it does not stand up to close scrutiny. Gray (1991) and Ostwald (2003b) have also reviewed Mandelbrot's assumptions and uncovered a range of political and philosophical problems in the aesthetic and cultural values embedded in his work. For example, Mandelbrot is highly critical of Modern architecture while praising Beaux-Arts or Baroque buildings. This is problematic for a range of reasons most notably because it places an undue positive emphasis on higher fractal dimensions (those closer to natural results like $D = 1.8$) while dismissing those that have relatively abstract or plain forms (say with a fractal $D = 1.1$) as being alienating. However, despite Mandelbrot's assertions, fractal dimension is not a determinant of good architecture, social responsibility or cultural meaning in the built environment. Fundamentally there is no direct correlation between fractal dimension and successful architecture. Fractal dimension in architecture is only useful as a comparative tool; it allows, for example, the visual complexity of a neighbourhood or street to be determined and this may then be compared with the visual complexity of a building that is proposed for this street. In this way, fractal geometry can suggest the extent to which a proposed building is "in-keeping with", or "sympathetic to" its visual environment.

2.2 The Mathematics of Box Counting

Bovill's method takes as its starting point a line drawing, say the façade of a building. A grid is then placed over the drawing and each square in the grid is analysed to determine whether any lines from the façade are present in it. Those grid boxes that have some detail in them are then marked. This data is then processed using the following numerical values;

$_{(s)}$	=	the size of the grid
$N_{(s)}$	=	the number of boxes containing some detail
$1/_{s}$	=	is the number of boxes at the base of the grid

Next, a grid of smaller scale is placed over the same façade and the same determination is made of whether detail is present in the boxes of the grid. A comparison is then made of the number of boxes with detail in the first grid ($N_{(s1)}$) and the number of boxes with detail in the second grid ($N_{(s2)}$). Such a comparison is made by plotting a log-log diagram ($\log[N_{(s)}]$ versus $\log[1/_{s}]$) for each grid size. This leads to the production of an estimate of the fractal dimension of the façade; actually it is an estimate of the box-counting dimension (D_b) which is sufficiently similar that most researchers don't differentiate between the two.

The slope of the line (D_b) is given by the following formula:

$$D_b = \frac{[\log(N_{(s2)}) - \log(N_{(s1)})]}{[\log_{(1/s2)} - \log_{(1/s1)}]}$$

where ($1/_{s}$) = the number of boxes across the bottom of the grid. (Bovill 1996: 42)

2.3 The Case Study Examples

In order to demonstrate the use of the box-counting method for determining fractal dimension in architecture Bovill offers several examples and a variation of the method. Before considering his case study examples, the variation is worthy of note even if only to observe that it is not repeated in any of the analyses inspired by, or directly adopting, Bovill's method. In explaining the variation Bovill rightly notes that architectural forms in general, and to a certain extent houses in particular, often have a striated form wherein the base and the top have difference visual complexities to that present in the central sections. In order to accommodate these differences, Bovill experiments with determinations of fractal dimension for the top, middle and bottom sections of buildings as well as comparisons between elements in the building on this basis. To a certain extent Bovill is right to realise that buildings necessarily have different visual complexities in different parts for a range of functional or largely pragmatic reasons. However, the percentage of examples of architecture that display the tripartite distribution of form is relatively small. Thus, in order for his method to be equally valid for a wide range of structures, the variation is not useful unless the characteristic forms being analysed possess clear horizontal or vertical striations. For example, certain Edwardian streetscapes display a range of characteristic horizontal bands and within such a historic neighbourhood the variation on Bovill's method may be useful. For the majority of other cases, a clear comparison, regardless of building type, is more important.

The first two of the examples considered by Bovill are Frank Lloyd Wright's Robie House and his Unity Temple. In the former case Bovill uses four grids, and three comparisons between the grids. As a result of this analysis he determines that the façade of the Robie House has a range of fractal dimensions from D = 1.645 to D = 1.441 although he proposes that the lower dimension is likely to be more accurate (see figure 2.1). In an interesting test of this result, Bovill then analyses a window detail from the Robie house and finds, in Wright's elaborate stained glass patterns, a slightly higher fractal dimension. Such a result would not be unexpected in one of Wright's houses of the era (Eaton 1998). These houses typically display more detail, up to a point at least, the closer the viewer comes to them. Mandelbrot refers to this point as the "scaling limit" and most real-world objects, including everything from buildings to trees, have such a limit. There is also an outer scaling limit; beyond which point an object blurs into a mass of points and looses any self similarity. Neither the inner nor the outer scaling limits are defined or discussed in detail by Bovill although a future determination of both would be useful for formalising Bovill's method and making it applicable to a wide range of cases.

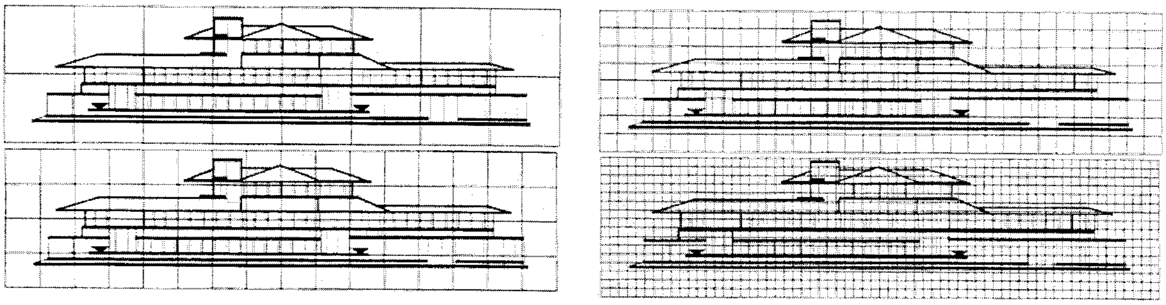


Figure 2.1: Bovill's box-counting method applied to the Robie House (Source: Bovill 1996: 120)

For his analysis of Wright's Unity Temple, Bovill uses three grids and two comparisons between grids. From this process Bovill determines that the façade of the Unity Temple has a fractal range of between D = 1.621 and D = 1.482 (see figure 2.2). Again, Bovill seeks validation of his result by analysing a smaller detail in the design; a planter box.

There are further case studies in Bovill's work, including Le Corbusier's Villa Savoye and several studies of elements in Alvar Aalto's work, but the method for considering the visual character and complexity of facades remains similar. Overall, the method delivers low fractal dimension figures for Le Corbusier's work ($1 < D < 1.5$) and higher ones for Frank Lloyd Wright's architecture ($1.5 < D < 2$).

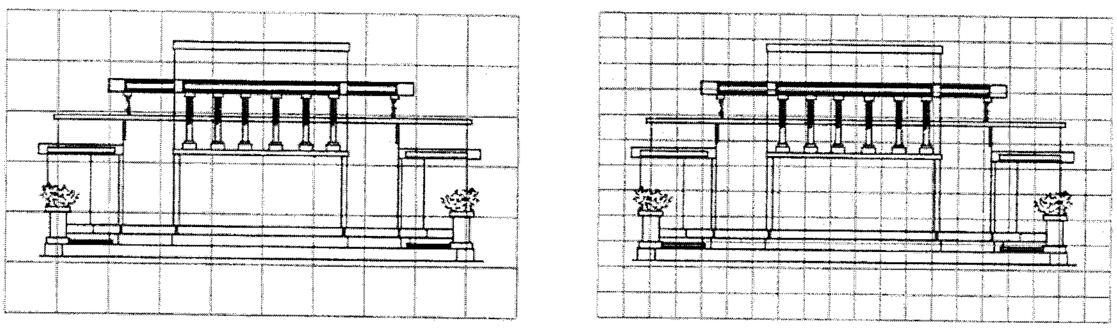


Figure 2.2: Bovill's box-counting method applied to the Unity Temple (Source: Bovill 1996: 129)

3.0 DISCUSSION: FRAMING THE IMAGE

At its core, Bovill's method analyses images, and yet, there is relatively little discussion in his work, and even less in the research that has followed his lead, about the type of image to use for fractal analysis. An obvious question would be, why use elevations? The human eye reads the world in perspective. It is impossible to experience an elevation; the problems of parallax ensure that in the "real world" no two lines are ever, perceptually at least, parallel. Why not then use perspective views? This question is even more compelling when you take into account the fact that fractal geometry is about a comparison between different scales of viewing. Bovill even argues, although it is partially refuted above, that a cascade of detail is critical for leading the eye closer to the building. Yet, Bovill's method doesn't rely on recording the change in detail as the eye comes closer to the building, instead it assumes that the eye (or viewing point) remains fixed while the amount of detail entering the eye increases. This is akin to placing a digital camera on a tripod and then, after manipulating the lens to perfectly correct the perspective, taking a 2 mega-pixel photo. Then, from the same position and after another level of parallax correction, a 4 mega-pixel image is taken and then an 8 mega-pixel image and so on. This differs from Bovill's view of the purpose of architecture. These are critical questions about the way in which the analysis of architecture is approached and they are all concerned with the framing of the raw image data.

The following, seemingly more realistic variations on Bovill's method, are alternative ways of framing the image that is analysed. Each variation uses a different combination of view points, perspective planes (and picture planes where the image is ultimately recorded). These variations also introduce the role of the cone of vision; something conspicuously lacking from much fractal analysis of architecture. In the following descriptions, for simplicity, the methods are described for orthogonal structures. Also, it is acknowledged that in order to determine the fractal dimension of an image, a comparison of two separate "grids" is required. For the purpose of considering alternatives, the following variations describe these paired grids typically as one conceptual view or picture plane.

3.1 Fixed position, one-point perspective

(see figure 3.1)
This variation involves a fixed viewpoint with the eye at right angles to the dominant surface of the façade, but with no correction for parallax. This variation suggests that all images are in one-point perspective, and that the gathering of data is analogous to increasing the mega-pixel value set in the camera. This variation has the advantage of a consistent rule for setting up the image composition (at right angles to the façade and a certain distance from it based on the dimensions of the building being considered and determined by a standard cone of vision).

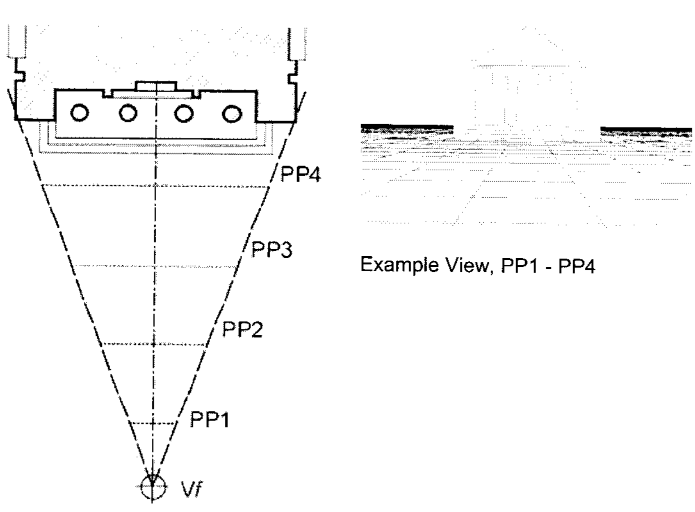


Figure 3.1: Fixed position, one-point perspective. *Vf* is a fixed viewpoint point and PP1-5 are picture planes where an image is recorded.

3.2 Fixed position, two-point perspective

(see figure 3.2)
A fixed viewpoint with the eye/camera not at right angles to the dominant surface of the façade, but with no correction for parallax. This suggests that all images are in at least two-point perspective and that the gathering of data is analogous to increasing the mega-pixel value set in the camera. This has the problem that there is no clear rule for setting the viewpoint even though the image is more natural (the fixed, one-point version above is relatively artificial for this reason).

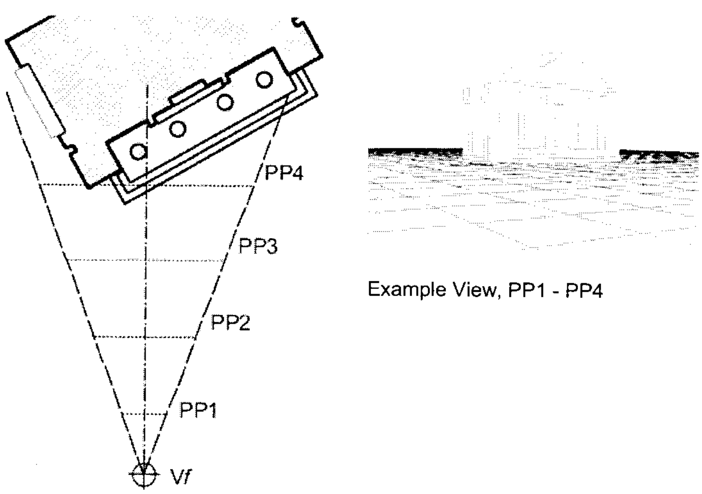


Figure 3.2: Fixed position, two-point perspective. *Vf* is a fixed viewpoint point and PP1-5 are picture planes where an image is recorded.

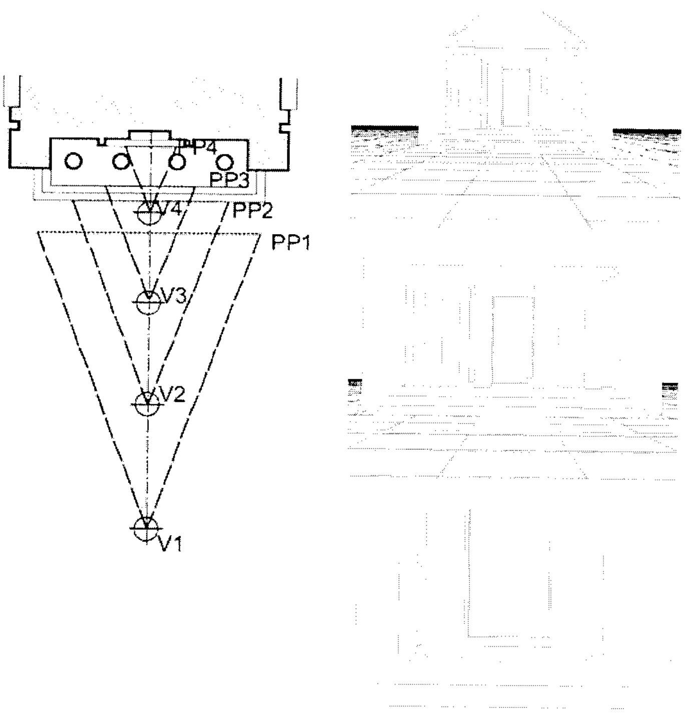
3.3 Variable position, one-point perspective (see figure 3.3)
This variation uses a range of viewpoints, starting further away from the façade and moving closer to it, but all at right angles to the dominant surface of the façade. At each viewpoint the standard cone of vision of the human eye determines the extent of the façade that is analysed. This means that, with each iteration, a reduced portion of the façade is considered. This is close to the way a human eye would operate if a person walked directly towards a façade. This variation can be refined to set a range of standard viewing distances along a line to the façade allowing it to be repeatable for a wide range of circumstances.

3.4 Variable position, two-point perspective (see figure 3.4)
A range of viewpoints positioned along a line, starting further away from the façade and moving closer to it, are used. None of these viewpoints are at right angles to the façade's geometry. At each viewpoint the standard cone of vision of the human eye determines the extent of the façade that is analysed.

3.5 Variable position, multiple-point perspective (see figure 3.5)
A range of viewpoints, starting further away from the façade and moving closer to it, are used. None of these viewpoints are at right angles to the façade's geometry and none are in a fixed line between the original view point and the final one (a characteristic of the other variations). At each viewpoint the standard cone of vision of the human eye determines the extent of the façade that is recorded.

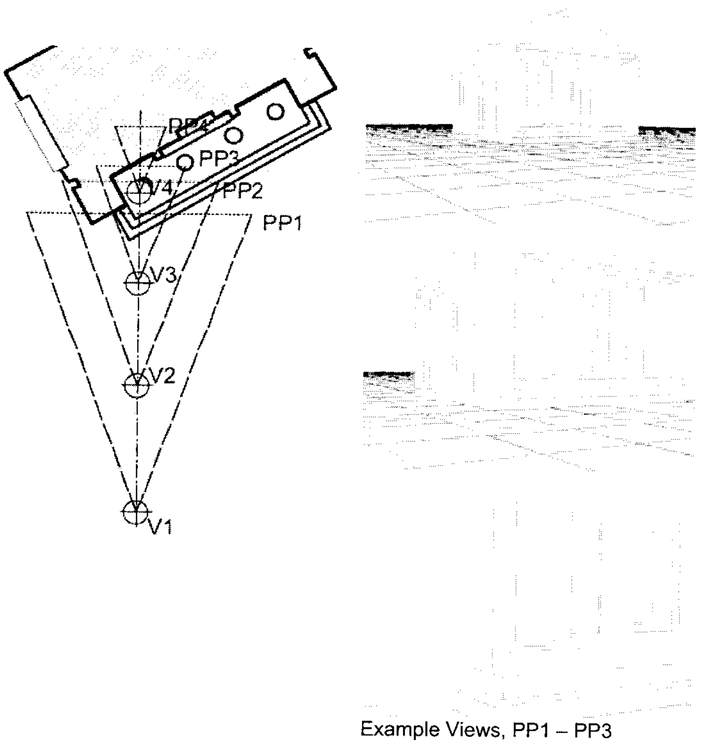
This is the closest of any of the methods to reality. It suggests that people rarely approach buildings along a single vector and it acknowledges the importance of the limits of human vision. However, despite these strengths it is hard to see how this could be repeated for multiple buildings (because fractal dimension in architecture is mostly useful, from an analytical point of view, as a comparative value). Notwithstanding this problem, there might be some ways to use this variation. For example, some houses can only be approached along a proscribed entry path; several of Wright's houses have hedges that line paths forcing a visitor to walk around to the rear to enter. In such cases, where the experience of the architecture is choreographed, it would be interesting to determine the fractal dimension of the architecture (or even the landscape framed in this way) along the entry route. Such an analysis would produce a valuable numerical expression of the way a designer intends to reveal the visual qualities of a building. An alternative way of using this variation might be to compare the fractal dimension of different approaches taken by people to a building. For example, imagine a civic building facing a piazza. A statistical analysis could be undertaken of the way in which many hundreds of people approach the building across the piazza. Imagine that, all things being equal in the piazza (ie. no physical or visual obstacles), there are three dominant paths taken by people. What would a fractal analysis of visual complexity of the environment along these three paths reveal? Would it suggest that people are drawn along similar or different visual paths? Are people drawn to the paths that maximise the visual complexity of the environment? This is certainly the untested assumption implicit in many applications of Mandelbrot's ideas in architecture.

An important characteristic of the five variations proposed is that, unlike Bovill's method, these will produce a wider range of D results. This is because there is an assumption implicit in the proposed variations, that a person's experience of a building changes as their perspective shifts, or distance from the building changes. This also suggests that the D value of a building will change, and sometimes substantially so, as a person's spatial relationship with the building alters. This mirrors Mandelbrot's early assertion that "scaling limits" apply to natural fractals; as a person comes closer to tree, sooner or later, their experience of it changes completely (and especially when they finally reach its trunk). In contrast, Bovill's method assumes that a person who is standing still will gain more information about the building the longer they look at it.



Example Views, PP1 – PP

Figure 3.3: Variable position, one-point Perspective. V1-5 are sequential viewing points. PP1-5 are picture planes where an image is recorded.



Example Views, PP1 – PP3

Figure 3.4: Variable position, two-point perspective. V1-5 are sequential viewing points. PP1-5 are picture planes where an image is recorded.

Both of these assumptions have some evidence for them and are at least partially true. Bovill is effectively advocating a mode of interaction with a building that is akin to scientific observation, whereas the five variations proposed in the present paper have more in common with the approaches of social scientists and anthropologists. Future research by the authors will provide comparative *D* values for Bovill's method and the new variations proposed.

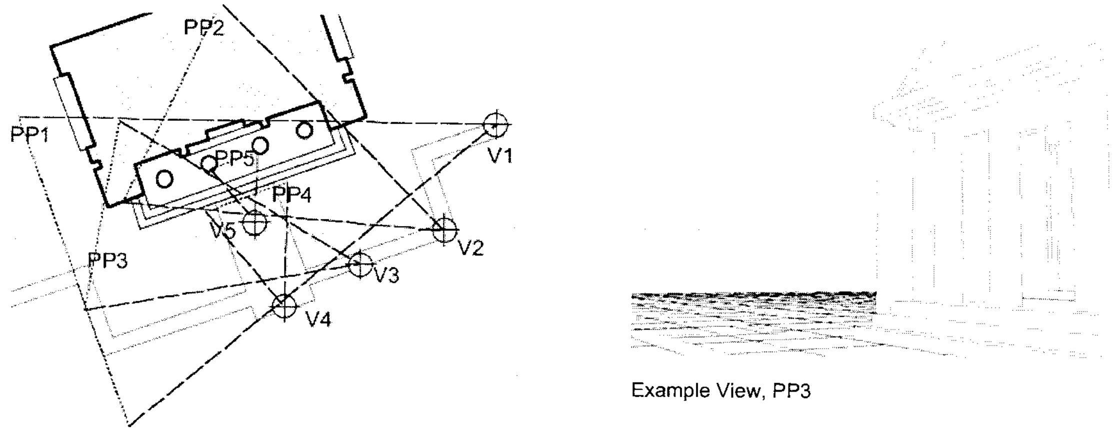


Figure 3.5: Variable position, multiple-point perspectives. V1-5 are sequential viewing points and PP1-5 are picture planes where an image is recorded.

CONCLUSION

This analysis of Bovill's method has begun to uncover a range of issues that should be refined, corrected or developed as a precursor to the method being tacitly accepted as a consistent tool to determine the visual character, of the built environment. Ultimately, the analysis of the assumptions implicit in Bovill's method does not undermine its importance, but it does reinforce the view that this method is not able to be used to make quality judgements about architecture. The method may be used to suggest the extent to which a design has multiple levels of detail, but this is not, in and of itself, any reflection of design quality, aesthetics or ethical values. Similarly, high fractal dimensions in architecture (where *D* approaches 2 in an image, or 3 in a form) are not any more "natural" than low fractal dimensions; fundamentally architecture is not nature and a higher *D* does not infer that a building is any closer to nature than a lower *D*. This is because the relationship between architectural form and nature is a symbolic or metaphoric one. Edward De Zurko famously criticised architects use of organic and natural descriptors for their buildings because "[t]he obvious truth of the matter is that buildings are not plants or animals [...] Architecture is not an organism; it is a product of the human will, the creative spirit of mankind." (1957: 3)The formal complexity of an architectural design may have similarities to the formal complexity of nature, but that does not make the building natural. At best, such a correlation suggests a similar visual quality which is, like military camouflage, artificial and hides substantial differences (Ostwald 2003a).

Consider Reading's argument that architectural designs that have been shaped in accordance to fractal geometry in general, and the golden section spiral in particular, (the golden section is regarded by mathematicians as a "trivial" fractal) will be environmentally beneficial to the planet (1994). As a detailed analysis of Reading shows, not only isn't this true—and especially not from a mathematical perspective—but it is likely to be the exact opposite case (Ostwald and Wassell 2002). First, buildings that are shaped around the golden section have a symbolic connection to nature. A symbolic quality may have some cultural or social benefit to the community, but it will not have any direct, or necessarily positive, impact on the ecology of its region. Next, the more fractal an object is, like a tree or plant, the more surface area it has. This is especially beneficial for the tree which requires surface area to assist photosynthesis. In contrast, buildings use more energy and material the larger their surface area. Buildings do not require complex surface profiles because they do not photosynthesise. They also do not need complex forms to encourage birds to be attracted to them to serve the needs of pollination, or to encourage the sacrificial erosion of limbs that promotes new growth. In many cases, the simpler the building form, the more likely it is to minimise environmental impact. This is because, at the very least, straightforward building forms tend to have less embodied energy invested in them which, in turn, means that they start their existence with a reduced ecological imprint. This does not mean that a building with a complex form, and associated high *D* value, cannot be ecologically sensitive. Rather, that such a building would be served by a wide range of passive and active measures and mechanisms that would promote its sustainable characteristics; few, if any of which, would be directly associated with the characteristic visual complexity of its form.

Returning to Bovill's method; in considering the way in which Bovill frames images for analysis a range of variations have been proposed. Each of these variations are more realistic in modelling the way in which humans experience architecture. They are superior to Bovill's method in all but one, important, way. Bovill's method, for all that it may be unrealistic, has the advantage that it is a straightforward, repeatable process. The method may not result in the most realistic or detailed results, but they are relatively consistent. Finally, the variations set out above, and especially the final one, suggest that there are powerful applications of fractal analysis that have not yet been developed or tested but which will be useful for producing a more nuanced, subtle or detailed, reading of visual complexity in the built environment. Future research from the authors will develop and test these ideas.

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